Numerical modeling of two-dimensional two-phase filtration under frontal drive

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Abstract. The problem associated with the development of gas fields with confined contour water is considered in the paper to increase gas recovery and determine the main indices of the reservoir, as well as the position of the moving boundaries for further development. To conduct a comprehensive study of the process under consideration, a computer model was developed described by a differential equation with the corresponding initial and boundary conditions. An algorithm was developed to solve the problem using the methods of longitudinal-transverse scheme and the flow version of the sweep method. The results are validated with a test example, and an example of a certain real object. The results are shown in the form of tables and isolines. Numerical experiments have shown that using this statement it is possible to apply the developed algorithms and software systems to solve the problem during field development.

Keywords: filtration, well, model, gas, oil, computational algorithm, numerical experiment, result, software package.

1. Introduction. The first steps in the development of the methods of filtration theory consisted in creating an algorithm and programs for solving prognostic problems, i.e. solving the corresponding hydrodynamic problems, mathematical study of which is reduced to the consideration of boundary-value problems described by the corresponding equations of the filtration theory in multiply coupled domains with inhomogeneous boundary conditions. These problems were successfully solved by analytical, approximate-analytical, variational and numerical methods.

The situation has changed dramatically with the beginning of the computer era simplifying the problems caused by large variety of oil and gas fields, and especially by the complexity of their geological structure. The methods of finite differences used to conduct large-scale mathematical experiments performed on modern computers, are widely used nowadays.

Groundwater monitoring is considered an important task of hydrodynamic study. It makes possible to judge the paths and filtration rates at water rise in the well location sites and possible drowning, the degree of development stability in the sites under consideration, etc.

Therefore, research on the effectiveness of the development of water-gas zones and gas deposits with contour water is relevant. To date, a considerable number of studies have been published in which the patterns of the drowning of wells and fields with contour and bottom waters have been revealed.

Nevertheless, there are currently objective reasons to conduct research in order to find the ways to increase the developing efficiency of water-gas zones and gas deposits with bottom water. This is due, firstly, to the fact that the well-known theory and practice of developing this type of deposits is based on the use of vertical production and injection well systems and, secondly, due to significant progress in creation and use of numerical algorithms and programs and modern powerful computers.

As a result, it became possible to set up large-scale mathematical experiments on development elements, taking into account the main determining parameters and factors. The results of many years of research to substantiate new principles and technologies for the development of oil and gas fields are summarized. To a large extent, their occurrence is associated with the current state in oil and gas industry and the achievements of scientific and technological progress [1,2].

With widespread occurrence of horizontal wells in oil and gas production systems, research on the problem of steady and unsteady inflow to this type of wells has significantly increased. A methodological basis for the interpretation of the research results of horizontal wells under unsteady conditions and for the issues of cone formation in the development of oil and gas or water-floating oil and gas deposits by a system of horizontal wells is given in [3].

The studies in [4] provide the materials of mathematical foundations for the theoretical gas dynamics. The principles of constructing a variety of gas-dynamic models are stated - from integrated conservation laws to specific formulas that describe a particular gas flow. Group-theoretical foundations of the derivation of differential equations that describe the classes of particular solutions are given. The methods of qualitative analysis are widely used when solving specific problems. To facilitate the material perception, the text includes graphic illustrations.

In [5-10], the problems of modeling the process of gas filtration in porous media are considered. A mathematical model of the object under study is developed, described by a nonlinear partial differential equation with constant coefficients; an ordinary sweep algorithm is used to solve the problem. The results for the model problem are obtained for reservoir parameters of constant values.

In [11], gas filtration problems with constant coefficients in a standard site with randomly located wells are presented. The algorithm for solving the problem is based on the flow version of the sweep method. Solution results for the model gas filtration problem are presented.

1.1. Statement of the problem. Let there be a productive formation (initially water-(gas-) saturated) confined to boundary water. The reservoir is developed using randomly located wells, with coordinates (x_i, y_i) in the mode of specified volume flow rates over time q_i. It is necessary to determine the time change of reservoir pressure and the position of the moving boundaries. For this, integrate the nonlinear partial differential equation of the parabolic type in the gas zone

$$\frac{\partial}{\partial x}\left[\frac{k}{\mu_g} p(x,y,t)\frac{\partial p\ (x,y,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{k}{\mu_g} p(x,y,t)\frac{\partial p\ (x,y,t)}{\partial y}\right]= \\ =\sigma m\frac{\partial p(x,y,t)}{\partial t}+F\ (x,y,t), \quad (x,y)\in G_1,\ \ t>0; \tag{1}$$

in the fluid zone

$$\frac{\partial}{\partial x}\left[\frac{k}{\mu_w}\frac{\partial p\ (x,y,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{k}{\mu_w}\frac{\partial p\ (x,y,t)}{\partial y}\right]= \\ =\beta_w^*\ m\frac{\partial p(x,y,t)}{\partial t}, \quad (x,y)\in G_2,\ \ t>0; \tag{2}$$

with initial

$$p(x,y,t)=p^0(x,y), \qquad t=0, \quad (x,y)\in\overline{G_1}\cup\overline{G_2}. \tag{3}$$

and boundary conditions

$$\frac{\partial p(x,y,t)}{\partial l_1}=0, \quad (x,y)\in\Gamma_2 \quad . \tag{4}$$

