Matter is normally classified as being in one of three states: solid, liquid, or gas. From everyday experience, we know that a solid has a definite volume and shape. A brick maintains its familiar shape and size day in and day out. We also know that a liquid has a definite volume but no definite shape. Finally, we know that an unconfined gas has neither a definite volume nor a definite shape. These definitions help us picture the states of matter, but they are somewhat artificial. For example, asphalt and plastics are normally considered solids, but over long periods of time they tend to flow like liquids. Likewise, most substances can be a solid, a liquid, or a gas (or a combination of any of these), depending on the temperature and pressure. In general, the time it takes a particular substance to change its shape in response to an external force determines whether we treat the substance as a solid, as a liquid, or as a gas.

A **fluid** is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by the walls of a container. Both liquids and gases are fluids.

In our treatment of the mechanics of fluids, we shall see that we do not need to learn any new physical principles to explain such effects as the buoyant force acting on a submerged object and the dynamic lift acting on an airplane wing. First, we consider the mechanics of a fluid at rest—that is, *fluid statics*—and derive an expression for the pressure exerted by a fluid as a function of its density and depth. We then treat the mechanics of fluids in motion—that is, *fluid dynamics.* We can describe a fluid in motion by using a model in which we make certain simplifying assumptions. We use this model to analyze some situations of practical importance. An analysis leading to *Bernoulli's equation* enables us to determine relationships between the pressure, density, and velocity at every point in a fluid.

15.1 PRESSURE

Fluids do not sustain shearing stresses or tensile stresses; thus, the only stress that can be exerted on an object submerged in a fluid is one that tends to compress the object. In other words, the force exerted by a fluid on an object is always perpendicular to the surfaces of the object, as shown in Figure 15.1.

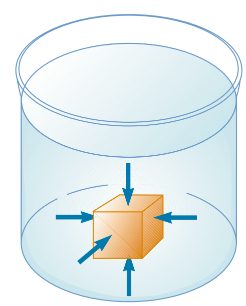


Figure 15.1 At any point on the surface of a submerged object, the force exerted by the fluid is perpendicular to the surface of the object. The force exerted by the fluid on the walls of the container is perpendicular to the walls at all points.

The pressure in a fluid can be measured with the device pictured in Figure 15.2. The device consists of an evacuated cylinder that encloses a light piston connected to a spring. As the device is submerged in a fluid, the fluid presses on the top of the piston and compresses the spring until the inward force exerted by the fluid is balanced by the outward force exerted by the spring. The fluid pressure can be measured directly if the spring is calibrated in advance. If F is the magnitude of the force exerted on the piston and A is the surface area of the piston,

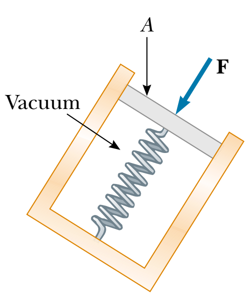


Figure 15.2 A simple device for measuring the pressure exerted by a fluid.

then the **pressure** P of the fluid at the level to which the device has been submerged is defined as the ratio F/A:

Definition of pressure

$$P \equiv \frac{F}{A} \tag{15.1}$$

Note that pressure is a scalar quantity because it is proportional to the magnitude of the force on the piston.

To define the pressure at a specific point, we consider a fluid acting on the device shown in Figure 15.2. If the force exerted by the fluid over an infinitesimal surface element of area dA containing the point in question is dF, then the pressure at that point is

$$P = \frac{dF}{dA} \tag{15.2}$$

As we shall see in the next section, the pressure exerted by a fluid varies with depth. Therefore, to calculate the total force exerted on a flat wall of a container, we must integrate Equation 15.2 over the surface area of the wall.

Because pressure is force per unit area, it has units of newtons per square meter (N/m^2) in the SI system. Another name for the SI unit of pressure is **pascal** (Pa):

$$1 \text{ Pa} \equiv 1 \text{ N/m}^2 \tag{15.3}$$

Snowshoes keep you from sinking into soft snow because they spread the downward force you exert on the snow over a large area, reducing the pressure on the snow's surface.

