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Numerical studies of long-wave processes in the reaches of hydrosystems and reservoirs

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Abstract. The problem of development of mathematical models and computer programs for calculation and forecast of various long-wave processes occurring in the reaches of reservoirs and hydro-systems is considered. The basic method to solve the problem is a mathematical modeling based on differential equations for the channel flow – the Saint-Venant equations using numerical methods. Conventional methods in hydraulics, methods for building mathematical models based on the laws of hydromechanics and their numerical calculations are also used. A mathematical and a computer program to carry out forecast calculations of long-wave processes occurring in the reaches of hydro-systems and reservoirs have been developed. The reliability of the results obtained is confirmed by rigorous mathematical statement, the use of well-known and tested equations and methods of hydraulics, and by the agreement of the results obtained in the work with the data obtained by other authors and available in literature.

1. Introduction

Broad-crested weirs are widely used in the practice of hydro-technical engineering; the flow through the weir can be considered as a long-wave process. These long-wave processes may be of natural origin due to freshets and floods; they may be the result of waves of release from overlying hydro-systems or they may be the breakthrough waves caused by hydrodynamic accidents. In this case, the role of a weir is played by a breakthrough in the dam body [1]. Breakthrough floods spread along the downstream at high speed and have an extremely high destructive force. The most severe consequences in the world practice of hydro-technical engineering have been caused by the breakthroughs of the pressure head of the dam banking up.

In solving practical problems of the hydraulics of open flows, one of the key points is the realization of the fact that two-dimensional unsteady Saint-Venant equations describe well enough both smoothly varying and sharply varying flows with the formation of breakthrough (circulation) zones. The derivation of the Saint-Venant equations without assumptions of smooth variability of the flow has been made by V.M. Lyatkher and A.N. Militeev in [2, 3], where it is shown that the pumping of energy into the circulation zone occurs due to pulsations at the border with the transit jet. Moreover, in numerical experiments, it is possible to obtain a spectrum of pulsations enriched by new harmonics when the computational grid is thickened, which, among other things, testifies to the high quality of the applied difference scheme.

There are a large number of publications devoted to the study of flow passage through a broad-crested weir. In [4], the influence of the Reynolds number on the coefficient of flow through the broad-crested weir is considered. It is shown that the coefficient of flow through the weir increases with increasing Reynolds number.

In [5] approach enabling the possibility to define the finite lifetime of a small earth dam is presented. The proposed approach does not require any variables monitoring. It is based on the definition of lifetime by assessing the water impact on the small earth dams by quantitative methods of system analysis.

In [6] there is theoretically shown that pressure is equal to half of flow depth at state of speed flow, i.e. it is equal to half of critical depth. Knowing it, authors offer a device that is designed to finding critical section and critical depth in open flows.

The study in [7] is devoted to the experimental investigation of a broad-crested weir. To determine the basic characteristics of the weir, generally accepted methods of hydraulic calculations are used. A refined correlation is obtained to determine the discharge coefficient of a broad-crested weir.

In [8], two mathematical models based on the Saint-Venant equations are considered for calculating unsteady flows in channels with floodplains. In the 1st model, the influence of the floodplain presence on the passage of flood or flush waves is taken into account through the morphometric and hydraulic characteristics, total for the channel and floodplain. In the 2nd model, based on the separation of the channel flow, the floodplain plays the role of a storage tank.

[9] is devoted to imitating the breakthrough of the Vakhdat dam (Iran) by integrating the ArcMap and HEC-RAS software and field observations. The Manning coefficient for the Geshlag River in the lower reaches of the dam is estimated using the Manning's ratio and field observations and measurements.

In [10], the problem of streamlining a weir crest of various designs is discussed. It is shown that when designing rectangular weirs without lateral compression, when no intake of atmospheric air under the jet from the downstream is provided, the flow measurement error may increase by 20–25 %. The best solution to this issue is, if possible, to project the weir with a small lateral compression, about 2–3 %.

In [11], a substantiation is given for choosing the optimal methods for organizing automated water flow accounting and collecting hydrological and climatic information for modeling the dynamics of water-balance elements, and to ensure adaptive land use and the needs of hydrological and geochemical monitoring.

In [12] is devoted to substantiating the conditions for the use of reserve weirs at hydro-systems with earth dams and to the development of the methods for calculating the basic parameters of such reserve weirs. As a result of research, it has been found that the advantages of the proposed structure compared to other weir structures are low construction costs, simplicity of the device, significantly lower total possible damage in the event of an accident at a dam, since the part of the excess flood will be discharged in advance through the reserve weir and will be distributed longer in time.

Stationary and non-stationary plane problems of hydromechanics for a semi-infinite fluid layer of finite depth are considered. Hydrodynamic pressure on the vertical upstream face of the dam under the horizontal seismic load with energy absorption on the reservoir bottom and a layer of sediment is determined in [13].

In [14] the methodology of the development of scenarios and the water overflow probability that is one of the basic causes of the emergency situations at the dams is given. The causes of the spillway failure and most frequent events leading to the failures or disturbances of the spillway structures are analyzed.

In [15] reliability problems for low pressure waterworks are considered. Earthen dams with long operational periods are shown to have substantial damage and are in unsatisfactory condition.

In [16] according to the proposed algorithm, a computer program was developed, and the calculation of the displacement of the crest of the dam Sayano-Shushenskaya HPP for 2001...2016 years (360 intervals). The estimation of accuracy of the received results is given.

