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# Mathematical modelling of bottom deformations in the kinematic wave approximation

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**Abstract.** Two possible approaches for generalizing the kinematic wave model for deformable channels are being discussed in this paper, including the approximation in which the law of conservation of fluid mass includes erosion and sedimentation; and approximation that the fluid flow is determined by the kinematic wave equation without taking into account bottom deformations.

Systematic comparison of the results of calculations of self-similar waves for different values of the change in water flow was carried out as well.

## 1. Introduction

For solving problems of flow transformation, the kinematic wave equation is often used: one of the simplest mathematical models of the channel flow, based on the use of the fluid mass conservation law with Chezy or Darcy–Weisbach equation [0, 0]. The use of such a mathematical model is permissible in the case when the linear and time scales of the consideration of phenomena are large enough, the effects associated with the inertia of the flow are insignificant; in addition, the difference between the slope of the water surface and the slope of the watercourse bottom is also insignificant. Such an approach is possible at large scales of consideration, but it may absolutely not correspond to reality at a smaller examination, since in a more detailed analysis, the force effect and inertness of the flow play the main role in the development of the wave process, and in some areas even a reverse slope of the bottom is possible. Despite the aforementioned significant limitations, estimates made using this mathematical model often give satisfactory results for estimating flow parameters in channels without retaining structures.

The kinematic wave equations are also of interest for understanding the processes occurring in rivers. For example, in [0], on the basis of analytical solutions of the kinematic wave equation, a theoretical explanation was found for the fact discovered earlier in the works of various authors - the level of flooding when the pressure front of the dam breaks at some distance from it ceases to depend on the form of the outflow hydrograph and is determined only by the volume of the emptied reservoir.



This paper discusses two possible approaches to generalizing the kinematic wave model for deformable channels (for wide rectangular channels):

- an approach that uses the laws of conservation of the mass of fluid and sediment in deformable channels, and the flow rate which determined by the Chezi or Darcy–Weisbach formulas,

- an even rougher approximation, in which it is assumed that the fluid flow is determined by the kinematic wave equation without taking into account bottom deformations.

## 2. Results and Discussion

The system of differential equations of fluid motion in a wide rectangular channel with a deformable bottom, provided that the flow is completely saturated with sediments in a coordinate system with the abscissa axis inclined to the horizontal at a slope  $I$ , has the following form  $[0, 0]$ :

$$\begin{cases} \frac{\partial Z_{rb}+h}{\partial t} + \frac{\partial q}{\partial x} = 0, \\ \frac{\partial hS+Z_{rb}S_{rb}}{\partial t} + \frac{\partial qS}{\partial x} = 0, \\ \frac{\partial q}{\partial t} + \frac{\partial qV+gh^2/2}{\partial x} + gh\left(\frac{\partial Z_{rb}}{\partial x} - I\right) + \frac{\lambda}{2}V|V| = 0, \end{cases} \quad (1)$$

here:  $t$  - time,  $x$  - length along the riverbed,  $h$  - depth,  $q$  - specific discharge,  $Z_{rb}$  - bed mark (over tilted axis  $X$ ),  $Z_{rb} = Z_{rb}+h$  - free water surface mark,  $V$  - velocity,  $V=q/h$ ,  $\lambda$  - hydraulic friction coefficient,  $S$ ,  $S_{rb}$  - volumetric concentrations of sediment in the flow and material of the channel bottom, respectively; further assume that  $S_{rb}$  is const and during sedimentation turns out to be equal to the same value that was in the channel before the start of the process of bottom deformations. It is also assumed that the volumetric sediment concentration  $S$  is a function of flow depth and flow rate:

$$S = S(h, V). \quad (2)$$

In this paper, channels are considered that have on average a constant bottom slope  $I$  along the length, and the slope of the  $X$  axis is taken to be equal to the bottom slope. In this case, it is assumed that both the liquid and the sediment material are incompressible, so that in the first and third equations, a reduction is made by the water density  $\rho$ , and in the second - on the sediment density  $\rho_s$ . It is assumed that the sediment concentration in the flow is low, which makes it possible to neglect the sediment momentum. Usually, for river flows, this hypothesis does not lead to significant errors, but it is unacceptable when modelling mudflows.

Equations (1) have trivial solutions:

- restrained fluid (for any shape of the longitudinal profile of the channel bottom  $Z_{rb} = Z_{rb}(x)$  and hydraulic roughness):

$$\begin{cases} Z_{rb} + h - Ix = const, \\ V = 0, \\ S = 0, \end{cases} \quad (3)$$

- uniform flow (in channels with a constant bottom slope and its roughness):

$$\begin{cases} q = const, \\ V = \sqrt{\frac{2ghI}{\lambda}} \\ S = S(h, V). \end{cases} \quad (4)$$

It is difficult to obtain other analytical solutions of system (1.1), and the main method of integrating system (1.1) is the numerical methods [0-11].

