

PUZZLER

When this honeybee gets back to its hive, it will tell the other bees how to return to the food it has found. By moving in a special, very precisely defined pattern, the bee conveys to other workers the information they need to find a flower bed. Bees communicate by “speaking in vectors.” What does the bee have to tell the other bees in order to specify where the flower bed is located relative to the hive? (*E. Webber/Visuals Unlimited*)



chapter

3

Vectors

Chapter Outline


- 3.1 Coordinate Systems
- 3.2 Vector and Scalar Quantities
- 3.3 Some Properties of Vectors
- 3.4 Components of a Vector and Unit Vectors

We often need to work with physical quantities that have both numerical and directional properties. As noted in Section 2.1, quantities of this nature are represented by vectors. This chapter is primarily concerned with vector algebra and with some general properties of vector quantities. We discuss the addition and subtraction of vector quantities, together with some common applications to physical situations.

Vector quantities are used throughout this text, and it is therefore imperative that you master both their graphical and their algebraic properties.

3.1 COORDINATE SYSTEMS

Many aspects of physics deal in some form or other with locations in space. In Chapter 2, for example, we saw that the mathematical description of an object's motion requires a method for describing the object's position at various times. This description is accomplished with the use of coordinates, and in Chapter 2 we used the cartesian coordinate system, in which horizontal and vertical axes intersect at a point taken to be the origin (Fig. 3.1). Cartesian coordinates are also called *rectangular coordinates*.

 Sometimes it is more convenient to represent a point in a plane by its *plane polar coordinates* (r, θ) , as shown in Figure 3.2a. In this *polar coordinate system*, r is the distance from the origin to the point having cartesian coordinates (x, y) , and θ is the angle between r and a fixed axis. This fixed axis is usually the positive x axis, and θ is usually measured counterclockwise from it. From the right triangle in Figure 3.2b, we find that $\sin \theta = y/r$ and that $\cos \theta = x/r$. (A review of trigonometric functions is given in Appendix B.4.) Therefore, starting with the plane polar coordinates of any point, we can obtain the cartesian coordinates, using the equations

$$x = r \cos \theta \quad (3.1)$$

$$y = r \sin \theta \quad (3.2)$$

Furthermore, the definitions of trigonometry tell us that

$$\tan \theta = \frac{y}{x} \quad (3.3)$$

$$r = \sqrt{x^2 + y^2} \quad (3.4)$$

These four expressions relating the coordinates (x, y) to the coordinates (r, θ) apply only when θ is defined, as shown in Figure 3.2a—in other words, when positive θ is an angle measured *counterclockwise* from the positive x axis. (Some scientific calculators perform conversions between cartesian and polar coordinates based on these standard conventions.) If the reference axis for the polar angle θ is chosen to be one other than the positive x axis or if the sense of increasing θ is chosen differently, then the expressions relating the two sets of coordinates will change.

Quick Quiz 3.1

Would the honeybee at the beginning of the chapter use cartesian or polar coordinates when specifying the location of the flower? Why? What is the honeybee using as an origin of coordinates?

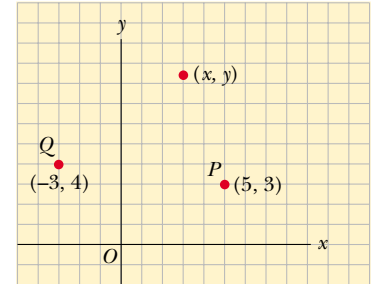
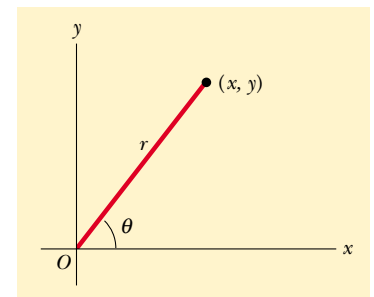
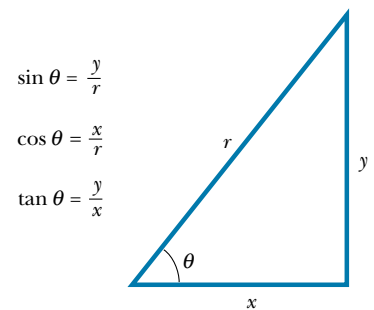


Figure 3.1 Designation of points in a cartesian coordinate system. Every point is labeled with coordinates (x, y) .