In [17], mathematical problem of calculating the unsteady flow of water is considered when regulating concentrated water discharge releases in watercourses in downstream spillways. An algorithm for the analytical solution of the problem is compiled, based on a hydraulic calculation of the process of propagation and transformation of long waves described by the Saint-Venant equations.

In [18], the effect of soaking of the upstream side of rectangular broad-crested weirs on discharge coefficient and flow characteristics is investigated. Five weirs have been developed with tilt angles of 15, 30, 45, 60 and 90° and a flow rate coefficient, negative velocity along the dam crest and water surface profiles along the weir crest have been estimated in laboratory hydraulic trays. The results obtained by reducing the inclined side upstream, increase the discharge efficiency and discharge capacity of the weir.

In [19] it is shown that in wide rectangular channels of flows for which the Manning formula is applicable, at constant roughness of the bottom along the length of the water channel, analytical solutions of the Saint-Venant equations can be constructed in the form of rising waves propagating down the slope of the channel without changing the shape. Such waves are called monoclinal ones.

It should be noted that the construction of solutions of the Saint-Venant equations in the form of a monoclinal wave is a rather complicated and time-consuming task [19]. The fact of the possibility of constructing such solutions is proved in [19], but the solutions themselves are not given. In [20], it is noted that the construction of such solutions for the propagation of a monoclinal wave along the initially dry channel is much simpler than in the general case with a filled channel, since in this case the water velocity is constant throughout the flow region and is equal to the wave propagation velocity. This is so because in initially dry bed there is no water flow through the

body of the wave. In [20], an analytical solution is constructed for a monoclinal wave propagating in a dry channel at a constant value of the hydraulic friction coefficient. The obtained analytical solution is compared to the results of numerical solution of the Saint-Venant equations using an explicit finite-difference scheme and the finite element method [21–23], showing a satisfactory agreement.

In [24], a new method for controlling unsteady flow in open channels is presented. The equations are derived from the differential form of complete shallow water equations in one dimension.

In [25] regularized equations describing hydrodynamic flows in the two-layer shallow water approximation are constructed. A conditionally stable finite-difference scheme based on the finite-volume method is proposed for the numerical solution of these equations.

In real long-wave processes in prismatic channels with a constant slope and roughness, a more complex flow pattern arises in which a quiet flow below the downstream floor at initial period of release wave passage can differ quite strongly from a monoclinal wave, and only gradually throughout the whole quiet flow section does the wave flow approach to it [26, 27].

Based on the above, the aim of this work is defined, which consists in determining the numerical value of the coefficient of flow through the broad-crested weir, which was used in the numerical study of long-wave processes.

2. Methods

Here we present the methodology for the end-to-end calculation of spillway dams, taking into account flooding and the possibility of maintaining a given (fixed or variable) level in the upstream (US) [28]. Figure 1 shows a diagram of river section with a diverting dam, which shows water flow through a weir.
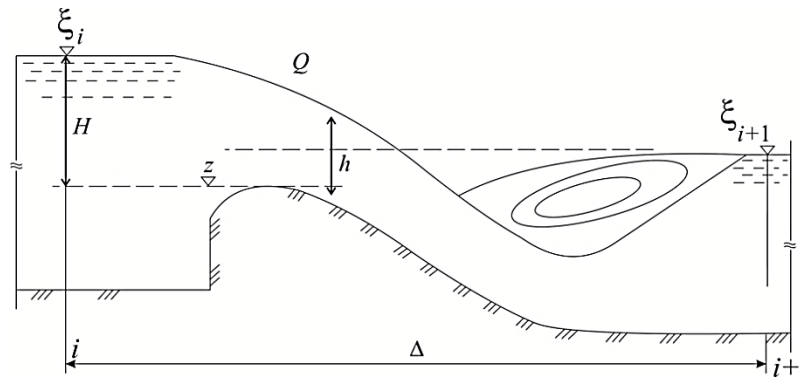

Figure 1. Pattern of water flow through the weir.

Here i, $i+1$ are the nodes of computational grid, where free surface levels are calculated ζ_i, ζ_{i+1}; Q is the flow rate through the dam; z is the top mark of the weir; $H = \zeta_i - z$ is the pressure over the crest of the weir; $h = \zeta_{i+1} - z$ is the depth of flooding from the downstream (DS); Δ is the distance between the nodes.

As a rule, a normal retaining level is maintained in the US by maneuvering dam shutters. However, as noted above, at high floods, accompanied by long-wave processes, the dam can be fully opened, and then there is a water flow through the weir, in conditions of flooding ($h > 0$) as well. Thus, at given water flow rate, three flow modes are possible: with a fixed level of US, through non-submerged and submerged weirs. Obviously, in the latter case, the DS level has an impact on the US level. The Saint-Venant equations do not describe the water flow through the submerged weir, therefore we modify them so that along with the flow in natural channel, to calculate end-to-end all the listed modes of water supporting structure. To do this, to the equation of motion on the segment [i, $i+1$] it is necessary to introduce an additional hydraulic resistance, similar to friction, which would provide the required water level difference at the dam, and to exclude convective terms. The equation of continuity does not change. Then, in the stationary case, the relationship of flow rates and levels is obtained in the form

$$g\omega \frac{\zeta_i - \zeta_{i+1}}{\Delta} = QF. \tag{1}$$

In the absence of a dam

$$F = F_0 = \frac{\lambda}{2} \frac{|Q|}{\omega R}, \tag{2}$$

where $\lambda = 2gn^2 R^{-1/3}$ is the hydraulic friction coefficient, other designations are similar to those given earlier;

ω is flow cross section;

R is hydraulic radius;

g is gravity relative to unit mass.