At moving boundary surfaces, the conditions of pressure continuity and flow continuity are fulfilled.

$$[p(x,y,t]_{(x,y)\in R(\mathrm{x,y,t})}=0, \\ \frac{k}{\mu_g}\ \frac{\partial p(x,y,t)}{\partial n}\Big|_{(x,y)\in R(\mathrm{x,y},t)-0}=\frac{k}{\mu_w}\ \frac{\partial p(x,y,t)}{\partial n}\Big|_{(x,y)\in R(\mathrm{x,y},t)+0},$$

The law of the movement of a flow is:

$$\frac{dR(x,y,t)}{dt}=-\frac{1}{m\sigma}\ \frac{k(x,y)}{\mu_w}\ \frac{\partial p(x,y,t)}{\partial n}\Big|_{(x,y)\in R(\mathrm{x,y},t)+0}, \\ R(x,y,t)\big|_{t=0}=R^0(x,y); \tag{5}$$

here $p(x,y,t)$ is the pressure; μ_g, μ_w are the coefficients of dynamic viscosity of gas and fluid, respectively; k is the permeability coefficient. $m(x,y)$ is the porosity coefficient; β_w^* is the coefficient of elastic capacity of formation fluid; t is time; p^0 is the initial reservoir pressure.

$F = \sum_{i=1}^{n_s} q_i(t)\delta(x-x_i, y-y_i)$, $q_i(t) = \oint_{S_i} \frac{k}{\mu_g}\frac{p}{p_{at}}\frac{\partial p}{\partial n_2} dS, \quad (x,y)\in S_i$ - flow rates of gas wells reduced to atmospheric pressure and formation temperature. $\delta = \begin{cases} 1, & x = x_i, y = y_i \\ 0, & x \neq x_i, y \neq y_i \end{cases}$ - Delta Dirac function. p_{at} - atmospheric pressure, S, G_2 - the contours of the wells and site, respectively; n_1, n_2 - the normals to the contours Γ_2, S, respectively; n_s - the number of wells; $[U(x)]|_{x=a} = U(a+0) - U(a-0);$ $R^0(x,y)$ - the initially set curve, showing the initial position of the moving boundary surface; $R(x,y,t)$ - the position of the moving boundary surface; n - the normal to the line of the moving boundary surface.

For the convenience, equation (1) and (2), can be written in the following form

$$\frac{\partial}{\partial x}\left[A(p)K\frac{\partial P(x,y,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[A(p)K\frac{\partial P(x,y,t)}{\partial y}\right] = \tag{6}$$
$$= M\frac{\partial P(x,y,t)}{\partial t} + F(x,y,t), \quad (x,y)\in(G_1\cup G_2), \quad t>0;$$

$$p(x,y,t) = p^0(x,y), \qquad t = 0, \quad (x,y)\in \overline{G_1}\cup\overline{G_2}. \tag{7}$$

$$\frac{\partial p(x,y,t)}{\partial l_1} = 0, \quad (x,y)\in \Gamma_2 \tag{8}$$

$$\frac{dR(x,y,t)}{dt} = -\frac{1}{m\sigma}\frac{k}{\mu_w}\frac{\partial P(x,y,t)}{\partial n}\Big|_{(x,y)\in R(x,y,t)+0}, \tag{9}$$
$$R(x,y,t)|_{t=0} = R^0(x,y);$$

here –

$$P(x,y,t), A(p), K, M = \begin{cases} p(x,y,t), & p(x,y,t), & \frac{k}{\mu_g}, & m\sigma & (x,y)\in G_1 \\ p(x,y,t), & 1, & \frac{k}{\mu_w}, & m\beta^* & (x,y)\in G_2 \end{cases}$$

2. Methods. To construct a numerical algorithm for problem solution using finite-difference methods [11-14], first turn to dimensionless variables. For this, we introduce some characteristic quantities

$$\bar{x} = \frac{x}{L_X}, \ \bar{y} = \frac{y}{L_y}, \ \bar{k} = \frac{k}{k_X}, \ \bar{\mu}_g = \frac{\mu_g}{\mu_X}, \bar{\mu}_w = \frac{\mu_w}{\mu_X}, \ \bar{p} = \frac{p}{p_X}, \ \bar{t} = \frac{t}{t_X},$$

where $L_X, L_y, k_X, \mu_X, p_X, t_X$ - are some given constants. Omitting the dashes over the letters and making some calculations, we obtain a dimensionless problem, which has the form similar to the problem given in (6) - (9).