In [0], a simplified mathematical model of flow in channels was considered, in which the third equation of system (1.1), which expresses the law of conservation of the momentum of a river flow, is replaced by the following relation following from the Darcy formula:

$$\begin{cases} \frac{\partial Z_{rb}+h}{\partial t} + \frac{\partial q}{\partial x} = 0 \\ \frac{\partial hS+Z_{rb}S_{rb}}{\partial t} + \frac{\partial qS}{\partial x} = 0 \\ q = \sqrt{\frac{2gh^3I}{\lambda}} \end{cases} \quad (5)$$

The same mathematical flow model is discussed below. In addition, an even rougher model was considered:

$$\begin{cases} \frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \\ \frac{\partial hS+Z_{rb}S_{rb}}{\partial t} + \frac{\partial qS}{\partial x} = 0 \\ q = \sqrt{\frac{2gh^3I}{\lambda}} \end{cases} \quad (6)$$

Systems of equations (1.5) and (1.6) cannot describe the structure of the transient process in the channel, but they allow one to assess some features of the flow without focusing on the details determined by the inertness of the flow and being sub-scale in this consideration. As noted above, such a mathematical model of the current is often used in hydrology in relation to unerodable channels. [0, 9, 12].

In this paper, the well-known solutions of system (6) are generalized for the class of formulas for the coefficient of hydraulic friction  $\lambda$  with its power-law dependence on the depth I:

$$\lambda = \frac{\Lambda}{h^\alpha}, \alpha > 0. \quad (7)$$

With  $\alpha=1/3$  и  $\Lambda = 2gn^2$ , here  $n$  – roughnessroughness, (7) is Manning's formula [0], and with  $\alpha=2/5$  и  $\Lambda = 2gn^2$  – Forchheimer [0] formula, with  $\alpha = 0$  a simplified technique is used, in which the depth in the computational domain changes slightly, so that the change in the coefficient of hydraulic friction  $\lambda$  can be neglected.

The main flow parameters, expressed through the specific water flow rate would have the form:

$$h = \left(\frac{\Lambda q^2}{2gl}\right)^{1/(3+\alpha)}, \quad V = \left(\frac{2gl}{\Lambda}\right)^{1/(3+\alpha)} q^{(1+\alpha)/(3+\alpha)}, \quad (8)$$

expressed through the depth

$$q = \sqrt{\frac{2glh^{3+\alpha}}{\Lambda}}, \quad V = \sqrt{\frac{2glh^{1+\alpha}}{\Lambda}}, \quad (9)$$

and expressed through the velocity:

$$h = \left(\frac{\Lambda V^2}{2gl}\right)^{1/(1+\alpha)}, \quad q = \left(\frac{\Lambda}{2gl}\right)^{1/(1+\alpha)} V^{(3+\alpha)/(1+\alpha)}. \quad (10)$$

### 2.1. Formulas for the weight-bearing capacity of the channel flow.

There are a large number of formulas of type (11) for the sediment concentration in the flow; some of them are given in [14], the analysis of their applicability in the conditions of some real rivers was carried out there. Here are some of these formulas that have found wide application in calculations for rivers and canals of Central Asia.

**R.A. Bagnold's formula:** In [0, 0], the Bagnold's formula is given in the following form:

$$Q_s = Q \frac{\rho \times \rho_s C_f V^2}{\rho_s - \rho gh} \left( \frac{0,13}{f-I} + \frac{0,01}{\frac{W}{V}-I} \right), \quad (11)$$

here:  $Q_s$  – mass flow capacity,  $Q$  – water discharge,  $\rho$  and  $\rho_s$  – density of water and soil mineral, respectively,  $C_f$  – channel's roughness coefficient,

$C_f = \frac{\lambda}{2}$ ,  $C_f V^2 = V_*^2$ ,  $V_* = \sqrt{\frac{\lambda}{2}} V$  – dynamic velocity,  $f = tg(\varphi)$  – internal sediment friction coefficient,  $\varphi$  – internal friction angle,  $W$  – settling velocity. Volume concentration of sediments corresponding to the formula (2.1)

$$S = \frac{\rho_s}{\rho_s - \rho} \frac{\lambda V^2}{2gh} \left( \frac{0,13}{f-I} + \frac{0,01}{\frac{W}{V}-I} \right), \quad (12)$$

For a uniform flow, the Bagnold formula is greatly simplified and, after substituting into it the second equation of the system (1.4)  $V = \sqrt{\frac{2ghI}{\lambda}}$  takes the following form:

$$S = \frac{\rho_s}{\rho_s - \rho} \frac{\lambda V^2}{2gh} \left( \frac{0,13}{f-I} + \frac{0,01}{\frac{W}{V}-I} \right) = \frac{\rho_s I}{\rho_s - \rho} \left( \frac{0,13}{f-I} + \frac{0,01}{\frac{W}{V}-I} \right) \quad (12a)$$