In the presence of a weir from a known ratio

$$Q = m\sigma b\sqrt{2g}\,H^{3/2},$$ (3)

where m is the coefficient of weir flow;

σ is the flooding coefficient;

b is the width of the weir; the following expression is obtained

$$H \equiv \zeta_i - z = \left(\frac{Q}{m\sigma\ b\sqrt{2g}}\right)^{2/3},$$ (4)

which is converted to the form (1) if assume that

$$F = \frac{g\,\omega(1-\xi)}{\Delta\left(m\,\sigma b\sqrt{2gQ}\right)^{2/3}} \equiv F_1(1-\xi).$$ (5)

where $\xi = \dfrac{h}{H} \equiv \dfrac{\zeta_{i+1}-z}{\zeta_i - z} \le 1$ is the degree of flooding.

The expression for the flooding coefficient σ depends on the type of weir, and, for the weirs of practical profile can be taken in the form:

$$\sigma = \begin{cases} (1-\xi)^{0.3}, & 0 \le \xi < 1; \\ 1, & \xi < 0. \end{cases}$$ (6)

Broad-crested weirs are widely used in the practice of hydraulic engineering, for example, in the inlet part of weir structures of dams with discharge facilities. One of the main hydrodynamic parameters of weirs are the flow carrying capacity, which can be determined using the calculation formula for broad-crest weirs in significantly narrowed channels, in dike dams, in the area of diverting dams, in narrow floodplains, in approaches to bridges, as well as in dams with partial passage of high floods on submerged floodplain. Since in various problems of computational hydrodynamics it is necessary to choose and adapt the above-mentioned computational formula, the identification of such an expression is the aim of this study.

Under conditions of sharp variability of hydrological processes, a large amount of water flow in a relatively short time, accompanied by long-wave processes, passes through rivers and canals. If these processes in the river beds have a natural origin (floods and freshets), then in hydro-systems and reservoirs they can be the flush waves from the overlying hydroelectric complex or the breakthrough waves caused by hydrodynamic accidents. In this case, the breakthrough in the dam body plays the role of a weir [29].

In numerical studies of long-wave processes, the inclusion to the program of an internal boundary condition that approximates a weir formula makes it very difficult. It should be noted that the calculation formula in a general form for a broad-crest weir can be obtained from the equation of the curve of free water surface in the channel, which is a special case of the Saint-Venant equation of motion for steady-state flow [29, 30]. It is easy to show from the equation of the curve of free surface that the transition of a steady-state river flow from quiet flow mode (where the Froude number and kinetic parameter is less than unity ($Fr < 1$ or $P_k < 1$)) to a turbulent mode ($Fr > 1$ or $P_k > 1$) is possible on the sections of river where the channel first narrows and then widens. This property of the channel flow is similar to the property of pressure gas flows in Laval nozzles; it is a striking example of a hydraulic-gas-dynamic analogy found by the classics in hydro-dynamics – N.Ye. Zhukovsky and D.P. Ryabushinsky [31].

To obtain an improved calculation formula for a non-submerged broad-crest weir using the formula for the curve of water free surface directly, without using the Belange hypothesis, the mathematical model is used to predict long-wave processes in the reaches of hydroelectric complexes and reservoirs; this model consists of hydrodynamic equations of water flow, presented in the form:

$$\begin{cases} \dfrac{\partial \omega}{\partial t} + \dfrac{\partial Q}{\partial x} = 0, \\[2mm] \dfrac{\partial Q}{\partial t} + \dfrac{\partial (V^2\omega + gS)}{\partial x} - g\dfrac{\partial S}{\partial x}\Big|_{Z_{fs}} = \text{const} + \dfrac{\lambda}{2}V^2\chi = 0, \end{cases}$$ (7)

where Z_{fs} is mark of free surface of flow.

Propagation velocity of waves of small amplitude along the channel, corresponding to (7) is

$$C = \sqrt{g \frac{\omega}{B}},$$

$$\omega \frac{\partial V}{\partial t} + V \frac{\partial \omega}{\partial t} + V \frac{\partial \omega V}{\partial x} + V \omega \frac{\partial V}{\partial x} + g \frac{\partial S}{\partial x} - g \frac{\partial S}{\partial x}\Big|_{Z_{fs}} = \text{const} + \frac{\lambda}{2} V^2 \chi = 0,$$

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + \frac{g}{\omega}\left(\frac{\partial S}{\partial x} - \frac{\partial S}{\partial x}\Big|_{Z_{fs}} = \text{const} \right) + \frac{\lambda}{2} \frac{V^2}{R} = 0, \tag{8}$$

$$S = \int_{Z_{rb}}^{Z_{fs}} B(Z_{fs} - z)dz, \cdot$$

where Z_{rb} is mark of the bottom of the channel.

