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Kinematic Flow Parameters During Liquid Movement in Pressurized Water Pipelines

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Abstract--- *In solving a number of practical problems associated with the flow of fluid in pipelines of various sections with steady and unsteady fluid motion, it becomes necessary to determine the kinematic parameters of the flow. In this paper, we consider solving a number of problems by the method of movable nodes. Using specific examples, the capabilities of this method are evaluated in comparison with other methods. The application of the method of moving nodes to the problems of flow in a flat pipe, flow in an ellipsoidal pipe, flow in a rectangular pipe are considered. A comparison of the exact and approximate solutions obtained on the basis of the method of moving nodes in the cross section* $x=0$ *at* $\sigma=1$ *is given. Comparison shows that the calculation by the above method gives a more accurate result. The maximum absolute difference between the exact and approximate solutions is 0.024. The problem of the transient flow of a viscous fluid between parallel walls is considered. The results show that partial approximations give better results compared to full approximations. Moreover, the best approximation in comparison with others gives at small* t *.*

Keywords---- *Method of Displaceable Nodes, Difference Method, Fluid Mechanics, Volume, Equations, Pipe, Speed, Pressure, Height, Viscosity.*

I. INTRODUCTION

There are various ways to solve problems associated with fluid flow in pressure pipelines, described by differential equations [1,2,3,4,5]. For example, when transporting a viscous fluid in pipelines of various sections [6,7,8,9,10], when using water for irrigation, water supply and other sectors [11,12].

When solving the problem of an unsteady flow of a viscous fluid between parallel walls, the Slezkina-Targ methods are widely used [1,2,3,4,5]. However, there is always a need for more accurate problem solving with the least effort [13,14,15].

In past research works [e.g., 11, 12], the method of movable nodes for the approximate solution of boundary value problems was derived. Discrete solutions of differential equations are averaged solutions of the original differential equation over control volumes. If the region under consideration is represented from one control volume, then it is possible to obtain an approximate analytical solution of the differential equation.

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II. Methodology

When solving the problems of fluid flow through pipes, in order to reduce the problem to a smaller dimension, the equation is averaged over the living section of the pipe. The method of moving nodes allows averaging over the "liquid volume".

If, for example, there is a range of variation with respect to a variable y is $0 \le y \le h$, then in order to obtain an averaged equation with respect to a variable y, one should pass in partial derivatives with respect to the variable y to a discrete with respect to the control volume $[y/2,(h-y)/2]$. In other words, we average the differential equation over the "liquid volume". This method of averaging allows you to go from ordinary differential equations to one algebraic equation.

In the case of partial differential equations, if averaging is performed over all variables, we obtain one algebraic equation; if averaging is performed over only one variable, then we obtain an equation whose independent variable decreases by one unit.

III. Results And Discussion

Let us consider the application of this averaging method to some model problems of fluid mechanics.

Flow in a flat pipe. The viscous fluid flow in a flat pipe in a one-dimensional formulation is described by the equation [14.15]:

$$\frac{d^2U}{dy^2} = -\frac{\Delta p}{\mu\, l} \tag{1}$$

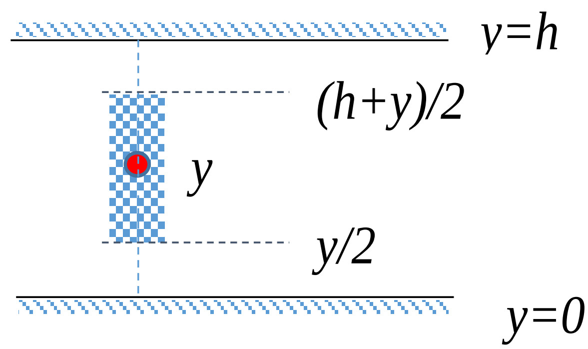

Fig. 1

where U – fluid velocity, y – vertical coordinate perpendicular to the flow, $\Delta p / l$ – pressure drop (const), μ – viscosity coefficient. Let $y = 0$ and $y = h$ motionless walls.

We average (1) over the liquid volume – $[y/2,(h-y)/2]$, Here the “y” is the node to be moved.