The first term in parentheses in the formula (2.2) -  $\frac{0,13}{f-I}$  reflects an important physical fact: a decrease in the stability of particles with an increase in the bottom slope  $I$ . With a large scale of consideration, it is natural to assume that the slope  $I$  much less than the coefficient of internal friction of the sediment material  $f$ , and as a first approximation it can be neglected. In this case, and this term can be viewed in a simplified form:  $\frac{0,13}{f}$ . Second term:  $\frac{0,01}{\frac{W}{V}-I}$ , apparently, should reflect the effect of a decrease in the probability of particle settling with an increase in the bottom slope. This effect is significant at local bottom depressions, at which the phenomenon of flow around the bottom dune appears and, possibly, a separated flow appears, but this effect is unlikely to be significant at currents close to uniform, even at relatively large slopes. As it seems to us, Probably, in practical engineering calculations, it is advisable to use the Bagnold formula in the following form:

$$S = \frac{\rho_s}{\rho_s - \rho} \frac{V_*^2}{gh} \left( \frac{0,13}{tg\varphi} + 0,01 \frac{V}{W} \right). \quad (13)$$

Below, Bagnold's formula is used in the form:

$$S = A_1 \frac{\lambda V^2}{gh} \left( A_2 + \frac{V}{W} \right), \quad A_1 = \frac{0,5\rho_s}{\rho_s - \rho} \times 10^{-3}, \quad A_2 = \frac{13}{tg\varphi} \quad (13a)$$

And with uniform water flow:

$$S = 2A_1 I \left( A_2 + \frac{V}{W} \right), \quad (13b)$$

**Kh. A. Ismagilov's formula**

According to [19 – 24], the formula is:

$$S = A_0 \frac{V^3}{ghW}, \quad (14)$$

here:  $A_0$  – coefficient, which depends on the watercourse; for Amu Darya River near the Cape Pulizindan:  $A_0 = 0.22$ , for canals in Central Asia  $A_0 = 0.18$ . For a simplified model of bottom deformations at  $\lambda = const$  Ismagilov's formula is a special case of Begnold's formula for  $A_1 = A_0/\lambda$ ,  $A_2 = 0$ .

### V.N. Goncharov's formula

According to [0] the formula is:

$$S = \begin{cases} 0 & \text{at } V \leq V_n, \\ \frac{1+\xi}{500} \frac{d}{h} \left( \frac{|V|^3}{V_n^2} - 1 \right) \left( 1 - \frac{V_n}{|V|} \right) & \text{at } V > V_n, \end{cases} \quad \xi = \frac{W_c}{W}, \quad W_c = \sqrt{\frac{2g}{1.75} \left( \frac{\rho_s}{\rho} - 1 \right) d} \quad (15)$$

here:  $V_n$  – noneroding velocity,  $d$  – diameter of soil particles of 50% size, that is, such a size that 50% of the volume of soil particles has a diameter greater than  $d$ , and 50% has a smaller diameter. The same applies to the settling velocity used in formulas (13) and (14). In this work, we will not focus on how soil heterogeneity affects the process of bottom deformations, and, for simplicity, we will assume that the soil is homogeneous and all particles have the same diameter  $d$ .

From our point of view, a significant defect of the Goncharov formula in comparison with the Begnold formula is the absence in it of an explicit dependence on the dynamic velocity associated with the turbulence energy of the flow, which is the primary cause of the suspension of soil particles and their transfer by the flow.

There are also a number of empirical formulas for non-blurring speeds. Here are the simplest ones:

### G/I/ Shamov's formula

$$V_n = 3,83 \sqrt[3]{d\sqrt{h}}, \quad (16)$$

### V.N. Goncharov's formula (2)

$$V_n = 0,06 \sqrt{gd^{0,4}(d + 0,014)^{0,6}} \sqrt[5]{\frac{h}{d}}, \quad (17)$$

Recommendations for determining the noneroding velocity are given in [0].

Begnold's formula is considered one of the most reliable and is often used in hydraulic calculations, while Goncharov's formula, according to [0], shows an unsatisfactory coincidence of the sediment concentration estimates obtained from it with field observations.