To differentiate the static moment of the section, the formula known from mathematical analysis is used, where there is a function:

$$F(x) = \int_{\alpha(x)}^{\beta(x)} \Phi(x, y)dy ; \quad \frac{dF}{dx} = \int_{\alpha}^{\beta} \frac{\partial \Phi}{\partial x} dy + \Phi_{|y=\beta} \frac{d\beta}{dx} - \Phi_{|y=\alpha} \frac{d\alpha}{dx} ;$$

$$\frac{\partial S}{\partial x} = \frac{\partial}{\partial x} \int_{Z_{rb}}^{Z_{fs}} \left(Z_{fs} - z \right) B dz = \int_{Z_{rb}}^{Z_{fs}} \frac{\partial \left(Z_{fs} - z \right) B}{\partial x} dz + \left(Z_{fs} - z \right) B \Big|_{z = Z_{fs}} \frac{\partial Z_{fs}}{\partial x} -$$

$$- \left(Z_{fs} - z \right) B \Big|_{z = Z_{Z_{rb}}} \frac{\partial Z_{rb}}{\partial x} = \int_{Z_{rb}}^{Z_{fs}} \frac{\partial Z_{fs}}{\partial x} B dz + \int_{Z_{rb}}^{Z_{fs}} Z_{fs} \frac{\partial B}{\partial x} dz - HB \frac{\partial Z_{rb}}{\partial x} =$$

$$= \frac{\partial Z_{fs}}{\partial x} \int_{Z_{rb}}^{Z_{fs}} B dz + Z_{fs} \int_{Z_{rb}}^{Z_{fs}} \frac{\partial B}{\partial x} dz - HB \frac{\partial Z_{rb}}{\partial x} = \omega \frac{\partial Z_{fs}}{\partial x} + Z_{fs} \int_{Z_{rb}}^{Z_{fs}} \frac{\partial B}{\partial x} dz - HB \frac{\partial Z_{rb}}{\partial x} ;$$

$$\frac{\partial S}{\partial x}\Big|_{Z_{fs}} = \text{const} = Z_{fs} \int_{Z_{rb}}^{Z_{fs}} \frac{\partial B}{\partial x} dz - HB \frac{\partial Z_{rb}}{\partial x} .$$

So,

$$\frac{\partial S}{\partial x} - \frac{\partial S}{\partial x}\Big|_{Z_{fs}} = \text{const} = \omega \frac{\partial Z_{fs}}{\partial x}$$

and

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial Z_{fs}}{\partial x} + \frac{\lambda}{2} \frac{V^2}{R} = 0 \tag{9}$$

hence

$$\frac{\partial V}{\partial t} + \frac{\partial \left(V^2 / 2 + g Z_{fs} \right)}{\partial x} + \frac{\lambda}{2} \frac{V^2}{R} = 0.. \tag{10}$$

Equation (10) at steady state flow, when $\frac{\partial V}{\partial t} = 0$, is the equation of curve of the free surface in the channel. When $\lambda = 0$, it turns into the Bernoulli equation

$$\frac{d \left(V^2 / 2 + g Z_{fs} \right)}{dx} + \frac{\lambda}{2} \frac{V^2}{R} = 0. \tag{11}$$

Given the stationary nature of the flow in the computational domain, we get

$$\frac{d\left(Q^2/2\omega^2 + gh\right)}{dx} = gI - \frac{\lambda}{2}\frac{V^2}{R}. \tag{12}$$

If to take a channel of rectangular cross section, but with varying width and bottom level along the flow, that is, at each location $\omega = Bh$, where B and h can vary in x, then instead of (12) we get

$$\left(g - \frac{Q^2}{\omega^3}\frac{\partial\omega}{\partial h}\right)\frac{dh}{dx} - \frac{Q^2}{\omega^3}\frac{\partial\omega}{\partial B}\frac{dB}{dx} = gI - \frac{\lambda}{2}\frac{V^2}{R}. \tag{13}$$

Since $\dfrac{\partial\omega}{\partial h} = B$.

$$\left(g - \frac{V^2 B}{Bh}\right)\frac{dh}{dx} - \frac{V^2 h}{Bh}\frac{dB}{dx} = gI - \frac{\lambda}{2}\frac{V^2}{R},$$

$$\left(1 - \frac{V^2}{gh}\right)\frac{dh}{dx} - \frac{V^2 h}{Bgh}\frac{dB}{dx} = gI - \frac{\lambda}{2}\frac{V^2}{hR},$$

$$R = Bh/(B + 2h).$$

As $Fr = \dfrac{V^2}{gh}$,

$$(1 - Fr)\frac{dh}{dx} = I + Fr\frac{h}{B}\frac{dB}{dx} - \frac{\lambda}{2}\frac{V^2}{gR},$$

$$\frac{dh}{dx} = \frac{I + Fr\dfrac{h}{B}\dfrac{dB}{dx} - \dfrac{\lambda}{2}\dfrac{V^2}{gR}}{1 - Fr}. \tag{14}$$

In cases where the influence of friction is small, it follows from (14) that at the narrowing of channel section ($I < 0$, $\dfrac{dB}{dx} < 0$): in a quiet flow state ($Fr < 1$, $Pk < 1$), the depth decreases, an increase in the average flow velocity is observed; in a turbulent state of flow ($Fr > 1$, $P_k > 1$) the depth of the flow increases, and the average velocity decreases;

at the widening of channel section ($I > 0$, $\dfrac{dB}{dx} > 0$): in a quiet flow ($Fr < 1$) the depth increases and the flow rate decreases, in a turbulent flow ($Fr > 1$) the depth decreases and the speed increases.

According to the above statements, it can be stated that the Froude number in a quiet flow increases with narrowing of the channel and decreases with its widening, and in a turbulent flow, on the contrary, it decreases with narrowing of the channel, and increases with its widening. According to the above, the transition of the flow from a quiet flow mode to a turbulent one can occur only when the channel form changes from narrowing to widening one, and in the narrowest section of the channel the values of the Froude number and the kinetic parameter will be equal to $Fr = P_k = 1.0$.

