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Numerical modeling of nonlinear viscoelastic vibrations of vehicles

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Abstract. Nonlinear viscoelastic oscillations of a vehicle in a vertical direction excited by static imbalance are considered. When taking into account the rheological properties of the suspension material and spring, the Boltzmann-Volterra principle was used. Mathematical models of the problem under consideration are obtained, which are described by systems of nonlinear integro-differential equations. A solution method based on quadrature formulas has been developed, and a computer program has been compiled on its basis. The results obtained are reflected in graphs. The influence of the material's rheological properties and the nonlinear characteristics of the suspension and spring on the amplitude and frequency of vertical vibrations of vehicles was investigated.

1 Introduction

Fluctuations in wheeled vehicle springing systems remain under the scrutiny of those skilled in the art, as they cause many undesirable manifestations during the operation of vehicles. Researchers are interested in the behavior of the sprung mass since the nature of the vibrations of the sprung mass most often affects the well-being of the driver and passengers and determines the possibilities of the safety of the transported cargo and traffic safety.

Forced oscillations of the car, excited by wheel imbalances, as well as variable circumferential thickness of the brake discs, relate to one of the theories of oscillations of the car. Meanwhile, the influence of these factors on the level of vibration, smoothness indicators, and, most importantly, on stability and active traffic safety is extremely significant, determining the overall level of vibration of the car.

In work [1], a study of energy consumption in an automobile suspension was carried out. The developed model considers suspensions with linear and nonlinear characteristics. A comparison of the results shows that there is little difference up to 30 km/h. When the speed exceeds 30 km/h, the nonlinearity characteristic must be considered. The amount of energy consumed during vibration damping is 0.06... 0.17 kWh.

The advantages of using an electromechanical converter on a car are the ability to easily control the damping force, recover mechanical energy into electrical energy, etc. Some

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design options exclude additional mechanical elements from the suspension. At that, the value of the unsprung weight of the car decreases, which reduces its dynamic load and increases reliability [2].

The principle of dividing the whole into composite elements simplifies the analysis and accompanying calculations. Still, at the same time, kinematic and dynamic connections that determine these elements' movement and the vibrational system are inevitably ignored. Simplified (partial) vibrational systems with a very limited number of degrees of freedom and, accordingly, generalized coordinates have the advantage that they significantly simplify the quantitative and qualitative analysis of the studied processes [3].

To eliminate or reduce the level of dangerous oscillations in the car springing system, numerous versions of automated suspension control systems are being developed.

1.1 Statement of problem

Setting the task. It should be borne in mind that the influence of static and moment imbalances on the car's dynamic characteristics is different. The nonlinear oscillations of the vehicle in the vertical direction excited by the static imbalance are considered here. For the analysis, the so-called flat model of the car was used, defined by generalized coordinates z_1, z_2, z_3 that is vertical movements of the wheels and the center of mass of the body, and the angle of rotation of the φ body (Fig. 1).

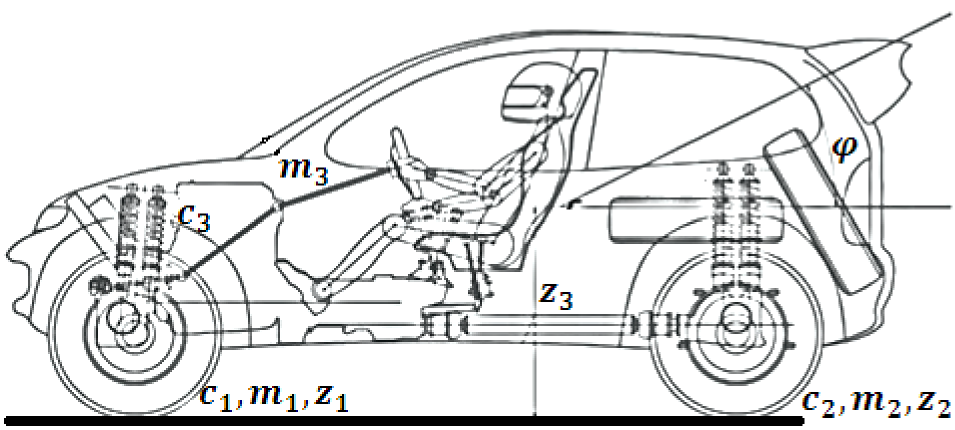

Fig.1. Flat car model

Using D'Alembert's principle and considering the fictitious equilibrium of the car, to which inertia forces are applied and excited by wheel imbalances, we obtain [2-4]:

$$\begin{cases} m_1\ddot{z}_1 + F(z_1) - F(z_3 - z_1 + a\varphi) = Q_1; \\ m_2\ddot{z}_2 + F(z_2) - F(z_3 - z_2 - b\varphi) = Q_2; \\ m_3\ddot{z}_2 + F(z_3 - z_1 + a\varphi) + F(z_3 - z_2 - b\varphi) = Q_3; \\ m_3\rho\ddot{\varphi} + aF(z_3 - z_1 + a\varphi) - bF(z_3 - z_2 - b\varphi) = Q_4. \end{cases}$$

The generalized forces are respectively equal to:

$$Q_1 = Q_2 = U\omega^2 sin\omega t;\ \ Q_3 = Q_4 = 0.$$

There are many rheological models describing the nonlinear properties of materials. According to the Volterra principle, the solution to the nonlinear viscoelasticity problem can be obtained from solving the corresponding problem of nonlinear elasticity theory by replacing elasticity constants in it with some operators.

For the function F (z), assume the expression [5-8]:

$$F(z) = \mathrm{c}(1 - R^*)z(1 + \gamma z^2)$$

where c is rigidity of suspension; γ is nonlinearity coefficient depending on physical properties of the suspension material; $R^* y(t) = \int_0^t R(t-\tau)y(\tau)d\tau$ are integral operators with cores of relaxation $R(t) = \varepsilon t^{\alpha-1} e^{-\beta t}$.

$$\begin{cases} m_1\ddot{z}_1 + \mathrm{c}_1(1 - R_1^*)z_1(1 + \gamma_1 z_1^2) - \\ -\mathrm{c}_3(1 - R_3^*)(z_3 - z_1 + a\varphi)[1 + \gamma_3(z_3 - z_1 + a\varphi)^2] = U\omega^2 sin\omega t \\ m_2\ddot{z}_2 + \mathrm{c}_2(1 - R_2^*)z_2(1 + \gamma_2 z_2^2) - \\ -\mathrm{c}_4(1 - R_4^*)(z_3 - z_2 - b\varphi)[1 + \gamma_4(z_3 - z_2 - b\varphi)^2] = U\omega^2 sin\omega t \\ m_3\ddot{z}_3 + \mathrm{c}_3(1 - R_3^*)(z_3 - z_1 + a\varphi)[1 + \gamma_3(z_3 - z_1 + a\varphi)^2] + \\ +\mathrm{c}_4(1 - R_4^*)(z_3 - z_2 - b\varphi)[1 + \gamma_4(z_3 - z_2 - b\varphi)^2] = 0 \\ m_3\rho^2\ddot{\varphi} + \mathrm{c}_3(1 - R_3^*)(az_3 - az_1 + a^2\varphi)[1 + \gamma_3(z_3 - z_1 + a\varphi)^2] - \\ -\mathrm{c}_4(1 - R_4^*)(bz_3 - bz_2 - b^2\varphi)[1 + \gamma_4(z_3 - z_2 - b\varphi)^2] = 0 \end{cases}$$

Here it is indicated: m_1, m_2 and c_1, c_2 are masses and stiffness factors of wheels; m_3, ρ_c are total mass and radius of inertia of sprung masses; c_3 is front suspension rigidity; c_4 is rigidity of rear suspension; a, b are distances from the front and rear wheels to the center of mass of the car; γ_i $(i = \overline{1,4})$ are nonlinearity coefficients.

2 Method of solution

Entering dimensionless t/τ parameters; $\frac{t}{\tau}$; $\frac{R_i}{\tau}$; $\frac{\omega}{\tau}$; $\frac{z_1}{l}$; $\frac{z_2}{l}$; $\frac{z_3}{l}$ and retaining the previous designations, we have:

$$\begin{cases} \ddot{z}_1 + k_1(1-R_1^*)z_1(1+\gamma_1 z_1^2) - \\ -k_2(1-R_3^*)(z_3-z_1+k_3\varphi)[1+\gamma_3(z_3-z_1+k_3\varphi)^2] = k_4 sin\omega t \\ \ddot{z}_2 + k_5(1-R_2^*)z_2(1+\gamma_2 z_2^2) - \\ -k_6(1-R_4^*)(z_3-z_2-k_7\varphi)[1+\gamma_4(z_3-z_2-k_7\varphi)^2] = k_8 sin\omega t \\ \ddot{z}_3 + k_9(1-R_3^*)(z_3-z_1+k_3\varphi)[1+\gamma_3(z_3-z_1+k_3\varphi)^2] + \\ +k_{10}(1-R_4^*)(z_3-z_2-k_7\varphi)[1+\gamma_4(z_3-z_2-k_7\varphi)^2] = 0 \\ \ddot{\varphi} + k_{11}(1-R_3^*)(z_3-z_1+k_3\varphi)[1+\gamma_3(z_3-z_1+k_3\varphi)^2] - \\ -k_{12}(1-R_4^*)(z_3-z_2-k_7\varphi)[1+\gamma_4(z_3-z_2-k_7\varphi)^2] = 0 \end{cases} \quad (1)$$

here

$$k_1 = \frac{c_1\tau^2}{m_1}; \quad k_2 = \frac{c_3\tau^2}{m_1}; \quad k_3 = \frac{a}{l}; \quad k_4 = \frac{U\omega^2\tau^2}{m_1 l};$$
$$k_5 = \frac{c_2\tau^2}{m_2}; \quad k_6 = \frac{c_4\tau^2}{m_2}; \quad k_7 = \frac{b}{l}; \quad k_8 = \frac{U\omega^2\tau^2}{m_2 l};$$
$$k_9 = \frac{c_3\tau^2}{m_3}; \quad k_{10} = \frac{c_4\tau^2}{m_3}; \quad k_{11} = \frac{c_3\tau^2 la}{m_3\rho^2}; \quad k_{12} = \frac{c_4\tau^2 lb}{m_3\rho^2}.$$

We accept that

$$z_1(0) = \dot{z}_1(0) = z_2(0) = \dot{z}_2(0) = z_3(0) = \dot{z}_3(0); \ \varphi(0) = \varphi_0 \text{ and } \dot{\varphi}(0) = 0. \quad (2)$$

The system of integro-differential equations (1) with initial conditions (2) is solved by a method based on the use of the quadrature formula [9-14]. Integrating twice by t system (1) in the interval [0; t] and assuming $t = t_n = n \cdot \Delta t$, $n = 0,1,2,3, \ldots$ (Δt -step in time) we have:

$$\begin{cases} z_{1n} + k_1 \int_0^{t_n} G_1(t_n - s) z_1(s)[1 + \gamma_1 z_1^2(s)]\, ds - \\ -k_2 \int_0^{t_n} G_3(t_n - s)[z_3(s) - z_1(s) + k_3\varphi(s)]\{1 + \gamma_3[z_3(s) - z_1(s) + k_3\varphi(s)]^2\} ds = \\ = k_4 \int_0^{t_n} (t_n - s)\, sin\omega s ds; \\ z_{2n} + k_5 \int_0^{t_n} G_2(t_n - s) z_2(s)[1 + \gamma_2 z_2^2(s)]\, ds - \\ -k_6 \int_0^{t_n} G_4(t_n - s)[z_3(s) - z_2(s) - k_7\varphi(s)]\{1 + \gamma_4[z_3(s) - z_2(s) - k_7\varphi(s)]^2\} ds = \\ = k_8 \int_0^{t_n} (t_n - s)\, sin\omega s ds; \\ z_{3n} + k_9 \int_0^{t_n} G_3(t_n - s)[z_3(s) - z_1(s) + k_3\varphi(s)]\{1 + \gamma_3[z_3(s) - z_1(s) + k_3\varphi(s)]^2\}\, ds + \\ +k_{10} \int_0^{t_n} G_4(t_n - s)[z_3(s) - z_2(s) - k_7\varphi(s)]\{1 + \gamma_4[z_3(s) - z_2(s) - k_7\varphi(s)]^2\} ds = 0; \\ \varphi_n - \varphi(0) + k_{11} \int_0^{t_n} G_3(t_n - s)[z_3(s) - z_1(s) + k_3\varphi(s)]\{1 + \gamma_3[z_3(s) - z_1(s) + k_3\varphi(s)]^2\}\, ds - \\ -k_{12} \int_0^{t_n} G_4(t_n - s)[z_3(s) - z_2(s) - k_7\varphi(s)]\{1 + \gamma_4[z_3(s) - z_2(s) - k_7\varphi(s)]^2\} ds = 0; \end{cases} \quad (3)$$

here

$$G_j(t_n - s) = t_n - s - \int_0^{t_n - s} (t_n - s - \tau) R_j(\tau)\, d\tau; \quad j = \overline{1,4}.$$