At the same time, apparently, Goncharov's formula reflects an important physical fact: at a flow velocity lower than the non-eroding velocity, the carrying capacity of the flow is equal to 0, and Begnold's formula does not have this property. Multiplier  $V_*^2 = \frac{\lambda}{2} V^2$  shows that with an increase in the energy of turbulence of the flow, its suspension capacity increases. It is natural to assume that the suspension capacity is not proportional to the square of the dynamic speed  $V_*^2 = \frac{\lambda}{2} V^2$ , but the difference between the squares of the dynamic speed and the dynamic noneroding velocity

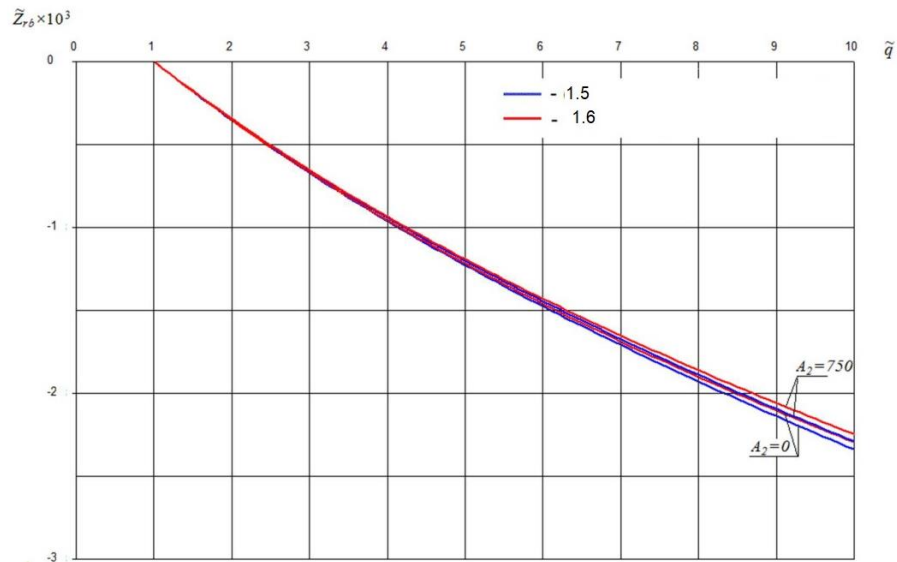
$$V_{*n}^2 = \frac{\lambda}{2} V_n^2.$$

$$S = A_1 \frac{V_*^2 - V_{*n}^2}{gh} \left( A_2 + \frac{V}{W} \right) \tag{18}$$

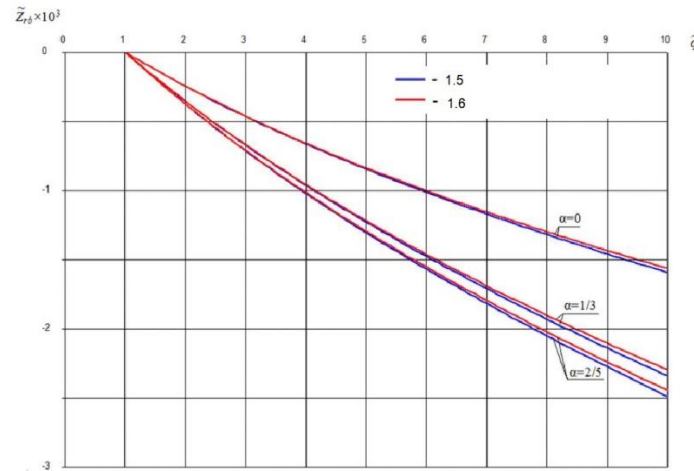
Formulas for estimating the suspension capacity of a flow, including the Begnold formula (13) and the modified Begnold formula (17), are suitable for low sediment concentrations. Formulas operating in an arbitrary range of concentrations, which can be used in mathematical modeling of mudflows, should not exceed the concentration  $S=1$  at any flow parameters (in fact, the concentration  $S_{max} < 1$ , corresponding dense packing of the soil material because pores between particles have a certain volume). Let us perform the following modification of the Begnold formula which does not allow exceeding the limiting concentration  $S_{max}$ :

$$S = \frac{2S_{max}}{\pi} \arctg \left[ \hat{A}_1 \frac{V_*^2 - V_{*n}^2}{gh} \left( A_2 + \frac{V}{W} \right) \right], \quad \hat{A}_1 = \frac{\pi A_1}{2S_{max}} \tag{19}$$

Figure 1 and 2 show graphs of functions  $\Delta \tilde{Z}_{rb} = F(\tilde{q})$  for various combinations of parameters for the system of equations (5) and (6)). Calculations were performed for  $1 < \tilde{q} \leq 10$  и  $2IA_1 = 4 \times 10^{-7}$ .