(a)



(b)

Figure 3.2 (a) The plane polar coordinates of a point are represented by the distance r and the angle θ , where θ is measured counterclockwise from the positive x axis. (b) The right triangle used to relate (x, y) to (r, θ) .

You may want to read *Talking Apes and Dancing Bees* (1997) by Betsy Wyckoff.

EXAMPLE 3.1 Polar Coordinates

The cartesian coordinates of a point in the xy plane are $(x, y) = (-3.50, -2.50)$ m, as shown in Figure 3.3. Find the polar coordinates of this point.

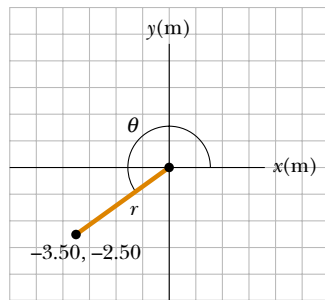


Figure 3.3 Finding polar coordinates when cartesian coordinates are given.

Solution

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50 \text{ m})^2 + (-2.50 \text{ m})^2} = 4.30 \text{ m}$$

$$\tan \theta = \frac{y}{x} = \frac{-2.50 \text{ m}}{-3.50 \text{ m}} = 0.714$$

$$\theta = 216^\circ$$

Note that you must use the signs of x and y to find that the point lies in the third quadrant of the coordinate system. That is, $\theta = 216^\circ$ and not 35.5° .

3.2 VECTOR AND SCALAR QUANTITIES

2.3 As noted in Chapter 2, some physical quantities are scalar quantities whereas others are vector quantities. When you want to know the temperature outside so that you will know how to dress, the only information you need is a number and the unit “degrees C” or “degrees F.” Temperature is therefore an example of a **scalar quantity**, which is defined as a quantity that is completely specified by a number and appropriate units. That is,

A scalar quantity is specified by a single value with an appropriate unit and has no direction.

Other examples of scalar quantities are volume, mass, and time intervals. The rules of ordinary arithmetic are used to manipulate scalar quantities.

If you are getting ready to pilot a small plane and need to know the wind velocity, you must know both the speed of the wind and its direction. Because direction is part of the information it gives, velocity is a **vector quantity**, which is defined as a physical quantity that is completely specified by a number and appropriate units plus a direction. That is,

A vector quantity has both magnitude and direction.

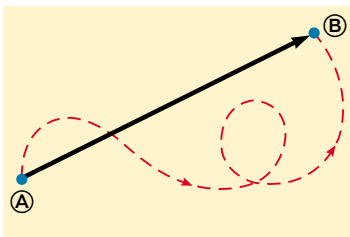


Figure 3.4 As a particle moves from \textcircled{A} to \textcircled{B} along an arbitrary path represented by the broken line, its displacement is a vector quantity shown by the arrow drawn from \textcircled{A} to \textcircled{B} .

Another example of a vector quantity is displacement, as you know from Chapter 2. Suppose a particle moves from some point \textcircled{A} to some point \textcircled{B} along a straight path, as shown in Figure 3.4. We represent this displacement by drawing an arrow from \textcircled{A} to \textcircled{B} , with the tip of the arrow pointing away from the starting point. The direction of the arrowhead represents the direction of the displacement, and the length of the arrow represents the magnitude of the displacement. If the particle travels along some other path from \textcircled{A} to \textcircled{B} , such as the broken line in Figure 3.4, its displacement is still the arrow drawn from \textcircled{A} to \textcircled{B} .