Replacing integrals with quadrature trapezoid formulas in the system (3), we have the following recompetitive ratios for determining the vertical movements of the wheels, the center of mass, and the angle of rotation of the body: $z_{1n} = z_1(t_n)$; $z_{2n} = z_2(t_n)$; $z_{3n} = z_3(t_n)$; $\varphi_n = \varphi(t_n)$.

$$
\begin{cases}
z_{1n} = -k_1 \sum_{i=0}^{n-1} A_i G_1(t_n - t_i) z_{1i}\left(1 + \gamma_1 z_{1i}^2\right) + \\
+k_2 \sum_{i=0}^{n-1} A_i G_3(t_n - t_i)(z_{3i} - z_{1i} + k_3\varphi_i)[1 + \gamma_3(z_{3i} - z_{1i} + k_3\varphi_i)^2] + \\
+k_4 \sum_{i=0}^{n-1} A_i (t_n - t_i) sin\omega t_i\,; \\
z_{2n} = -k_5 \sum_{i=0}^{n-1} A_i G_2(t_n - t_i) z_{2i}\left(1 + \gamma_2 z_{2i}^2\right) + \\
+k_6 \sum_{i=0}^{n-1} A_i G_4(t_n - t_i)(z_{3i} - z_{2i} - c_7\varphi_i)[1 + \gamma_4(z_{3i} - z_{1i} - k_7\varphi_i)^2] + \\
+k_8 \sum_{i=0}^{n-1} A_i (t_n - t_i) sin\omega t_i\,; \\
z_{3n} = -k_9 \sum_{i=0}^{n-1} A_i G_3(t_n - t_i)(z_{3i} - z_{1i} + k_3\varphi_i)[1 + \gamma_3(z_{3i} - z_{1i} + k_3\varphi_i)^2] - \\
-k_{10} \sum_{i=0}^{n-1} A_i G_4(t_n - t_i)(z_{3i} - z_{2i} - k_7\varphi_i)[1 + \gamma_4(z_{3i} - z_{2i} - k_7\varphi_i)^2]; \\
\varphi_n = \varphi_0 - c_{11} \sum_{i=0}^{n-1} A_i G_3(t_n - t_i)(z_{3i} - z_{1i} + k_3\varphi_i)[1 + \gamma_3(z_{3i} - z_{1i} + k_3\varphi_i)^2] + \\
+k_{12} \sum_{i=0}^{n-1} A_i G_4(t_n - t_i)(z_{3i} - z_{2i} - c_7\varphi_i)[1 + \gamma_4(z_{3i} - z_{2i} - k_7\varphi_i)^2].
\end{cases} \quad (4)
$$

here $A_0 = \frac{\Delta t}{2};\ A_j = \Delta t, j = \overline{1, n-1}.$

3 Results and Discussion

To carry out a computational experiment, a computer program was compiled, the numerical results of which are presented in the form of graphs. Numerical calculations were performed. The following initial data were used:
$k_1 = 571{,}33$; $k_2 = 472{,}87$; $k_3 = 0{,}6$; $k_4 = 0{,}0004$; $k_5 = 571{,}33$; $k_6 = 545{,}73$; $k_7 = 0{,}4$; $k_8 = 0{,}0004$; $k_9 = 56{,}74$; $k_{10} = 65{,}49$; $k_{11} = 478{,}78$; $k_{12} = 368{,}37$; $\varphi_0 = 0{,}8$; $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0{,}25$; $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0{,}05$; $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 0{,}05$; $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0{,}8$.

Figures 2,3,4,5 show the shape of the vertical vibrations of the wheels, the center of mass, and the body's rotation angle, respectively. Here, the solid line is indicated for elastic (ε_i=0) and the dotted line (ε_i=0,05) for viscoelastic suspension and spring. Taking into account the rheological properties of the suspension material and the spring leads to a decrease in the amplitude of vertical vibrations of the wheels, the center of mass, and the angle of rotation of the body. A decrease in the frequency of oscillations leads to a phase shift to the right. Over time, the viscoelastic properties of the suspension and spring significantly affect amplitude and frequency.