Fundamental importance of the hydraulic-gas-dynamic analogy of hydrodynamics found by N.E. Zhukovsky and D.P. Ryabushinsky, similar to the theory of the Laval nozzle widely known in technical gas dynamics has already been noted. In accordance with this analogy, the depth of the flow is similar to gas density, the pressure forces for the channel flow and gas flow in a pipe are similar, the propagation velocity of waves of small amplitude in the channel is similar to the sound velocity in a gas. The analogue of the Froude number is the square of the Mach number. A detailed description of the hydraulic-gas-dynamic analogy and its use in engineering is given in literature [31].

Further, the minimum area in the dam location, in which the Froude number $Fr = 1$, will be called critical one, and the flow parameters – critical parameters; will mark it by an index with an asterisk. Naturally, the transition from the quiet mode of the flow of a channel stream to a turbulent mode is possible only under the condition that the critical depth of the flow is not flooded from the downstream.

So,

$$Fr = \frac{V_*^2}{gh_*} = 1 .$$ (15)

These considerations have been carried out under the insignificant influence of hydraulic friction along the channel, where the basic conditions for the application of the Bernoulli equation are satisfied.

Suppose that the upstream channel becomes very broad (a reservoir is located there), so that the velocity pressure in that zone can be neglected; let the water depth in the reservoir above the bottom of the critical section of the channel be equal to N. Then the Bernoulli equation will look like:

$$H = h_* + \frac{V_*^2}{2g} ,$$ (16)

or in accordance with (15),

$$V_*^2 = gh_* ,$$

$$H = \frac{3}{2} h_* .$$ (17)

From the above follows the well-known in hydraulics formula of a broad-crested weir

$$Q = B_* h_* V_* = B_* h_* \sqrt{gh_*} = B_* \sqrt{g} h_*^{3/2} = B_* \sqrt{g} \left(\frac{2}{3} H \right)^{3/2} = m B_* \sqrt{2g} H^{3/2} ,$$ (18)

where m is the discharge coefficient to determine the flow rate of water through a non-submerged broad-crested weir (18); it is a result of some mathematical transformations of equation (11) directly, without using the Belange hypothesis.

$$m = 2 / 3^{3/2} \approx 0.385 .$$ (19)

It should be noted that the well-known formula of Saint-Venant-Wanzel [31] is an analogue of formula (18) for pressure flows, which allows determining the flow rate of gas flowing out of a pressure tank through nozzles (assuming the process is adiabatic); it is widely used in engineering.

A numerical experiment has been conducted on flow passage through a non-submerged broad-crested weir. The A.N. Militeev explicit finite-difference scheme, adapted for channels of arbitrary shape, was used [32]. In the experiment, the channel of a rectangular cross section without a slope and friction has been suddenly narrowed 100 times, and then suddenly widened 100 times. On the finite-difference grid, the weir was modeled by two narrowing locations. As a boundary condition at the inlet to computational domain, the flow rate was set, at the outlet – the Froude number Fr. Generally speaking, it could be set greater than 1, but in the study the Froude number was taken as $Fr = 0.12$. As an initial condition, the water flow over the entire area was assumed to be 0, a depth was set above the narrowed area, and below – the depth corresponding to the inlet flow rate and the Froude number at the outlet from the area.

In a numerical experiment, after a certain period, a flow mode was established in which the depth in the upstream significantly exceeded the depth in the downstream; below the narrowed zone, a turbulent flow area, a hydraulic jump and a quiet flow area appeared. In the absence of friction with the bottom, the problem of numerical simulation of the section of displacement of hydraulic jump is impossible, but this was not the subject of this experiment. Figure 2 presents a graph of the value $m = \frac{Q}{B_* \sqrt{2g} H^{3/2}}$ obtained in numerical experiment, which, in a steady mode is the weir coefficient. As a result of the experiment found that $m \approx 0.394$, it slightly differs from the theoretical value of $m \approx 0.385$.

When setting the hydraulic friction on the section of narrowing, the coefficient of weir flow drops. According to the above, it can be concluded that the equation of curve of water free surface is a result of the Saint-Venant equations; and in numerical calculations of flows on broad-crested weirs (and polygonal weirs of a spread out profile) it is possible to use them directly, end-to-end, without inserting the formula of weir as an internal boundary condition. Naturally, this does not apply to calculations of weirs of practical profile or the ones with a thin wall, in which there is a greater curvature of jets and the pressure distribution over the depth differs greatly from the hydrostatic one; this excludes the use of the Saint-Venant equations. For flows with