(a)



(b)



(c)

(a) The number of apples in the basket is one example of a scalar quantity. Can you think of other examples? (*Superstock*) (b) Jennifer pointing to the right. A vector quantity is one that must be specified by both magnitude and direction. (*Photo by Ray Serway*) (c) An anemometer is a device meteorologists use in weather forecasting. The cups spin around and reveal the magnitude of the wind velocity. The pointer indicates the direction. (*Courtesy of Peet Bros. Company, 1308 Doris Avenue, Ocean, NJ 07712*)

In this text, we use a boldface letter, such as \mathbf{A} , to represent a vector quantity. Another common method for vector notation that you should be aware of is the use of an arrow over a letter, such as \vec{A} . The magnitude of the vector \mathbf{A} is written either A or $|\mathbf{A}|$. The magnitude of a vector has physical units, such as meters for displacement or meters per second for velocity.

3.3 SOME PROPERTIES OF VECTORS

Equality of Two Vectors

For many purposes, two vectors \mathbf{A} and \mathbf{B} may be defined to be equal if they have the same magnitude and point in the same direction. That is, $\mathbf{A} = \mathbf{B}$ only if $A = B$ and if \mathbf{A} and \mathbf{B} point in the same direction along parallel lines. For example, all the vectors in Figure 3.5 are equal even though they have different starting points. This property allows us to move a vector to a position parallel to itself in a diagram without affecting the vector.

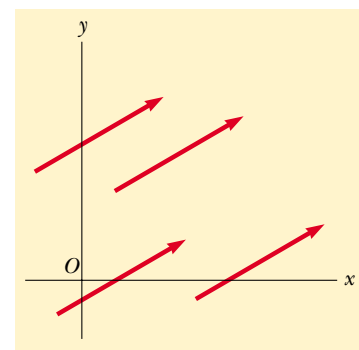


Figure 3.5 These four vectors are equal because they have equal lengths and point in the same direction.

Adding Vectors

2.4 The rules for adding vectors are conveniently described by geometric methods. To add vector \mathbf{B} to vector \mathbf{A} , first draw vector \mathbf{A} , with its magnitude represented by a convenient scale, on graph paper and then draw vector \mathbf{B} to the same scale with its tail starting from the tip of \mathbf{A} , as shown in Figure 3.6. The **resultant vector** $\mathbf{R} = \mathbf{A} + \mathbf{B}$ is the vector drawn from the tail of \mathbf{A} to the tip of \mathbf{B} . This procedure is known as the **triangle method of addition**.

For example, if you walked 3.0 m toward the east and then 4.0 m toward the north, as shown in Figure 3.7, you would find yourself 5.0 m from where you

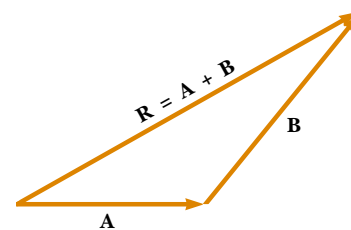


Figure 3.6 When vector \mathbf{B} is added to vector \mathbf{A} , the resultant \mathbf{R} is the vector that runs from the tail of \mathbf{A} to the tip of \mathbf{B} .

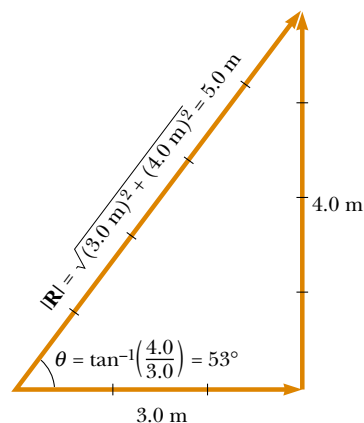


Figure 3.7 Vector addition. Walking first 3.0 m due east and then 4.0 m due north leaves you $|\mathbf{R}| = 5.0$ m from your starting point.

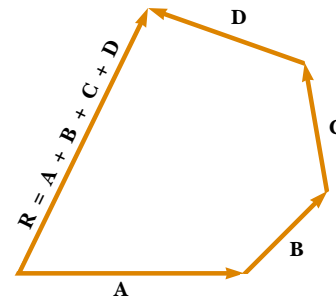


Figure 3.8 Geometric construction for summing four vectors. The resultant vector \mathbf{R} is by definition the one that completes the polygon.

started, measured at an angle of 53° north of east. Your total displacement is the vector sum of the individual displacements.