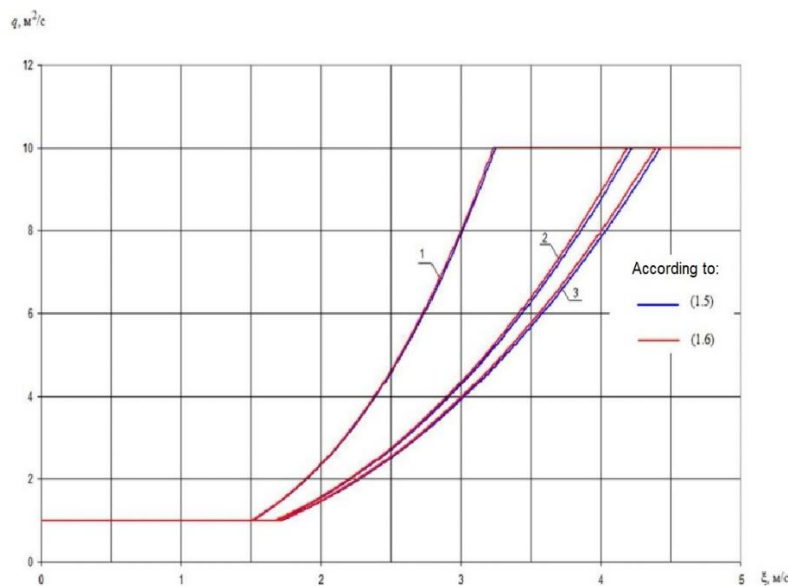


**Figure 1.** Graphs of the relationship between the depth of erosion and the coefficient of increase in water discharge during the passage of the bursting wave of increase at various values of the parameter  $A_2 = \frac{13}{tg(\varphi)}$ , here  $\varphi$  – internal friction angle of sediment material ( $A_2=750$  with  $\varphi \approx 1^\circ$ ).



**Figure 2.** Graphs of the relationship between the depth of erosion and the coefficient of increase in water consumption during the passage of the bursting wave of the increase at various values of the exponent in the formula (3): Legend:  $\alpha=1/3$  – Manning’s formula,  $\alpha=2/5$  – Forchheimer’s formula.

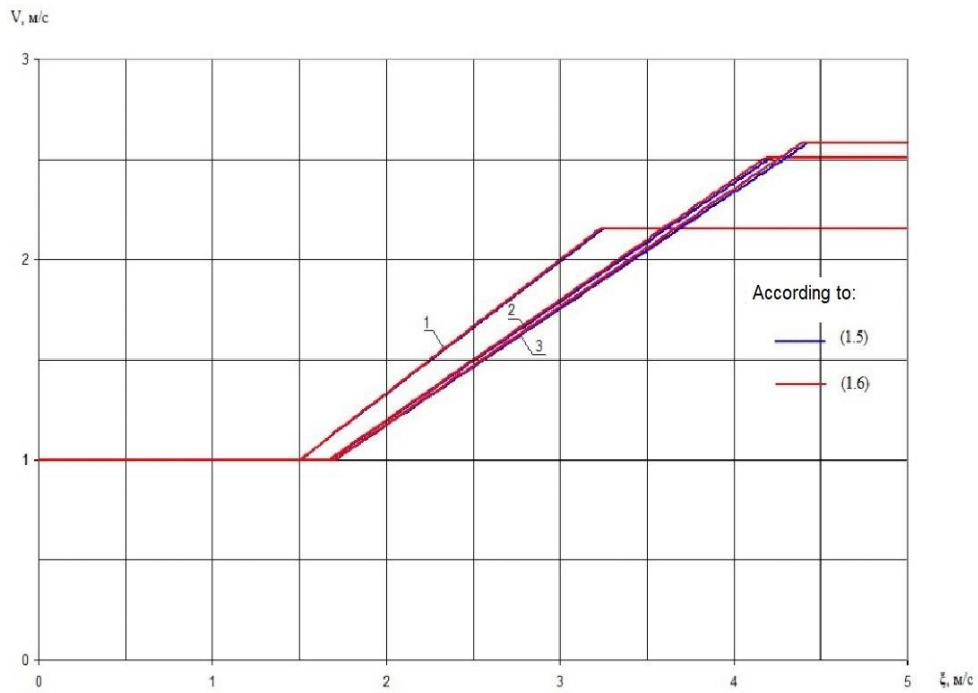
In figure 1-4 presents the results of calculating the parameters of a simple depression wave obtained using the numerical finite-difference method for equations (5) and analytical formulas for equations (6). Specific discharge decreased in the calculations from 10 m<sup>2</sup>/sec to 1 m<sup>2</sup>/sec. The calculations were carried out for three values of the exponent  $\alpha$  in the formula (3):  $\alpha=0$ ,  $\alpha=1/3$  (Manning’s formula),  $\alpha=2/5$  (Forchheimer’s formula). parametric variable  $\lambda=2gn^2=2\times 10^{-3}$ ,  $2\times I\times A_1=4\times 10^{-7}$ ,  $A_2=0$  (soil with zero angle of internal friction).



