Motion of Particles in Shear Flows Taking into Account Connected Mass

Umriddin Dalabaev, Aibek Arifjanov, Dilbar Abduraimova and Askar Babaev

Abstract--- *Solve number of practical problems associated with the movement of particles in a fluid flow, it becomes necessary to search for the nature of the movement of particles, taking into account a number of factors. Such tasks are gaining relevance when irrigation with a flexible hose in the conditions of Uzbekistan. Since the Amu Darya and Syr Darya rivers are the source of irrigated water, which bring a large amount of suspended particles in the water stream with water. In this research paper, we consider the motion of particles in flows, taking into account the attached mass. When the concentration of dispersed particles is insignificant, the mutual influence of the particles is negligible, the movement of each particle in the liquid can be considered as occurring regardless of the presence of other particles. Therefore, the results that will be obtained for the case of movement of a single particle in a liquid are applicable. This makes it possible to study the laws of particle behavior based only on the equation for particles. Here, the behavior of particles is studied taking into account the influence of the added mass effect and the Safman forces. According to the results of the study in the conditions. particle motion in an upward flow, the effect of the attached mass effect is comparable to the Stokes hydrodynamic force at moderate. It is established that the separation of the Safman force to the Stokes force depends on the dimensionless parameter. The results show that for currents, the effect of the attached mass significantly affects the regime of particle flow in the water stream*.

Keywords--- *Density, Hydrodynamics, Flow, Flow Regime, Reynolds Number, Mass, Particles, Trajectory, Velocity, Viscosity.*

I. INTRODUCTION

When the concentration of dispersed particles is insignificant, the mutual influence of the particles is negligible, the movement of each particle in the liquid can be considered as occurring regardless of the presence of other particles. Therefore, the results that will be obtained for the case of movement of a single particle in a liquid are applicable. This makes it possible to study the laws of particle behavior based only on the equation for particles. Here, the behavior of particles is studied taking into account the influence of the added mass effect and the Safman forces.

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II. Methods of Research

1. Particle Motion in Upstream

When moving in a fluid stream, forces of various nature act on it [1,2,3]. Here we consider the effect of the attached mass effect on the motion of a single particle. The system of equations of particle motion, taking into account the attached mass and gravity in dimensionless variables, has the form:

$$\frac{dx}{dt}=u,\quad \frac{dy}{dt}=v,\quad \frac{du}{dt}=D(u_l-u)-DW,\quad \frac{dv}{dt}=-Dv \tag{1}$$

where x, y particle trajectory, u, v particle velocity, u_l carrier fluid velocity (Poiseuille flow), with the axis x directed against gravity in the middle of the channel, and the axis y perpendicular to it; $D=\frac{A}{\rho+0{,}5}$, $A=\frac{9}{2\varepsilon^2 R}$, $W=\frac{2r^2(\rho_p-\rho_l)g}{9\mu U}$. Here r - particle size, R - Reynolds number $(\varepsilon=r/L,\ R=UL/\nu)$, U - characteristic speed, $L-$ characteristic size, ν - kinematic viscosity of the carrier fluid, $\rho=\rho_p/\rho_l$, ρ_p - particle density, ρ_l - the density of the carrier medium.

The number D is the inverse of the Stokes number ($D=1/Stk$). Solution of systems of equations (1) under initial conditions $y(0)=y_0,\ x(0)=0,\ u(0)=u_0,\ v(0)=v_0$ has the form:

$$x(t)=\frac{1}{D}\left(u_0-1{,}5+6y_0^2+12y_0\frac{v_0}{D}+W\right)\left(1-\exp(-Dt)\right)+\left[1{,}5-6\left(y_0+\frac{v_0}{D}\right)^2-W\right]t+$$

$$+12\left(y_0+\frac{v_0}{D}\right)\left[\frac{1}{D^2}-\frac{t\exp(-Dt)}{D}-\frac{\exp(-Dt)}{D^2}\right]+\frac{6v_0^2}{2D^3}\left(1-\exp(-2Dt)\right),$$

$$y(t)=-\frac{v_0}{D}\exp(-Dt)+y_0+\frac{v_0}{D},\quad v(t)=v_0\exp(-Dt),$$

$$u(t)=1{,}5-6\left(y_0+\frac{v_0}{D}\right)^2-W-\left[12\left(y_0+\frac{v_0}{D}\right)v_0t+u_0-1{,}5+6y_0\frac{v_0}{D}\right]\exp(-Dt)+\frac{6v_0^2}{D^2}\exp(-2Dt).$$