Quick Quiz 15.1

Suppose you are standing directly behind someone who steps back and accidentally stomps on your foot with the heel of one shoe. Would you be better off if that person were a professional basketball player wearing sneakers or a petite woman wearing spike-heeled shoes? Explain.

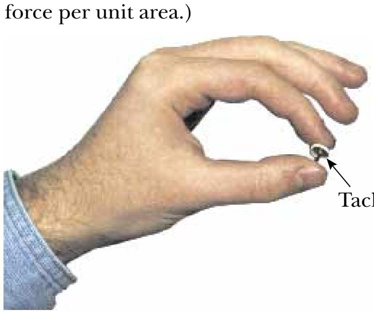
Quick Quiz 15.2

After a long lecture, the daring physics professor stretches out for a nap on a bed of nails, as shown in Figure 15.3. How is this possible?

Figure 15.3

QuickLab

Place a tack between your thumb and index finger, as shown in the figure. Now very gently squeeze the tack and note the sensation. The pointed end of the tack causes pain, and the blunt end does not. According to Newton's third law, the force exerted by the tack on the thumb is equal in magnitude and opposite in direction to the force exerted by the tack on the index finger. However, the pressure at the pointed end of the tack is much greater than the pressure at the blunt end. (Remember that pressure is force per unit area.)



EXAMPLE 15.1 The Water Bed

The mattress of a water bed is 2.00 m long by 2.00 m wide and 30.0 cm deep. (a) Find the weight of the water in the mattress.

Solution The density of water is 1 000 kg/m^3 (Table 15.1), and so the mass of the water is

$$M = \rho V = (1\,000 \text{ kg/m}^3)(1.20 \text{ m}^3) = 1.20 \times 10^3 \text{ kg}$$

and its weight is

$$Mg = (1.20 \times 10^3 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{1.18 \times 10^4 \text{ N}}$$

This is approximately 2 650 lb. (A regular bed weighs approximately 300 lb.) Because this load is so great, such a water bed is best placed in the basement or on a sturdy, well-supported floor.

(b) Find the pressure exerted by the water on the floor when the bed rests in its normal position. Assume that the entire lower surface of the bed makes contact with the floor.

Solution When the bed is in its normal position, the cross-sectional area is 4.00 m^2; thus, from Equation 15.1, we find that

$$P = \frac{1.18 \times 10^4 \text{ N}}{4.00 \text{ m}^2} = \boxed{2.95 \times 10^3 \text{ Pa}}$$

TABLE 15.1 Densities of Some Common Substances at Standard Temperature (0°C) and Pressure (Atmospheric)

Substance	ρ (kg/m^3)	Substance	ρ (kg/m^3)
Air	1.29	Ice	0.917×10^3
Aluminum	2.70×10^3	Iron	7.86×10^3
Benzene	0.879×10^3	Lead	11.3×10^3
Copper	8.92×10^3	Mercury	13.6×10^3
Ethyl alcohol	0.806×10^3	Oak	0.710×10^3
Fresh water	1.00×10^3	Oxygen gas	1.43
Glycerine	1.26×10^3	Pine	0.373×10^3
Gold	19.3×10^3	Platinum	21.4×10^3
Helium gas	1.79×10^{-1}	Seawater	1.03×10^3
Hydrogen gas	8.99×10^{-2}	Silver	10.5×10^3

15.2 VARIATION OF PRESSURE WITH DEPTH

As divers well know, water pressure increases with depth. Likewise, atmospheric pressure decreases with increasing altitude; it is for this reason that aircraft flying at high altitudes must have pressurized cabins.

We now show how the pressure in a liquid increases linearly with depth. As Equation 1.1 describes, the *density* of a substance is defined as its mass per unit volume: $\rho \equiv m/V$. Table 15.1 lists the densities of various substances. These values vary slightly with temperature because the volume of a substance is temperature dependent (as we shall see in Chapter 19). Note that under standard conditions (at 0°C and at atmospheric pressure) the densities of gases are about 1/1 000 the densities of solids and liquids. This difference implies that the average molecular spacing in a gas under these conditions is about ten times greater than that in a solid or liquid.