small curvature of jets the Saint-Venant equations are suitable, the flow coefficient $m = 2/3^{3/2}$ is a maximum and suits only in the absence of any additional hydraulic losses in the inlet section.
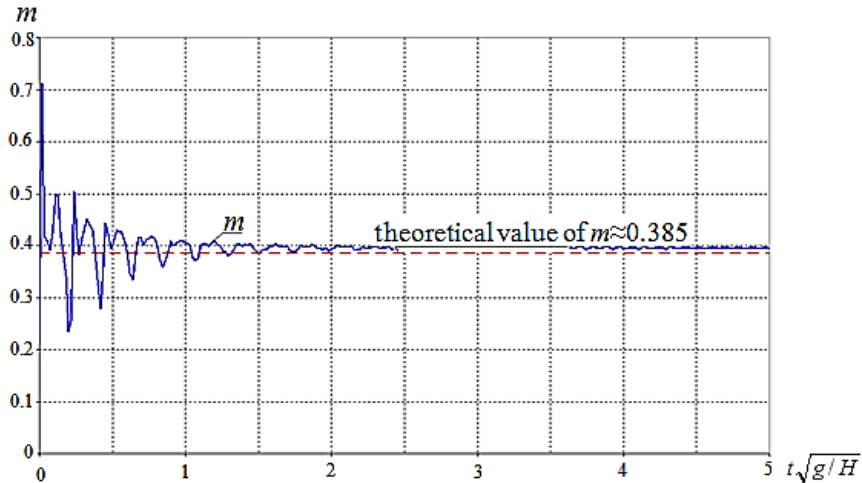

Figure 2. Change of value $m = \dfrac{Q}{B_* \sqrt{2g}\,H^{3/2}}$ **in time obtained in numerical experiment**

(in the steady state, m is the coefficient of weir flow).

3. Results and Discussion

To study the course of the long-wave process in the reaches of hydro-systems and reservoirs, take the river basin to the plain part of the region with real hydrological, hydraulic and morphometric parameters. In the considered section of the river basin, there is a cascade with several reservoirs for irrigation and hydropower purposes. An example of a cascade accident on a river in a plain terrain is shown in Figure 3.
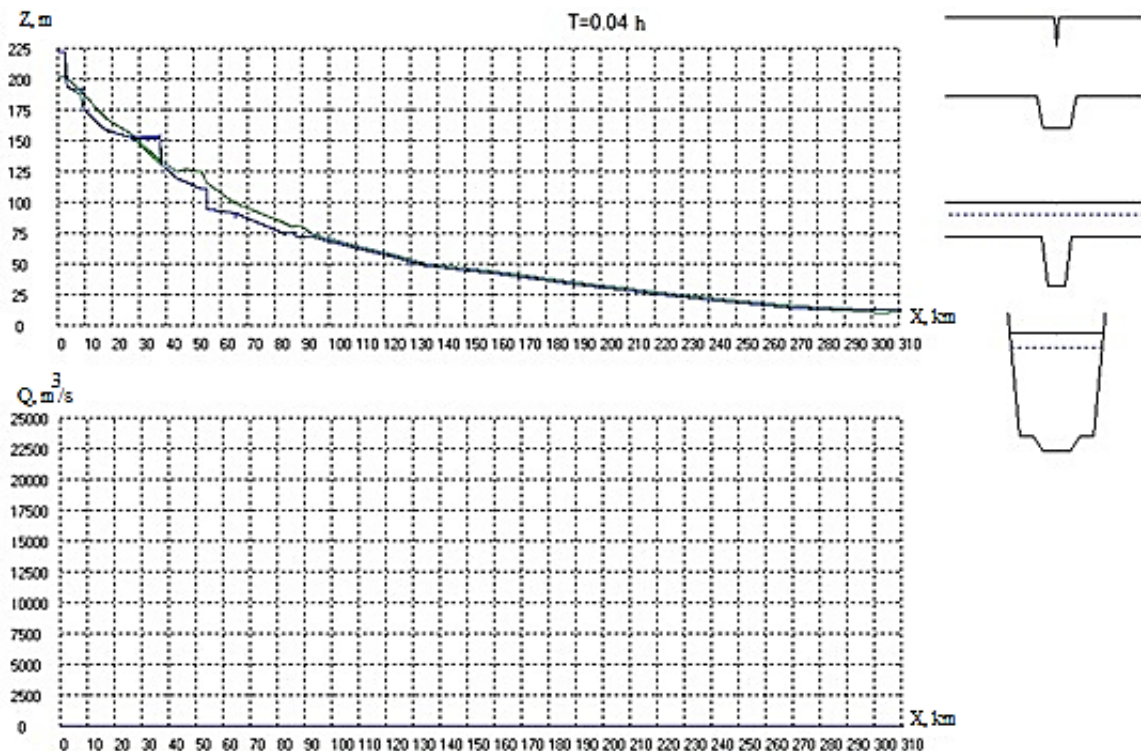

**Figure 3. Beginning of a numerical study of a cascade of structures
in a plain river with three reservoirs over a length of 300 km.**

A cascade consists of 3 relatively small reservoirs (upper one – 100 million m³, medium – 4 million m³, lower – 100 million m³). With a sharp increase in water volume in the first reservoir, the long-wave process begins with the passage of a large flow of water through the dam. According to the course of calculation, the long-wave flow destroys at high velocity the first dam and makes sharply varying movements in the form of a wave after $t = 1.1$h of the estimated time, reaches the dam of the second reservoir (Figure 4).