Based on the solution obtained, it is not difficult to determine the effect of the added mass effect. Let be $v,\ v_m$ represents the transverse velocity of the particles without and taking into account the attached mass, respectively. Then it's easy to find

$$\max_t\left|v-v_m\right|=\left|\frac{v_0}{2\rho}\exp\left((2\rho+1)\ln\frac{\rho}{\rho+0{,}5}\right)\right|.$$

For longitudinal velocity, we have

$$\max_t\left|u-u_m\right|=\left|\frac{1}{2\rho}(u_0-1{,}5+6y_0^2+W)\exp\left((2\rho+1)\ln\frac{\rho}{\rho+0{,}5}\right)\right|.$$

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Note that the maximum difference for the longitudinal and transverse velocities takes place at $t_* = -\frac{2\rho+1}{A}\ln\left(\frac{\rho}{\rho+0,5}\right)$.

Thus, the effect of the added mass is essential for moderate ρ. It is also easy to identify the ratio of the force of the attached mass to the Stokes force at $W = 0$:

$$\frac{|F_m|}{|F_h|} = \frac{1}{2\rho+1}. \qquad (2)$$

This ratio shows that the effect of the added mass effect is comparable with the Stokes hydrodynamic force at moderate. It is easy to determine the condition for which the particle does not leave the layer: without taking into account the attached mass

$\left(-\frac{1}{2}-y_0\right)\frac{A}{\rho} \le v_0 \le \left(\frac{1}{2}-y_0\right)\frac{A}{\rho}$, taking into account the attached mass $\left(-\frac{1}{2}-y_0\right)\frac{A}{\rho+0,5} \le v_0 \le \left(\frac{1}{2}-y_0\right)\frac{A}{\rho+0,5}$.

Thus, when taking into account the effect of the attached mass, the range of variation of the initial transverse velocity narrows, and the degree of compression is the more noticeable the smaller.

Figure-1.a shows the changes in the difference in longitudinal velocities related to $u_0 - 1,5 + 6y_0^2 + W$: $Ur = \left|(u-u_m)/(u_0 - 1,5 + 6y_0^2 + W)\right|$, where u longitudinal speed excluding attached mass, a u_m taking into account the attached mass; curves with numbers 1, 2, 3 correspond to the values $\rho = 0,5,$ $\rho = 1,$ and $\rho = 2,$ respectively for $A = 1.$

Note that the above conclusion takes place subject to the Stokes flow regime.

2. Particle during the Couette

Let us consider the behavior of a single particle in the field of the Couette flow taking into account the Safman forces. The system of dimensionless equations describing the behavior of a particle taking into account the Safman forces is presented in the form:

$$\frac{dx}{dt} = u, \quad \frac{dy}{dt} = v, \quad \frac{du}{dt} = D(y-u), \quad \frac{dv}{dt} = -Dv + a(y-u), \qquad (3)$$

where $a = \frac{a_*\sqrt{A}}{\rho+0,5}$ $\left(a_* = \frac{1,615}{\pi}\sqrt{2}\right)$ dimensionless parameter. Solution of systems of equations (3) under initial conditions $y(0) = y_0$, $x(0) = 0$, $u(0) = u_0$, $v(0) = v_0$ for case $a \ne D^2$ has the form:

$$v = C_1\exp((-D+\sqrt{a})t) + C_2\exp((-D-\sqrt{a})t),$$

$$y = \frac{C_1}{-D+\sqrt{a}}\exp((-D+\sqrt{a})t) - \frac{C_2}{D+\sqrt{a}}\exp((-D-\sqrt{a})t) + C_3,$$

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$$u = y - \frac{1}{\sqrt{a}}\left[C_1 \exp((-D+\sqrt{a})t) - C_2 \exp((-D-\sqrt{a})t)\right],$$

Where $C_1 = 0{,}5\left[v_0 - \sqrt{a}(u_0 - y_0)\right]$, $C_2 = 0{,}5\left[v_0 + \sqrt{a}(u_0 - y_0)\right]$, $C_3 = y_0 + \frac{C_1}{D-\sqrt{a}} + \frac{C_2}{D+\sqrt{a}}$.