Now let us consider a fluid of density ρ at rest and open to the atmosphere, as shown in Figure 15.4. We assume that ρ is constant; this means that the fluid is incompressible. Let us select a sample of the liquid contained within an imaginary cylinder of cross-sectional area A extending from the surface to a depth h. The

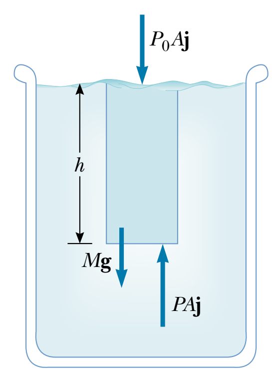


Figure 15.4 How pressure varies with depth in a fluid. The net force exerted on the volume of water within the darker region must be zero.

QuickLab

Poke two holes in the side of a paper or polystyrene cup—one near the top and the other near the bottom. Fill the cup with water and watch the water flow out of the holes. Why does water exit from the bottom hole at a higher speed than it does from the top hole?

pressure exerted by the outside liquid on the bottom face of the cylinder is P, and the pressure exerted on the top face of the cylinder is the atmospheric pressure P_0. Therefore, the upward force exerted by the outside fluid on the bottom of the cylinder is PA, and the downward force exerted by the atmosphere on the top is P_0A. The mass of liquid in the cylinder is $M = \rho V = \rho Ah$; therefore, the weight of the liquid in the cylinder is $Mg = \rho Ahg$. Because the cylinder is in equilibrium, the net force acting on it must be zero. Choosing upward to be the positive y direction, we see that

$$\sum F_y = PA - P_0A - Mg = 0$$

or

$$PA - P_0A - \rho Ahg = 0$$

$$PA - P_0A = \rho Ahg$$

Variation of pressure with depth

$$P = P_0 + \rho gh \qquad \textbf{(15.4)}$$

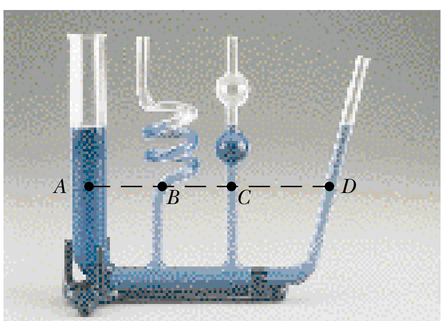
That is, **the pressure P at a depth h below the surface of a liquid open to the atmosphere is *greater* than atmospheric pressure by an amount ρgh.** In our calculations and working of end-of-chapter problems, we usually take atmospheric pressure to be

$$P_0 = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

Equation 15.4 implies that the pressure is the same at all points having the same depth, independent of the shape of the container.

Quick Quiz 15.3

In the derivation of Equation 15.4, why were we able to ignore the pressure that the liquid exerts on the sides of the cylinder?



