The concept of potential energy was introduced in Chapter 8 in connection with such conservative forces as the force of gravity and the elastic force exerted by a spring. By using the law of conservation of energy, we were able to avoid working directly with forces when solving various problems in mechanics. In this chapter we see that the concept of potential energy is also of great value in the study of electricity. Because the electrostatic force given by Coulomb's law is conservative, electrostatic phenomena can be conveniently described in terms of an electric potential energy. This idea enables us to define a scalar quantity known as *electric potential.* Because the electric potential at any point in an electric field is a scalar function, we can use it to describe electrostatic phenomena more simply than if we were to rely only on the concepts of the electric field and electric forces. In later chapters we shall see that the concept of electric potential is of great practical value.

25.1 POTENTIAL DIFFERENCE AND ELECTRIC POTENTIAL

11.8 When a test charge q_0 is placed in an electric field $\mathbf{E}$ created by some other charged object, the electric force acting on the test charge is $q_0\mathbf{E}$. (If the field is produced by more than one charged object, this force acting on the test charge is the vector sum of the individual forces exerted on it by the various other charged objects.) The force $q_0\mathbf{E}$ is conservative because the individual forces described by Coulomb's law are conservative. When the test charge is moved in the field by some external agent, the work done by the field on the charge is equal to the negative of the work done by the external agent causing the displacement. For an infinitesimal displacement $d\mathbf{s}$, the work done by the electric field on the charge is $\mathbf{F}\cdot d\mathbf{s} = q_0\mathbf{E}\cdot d\mathbf{s}$. As this amount of work is done by the field, the potential energy of the charge–field system is decreased by an amount $dU = -q_0\mathbf{E}\cdot d\mathbf{s}$. For a finite displacement of the charge from a point A to a point B, the change in potential energy of the system $\Delta U = U_B - U_A$ is

$$\Delta U = -q_0 \int_A^B \mathbf{E}\cdot d\mathbf{s} \qquad \textbf{(25.1)}$$

Change in potential energy

The integration is performed along the path that q_0 follows as it moves from A to B, and the integral is called either a *path integral* or a *line integral* (the two terms are synonymous). Because the force $q_0\mathbf{E}$ is conservative, **this line integral does not depend on the path taken from *A* to *B*.**

Quick Quiz 25.1

If the path between A and B does not make any difference in Equation 25.1, why don't we just use the expression $\Delta U = -q_0 Ed$, where d is the straight-line distance between A and B?

The potential energy per unit charge U/q_0 is independent of the value of q_0 and has a unique value at every point in an electric field. This quantity U/q_0 is called the **electric potential** (or simply the **potential**) V. Thus, the electric potential at any point in an electric field is

$$V = \frac{U}{q_0} \qquad \textbf{(25.2)}$$

The fact that potential energy is a scalar quantity means that electric potential also is a scalar quantity.

The **potential difference** $\Delta V = V_B - V_A$ between any two points A and B in an electric field is defined as the change in potential energy of the system divided by the test charge q_0:

Potential difference

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \tag{25.3}$$

Potential difference should not be confused with difference in potential energy. The potential difference is proportional to the change in potential energy, and we see from Equation 25.3 that the two are related by $\Delta U = q_0 \Delta V$.

Electric potential is a scalar characteristic of an electric field, independent of the charges that may be placed in the field. However, when we speak of potential energy, we are referring to the charge–field system. Because we are usually interested in knowing the electric potential at the location of a charge and the potential energy resulting from the interaction of the charge with the field, we follow the common convention of speaking of the potential energy as if it belonged to the charge.

Because the change in potential energy of a charge is the negative of the work done by the electric field on the charge (as noted in Equation 25.1), the potential difference ΔV between points A and B equals the work per unit charge that an external agent must perform to move a test charge from A to B without changing the kinetic energy of the test charge.