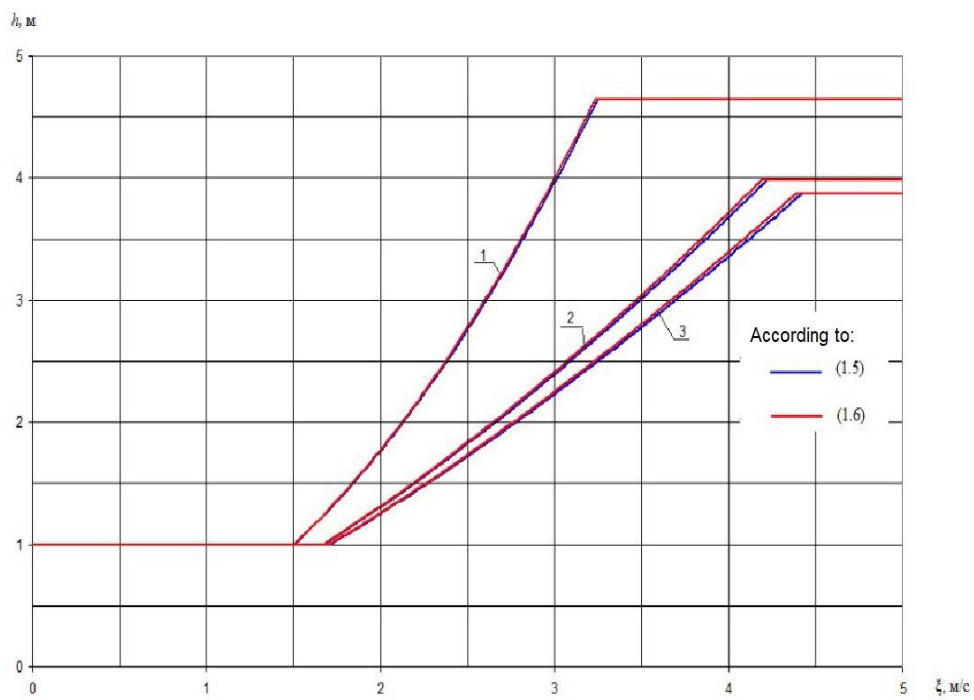
**Figure 3.** Graphs of changes in water flow along the channel during the passage of a simple depression wave with different formulas specifying hydraulic friction.

Legend: 1 -  $\alpha=0$ , 2;  $\alpha=1/3$  (Mannig’s formula), 3 -  $\alpha=2/5$  (Forchheimer’s formula).