Then

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$$\int_{y/2}^{(h+y)/2} \frac{d^2U}{dy^2} dy = \int_{y/2}^{(h+y)/2} \left(-\frac{\Delta p}{\mu\ l} \right) dy$$

Then

$$\left.\frac{dU}{dy}\right|_{(h+y)/2} - \left.\frac{dU}{dy}\right|_{y/2} = \left(-\frac{\Delta p}{\mu\ l} \right) \frac{h}{2} \qquad (2)$$

Replace the derivatives in (2) by the difference relation:

$$\left.\frac{dU}{dy}\right|_{(h+y)/2} \approx \frac{u(h) - u(y)}{h - y}, \ \left.\frac{dU}{dy}\right|_{y/2} \approx \frac{u(y) - u(0)}{y}. \qquad (3)$$

Here $u(y)$ is an approximate value $U(y)$. Thus, the approximation (1) over the moved node has the form:

$$\frac{u(h) - u(y)}{h - y} - \frac{u(y) - u(0)}{y} = \left(-\frac{\Delta p}{\mu\ l} \right) \frac{h}{2}. \qquad (4)$$

And then, taking into account the sticking condition

$$u(y) = -\frac{\Delta p}{2\mu\ l} y(h - y).$$

Here $u(y)$ is an averaged solution. For this problem, the averaged solution coincides with the exact solution.

Flow in an ellipsoidal tube. The equation describing the one-dimensional flow in an ellipsoidal tube of a viscous fluid has the form [4]:

$$\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = -\frac{\Delta p}{\mu l} \qquad (5)$$

u - is the flow rate, μ - flow viscosity, Δp/l (Δp/l =const) pressure drop. Equation (5) is considered in the region $\frac{y^2}{a^2} + \frac{z^2}{b^2} \le 1$ (cross section of an ellipsoidal pipe, Fig. 2), the boundary condition is the adhesion conditions (U=0).

Equation (2) is replaced by the difference

$$\frac{2}{y_E - y_W}\left(\frac{U_E - u}{y_E - y} - \frac{u - U_W}{y - y_W} \right) + \frac{2}{z_N - z_S}\left(\frac{U_N - u}{z_N - z} - \frac{u - U_S}{z - z_S} \right) = -\frac{\Delta p}{\mu l}.$$

Hence, given that

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$z_N = b\sqrt{1-y^2/a^2}$, $z_S = -b\sqrt{1-y^2/a^2}$, $y_E = a\sqrt{1-z^2/b^2}$, $y_W = -a\sqrt{1-z^2/b^2}$. we get

$$u = \frac{a^2b^2}{2(a^2+b^2)}\left(1-\frac{y^2}{a^2}-\frac{z^2}{b^2}\right)\frac{\Delta p}{\mu l}$$

coinciding with the exact solution.

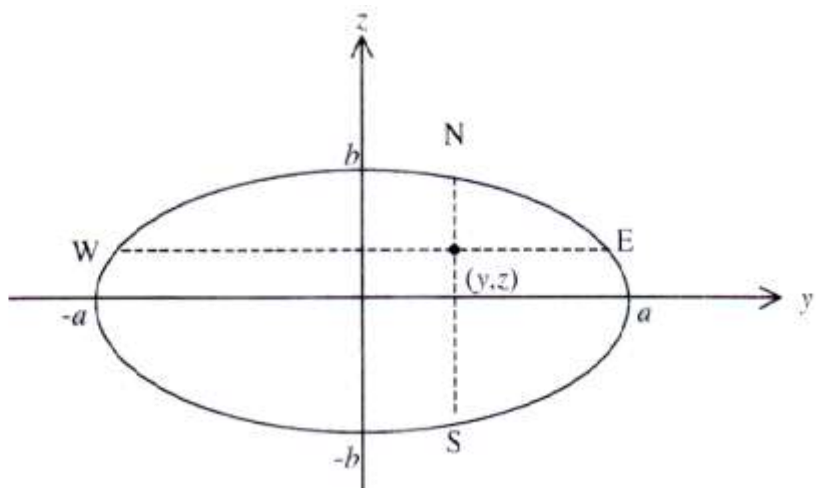

Fig. 2: Ellipsoidal Tube Section

Flow in a rectangular pipe. Equation (5) also describes the flow of an incompressible viscous fluid into rectangular pipes [2]. Let us denote the height of the rectangle parallel to the axis by Oz, and the base parallel to the axis by Oy, by $2\sigma h$, where σ – any positive constant. We draw axis Ox through the center of the rectangle and direct it downstream.