Just as with potential energy, only *differences* in electric potential are meaningful. To avoid having to work with potential differences, however, we often take the value of the electric potential to be zero at some convenient point in an electric field. This is what we do here: arbitrarily establish the electric potential to be zero at a point that is infinitely remote from the charges producing the field. Having made this choice, we can state that the **electric potential at an arbitrary point in an electric field equals the work required per unit charge to bring a positive test charge from infinity to that point.** Thus, if we take point A in Equation 25.3 to be at infinity, the electric potential at any point P is

$$V_P = -\int_\infty^P \mathbf{E} \cdot d\mathbf{s} \tag{25.4}$$

In reality, V_P represents the potential difference ΔV between the point P and a point at infinity. (Eq. 25.4 is a special case of Eq. 25.3.)

Because electric potential is a measure of potential energy per unit charge, the SI unit of both electric potential and potential difference is joules per coulomb, which is defined as a **volt** (V):

Definition of volt

$$1\ \text{V} \equiv 1\ \frac{\text{J}}{\text{C}}$$

That is, 1 J of work must be done to move a 1-C charge through a potential difference of 1 V.

Equation 25.3 shows that potential difference also has units of electric field times distance. From this, it follows that the SI unit of electric field (N/C) can also be expressed in volts per meter:

$$1\ \frac{\text{N}}{\text{C}} = 1\ \frac{\text{V}}{\text{m}}$$

A unit of energy commonly used in atomic and nuclear physics is the **electron volt** (eV), which is defined as **the energy an electron (or proton) gains or loses by moving through a potential difference of 1 V.** Because 1 V = 1 J/C and because the fundamental charge is approximately 1.60×10^{-19} C, the electron volt is related to the joule as follows:

$$1\ \text{eV} = 1.60 \times 10^{-19}\ \text{C} \cdot \text{V} = 1.60 \times 10^{-19}\ \text{J} \qquad \textbf{(25.5)}$$

The electron volt

For instance, an electron in the beam of a typical television picture tube may have a speed of 3.5×10^7 m/s. This corresponds to a kinetic energy of 5.6×10^{-16} J, which is equivalent to 3.5×10^3 eV. Such an electron has to be accelerated from rest through a potential difference of 3.5 kV to reach this speed.

25.2 POTENTIAL DIFFERENCES IN A UNIFORM ELECTRIC FIELD

Equations 25.1 and 25.3 hold in all electric fields, whether uniform or varying, but they can be simplified for a uniform field. First, consider a uniform electric field directed along the negative y axis, as shown in Figure 25.1a. Let us calculate the potential difference between two points A and B separated by a distance d, where d is measured parallel to the field lines. Equation 25.3 gives

$$V_B - V_A = \Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\int_A^B E \cos 0° \, ds = -\int_A^B E \, ds$$

Because E is constant, we can remove it from the integral sign; this gives

$$\Delta V = -E \int_A^B ds = -Ed \qquad \textbf{(25.6)}$$

Potential difference in a uniform electric field

The minus sign indicates that point B is at a lower electric potential than point A; that is, $V_B < V_A$. **Electric field lines always point in the direction of decreasing electric potential,** as shown in Figure 25.1a.

Now suppose that a test charge q_0 moves from A to B. We can calculate the change in its potential energy from Equations 25.3 and 25.6:

$$\Delta U = q_0 \, \Delta V = -q_0 Ed \qquad \textbf{(25.7)}$$

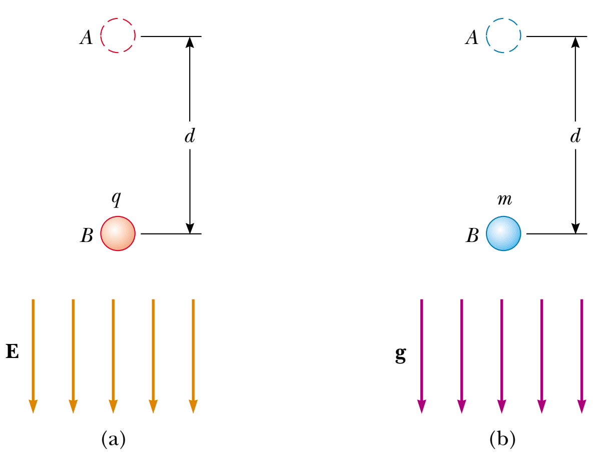

Figure 25.1 (a) When the electric field **E** is directed downward, point B is at a lower electric potential than point A. A positive test charge that moves from point A to point B loses electric potential energy. (b) A mass m moving downward in the direction of the gravitational field **g** loses gravitational potential energy.