The dynamics of water level in the river basin and of the flow of water with characteristic cross sections in the locations of the cascade reservoirs are presented.
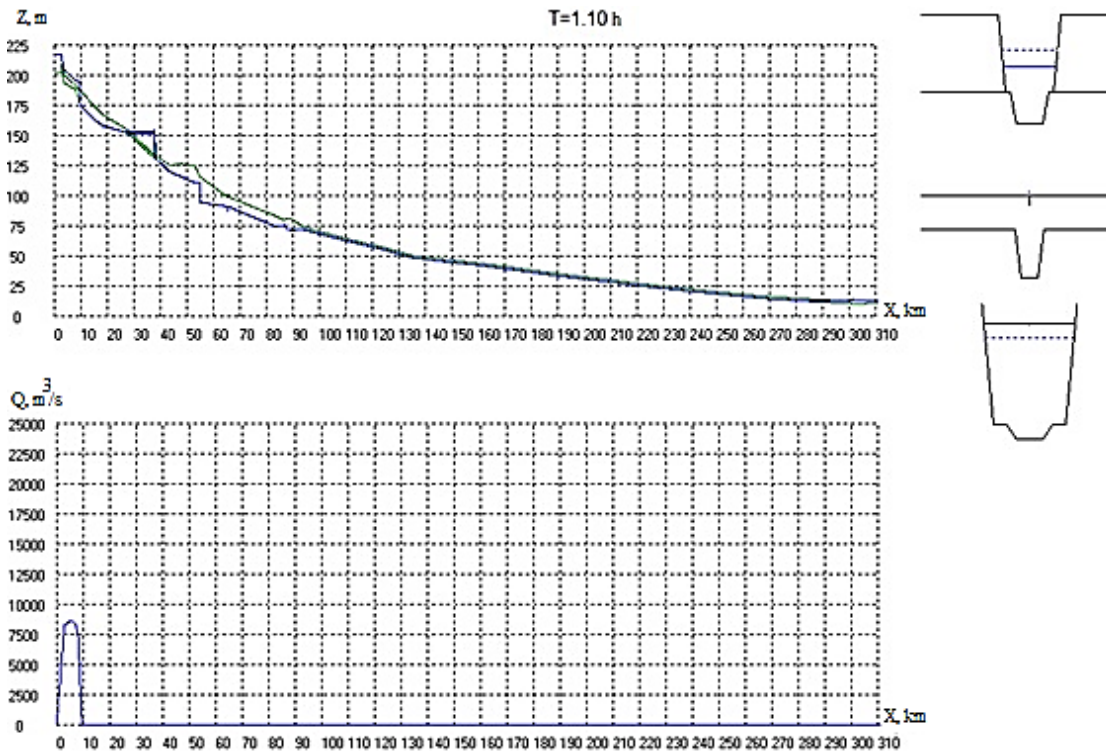

Figure 4. Results of numerical calculation of sharply varying flow after *t* = 1.1 h.

The intensively filled second reservoir almost instantly transmits a long wave in the downstream of the cascade reservoir. As the calculation results show, it is possible to hold the accident for a certain time, about 3 hours, when a breakthrough wave reaches the lower reservoir (Figure 5).
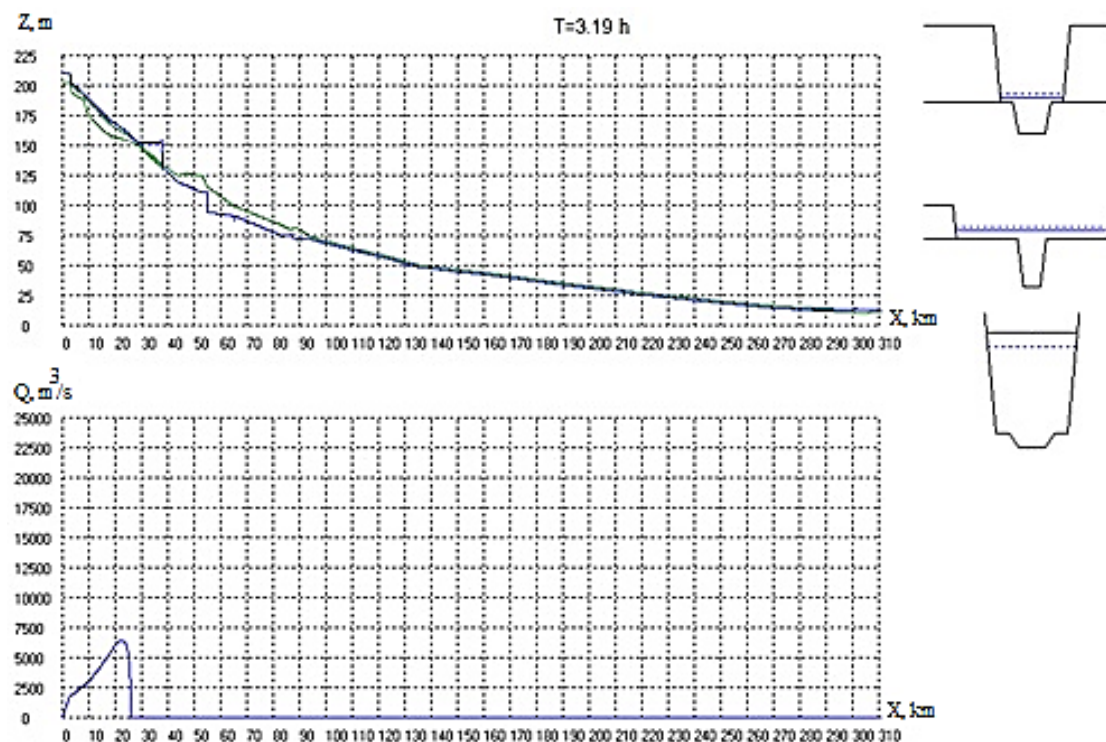

Figure 5. Results of numerical calculation of sharply varying flow in a cascade after *t* = 3.19 hours.

The dynamics of the water level in the river basin and of the flow of water with characteristic cross sections in the locations of the cascade reservoirs are presented.