We transform equation (5) to a dimensionless form. For the scale of lengths we take the height *h*, and for the scale of speeds - the value $h^2 / \mu \cdot \Delta p / l$. We introduce the following dimensionless quantities:

$$Y = y/h, \quad Z = z/h, \quad V = U\mu l/(h^2\Delta p)$$

Substituting in (5), we obtain

$$\frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial Z^2} = -1 \qquad (6)$$

Boundary conditions for (6)

$$V(Y,-1)=0, \;\; V(Y,1)=0, \;\; V(-\sigma,Z)=0, \;\; V(\sigma,Z)=0 \qquad (7)$$

Equation (6) is replaced by difference, and taking into account the boundary condition (7), we have

$$\frac{2}{2\sigma}\left(\frac{-V}{\sigma-Y}-\frac{V}{Y+\sigma}\right)+\frac{2}{1+1}\left(\frac{-V}{1-Z}-\frac{V}{Z+1}\right)=-1.$$

From here we determine the approximate analytical solution:

$$V=\frac{1}{2}\frac{(\sigma^2-Y^2)(1-Z^2)}{1-Z^2+\sigma^2-y^2} \tag{8}$$

The exact solution to the problem is:

$$u=\frac{16\sigma^2}{\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3}\left[1-\frac{ch\left(\frac{2n+1}{2}\frac{\pi}{\sigma}Y\right)}{ch\left(\frac{2n+1}{2}\frac{\pi}{\sigma}\right)}\right]\cos\left(\frac{2n+1}{2}\frac{\pi}{\sigma}Z\right)$$

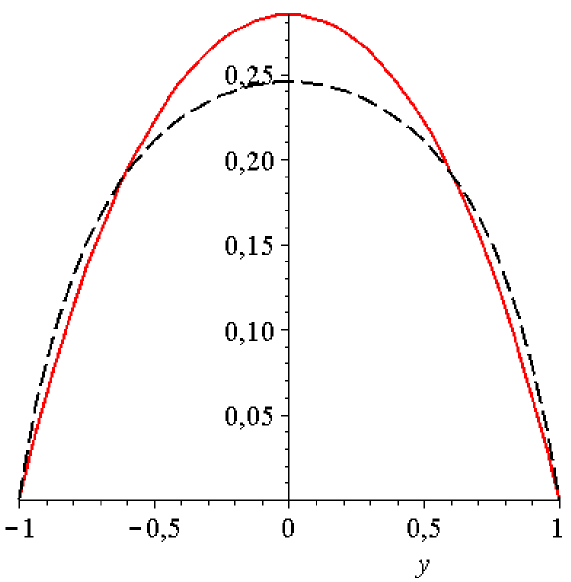

Fig.3: Cross Section Solution $x=0$ for (8).

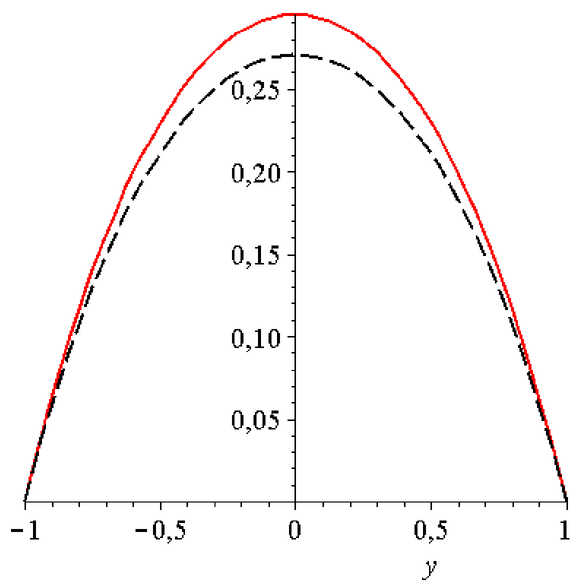

Fig.4: Cross Section Solution $x=0$ for (10)

Figure 3 shows a comparison of the exact and approximate solutions in the cross section $x=0$ for $\sigma=1$. The maximum absolute difference between the exact and approximate solutions is 0.045.