QuickLab

It takes an electric field of about 30 000 V/cm to cause a spark in dry air. Shuffle across a rug and reach toward a doorknob. By estimating the length of the spark, determine the electric potential difference between your finger and the doorknob after shuffling your feet but before touching the knob. (If it is very humid on the day you attempt this, it may not work. Why?)

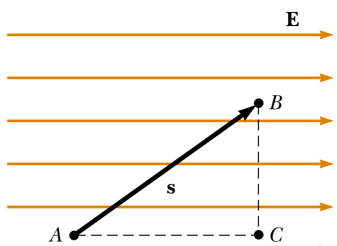

Figure 25.2 A uniform electric field directed along the positive x axis. Point B is at a lower electric potential than point A. Points B and C are at the *same* electric potential.

From this result, we see that if q_0 is positive, then ΔU is negative. We conclude that **a positive charge loses electric potential energy when it moves in the direction of the electric field.** This means that an electric field does work on a positive charge when the charge moves in the direction of the electric field. (This is analogous to the work done by the gravitational field on a falling mass, as shown in Figure 25.1b.) If a positive test charge is released from rest in this electric field, it experiences an electric force $q_0\mathbf{E}$ in the direction of $\mathbf{E}$ (downward in Fig. 25.1a). Therefore, it accelerates downward, gaining kinetic energy. **As the charged particle gains kinetic energy, it loses an equal amount of potential energy.**

If q_0 is negative, then ΔU is positive and the situation is reversed: **A negative charge gains electric potential energy when it moves in the direction of the electric field.** If a negative charge is released from rest in the field $\mathbf{E}$, it accelerates in a direction opposite the direction of the field.

Now consider the more general case of a charged particle that is free to move between any two points in a uniform electric field directed along the x axis, as shown in Figure 25.2. (In this situation, the charge is not being moved by an external agent as before.) If $\mathbf{s}$ represents the displacement vector between points A and B, Equation 25.3 gives

$$\Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\mathbf{E} \cdot \int_A^B d\mathbf{s} = -\mathbf{E} \cdot \mathbf{s} \qquad \textbf{(25.8)}$$

where again we are able to remove $\mathbf{E}$ from the integral because it is constant. The change in potential energy of the charge is

$$\Delta U = q_0 \, \Delta V = -q_0 \, \mathbf{E} \cdot \mathbf{s} \qquad \textbf{(25.9)}$$

11.9 Finally, we conclude from Equation 25.8 that all points in a plane perpendicular to a uniform electric field are at the same electric potential. We can see this in Figure 25.2, where the potential difference $V_B - V_A$ is equal to the potential difference $V_C - V_A$. (Prove this to yourself by working out the dot product $\mathbf{E} \cdot \mathbf{s}$ for $\mathbf{s}_{A \to B}$, where the angle θ between $\mathbf{E}$ and $\mathbf{s}$ is arbitrary as shown in Figure 25.2, and the dot product for $\mathbf{s}_{A \to C}$, where $\theta = 0$.) Therefore, $V_B = V_C$. **The name equipotential surface is given to any surface consisting of a continuous distribution of points having the same electric potential.**

An equipotential surface

Note that because $\Delta U = q_0 \Delta V$, no work is done in moving a test charge between any two points on an equipotential surface. The equipotential surfaces of a uniform electric field consist of a family of planes that are all perpendicular to the field. Equipotential surfaces for fields with other symmetries are described in later sections.

Quick Quiz 25.2

The labeled points in Figure 25.3 are on a series of equipotential surfaces associated with an electric field. Rank (from greatest to least) the work done by the electric field on a positively charged particle that moves from A to B; from B to C; from C to D; from D to E.

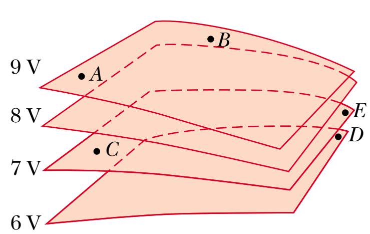

Figure 25.3 Four equipotential surfaces.