A geometric construction can also be used to add more than two vectors. This is shown in Figure 3.8 for the case of four vectors. The resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D}$ is the vector that completes the polygon. In other words, **\mathbf{R} is the vector drawn from the tail of the first vector to the tip of the last vector.**

An alternative graphical procedure for adding two vectors, known as the **parallelogram rule of addition**, is shown in Figure 3.9a. In this construction, the tails of the two vectors \mathbf{A} and \mathbf{B} are joined together and the resultant vector \mathbf{R} is the diagonal of a parallelogram formed with \mathbf{A} and \mathbf{B} as two of its four sides.

When two vectors are added, the sum is independent of the order of the addition. (This fact may seem trivial, but as you will see in Chapter 11, the order is important when vectors are multiplied). This can be seen from the geometric construction in Figure 3.9b and is known as the **commutative law of addition**:

Commutative law

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} \quad (3.5)$$

When three or more vectors are added, their sum is independent of the way in which the individual vectors are grouped together. A geometric proof of this rule

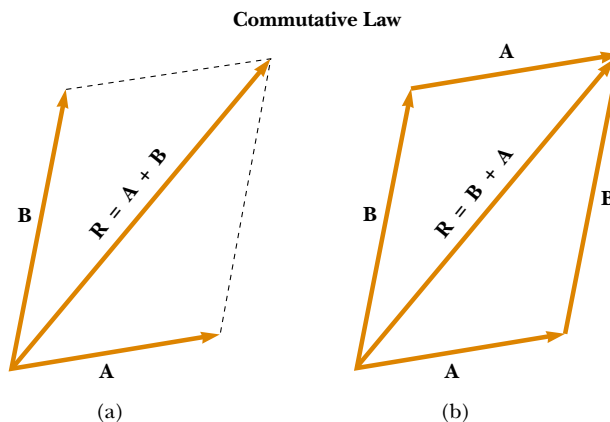


Figure 3.9 (a) In this construction, the resultant \mathbf{R} is the diagonal of a parallelogram having sides \mathbf{A} and \mathbf{B} . (b) This construction shows that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$ —in other words, that vector addition is commutative.

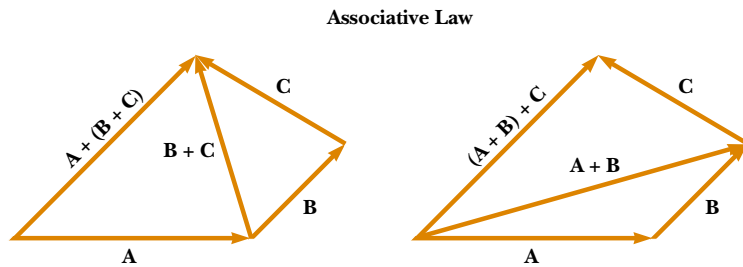


Figure 3.10 Geometric constructions for verifying the associative law of addition.

for three vectors is given in Figure 3.10. This is called the **associative law of addition**:

$$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C} \quad (3.6)$$

In summary, a **vector quantity has both magnitude and direction and also obeys the laws of vector addition** as described in Figures 3.6 to 3.10. When two or more vectors are added together, *all* of them *must* have the same units. It would be meaningless to add a velocity vector (for example, 60 km/h to the east) to a displacement vector (for example, 200 km to the north) because they represent different physical quantities. The same rule also applies to scalars. For example, it would be meaningless to add time intervals to temperatures.

Negative of a Vector

The negative of the vector \mathbf{A} is defined as the vector that when added to \mathbf{A} gives zero for the vector sum. That is, $\mathbf{A} + (-\mathbf{A}) = 0$. The vectors \mathbf{A} and $-\mathbf{A}$ have the same magnitude but point in opposite directions.

Subtracting Vectors

The operation of vector subtraction makes use of the definition of the negative of a vector. We define the operation $\mathbf{A} - \mathbf{B}$ as vector $-\mathbf{B}$ added to vector \mathbf{A} :

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) \quad (3.7)$$

The geometric construction for subtracting two vectors in this way is illustrated in Figure 3.11a.