To increase the accuracy of the approximate solution in equation (6), we approximate only the term. For example, we approximate equation (6) as follows:

$$\frac{2}{2\sigma}\left(\frac{-V}{\sigma-Y}-\frac{V}{Y+\sigma}\right)+\frac{\partial^2 V}{\partial Z^2}=-1 \tag{9}$$

We got the ordinary differential equation, we consider the variable in equation (9) as a parameter. We solve equation (9) with constant coefficients, taking into account the boundary conditions, we find an approximate solution

$$V = C_1 \exp\left(\sqrt{k}Z\right) + C_2 \exp\left(-\sqrt{k}Z\right) + \frac{1}{k}. \tag{10}$$

Here $k = 2/\left((\sigma - Y)(Y+\sigma)\right)$, $C_2 = -\frac{1}{k}\frac{\exp(\sqrt{k}) - \exp(-\sqrt{k})}{\exp(2\sqrt{k}) - \exp(-2\sqrt{k})}$,

$$C_1 = -C_2 - \exp(2\sqrt{k}) - \frac{1}{k}\exp(-\sqrt{k}).$$

Figure 4 shows a comparison of the exact approximate solution obtained on the basis of (10) in the cross section $x = 0$ for $\sigma = 1$.. A comparison of Fig. 3 and Fig. 4 shows that the calculation by formula (10) gives a more accurate result. The maximum absolute difference between the exact and approximate solutions is equal to that obtained by (10) and is equal to 0.024. The solid curves in Fig. 3 and 4 represent an exact solution.

Unsteady flow of viscous fluid between parallel walls. Let a viscous fluid fill the entire space between horizontal planes located at a certain distance from each other. Let the lower plane be motionless all the time, and let the upper one begin moving at the moment $t = 0$ right at a constant speed. We neglect the action of gravity and consider the pressure everywhere constant. We assume that the flow is directed parallel to the axis *ox*. Then the equation of motion of a viscous fluid in dimensionless variables has the form [3].

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2}$$

The exact solution to the equation under the conditions:

$$u(0, y) = 0,\ u(t,0) = 0,\ u(t,1) = 1$$

Has the form:

$$u = y + \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^k}{k}\sin(k\pi y)\exp(-k^2\pi^2 t)$$

If we average over two variables, we get an approximate solution in the form:

$$u_1 = \frac{2ty}{y(1-y) + 2t}$$

Here $\max\left|\bar{u}_1 - u\right| \cong 0{,}2.$

If averaging is performed only over a variable y, then we obtain an approximate solution in the form:

$$\bar{u}_2 = y\left(1 - \exp\left(-\frac{2t}{y(1-y)}\right)\right)$$

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In this case, $\max|\bar{u}_2 - u| \cong 0{,}08$ and also shows that partial approximation gives the best result.

If averaging is carried out only over a variable t, then we obtain an approximate solution in the form:

$$\bar{u}_3 = \frac{\exp(y/\sqrt{t}) - \exp(-y/\sqrt{t})}{\exp(1/\sqrt{t}) - \exp(-1/\sqrt{t})}$$

For this method of averaging, we have $\max|\bar{u}_3 - u| \cong 0{,}12$

The results obtained show that partial approximations give better results compared to full approximations. It should be noted that the approximation $\bar{u}_3$ gives the best approximation in comparison with others at small t.

Let us now compare the proposed solution method with the Slezkin-Targ method widely used in fluid mechanics. Averaging by the Slezkin-Targ method to the original equation gives

$$\bar{u}_4 = \frac{y(y-1)}{1+y}\exp(-6t) + y$$

For this method of averaging, we have $\max|\bar{u}_4 - u| \cong 0{,}7$. Comparison $\bar{u}_2$ and $\bar{u}_4$ under $t \geq 0{,}5$ gives a close result. However, at the initial moment of time $(t < 0{,}5)$, the solution according to the Slezkin – Targ method [2] gives the worst result compared to the three averagings considered.

IV. CONCLUSIONS

When determining the parameters of the motion of a viscous fluid in flat, ellipsoidal, and rectangular pipes, the application of the method of moving nodes gives more accurate results compared to existing ones. The considered examples show that the proposed method makes it possible to obtain solutions of equations suitable for practical problems.

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