CHDMni simpleks usulida yechish

Reja:

1. CHDMni simpleks usulida yechish.
2. CHDM ni amaliy dasturlar yordamida yechish.

8.1. CHDMni simpleks usulida yechish.

Ayrim agroinjeneriya masalalarini yechish, shu jumladan gidromelioratsiya masalalari chiziqli dasturlash masalalarini yechishga keltiriladi. Chiziqli dasturlash masalasi umumiy holda quyidagi ko'rinishda bo'ladi:

$$z = c_1x_1 + c_2x_2 ... + c_nx_n \to max(min) \tag{8.1}$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 ... + a_{1n}x_n \le a_1 \\ a_{21}x_1 + a_{22}x_2 ... + a_{2n}x_3 \le a_2 \\ \text{..........} \\ a_{m1}x_1 + a_{m2}x_2 ... + a_{mn}x_3 \le a_n \end{cases} \tag{8.2}$$

$$x_1 \ge 0,\ x_2 \ge 0, ..., x_n \ge 0 \tag{8.3}$$

Bu yerda (8.1) maqsad funktsiyasi, (8.2) cheklanishlar sistemasi, (8.3) nomanfiylik sharti deyiladi. Masalada $x_1, x_2, \ldots, x_n$ o'zgaruvchilarning shunday qiymatlarini topish kerakki, ular (8.2) va (8.3) shartlarni qanoatlantirsin hamda (8.1) funktsiya maksimal (minimal) qiymatni qabul qilsin.

Ushbu masalani umumiy holda simpleks usulda, o’zgaruvchilar soni ikkita bo’lgan holda esa, grafik usulda yechish mumkin.

8.2. Chiziqli dasturlash masalasini simpleks usulida yechish

Ma’lumki, chiziqli dasturlash masalasi umumiy holda simpleks usulda yechiladi. Chiziqli dasturlash masalasini simpleks usulda yechish ikki bosqichdan iborat bo’lib, birinchi bosqichda masalaning tayanch yechimi , ikkinchi bosqichda esa optimal yechim topiladi.

Tayanch yechimni topish qoidasi quyidagicha:

1) (8.1)-(8.3) masalani quyidagi

$$z = c_1x_1 + c_2x_2 ... + c_nx_n \to max(min)$$

$$\begin{cases} y_1 = -a_{11}x_1 - a_{12}x_2 ... - a_{1n}x_n + a_1 \geq 0 \\ y_2 = -a_{21}x_1 - a_{22}x_2 ... - a_{2n}x_n + a_2 \geq 0 \\ .... \\ y_n = -a_{m1}x_1 - a_{m2}x_2 ... - a_{mn}x_n + a_m \geq 0 \end{cases}$$

$$x_1 \geq 0,\ x_2 \geq 0, ..., x_n \geq 0$$

ko'rinishga keltiramiz.

2) Yuqoridagi munosabatlardan quyidagi simpleks jadvalini tuzamiz:

	$-x_1$	$-x_2$	...	$-x_n$	Ozod sonlar
$y_1 =$	a_{11}	a_{12}	...	a_{1n}	a_1
$y_2 =$	a_{21}	a_{22}	...	a_{2n}	a_2
...	...	...	...	...	...
$y_m =$	a_{m1}	a_{m2}	...	a_{mn}	a_m
$z =$	c_1	c_2	...	c_n	

Ozod sonlar ustunidagi manfiy sonlarni qaraymiz. Agar ushbu sonlarning hammasi musbat bo’lsa, u holda masalaning tayanch yechimi topilgan xisoblanadi. Agar ozod sonlar orasida bir nechta manfiy sonlar mavjud bo’lsa, ulardan birini tanlaymiz. Faraz qilaylik – l satrdagi a1< 0 ozod sonni tanlab olaylik.

	$-x_1$	$-x_2$	$\cdots$	$-x_k$	$-x_{k+1}$	$\cdots$	$-x_n$	Ozod sonlar
$y_1 =$	a_{11}	a_{12}	$\cdots$	a_{1k}	a_{1k+1}	$\cdots$	a_{1n}	a_1
$y_2 =$	a_{21}	a_{22}	$\cdots$	a_{2k}	a_{2k+1}	$\cdots$	a_{2n}	a_2
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$y_i =$	a_{i1}	a_{i2}	$\cdots$	a_{ik}	a_{ik+1}	$\cdots$	a_{in}	a_i
$y_{i+1} =$	a_{i+11}	a_{i+12}	$\cdots$	a_{i+1k}	a_{i+1k+1}	$\cdots$	a_{i+1n}	a_{i+1}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$y_j =$	a_{jk}	a_{j2}	$\cdots$.	a_{jk}	a_{jk+1}	$\cdots$	a_{jn}	a_j
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$y_m =$	a_{mk}	a_{m2}	$\cdots$.	a_{mk}	a_{mk+1}	$\cdots$	a_{mn}	a_m
$z =$	c_1	c_2	$\cdots$	c_k	c_{k+1}	$\cdots$	c_n	0