Another way of looking at vector subtraction is to note that the difference $\mathbf{A} - \mathbf{B}$ between two vectors \mathbf{A} and \mathbf{B} is what you have to add to the second vector to obtain the first. In this case, the vector $\mathbf{A} - \mathbf{B}$ points from the tip of the second vector to the tip of the first, as Figure 3.11b shows.

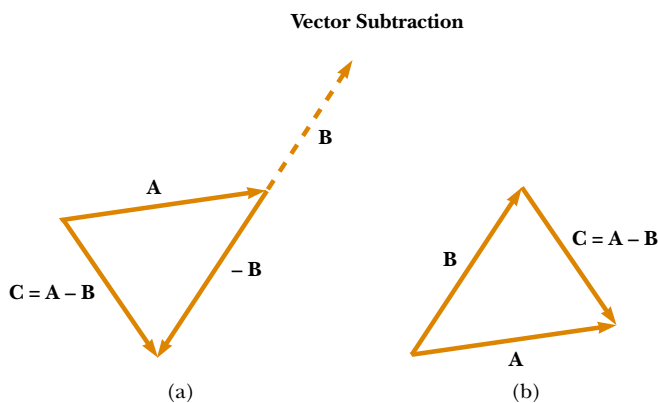


Figure 3.11 (a) This construction shows how to subtract vector \mathbf{B} from vector \mathbf{A} . The vector $-\mathbf{B}$ is equal in magnitude to vector \mathbf{B} and points in the opposite direction. To subtract \mathbf{B} from \mathbf{A} , apply the rule of vector addition to the combination of \mathbf{A} and $-\mathbf{B}$: Draw \mathbf{A} along some convenient axis, place the tail of $-\mathbf{B}$ at the tip of \mathbf{A} , and \mathbf{C} is the difference $\mathbf{A} - \mathbf{B}$. (b) A second way of looking at vector subtraction. The difference vector $\mathbf{C} = \mathbf{A} - \mathbf{B}$ is the vector that we must add to \mathbf{B} to obtain \mathbf{A} .

EXAMPLE 3.2 A Vacation Trip

A car travels 20.0 km due north and then 35.0 km in a direction 60.0° west of north, as shown in Figure 3.12. Find the magnitude and direction of the car's resultant displacement.

Solution In this example, we show two ways to find the resultant of two vectors. We can solve the problem geometrically, using graph paper and a protractor, as shown in Figure 3.12. (In fact, even when you know you are going to be carry-

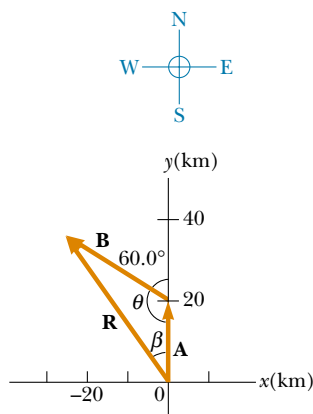


Figure 3.12 Graphical method for finding the resultant displacement vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$.

ing out a calculation, you should sketch the vectors to check your results.) The displacement \mathbf{R} is the resultant when the two individual displacements \mathbf{A} and \mathbf{B} are added.

To solve the problem algebraically, we note that the magnitude of \mathbf{R} can be obtained from the law of cosines as applied to the triangle (see Appendix B.4). With $\theta = 180^\circ - 60^\circ = 120^\circ$ and $R^2 = A^2 + B^2 - 2AB \cos \theta$, we find that

$$\begin{aligned} R &= \sqrt{A^2 + B^2 - 2AB \cos \theta} \\ &= \sqrt{(20.0 \text{ km})^2 + (35.0 \text{ km})^2 - 2(20.0 \text{ km})(35.0 \text{ km}) \cos 120^\circ} \\ &= 48.2 \text{ km} \end{aligned}$$

The direction of \mathbf{R} measured from the northerly direction can be obtained from the law of sines (Appendix B.4):

$$\begin{aligned} \frac{\sin \beta}{B} &= \frac{\sin \theta}{R} \\ \sin \beta &= \frac{B}{R} \sin \theta = \frac{35.0 \text{ km}}{48.2 \text{ km}} \sin 120^\circ = 0.629 \\ \beta &= 38.9^\circ \end{aligned}$$

The resultant displacement of the car is 48.2 km in a direction 38.9° west of north. This result matches what we found graphically.