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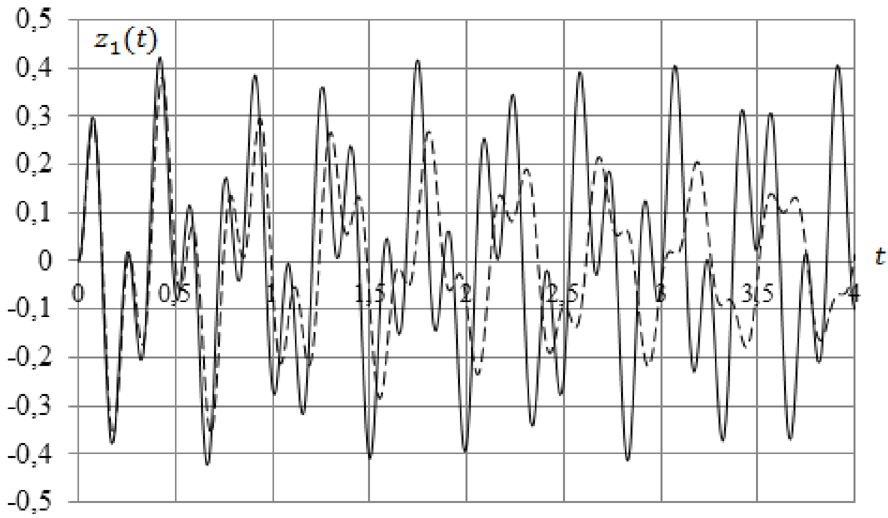

Fig.2. Front axle oscillation shape.

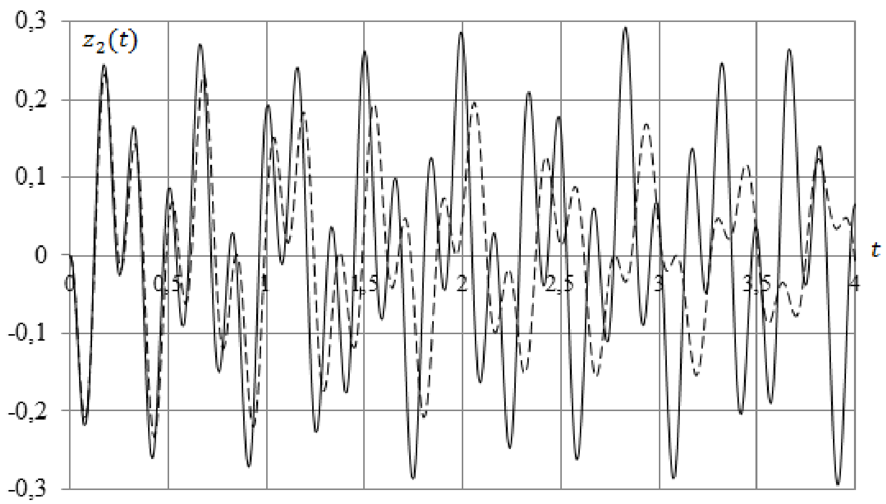

Fig.3. The form of oscillation of the rear axle.

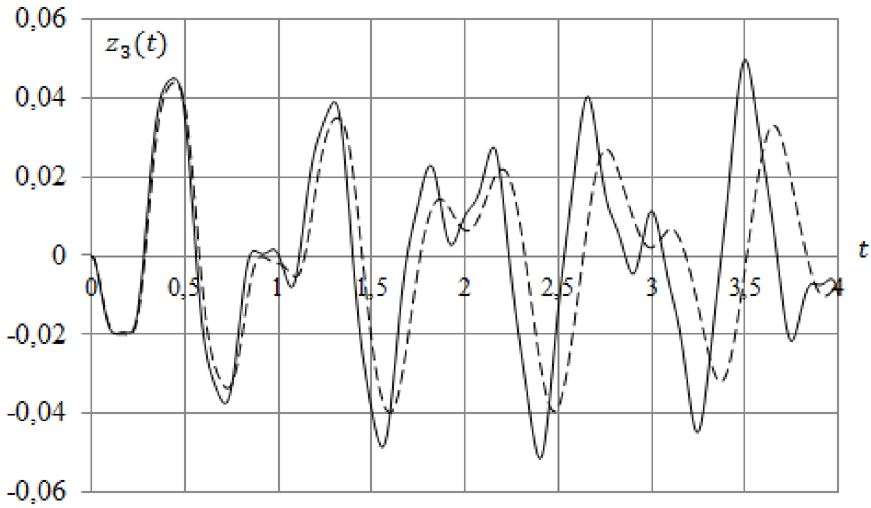

Fig.4. Shape of oscillation of body center of mass.