4) *l*- satrdagi manfiy sonlarni qaraymiz. Agar ushbu satrda manfiy sonlar bo'lmasa, masala yechimga ega bo'lmaydi, agar manfiy sonlar bir nechta bo'lsa, ulardan birini tanlaymiz. Masalan, k- ustundagi alk < 0 sonni tanlab olaylik. k - ustun hal qiluvchi ustun deyiladi.

5) Ozod sonlar va k-ustundagi mos koeffitsiyentlar juftliklarini qaraymiz. Agar ularning ishoralari bir xil bo'lsa, ozod sonlarni mos koeffitsiyentlarga bo'lamiz.

6) Hosil bo'lgan nisbatlarning eng kichigini tanlab olamiz: .

$$\theta = min\left(\frac{c_i}{a_{ik}}\right)_{i=\overline{1,p}}$$

Bu yerda p- tanlab olingan juftliklar soni. 0– hal qiluvchi element deyiladi. Agar 0 j - satrga mos kelsa, j- satr hal qiluvchi satr deyiladi. Jadval quyidagi ko'rinishga keladi:

	$-x_1$	$-x_2$	$\cdots$	$-x_k$	$-x_{k+1}$	$\cdots$	$-x_n$	Ozod sonlar
$y_1 =$	a_{11}	a_{12}	$\cdots$	a_{1k}	a_{1k+1}	$\cdots$	a_{1n}	a_1
$y_2 =$	a_{21}	a_{22}	$\cdots$	a_{2k}	a_{2k+1}	$\cdots$	a_{2n}	a_2
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$y_i =$	a_{i1}	a_{i2}	$\cdots$	a_{ik}	a_{ik+1}	$\cdots$	a_{in}	a_i
$y_{i+1} =$	a_{i+11}	a_{i+12}	$\cdots$	a_{i+1k}	a_{i+1k+1}	$\cdots$	a_{i+1n}	a_{i+1}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$y_j =$	a_{jk}	a_{j2}	$\cdots .$	a_{jk}	a_{jk+1}	$\cdots$	a_{jn}	a_j
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$y_m =$	a_{mk}	a_{m2}	$\cdots .$	a_{mk}	a_{mk+1}	$\cdots$	a_{mn}	a_m
$z =$	c_1	c_2	$\cdots$	c_k	c_{k+1}	$\cdots$	c_n	0

7) a_{jk} elementga nisbatan modifikatsiyalangan Jordan almashtirishlarini, boshqacha aytganda simpleks almashtirishlarni bajarib navbatdagi jadvalni to'ldiramiz;

6.1) Hal qiluvchi satr va ustundagi o'zgaruvchilar o'rni almashtiriladi;

7.2) Hal qiluvchi element o'rniga unga teskari sonni yozamiz;

7.3) Hal qiluvchi satr elementlarini hal qiluvchi elementga bo'lib, mos kataklarga yozamiz;

7.4) Hal qiluvchi ustun elementlarini hal qiluvchi elementga bo'lib, ishorasini o'zgartiramiz va mos kataklarga yozamiz;

7.5) Qolgan kataklar to'rtburchak qoidasi bo'yicha to'ldiriladi.

Masalan, (2.2) katakni to'ldirish uchun quyidagi xisoblash bajariladi:

$$a_{22}^{1} = \frac{a_{jk} \cdot a_{22} - a_{2k} \cdot a_{j2}}{a_{jk}}$$

Natijada jadval quyidagi ko'rinishga keladi:

	$-x_1$	$-x_2$	$\cdots$	$-y_j$	$-x_{k+1}$	$\cdots$	$-x_n$	Ozod sonlar
$y_1 =$	a'_{11}	a'_{12}	$\cdots$	$-\frac{a_{1k}}{a_{jk}}$	a'_{1k+1}	$\cdots$	a'_{1n}	a'_1
$y_2 =$	a'_{21}	a'_{22}	$\cdots$	$-\frac{a_{2k}}{a_{jk}}$	a'_{2k+1}	$\cdots$	a'_{2n}	a'_2
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$y_l =$	a'_{l1}	a'_{l2}	$\cdots$	$-\frac{a_{lk}}{a_{jk}}$	a'_{lk+1}	$\cdots$	a'_{ln}	a'_l
$y_{l+1} =$	a'_{l+11}	a'_{l+12}	$\cdots$	$-\frac{a_{l+1k}}{a_{jk}}$	a'_{l+1k+1}	$\cdots$	a'_{l+1n}	a'_{l+1}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$x_k =$	$\frac{a_{j1}}{a_{jk}}$	$\frac{a_{j2}}{a_{jk}}$	$\cdots$	$\frac{1}{a_{jk}}$	$\frac{a_{jk+1}}{a_{jk}}$	$\cdots$	$\frac{a_{jn}}{a_{jk}}$	$\frac{a_j}{a_{jk}}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$y_m =$	a'_{mk}	a'_{m2}	$\cdots$	$-\frac{a_{mk}}{a_{jk}}$	a'_{mk+1}	$\cdots$	a'_{mn}	a'_m
$z =$	c'_1	c'_2	$\cdots$	$-\frac{c_k}{a_{jk}}$	c'_{k+1}	$\cdots$	c'_n	z'_{max}

8) So’ngra 3)-6) punktlar barcha ozod sonlar musbat bo’lguncha yoki masalaning yechimi mavjud emasligi aniqlangunga qadar takrorlanadi.

Tayanch yechim topilgach optimal yechimni topishga o’tish mumkin. Buning uchun quyidagi amallar bajariladi:

1) qatordagi manfiy sonlar qaraladi. Agar manfiy sonlar bo’lmasa, optimal yechim topilgan xisoblanadi va 1 - ustundagi o’zgaruvchilar va ularga mos ozod sonlarga, 1-satrdagi o’zgaruvchilar esa nolga tenglashtiriladi. Agar ushbu satrda bir nechta manfiy sonlar bo’lsa, ulardan eng kichigi tanlanadi. Masalan eng kichik manfiy koeffitsiyent bo’lsin.

	$-x_1$	$-x_2$	$\cdots$	$-y_i$	$-x_{k+1}$	$\cdots$	$-x_n$	Ozod sonlar
$y_1 =$	a'_{11}	a'_{12}	$\cdots$	a'_{1k}	a'_{1k+1}	$\cdots$	a'_{1n}	a'_1
$y_2 =$	a'_{21}	a'_{22}	$\cdots$	a'_{2k}	$a'_{2\,k+1}$	$\cdots$	a'_{2n}	a'_2
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$y_l =$	a'_{l1}	a'_{l2}	$\cdots$	a'_{lk}	a'_{lk+1}	$\cdots$	a'_{ln}	a'_l
$y_{l+1} =$	$a'_{l+1\,1}$	$a'_{l+1\,2}$	$\cdots$	$a'_{l+1\,k}$	$a'_{l+1\,k+1}$	$\cdots$	$a'_{l+1\,n}$	a'_{l+1}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$x_k =$	a'_{i1}	a'_{i2}	$\cdots$	a'_{ik}	a'_{ik+1}	$\cdots$	a'_{in}	a'_i
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$y_m =$	a'_{mk}	a'_{m2}	$\cdots$	a'_{mk}	a'_{mk+1}	$\cdots$	a'_{mn}	a'_m
$z =$	c'_1	c'_2	$\cdots$	c'_k	c'_{k+1}	$\cdots$	c'_n	z'_{max}

2) 2- ustundagi musbat sonlarni tanlaymiz. Agar ushbu ustunda musbat sonlar bo'lmasa, masalaning optimal yechimi cheksizlikka intiladi. Agar ustunda musbat sonlar bo'lsa, ularga mos ozod sonlarni bo'lib, eng kichik nisbatni tanlab olamiz: .

$$\theta = min\left(\frac{a_2}{a_{j2}}\right)_{j=\overline{1,k}}$$

Bu yerda - tanlab olingan juftliklar soni.

3) Eng kichik nisbatga mos element hal qiluvchi element xisoblanadi va unga nisbatan simpleks almashtirishlari bajariladi.

4) 1)-3) punktlar qatordagi barcha sonlar musbat bo'lguncha yoki masalaning yechimi yuqoridan chegaralamaganligi aniqlanguncha davom ettiriladi.

Agar maqsad funktsiyasida

$$z = c_1 x_1 + c_2 x_2 \ldots + c_n x_n \rightarrow min$$

bo’lsa, u holda masala koeffitsiyentlar ishoralari o’zgartirilib, maksimumga keltiriladi:

$$Z_{min} = -Z_{max}$$

$$z = -c_1 x_1 - c_2 x_2 - \ldots - c_n x_n \rightarrow max$$

va masala yuqoridagi usul bilan yechiladi. Oxirgi natijada bo’ladi.