Multiplying a Vector by a Scalar

If vector \mathbf{A} is multiplied by a positive scalar quantity m , then the product $m\mathbf{A}$ is a vector that has the same direction as \mathbf{A} and magnitude mA . If vector \mathbf{A} is multiplied by a negative scalar quantity $-m$, then the product $-m\mathbf{A}$ is directed opposite \mathbf{A} . For example, the vector $5\mathbf{A}$ is five times as long as \mathbf{A} and points in the same direction as \mathbf{A} ; the vector $-\frac{1}{3}\mathbf{A}$ is one-third the length of \mathbf{A} and points in the direction opposite \mathbf{A} .

Quick Quiz 3.2

If vector \mathbf{B} is added to vector \mathbf{A} , under what condition does the resultant vector $\mathbf{A} + \mathbf{B}$ have magnitude $A + B$? Under what conditions is the resultant vector equal to zero?

3.4 COMPONENTS OF A VECTOR AND UNIT VECTORS

The geometric method of adding vectors is not recommended whenever great accuracy is required or in three-dimensional problems. In this section, we describe a method of adding vectors that makes use of the *projections* of vectors along coordinate axes. These projections are called the **components** of the vector. Any vector can be completely described by its components.

Consider a vector \mathbf{A} lying in the xy plane and making an arbitrary angle θ with the positive x axis, as shown in Figure 3.13. This vector can be expressed as the

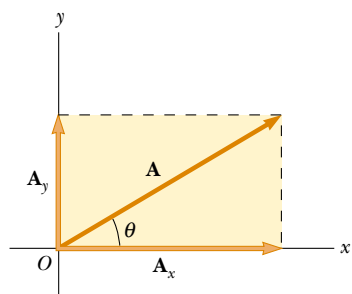


Figure 3.13 Any vector \mathbf{A} lying in the xy plane can be represented by a vector \mathbf{A}_x lying along the x axis and by a vector \mathbf{A}_y lying along the y axis, where $\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y$.

sum of two other vectors \mathbf{A}_x and \mathbf{A}_y . From Figure 3.13, we see that the three vectors form a right triangle and that $\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y$. (If you cannot see why this equality holds, go back to Figure 3.9 and review the parallelogram rule.) We shall often refer to the “components of a vector \mathbf{A} ,” written A_x and A_y (without the boldface notation). The component A_x represents the projection of \mathbf{A} along the x axis, and the component A_y represents the projection of \mathbf{A} along the y axis. These components can be positive or negative. The component A_x is positive if \mathbf{A}_x points in the positive x direction and is negative if \mathbf{A}_x points in the negative x direction. The same is true for the component A_y .

From Figure 3.13 and the definition of sine and cosine, we see that $\cos \theta = A_x/A$ and that $\sin \theta = A_y/A$. Hence, the components of \mathbf{A} are

$$A_x = A \cos \theta \quad (3.8)$$

$$A_y = A \sin \theta \quad (3.9)$$

These components form two sides of a right triangle with a hypotenuse of length A . Thus, it follows that the magnitude and direction of \mathbf{A} are related to its components through the expressions

$$A = \sqrt{A_x^2 + A_y^2} \quad (3.10)$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \quad (3.11)$$

Note that **the signs of the components A_x and A_y depend on the angle θ** . For example, if $\theta = 120^\circ$, then A_x is negative and A_y is positive. If $\theta = 225^\circ$, then both A_x and A_y are negative. Figure 3.14 summarizes the signs of the components when \mathbf{A} lies in the various quadrants.

When solving problems, you can specify a vector \mathbf{A} either with its components A_x and A_y or with its magnitude and direction A and θ .

Quick Quiz 3.3

Can the component of a vector ever be greater than the magnitude of the vector?