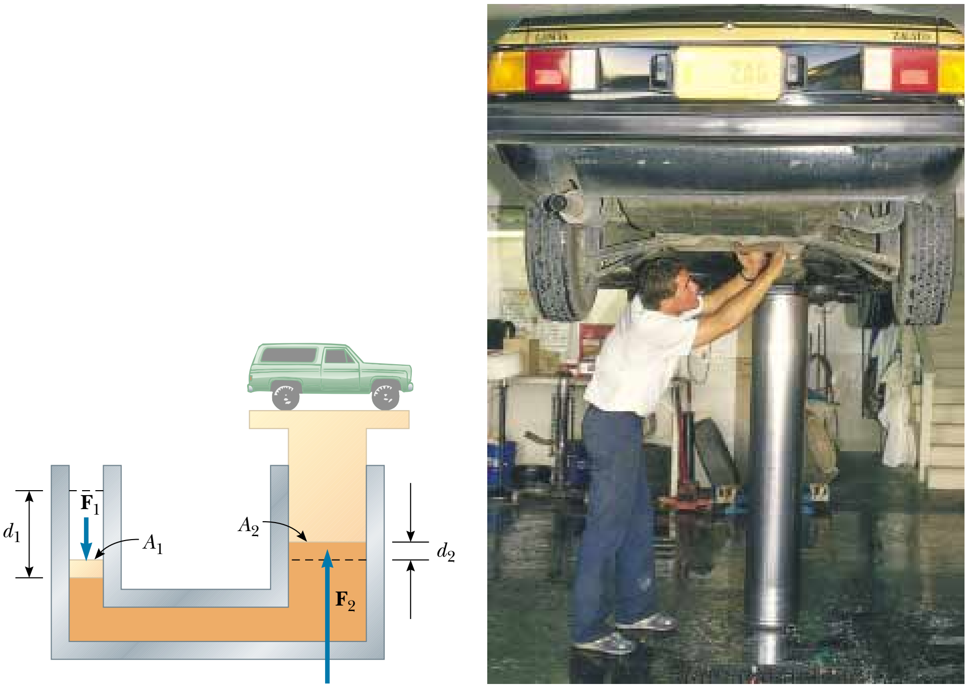
This arrangement of interconnected tubes demonstrates that the pressure in a liquid is the same at all points having the same elevation. For example, the pressure is the same at points A, B, C, and D.

In view of the fact that the pressure in a fluid depends on depth and on the value of P_0, any increase in pressure at the surface must be transmitted to every other point in the fluid. This concept was first recognized by the French scientist Blaise Pascal (1623–1662) and is called **Pascal's law: A change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.**

An important application of Pascal's law is the hydraulic press illustrated in Figure 15.5a. A force of magnitude F_1 is applied to a small piston of surface area A_1. The pressure is transmitted through a liquid to a larger piston of surface area A_2. Because the pressure must be the same on both sides, $P = F_1/A_1 = F_2/A_2$. Therefore, the force F_2 is greater than the force F_1 by a factor A_2/A_1, which is called the *force-multiplying factor.* Because liquid is neither added nor removed, the volume pushed down on the left as the piston moves down a distance d_1 equals the volume pushed up on the right as the right piston moves up a distance d_2. That is, $A_1d_1 = A_2d_2$; thus, the force-multiplying factor can also be written as d_1/d_2. Note that $F_1d_1 = F_2d_2$. Hydraulic brakes, car lifts, hydraulic jacks, and forklifts all make use of this principle (Fig. 15.5b).

Quick Quiz 15.4

A grain silo has many bands wrapped around its perimeter (Fig. 15.6). Why is the spacing between successive bands smaller at the lower portions of the silo, as shown in the photograph?



(a) (b)

Figure 15.5 (a) Diagram of a hydraulic press. Because the increase in pressure is the same on the two sides, a small force $\mathbf{F}_1$ at the left produces a much greater force $\mathbf{F}_2$ at the right. (b) A vehicle undergoing repair is supported by a hydraulic lift in a garage.

Figure 15.6

EXAMPLE 15.2 The Car Lift

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section and a radius of 5.00 cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0 cm. What force must the compressed air exert to lift a car weighing 13 300 N? What air pressure produces this force?

Solution Because the pressure exerted by the compressed air is transmitted undiminished throughout the liquid, we have

$$F_1 = \left(\frac{A_1}{A_2}\right) F_2 = \frac{\pi(5.00 \times 10^{-2}\ \text{m})^2}{\pi(15.0 \times 10^{-2}\ \text{m})^2}(1.33 \times 10^4\ \text{N})$$
$$= 1.48 \times 10^3\ \text{N}$$

The air pressure that produces this force is

$$P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3\ \text{N}}{\pi(5.00 \times 10^{-2}\ \text{m})^2} = \boxed{1.88 \times 10^5\ \text{Pa}}$$

This pressure is approximately twice atmospheric pressure.