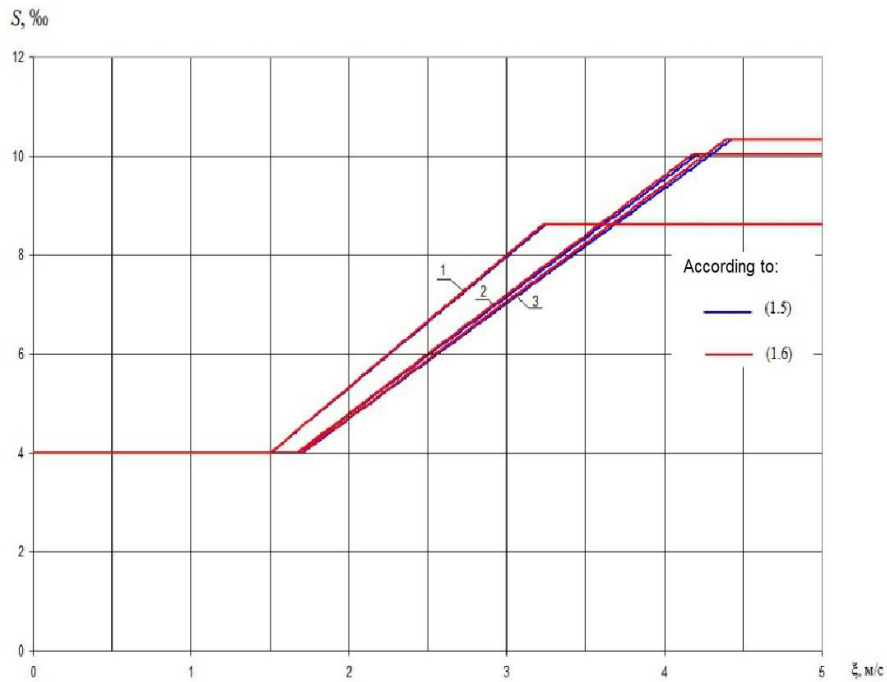




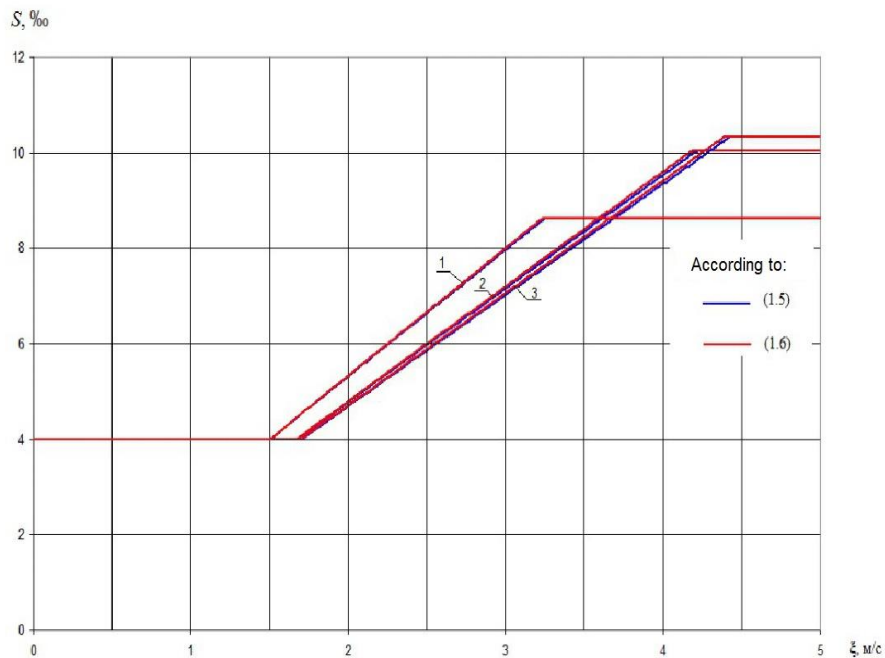
**Figure 4.** Graphs of the change in the velocity of the current along the channel during the passage of a simple sinking wave with various formulas specifying hydraulic friction (Legend see at Fig. 3).



**Figure 5.** Graphs of the change in the depth of the current along the channel during the passage of a simple sinking wave with various formulas specifying hydraulic friction (Legend see at Fig. 3).

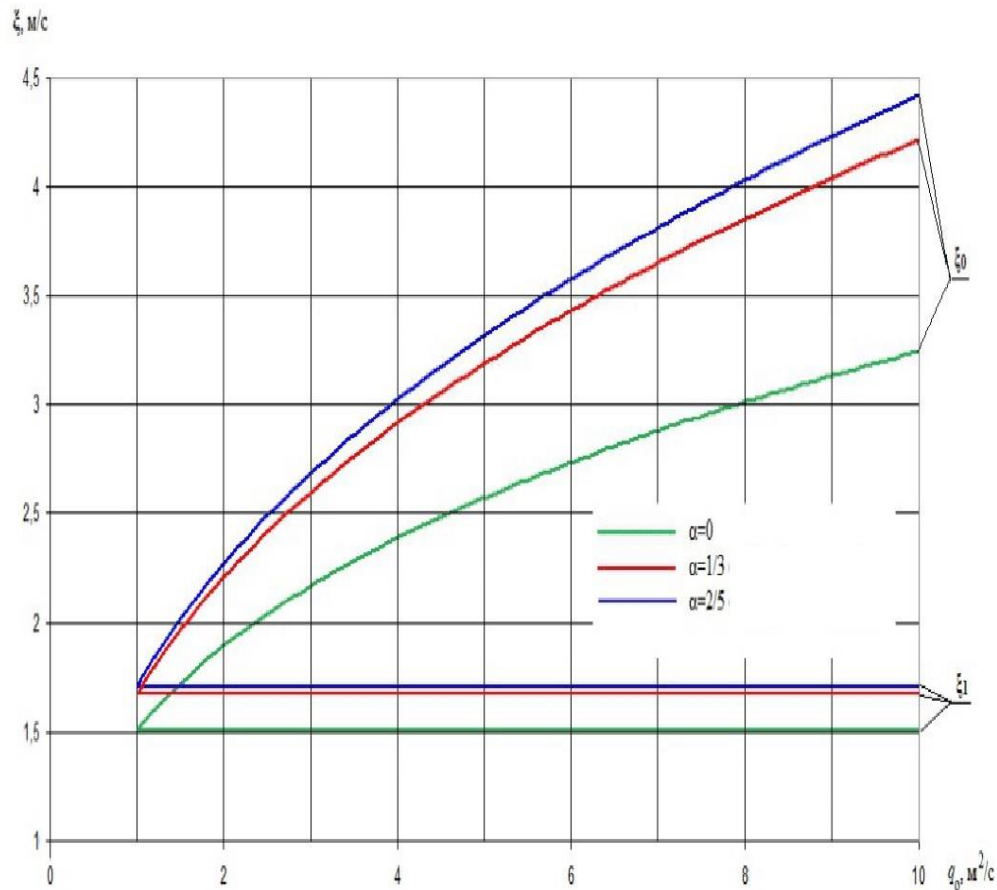


**Figure 6.** Graphs of the change in the volumetric sediment concentration of the current along the channel during the passage of a simple sinking wave with various formulas specifying hydraulic friction (Legend see at Fig. 3).