Cover the given domain $\Omega = \{0 \le x \le 1; \ 0 \le y \le 1\}$ with a uniform grid

$$\bar{\omega}_{xy} = \{x_i = ih_x, h_x = 1/N_x, i = \overline{0,N_x}, \ y_j = jh_y, \ h_y = 1/N_y, \ j = \overline{1,N_y},\}.$$

For the time step we take the grid

$$\bar{\omega}_\tau = \{t_k = k\tau, \ \tau = 1/\mathrm{T}, \ k = \overline{0,\mathrm{T}}\}.$$

By entering the notation

$$W_x = K\frac{\partial P}{\partial x}, \quad W_y = K\frac{\partial P}{\partial y}$$

and approximating the dimensionless problem using the Samarsky longitudinal-transverse scheme [12], without intermediate calculations, we obtain a chain of one-dimensional difference problems of the form

$$
\begin{cases}
A_{i+1/2,j}^{k+1/2} W_{i+1/2,j}^{k+1/2} - A_{i-1/2,j}^{k+1/2} W_{i-1/2,j}^{k+1/2} = \dfrac{h_x}{0.5\tau} M_{i,j} P_{i,j}^{k+1/2} + \dfrac{h_x}{0.5\tau} \Phi_{i,j}^{k+1/2}, \\
\Phi_{i,j}^{k+1/2} = -M_{i,j} P_{i,j}^{k} + 0.5\tau F_{i,j}^{k+1} - (A_{i,j+1/2}^{k} W_{i,j+1/2}^{k} - A_{i,j-1/2}^{k} W_{i,j-1/2}^{k}) / h_y, \\
W_{i+1/2,j}^{k+1/2} = \dfrac{1}{h_x} K_{i+1/2,j} (P_{i+1,j}^{k+1/2} - P_{i,j}^{k+1/2}), \\
A_{0,\mathrm{j}}^{k+1/2} W_{0,\mathrm{j}}^{k+1/2} = 0, \qquad A_{\mathrm{N_1,j}}^{k+1/2} W_{\mathrm{N_1,j}}^{k+1/2} = 0, \\
R_{l_1,l_2}^{k+1/2} = R_{l_1,l_2}^{k} - C_{l_1,l_2} W_{l_1,l_2}^{k+1/2}
\end{cases} \tag{4}
$$

$$
\begin{cases}
A_{i,j+1/2}^{k+1} W_{i,j+1/2}^{k+1} - A_{i,j-1/2}^{k+1} W_{i,j-1/2}^{k+1} = \dfrac{h_y}{0.5\tau} M_{i,j} P_{i,j}^{k+1} + \dfrac{h_y}{0.5\tau} \Phi_{i,j}^{k+1}, \\
\Phi_{i,j}^{k+1} = -M_{i,j} P_{i,j}^{k+1/2} + 0.5\tau F_{i,j}^{k+1} - (A_{i+1/2,j}^{k+1/2} W_{i+1/2,j}^{k+1/2} - A_{i-1/2,j}^{k+1/2} W_{i-1/2,j}^{k+1/2}) / h_x, \\
W_{i,j+1/2}^{k+1} = \dfrac{1}{h_y} K_{i,j+1/2} (P_{i,j+1}^{k+1} - P_{i,j}^{k+1}), \\
A_{i,0}^{k+1} W_{i,0}^{k+1} = 0, \qquad A_{i,N_2}^{k+1} W_{i,N_2}^{k+1} = 0, \\
R_{l_1,l_2}^{k+1} = R_{l_1,l_2}^{k+1/2} - C_{l_1,l_2} W_{l_1,l_2}^{k+1}
\end{cases} \tag{5}
$$

The resulting chain of one-dimensional difference problems (4) - (5) is solved by the flow sweep method [11].

3. Results and Discussion.

To validate the reliability of the results obtained using the above computational algorithms, we take a circular region confined to the boundary water as a test example (Fig. 1 a). In the center of the region, there is one pumping well with a constant flow rate, its value is $q_0 = 10^4 m^3 / days$, and with a thickness of 10 m. The example is solved to compare the results with a one-dimensional flat-radial filtration under the same initial data (Fig. 1 b).