According to the results of numerical study of the developed model, after 8 hours, the third reservoir overflows and a breakthrough occurs, accompanied by long waves, and the throughput capacity of the water

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discharge and weir structures of the last hydro-system turns out to be insufficient to prevent a hydrodynamic accident during large floods (Figure 6).
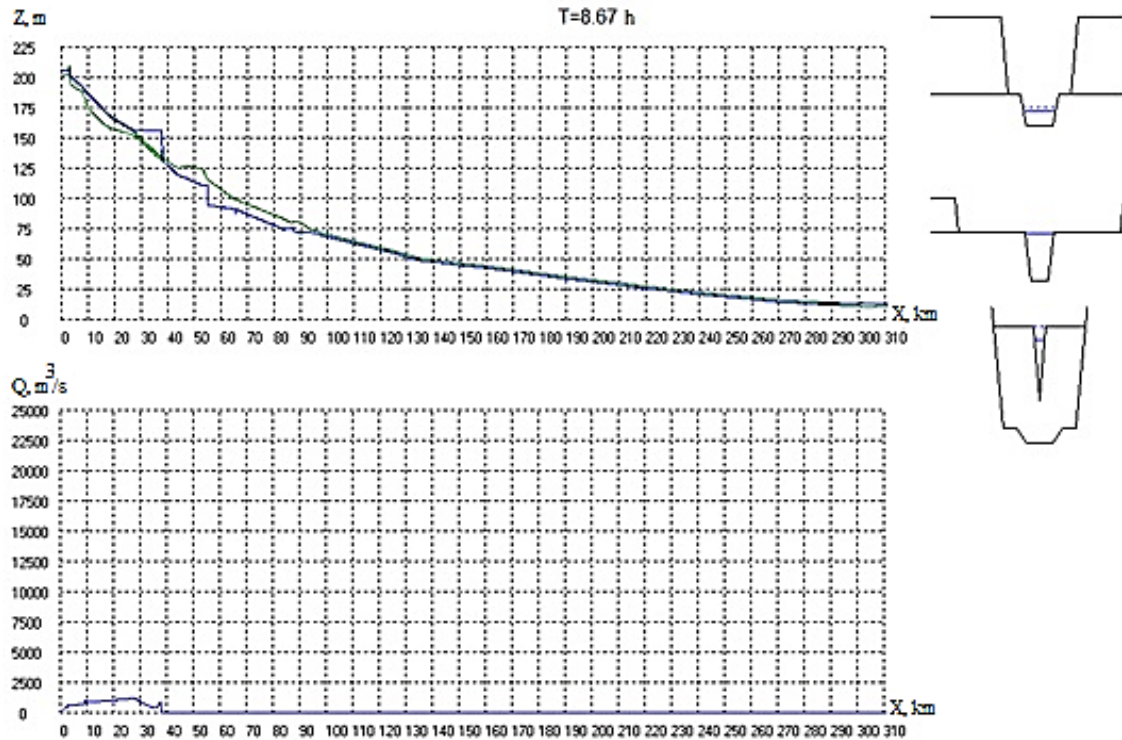

Figure 6. Results of numerical calculation of sharply varying flow in a cascade after *t* = 8 h.

The dynamics of the water level in the river basin and the flow of water with characteristic cross sections in the locations of the cascade reservoirs are presented. Beginning of the fourth process of long waves propagation through the dam of the third reservoir.

Further, the breakthrough wave, gradually weakening (but for a small river remaining catastrophic along the entire length of the river), spreads downstream till the place where the river flows into a large lake.

The result of calculation allows us to determine the characteristic of the flow dynamics in the river bed and in floodplain and flooding zone of the adjacent territories of the studied river basin. These data can be used in determining the extent of possible damage and the losses resulting from the passage of flood flows, accompanied by long-wave processes. The description of the dynamics of hydrodynamic parameters of the flow and the morphometry of the riverbed in the developed model is the next step in numerical studies of the current investigation. A cascade of hydro-technical structures on the Chirchik river basin has been chosen for the numerical study.

4. Conclusions

1. Using the vector flow equation and the scalar equation of continuity, a system of equations describing the flow of water is obtained.

2. The value of the coefficient of flow is calculated, where the input part of overflow to the broad-crested weir is taken.

3. Numerical studies of the passage of long waves through the dams of the reservoirs of the Chirchik river basin have been carried out.

4. Numerical studies of the passage of long waves through the dam of a real object have been carried out.

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Численные исследования длинноволновых процессов в бьефах гидроузлов и водохранилищ

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Ключевые слова: длинноволновые процессы, проран, уравнения движения Сен-Венана, математическое моделирование, численный метод.

Аннотация. Рассматривается проблема разработки математических моделей и компьютерных программ для расчетов и прогнозов различных длинноволновых процессов, происходящих в бьефах водохранилищ и гидроузлов. Основным методом решения задач является математическое моделирование на основе дифференциальных уравнений для руслового потока – уравнений Сен-Венана, с использованием численных методов. Также используются общепринятые методы в гидравлике, методы составления математических моделей на основе законов гидромеханики и их численных расчетов. Разработана математическая модель и компьютерная программа для проведения прогнозных расчетов длинноволновых процессов происходящих в бьефах гидроузлов и водохранилищ. Достоверность полученных результатов подтверждается достаточно строгой математической постановкой, применением известных и апробированных уравнений и методов гидравлики, а также соответствием полученных в работе результатов с имеющимися в литературе данными других авторов.

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