Using the obtained solution, some conclusions can be drawn. Under the condition $D^2 > a$ asymptotic solution $y(t)$ at $t \to \infty$ becomes finite. We calculate the difference of the asymptotic solution:

$$y_{m\infty} - y_{\infty} = \left[v_0\sqrt{A} - a_*(u_0 - y_0)\right]\frac{0{,}5A^{1,5}}{(A^{1,5} - a_*(\rho+0{,}5))(A^{1,5} - a_*\rho)},$$

Where $y_{m\infty}, y_{\infty}$ asymptotic solution with and without added mass effect, respectively. It can be seen that the stabilization level of the particle is noticeably different when the attached mass is taken into account, especially for small values of the difference $A^{1,5} - a_*(\rho+0{,}5)$. Note that, under the condition $a_*\rho < A^{1,5} < a_*(\rho+0{,}5)$ values $y_{m\infty}$ will be final y_{∞} will be endless. We find the condition under which the particle does not leave the layer:

without added mass

$$\frac{a_* u_0}{\sqrt{A}} - y_0\frac{A}{\rho} \le v_0 \le \frac{A}{\rho}(1-y_0) + \frac{a_*(v_0-1)}{\sqrt{A}},$$

taking into account the attached mass

$$\frac{a_* u_0}{\sqrt{A}} - y_0\frac{A}{\rho+0{,}5} \le v_0 \le \frac{A}{\rho+0{,}5}(1-y_0) + \frac{a_*(v_0-1)}{\sqrt{A}},$$

Thus, the condition under which the particle does not leave the layer, if the attached mass is taken into account, narrows.

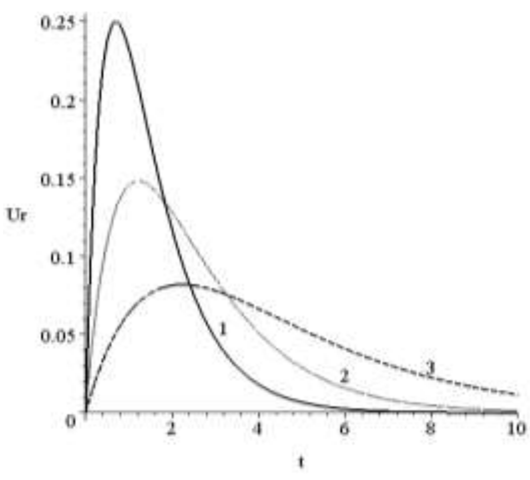

Fig. 1.a: The effect of the attached mass in the upstream

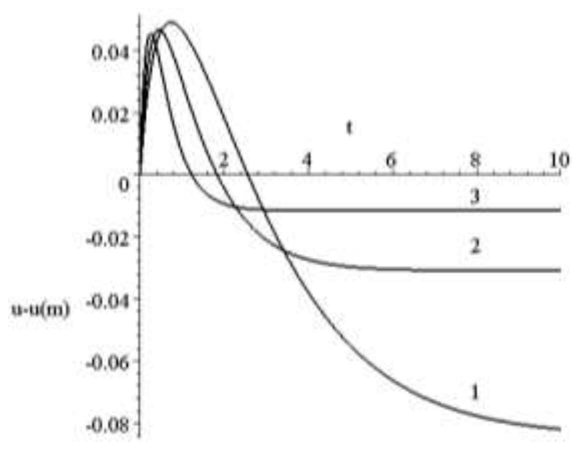

Fig. 1.b: The effect of the added mass effect during the Couette

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Figure-1b. shows the change in the difference in the longitudinal velocity without taking into account (*u*) and taking into account (*u* (*m*)) the attached mass, the curves with numbers 1, 2, 3 correspond to the values and, respectively, for $v_0 = 0,\ u_0 = 0,\ y_0 = 0,$ and $\rho = 1.$

It is easy to establish that the ratio of the force due to the attached mass to the Stokes force is represented in the form:

$$\frac{|F_m|}{|F_h|} = \frac{1}{2\rho + 1} \frac{\sqrt{(y-u)^2 + (v-\lambda)^2}}{\sqrt{(y-u)^2 + v^2}},$$

where $\lambda = \frac{a_*}{\sqrt{A}}$. This shows that when $A >> 1$ the effect of the added mass effect is commensurate with the forces in the case of a flow without the Safman forces during the motion of particles in a Pusel field. Calculation of the ratio of the Safman force to the Stokes force gives:

$\max \frac{|F_S|}{|F_h|} = \lambda.$ Thus, the ratio of the Saffman force to the Stokes force depends on the dimensionless parameter $\sqrt{A}$.