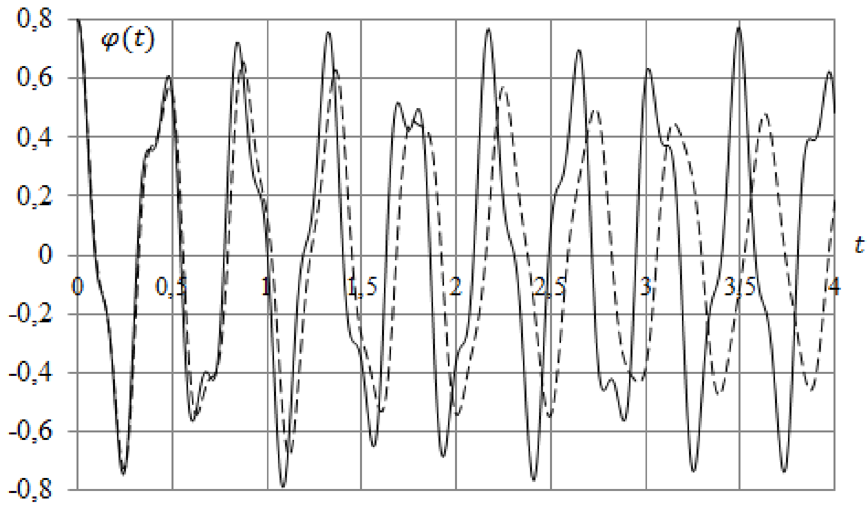

Fig.5. Change the angle of rotation of the body.

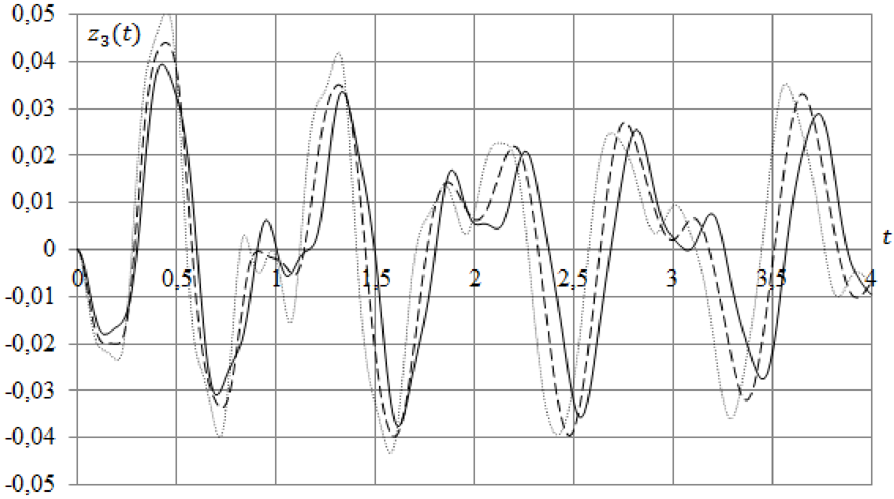

Fig.6. Changes the body center of mass at $\gamma_i = 0$ (solid line), $\gamma_i = 0{,}8$ (dashed line), and $\gamma_i = 1{,}6$ (dot line).

Figure 6 shows the effect of the nonlinearity factor on the shape of the body center of mass oscillations. An increase in the nonlinearity factor leads to an increase in the frequency of oscillations. The amplitude change is insignificant.

4 Conclusions

The forced oscillations of the vehicle excited by the moment imbalances of the wheels have a significantly more complex nature than the oscillations excited by the static imbalances. Angular oscillations of the controlled wheels around the axes of rotation, excited by moment imbalances, in the presence of a "rolling arm" and due to the effect of gyroscopic moments of the wheels, involve the entire structure of the car in a single oscillatory process. Therefore, to describe this resulting process, it is necessary to use the generalized system of integro-differential equations given above. The use of schemes capable of solving the problem in question in a closed form or using well-studied algorithms of type (4) is of great interest. The obtained results allow us to conclude that it is advisable to take into account the hereditary deformable properties of the suspension and the spring to reduce the amplitude of vibrations of the vehicle during transient processes.

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