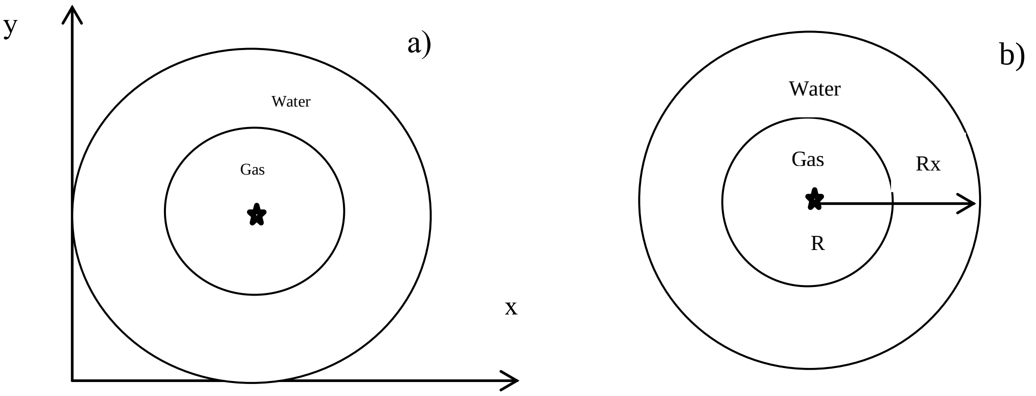

Fig. 1. Results of Testing Example.

Table 1 shows the results of calculations obtained for both statements, for a time of 180 days. The number of iterations with the refined nonlinear term does not exceed two steps for the total time of solution. Based on comparative analysis of results, it should be noted that the results obtained with developed algorithms in numerical determination of the pressure field and the position of the moving two-phase boundaries in a two-dimensional statement correctly reflect the physical pattern of the process.

Table 1

One-dimensional statement							
	0.930	0.931	0.933	0.935	0.936	0.937	0.937
Two-dimensional statement							
y \ x	0.50	0.55	0.65	0.75	0.85	0.95	1.0
0.50	0.931	0.933	0.933	0.934	0.935	0.936	0.936
0.55	0.933	0.933	0.933	0.934	0.935	0.936	
0.65	0.933	0.933	0.934	0.935	0.936	0.936	

0.75	0.934	0.934	0.935	0.936	0.936		
0.85	0.935	0.934	0.936	0.936			
0.95	0.936	0.936	0.936				
1.0	0.936						

Now consider the solution of the problem in the region of complex shape confined to the boundary water (Fig. 2) with real data for a certain field under water pressure conditions. In the region there are 14 wells, their numbers, coordinates and flow rates are given in Table 2. Figure 3 shows the changes in the pressure field obtained in 360 days.

Table 2

1	201	11	6	63722.0
2	202	9	18	45885.0
3	203	20	20	9958.0
4	204	22	14	699710.0
5	205	17	14	669510.0
6	206	29	19	237486.0
7	207	6	10	644090.0
8	208	32	17	224950.0
9	209	6	15	183475.0
10	210	29	15	724713.0
11	211	18	8	6800.0
12	212	11	14	714963.0
13	213	22	10	59100.0
14	216	27	16	436381.0

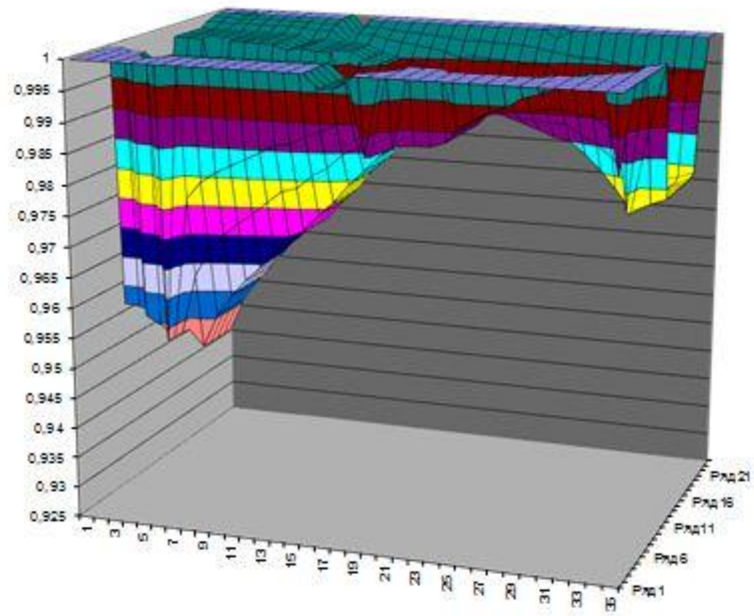

Fig. 2. Isoline of the field of pressure.

Conclusions

From the above results, we can conclude that the developed algorithm is applicable for solving the problem of gas filtration during frontal drive in a porous medium. The symmetry of the obtained results was validated using a test example both in two-dimensional and one-dimensional plane radial statement under the same initial data. An example of randomly spaced several wells was also considered. From the results obtained, it can be said that the developed algorithm and software product can be applied to determine the performance of real deposits.

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