Suppose you are working a physics problem that requires resolving a vector into its components. In many applications it is convenient to express the components in a coordinate system having axes that are not horizontal and vertical but are still perpendicular to each other. If you choose reference axes or an angle other than the axes and angle shown in Figure 3.13, the components must be modified accordingly. Suppose a vector \mathbf{B} makes an angle θ' with the x' axis defined in Figure 3.15. The components of \mathbf{B} along the x' and y' axes are $B_{x'} = B \cos \theta'$ and $B_{y'} = B \sin \theta'$, as specified by Equations 3.8 and 3.9. The magnitude and direction of \mathbf{B} are obtained from expressions equivalent to Equations 3.10 and 3.11. Thus, we can express the components of a vector in *any* coordinate system that is convenient for a particular situation.

Unit Vectors

Vector quantities often are expressed in terms of unit vectors. **A unit vector is a dimensionless vector having a magnitude of exactly 1.** Unit vectors are used to specify a given direction and have no other physical significance. They are used solely as a convenience in describing a direction in space. We shall use the symbols

Components of the vector \mathbf{A}

Magnitude of \mathbf{A}

Direction of \mathbf{A}

A_x negative	A_x positive
A_y positive	A_y positive
A_x negative	A_x positive
A_y negative	A_y negative

Figure 3.14 The signs of the components of a vector \mathbf{A} depend on the quadrant in which the vector is located.

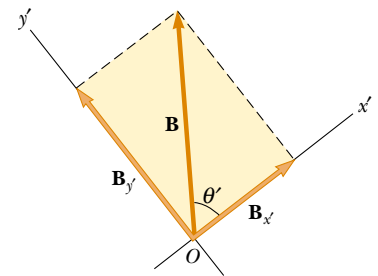


Figure 3.15 The component vectors of \mathbf{B} in a coordinate system that is tilted.

\mathbf{i} , \mathbf{j} , and \mathbf{k} to represent unit vectors pointing in the positive x , y , and z directions, respectively. The unit vectors \mathbf{i} , \mathbf{j} , and \mathbf{k} form a set of mutually perpendicular vectors in a right-handed coordinate system, as shown in Figure 3.16a. The magnitude of each unit vector equals 1; that is, $|\mathbf{i}| = |\mathbf{j}| = |\mathbf{k}| = 1$.

Consider a vector \mathbf{A} lying in the xy plane, as shown in Figure 3.16b. The product of the component A_x and the unit vector \mathbf{i} is the vector $A_x\mathbf{i}$, which lies on the x axis and has magnitude $|A_x|$. (The vector $A_x\mathbf{i}$ is an alternative representation of vector \mathbf{A}_x .) Likewise, $A_y\mathbf{j}$ is a vector of magnitude $|A_y|$ lying on the y axis. (Again, vector $A_y\mathbf{j}$ is an alternative representation of vector \mathbf{A}_y .) Thus, the unit-vector notation for the vector \mathbf{A} is

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} \quad (3.12)$$

For example, consider a point lying in the xy plane and having cartesian coordinates (x, y) , as in Figure 3.17. The point can be specified by the **position vector** \mathbf{r} , which in unit-vector form is given by

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} \quad (3.13)$$

This notation tells us that the components of \mathbf{r} are the lengths x and y .

Now let us see how to use components to add vectors when the geometric method is not sufficiently accurate. Suppose we wish to add vector \mathbf{B} to vector \mathbf{A} , where vector \mathbf{B} has components B_x and B_y . All we do is add the x and y components separately. The resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$ is therefore

$$\mathbf{R} = (A_x\mathbf{i} + A_y\mathbf{j}) + (B_x\mathbf{i} + B_y\mathbf{j})$$

or

$$\mathbf{R} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} \quad (3.14)$$

Because $\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$, we see that the components of the resultant vector are

$$\begin{aligned} R_x &= A_x + B_x \\ R_y &= A_y + B_y \end{aligned} \quad (3.15)$$