The input work (the work done by $\mathbf{F}_1$) is equal to the output work (the work done by $\mathbf{F}_2$), in accordance with the principle of conservation of energy.

EXAMPLE 15.3 A Pain in the Ear

Estimate the force exerted on your eardrum due to the water above when you are swimming at the bottom of a pool that is 5.0 m deep.

Solution First, we must find the unbalanced pressure on the eardrum; then, after estimating the eardrum's surface area, we can determine the force that the water exerts on it.

The air inside the middle ear is normally at atmospheric pressure P_0. Therefore, to find the net force on the eardrum, we must consider the difference between the total pressure at the bottom of the pool and atmospheric pressure:

$$P_{bot} - P_0 = \rho g h$$
$$= (1.00 \times 10^3\ kg/m^3)(9.80\ m/s^2)(5.0\ m)$$
$$= 4.9 \times 10^4\ Pa$$

We estimate the surface area of the eardrum to be approximately 1 $cm^2 = 1 \times 10^{-4}\ m^2$. This means that the force on it is $F = (P_{bot} - P_0)A \approx 5$ N. Because a force on the eardrum of this magnitude is extremely uncomfortable, swimmers often "pop their ears" while under water, an action that pushes air from the lungs into the middle ear. Using this technique equalizes the pressure on the two sides of the eardrum and relieves the discomfort.

EXAMPLE 15.4 The Force on a Dam

Water is filled to a height H behind a dam of width w (Fig. 15.7). Determine the resultant force exerted by the water on the dam.

Solution Because pressure varies with depth, we cannot calculate the force simply by multiplying the area by the pressure. We can solve the problem by finding the force dF exerted on a narrow horizontal strip at depth h and then integrating the expression to find the total force. Let us imagine a vertical y axis, with $y = 0$ at the bottom of the dam and our strip a distance y above the bottom.

We can use Equation 15.4 to calculate the pressure at the depth h; we omit atmospheric pressure because it acts on both sides of the dam:

$$P = \rho g h = \rho g(H - y)$$

Using Equation 15.2, we find that the force exerted on the shaded strip of area $dA = w\,dy$ is

$$dF = P\,dA = \rho g(H - y)w\,dy$$

Therefore, the total force on the dam is

$$F = \int P\,dA = \int_0^H \rho g(H - y)w\,dy = \boxed{\tfrac{1}{2}\rho g w H^2}$$

Note that the thickness of the dam shown in Figure 15.7 increases with depth. This design accounts for the greater and greater pressure that the water exerts on the dam at greater depths.

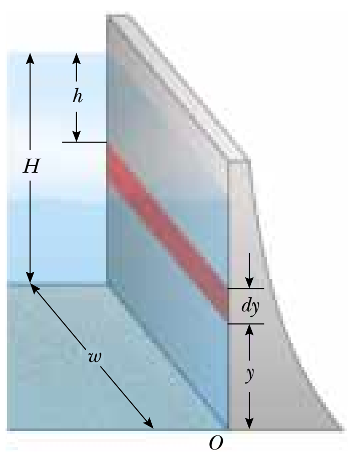


Figure 15.7 Because pressure varies with depth, the total force exerted on a dam must be obtained from the expression $F = \int P\,dA$, where dA is the area of the dark strip.

Exercise Find an expression for the average pressure on the dam from the total force exerted by the water on the dam.

Answer $\frac{1}{2}\rho g H$.

15.3 PRESSURE MEASUREMENTS

One simple device for measuring pressure is the open-tube manometer illustrated in Figure 15.8a. One end of a U-shaped tube containing a liquid is open to the atmosphere, and the other end is connected to a system of unknown pressure P. The difference in pressure $P - P_0$ is equal to $\rho g h$; hence, $P = P_0 + \rho g h$. The pressure P is called the **absolute pressure,** and the difference $P - P_0$ is called the **gauge pressure.** The latter is the value that normally appears on a pressure gauge. For example, the pressure you measure in your bicycle tire is the gauge pressure.