EXAMPLE 25.1 The Electric Field Between Two Parallel Plates of Opposite Charge

A battery produces a specified potential difference between conductors attached to the battery terminals. A 12-V battery is connected between two parallel plates, as shown in Figure 25.4. The separation between the plates is $d = 0.30$ cm, and we assume the electric field between the plates to be uniform. (This assumption is reasonable if the plate separation is small relative to the plate dimensions and if we do not consider points near the plate edges.) Find the magnitude of the electric field between the plates.

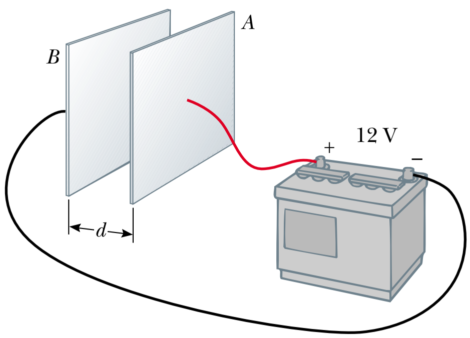

Figure 25.4 A 12-V battery connected to two parallel plates. The electric field between the plates has a magnitude given by the potential difference ΔV divided by the plate separation d.

Solution The electric field is directed from the positive plate (A) to the negative one (B), and the positive plate is at a higher electric potential than the negative plate is. The potential difference between the plates must equal the potential difference between the battery terminals. We can understand this by noting that all points on a conductor in equilibrium are at the same electric potential[1]; no potential difference exists between a terminal and any portion of the plate to which it is connected. Therefore, the magnitude of the electric field between the plates is, from Equation 25.6,

$$E = \frac{|V_B - V_A|}{d} = \frac{12\text{ V}}{0.30 \times 10^{-2}\text{ m}} = \boxed{4.0 \times 10^3\text{ V/m}}$$

This configuration, which is shown in Figure 25.4 and called a *parallel-plate capacitor,* is examined in greater detail in Chapter 26.

EXAMPLE 25.2 Motion of a Proton in a Uniform Electric Field

A proton is released from rest in a uniform electric field that has a magnitude of 8.0×10^4 V/m and is directed along the positive x axis (Fig. 25.5). The proton undergoes a displacement of 0.50 m in the direction of **E**. (a) Find the change in electric potential between points A and B.

Solution Because the proton (which, as you remember, carries a positive charge) moves in the direction of the field, we expect it to move to a position of lower electric potential.

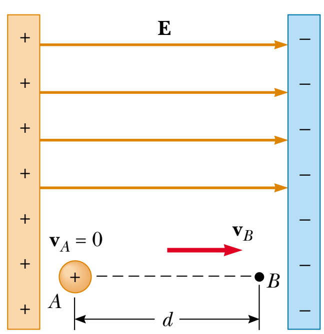

Figure 25.5 A proton accelerates from A to B in the direction of the electric field.

From Equation 25.6, we have

$$\Delta V = -Ed = -(8.0 \times 10^4\text{ V/m})(0.50\text{ m})$$
$$= \boxed{-4.0 \times 10^4\text{ V}}$$

(b) Find the change in potential energy of the proton for this displacement.

Solution

$$\Delta U = q_0\,\Delta V = e\,\Delta V$$
$$= (1.6 \times 10^{-19}\text{ C})(-4.0 \times 10^4\text{ V})$$
$$= \boxed{-6.4 \times 10^{-15}\text{ J}}$$

The negative sign means the potential energy of the proton decreases as it moves in the direction of the electric field. As the proton accelerates in the direction of the field, it gains kinetic energy and at the same time loses electric potential energy (because energy is conserved).

Exercise Use the concept of conservation of energy to find the speed of the proton at point B.

Answer 2.77×10^6 m/s.

[1] The electric field vanishes within a conductor in electrostatic equilibrium; thus, the path integral $\int \mathbf{E} \cdot d\mathbf{s}$ between any two points in the conductor must be zero. A more complete discussion of this point is given in Section 25.6.