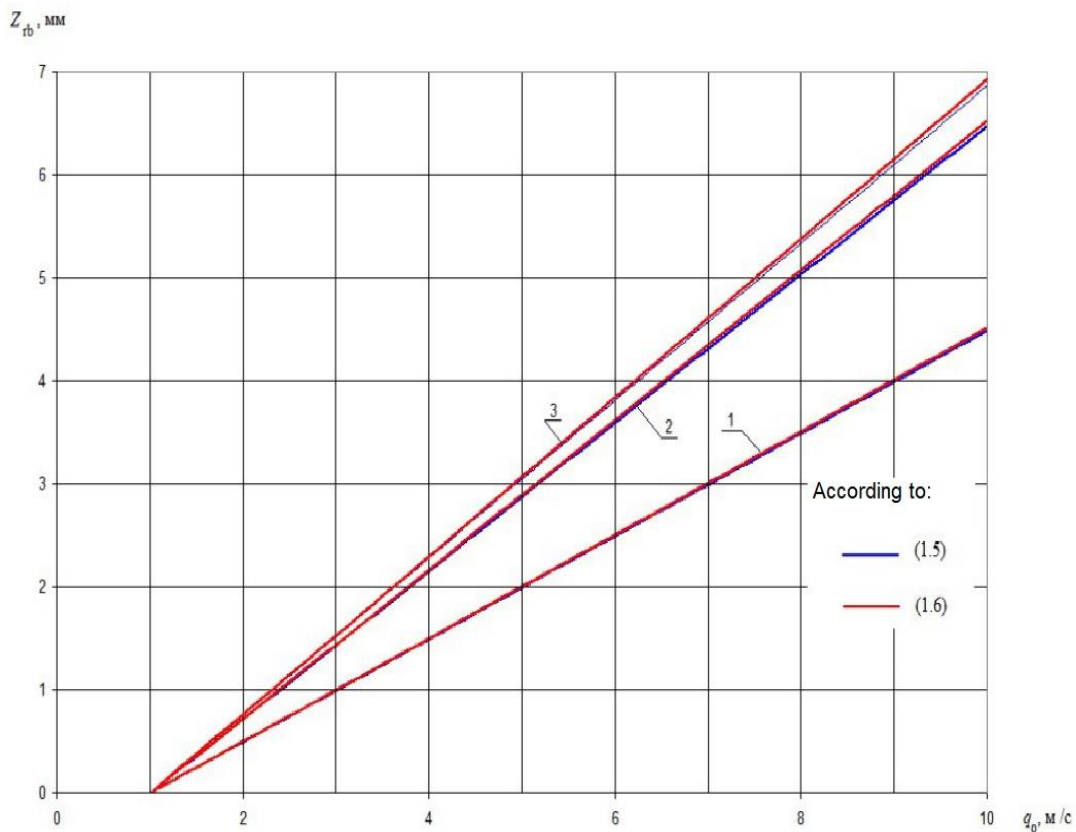


**Figure 7.** Graphs of the change in the sediment layer of the current along the channel during the passage of a simple sinking wave with various formulas specifying hydraulic friction (Legend see at Fig. 3).

The relationship between the limiting values of the self-similar argument  $\xi = \frac{x}{t}$  and the initial water flow rate in the watercourse during the passage of a simple depression wave with various formulas specifying hydraulic friction is at Fig. 8.



**Figure 8.** Relationship plots of values of a self-similar argument  $\xi = \frac{x}{t}$  and the initial flow rate of water in a watercourse during the passage of a simple depression wave with various formulas specifying hydraulic friction. Legend:  $\xi_0$  – self-similar variable value  $\xi = \frac{x}{t}$  at the boundary between the undisturbed flow and a simple depression wave,  $\xi_1$  – value of the self-similar variable at the boundary between a simple depression wave and the flow established after its passage.



**Figure 9.** Graphs of the relationship between the sediment layer and the initial specific water flow rate  $q_0$  using the passage of a simple downward wave (the minimum value of the specific water flow rate after establishing  $q_1 = 1 \text{ m/sec}^2$  with various formulas specifying hydraulic friction (Legend see at Fig. 3).

### 3. Conclusion

The paper considers two versions of kinematic wave equations in a broad rectangular bed assuming total saturation flux sediments:

- Version 1: equations in which the law of conservation of fluid mass includes erosion and sedimentation,
- Version 2: equations in which the motion of the liquid phase is considered to be described by the usual equation of a kinematic wave, bottom deformations are assumed to be small and do not affect the hydrodynamic picture.

For Version 1, the simulation of the equations of a kinematic wave taking into account bottom deformations there was an algorithm for the numerical solution and a program in Pascal language in the Delphi programming environment have been developed.

For Version 2, the well-known analytical solutions of hydraulic parameters were generalized for the case of a power-law dependence of the hydraulic friction coefficient on the flow depth, including the Manning and Forchheimer formulas.

Systematic comparison of the results of calculations of self-similar waves for different values of the change in water flow was carried out.

The possibility of generalizing the Begnold formula for high sediment concentrations is discussed.

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