Another instrument used to measure pressure is the common *barometer,* which was invented by Evangelista Torricelli (1608–1647). The barometer consists of a

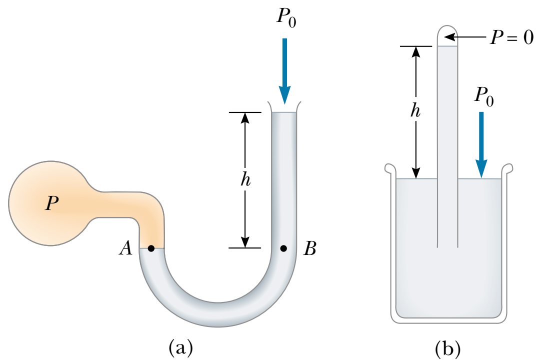


Figure 15.8 Two devices for measuring pressure: (a) an open-tube manometer and (b) a mercury barometer.

long, mercury-filled tube closed at one end and inverted into an open container of mercury (Fig. 15.8b). The closed end of the tube is nearly a vacuum, and so its pressure can be taken as zero. Therefore, it follows that $P_0 = \rho g h$, where h is the height of the mercury column.

One atmosphere ($P_0 = 1$ atm) of pressure is defined as the pressure that causes the column of mercury in a barometer tube to be exactly 0.760 0 m in height at 0°C, with $g = 9.806\ 65\ \text{m/s}^2$. At this temperature, mercury has a density of $13.595 \times 10^3\ \text{kg/m}^3$; therefore,

$$\begin{aligned} P_0 &= \rho g h = (13.595 \times 10^3\ \text{kg/m}^3)(9.806\ 65\ \text{m/s}^2)(0.760\ 0\ \text{m}) \\ &= 1.013 \times 10^5\ \text{Pa} = 1\ \text{atm} \end{aligned}$$

Quick Quiz 15.5

Other than the obvious problem that occurs with freezing, why don't we use water in a barometer in the place of mercury?

15.4 BUOYANT FORCES AND ARCHIMEDES'S PRINCIPLE

Have you ever tried to push a beach ball under water? This is extremely difficult to do because of the large upward force exerted by the water on the ball. The upward force exerted by water on any immersed object is called a **buoyant force.** We can determine the magnitude of a buoyant force by applying some logic and Newton's second law. Imagine that, instead of air, the beach ball is filled with water. If you were standing on land, it would be difficult to hold the water-filled ball in your arms. If you held the ball while standing neck deep in a pool, however, the force you would need to hold it would almost disappear. In fact, the required force would be zero if we were to ignore the thin layer of plastic of which the beach ball is made. Because the water-filled ball is in equilibrium while it is submerged, the magnitude of the upward buoyant force must equal its weight.

If the submerged ball were filled with air rather than water, then the upward buoyant force exerted by the surrounding water would still be present. However, because the weight of the water is now replaced by the much smaller weight of that volume of air, the net force is upward and quite great; as a result, the ball is pushed to the surface.

Archimedes's principle

The manner in which buoyant forces act is summarized by **Archimedes's principle,** which states that **the magnitude of the buoyant force always equals the weight of the fluid displaced by the object.** The buoyant force acts vertically upward through the point that was the center of gravity of the displaced fluid.

Note that Archimedes's principle does not refer to the makeup of the object experiencing the buoyant force. The object's composition is not a factor in the buoyant force. We can verify this in the following manner: Suppose we focus our attention on the indicated cube of liquid in the container illustrated in Figure 15.9. This cube is in equilibrium as it is acted on by two forces. One of these forces is the gravitational force $\mathbf{F}_g$. What cancels this downward force? Apparently, the rest of the liquid in the container is holding the cube in equilibrium. Thus, the magnitude of the buoyant force $\mathbf{B}$ exerted on the cube is exactly equal to the magnitude of $\mathbf{F}_g$, which is the weight of the liquid inside the cube:

$$B = F_g$$

Now imagine that the cube of liquid is replaced by a cube of steel of the same dimensions. What is the buoyant force acting on the steel? The liquid surrounding a cube behaves in the same way no matter what the cube is made of. Therefore, **the buoyant force acting on the steel cube is the same as the buoyant force acting on a cube of liquid of the same dimensions.** In other words, the magnitude of the buoyant force is the same as the weight of the *liquid* cube, not the steel cube. Although mathematically more complicated, this same principle applies to submerged objects of any shape, size, or density.