3. Particle during Poiseuille taking into account the Safman Force

Consider the particle flow in an upward flow taking into account the Safman force. We consider the fluid flow to be Poiseuly. Then the system of dimensionless equations has the form.

$$\frac{dx}{dt} = u,\quad \frac{dy}{dt} = v,\quad \frac{du}{dt} = \varphi D(u_l - u) - DW,\quad \frac{dv}{dt} = -\varphi Dv + a\sqrt{|12y|}(u_l - u). \qquad (4)$$

Solution of systems of equations (4) under initial conditions $y(0) = y_0,\ x(0) = 0,\ u(0) = u_0,\ v(0) = v_0$ solved numerically ($\varphi = 1 + 0{,}15\,\mathrm{Re}^{0{,}687}$, Re Reynolds number of particles).

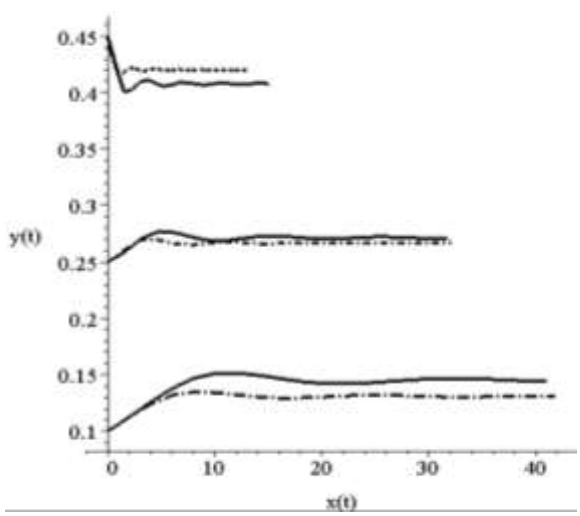

Fig. 2a. The effect of the attached mass effect on the particle trajectory. $(u_0 = 1,\ v_0 = 0,\ \varepsilon = 0{,}01)$

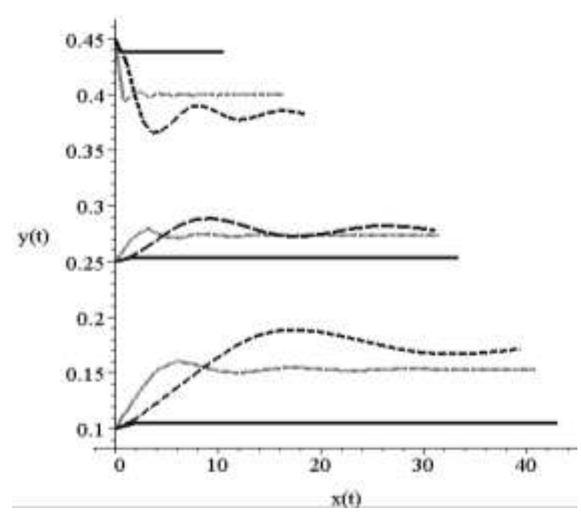

Fig. 2b. The influence of coefficient A on the particle trajectory $(u_0 = 1,\ v_0 = 0,\ \varepsilon = 0{,}01)$.

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III. RESULTS

In fig. 2a, particle trajectories, in the case of different initial positions of the particle, the dashed lines correspond without taking into account the effect of the attached mass $(A=0{,}01;\ \rho=0{,}5?\ W=0)$. Accounting for the added mass stimulates the Segre-Zilberberg effect. In fig.2b shows graphs of the effect of the coefficient on the particle trajectory. $(\rho=1, W=0;, \varepsilon=0{,}01)$. The solid lines correspond to $A=10$, dotted lines refer to $A=0{,}1$, and the dotted ones are $A=0{,}001$. A decrease in the coefficient leads to the stimulation of the Segre-Zilberberg effect.

IV. CONCLUSION

Certainly, it is easy to establish that the ratio of forces due to the attached mass to the hydrodynamic force at $W=0$ there is

$$\frac{|F_m|}{|F_h|}=\frac{1}{2\rho+1}\frac{\sqrt{(u_l-u)^2+(v-\lambda\sqrt{12|y|}(u_l-u)/\sqrt{\varphi})^2}}{\sqrt{(u_l-u)^2+v^2}}.$$

This shows that, when $A>>1$, but finite ρ force ratio is determined similarly (2). Thus, in shear flows, the added mass effect significantly affects the flow regime.

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