Position vector

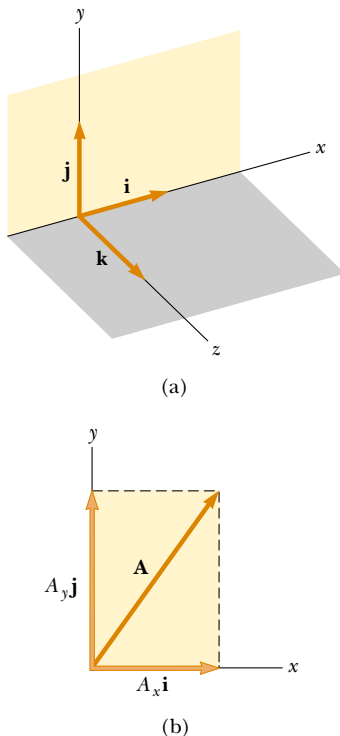


Figure 3.16 (a) The unit vectors \mathbf{i} , \mathbf{j} , and \mathbf{k} are directed along the x , y , and z axes, respectively. (b) Vector $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j}$ lying in the xy plane has components A_x and A_y .

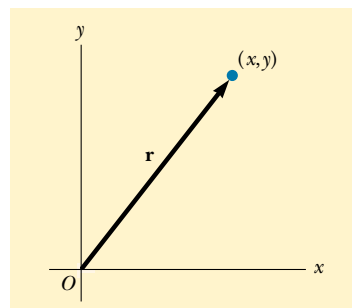


Figure 3.17 The point whose cartesian coordinates are (x, y) can be represented by the position vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$.

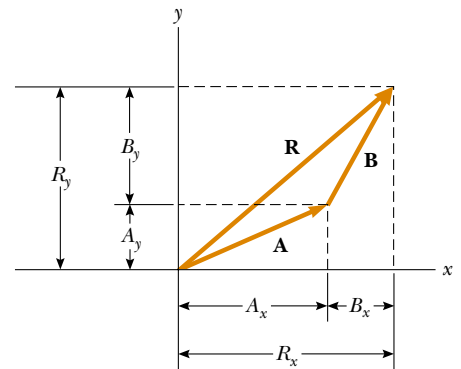


Figure 3.18 This geometric construction for the sum of two vectors shows the relationship between the components of the resultant \mathbf{R} and the components of the individual vectors.

We obtain the magnitude of \mathbf{R} and the angle it makes with the x axis from its components, using the relationships

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2} \quad (3.16)$$

$$\tan \theta = \frac{R_y}{R_x} = \frac{A_y + B_y}{A_x + B_x} \quad (3.17)$$

We can check this addition by components with a geometric construction, as shown in Figure 3.18. Remember that you must note the *signs* of the components when using either the algebraic or the geometric method.

At times, we need to consider situations involving motion in three component directions. The extension of our methods to three-dimensional vectors is straightforward. If \mathbf{A} and \mathbf{B} both have x , y , and z components, we express them in the form

$$\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k} \quad (3.18)$$

$$\mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k} \quad (3.19)$$

The sum of \mathbf{A} and \mathbf{B} is

$$\mathbf{R} = (A_x + B_x) \mathbf{i} + (A_y + B_y) \mathbf{j} + (A_z + B_z) \mathbf{k} \quad (3.20)$$

Note that Equation 3.20 differs from Equation 3.14: in Equation 3.20, the resultant vector also has a z component $R_z = A_z + B_z$.

QuickLab

Write an expression for the vector describing the displacement of a fly that moves from one corner of the floor of the room that you are in to the opposite corner of the room, near the ceiling.

Quick Quiz 3.4

If one component of a vector is not zero, can the magnitude of the vector be zero? Explain.

Quick Quiz 3.5

If $\mathbf{A} + \mathbf{B} = 0$, what can you say about the components of the two vectors?

Problem-Solving Hints

Adding Vectors

When you need to add two or more vectors, use this step-by-step procedure:

- Select a coordinate system that is convenient. (Try to reduce the number of components you need to find by choosing axes that line up with as many vectors as possible.)
- Draw a labeled sketch of the vectors described in the problem.
- Find the x and y components of all vectors and the resultant components (the algebraic sum of the components) in the x and y directions.
- If necessary, use the Pythagorean theorem to find the magnitude of the resultant vector and select a suitable trigonometric function to find the angle that the resultant vector makes with the x axis.