Although we have described the magnitude and direction of the buoyant force, we still do not know its origin. Why would a fluid exert such a strange force, almost as if the fluid were trying to expel a foreign body? To understand why, look again at Figure 15.9. The pressure at the bottom of the cube is greater than the pressure at the top by an amount ρgh, where h is the length of any side of the cube. The pressure difference ΔP between the bottom and top faces of the cube is equal to the buoyant force per unit area of those faces—that is, $\Delta P = B/A$. Therefore, $B = (\Delta P)A = (\rho gh)A = \rho gV$, where V is the volume of the cube. Because the mass of the fluid in the cube is $M = \rho V$, we see that

$$B = F_g = \rho Vg = Mg \qquad \textbf{(15.5)}$$

where Mg is the weight of the fluid in the cube. Thus, the buoyant force is a result of the pressure differential on a submerged or partly submerged object.

Before we proceed with a few examples, it is instructive for us to compare the forces acting on a totally submerged object with those acting on a floating (partly submerged) object.

Case 1: Totally Submerged Object When an object is totally submerged in a fluid of density ρ_f, the magnitude of the upward buoyant force is $B = \rho_f V_o g$, where V_o is the volume of the object. If the object has a mass M and density ρ_o, its weight is equal to $F_g = Mg = \rho_o V_o g$, and the net force on it is $B - F_g = (\rho_f - \rho_o) V_o g$. Hence, if the density of the object is less than the density of the fluid, then the downward force of gravity is less than the buoyant force, and the unconstrained object accelerates upward (Fig. 15.10a). If the density of the object is greater than the density of the fluid, then the upward buoyant force is less than the downward force of gravity, and the unsupported object sinks (Fig. 15.10b).

Case 2: Floating Object Now consider an object of volume V_o in static equilibrium floating on a fluid—that is, an object that is only partially submerged. In this

Archimedes (c. 287–212 B.C.) Archimedes, a Greek mathematician, physicist, and engineer, was perhaps the greatest scientist of antiquity. He was the first to compute accurately the ratio of a circle's circumference to its diameter, and he showed how to calculate the volume and surface area of spheres, cylinders, and other geometric shapes. He is well known for discovering the nature of the buoyant force.

Archimedes was also a gifted inventor. One of his practical inventions, still in use today, is Archimedes's screw—an inclined, rotating, coiled tube originally used to lift water from the holds of ships. He also invented the catapult and devised systems of levers, pulleys, and weights for raising heavy loads. Such inventions were successfully used to defend his native city Syracuse during a two-year siege by the Romans.

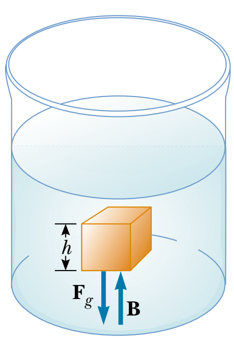


Figure 15.9 The external forces acting on the cube of liquid are the force of gravity $\mathbf{F}_g$ and the buoyant force $\mathbf{B}$. Under equilibrium conditions, $B = F_g$.

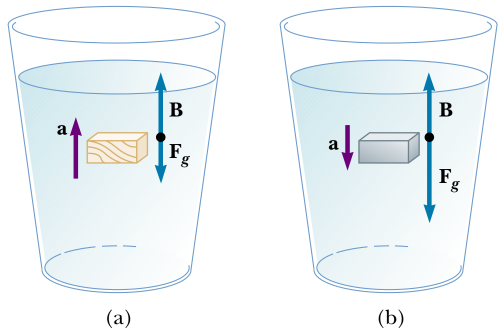


Figure 15.10 (a) A totally submerged object that is less dense than the fluid in which it is submerged experiences a net upward force. (b) A totally submerged object that is denser than the fluid sinks.

Hot-air balloons. Because hot air is less dense than cold air, a net upward force acts on the balloons.

case, the upward buoyant force is balanced by the downward gravitational force acting on the object. If V_f is the volume of the fluid displaced by the object (this volume is the same as the volume of that part of the object that is beneath the fluid level), the buoyant force has a magnitude $B = \rho_f V_f g$. Because the weight of the object is $F_g = Mg = \rho_o V_o g$, and because $F_g = B$, we see that $\rho_f V_f g = \rho_o V_o g$, or

$$\frac{\rho_o}{\rho_f} = \frac{V_f}{V_o} \qquad \textbf{(15.6)}$$

Under normal conditions, the average density of a fish is slightly greater than the density of water. It follows that the fish would sink if it did not have some mechanism for adjusting its density. The fish accomplishes this by internally regulating the size of its air-filled swim bladder to balance the change in the magnitude of the buoyant force acting on it. In this manner, fish are able to swim to various depths. Unlike a fish, a scuba diver cannot achieve neutral buoyancy (at which the buoyant force just balances the weight) by adjusting the magnitude of the buoyant force B. Instead, the diver adjusts F_g by manipulating lead weights.

Quick Quiz 15.6

Steel is much denser than water. In view of this fact, how do steel ships float?

Quick Quiz 15.7

A glass of water contains a single floating ice cube (Fig. 15.11). When the ice melts, does the water level go up, go down, or remain the same?

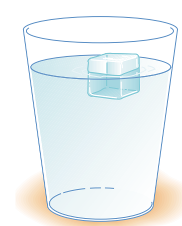


Figure 15.11

Quick Quiz 15.8

When a person in a rowboat in a small pond throws an anchor overboard, does the water level of the pond go up, go down, or remain the same?

Example 15.5 Eureka!

Archimedes supposedly was asked to determine whether a crown made for the king consisted of pure gold. Legend has it that he solved this problem by weighing the crown first in air and then in water, as shown in Figure 15.12. Suppose the scale read 7.84 N in air and 6.86 N in water. What should Archimedes have told the king?

Solution When the crown is suspended in air, the scale reads the true weight $T_1 = F_g$ (neglecting the buoyancy of air). When it is immersed in water, the buoyant force $\mathbf{B}$ reduces the scale reading to an apparent weight of $T_2 = F_g - B$. Hence, the buoyant force exerted on the crown is the difference between its weight in air and its weight in water:

$$B = F_g - T_2 = 7.84 \text{ N} - 6.86 \text{ N} = 0.98 \text{ N}$$

Because this buoyant force is equal in magnitude to the weight of the displaced water, we have $\rho_w g V_w = 0.98$ N, where V_w is the volume of the displaced water and ρ_w is its density. Also, the volume of the crown V_c is equal to the volume of the displaced water because the crown is completely submerged. Therefore,

$$V_c = V_w = \frac{0.98 \text{ N}}{g\rho_w} = \frac{0.98 \text{ N}}{(9.8 \text{ m/s}^2)(1\,000 \text{ kg/m}^3)}$$
$$= 1.0 \times 10^{-4} \text{ m}^3$$

Finally, the density of the crown is

$$\rho_c = \frac{m_c}{V_c} = \frac{m_c g}{V_c g} = \frac{7.84 \text{ N}}{(1.0 \times 10^{-4} \text{ m}^3)(9.8 \text{ m/s}^2)}$$
$$= 8.0 \times 10^3 \text{ kg/m}^3$$

From Table 15.1 we see that the density of gold is 19.3×10^3 kg/m^3. Thus, Archimedes should have told the king that he had been cheated. Either the crown was hollow, or it was not made of pure gold.

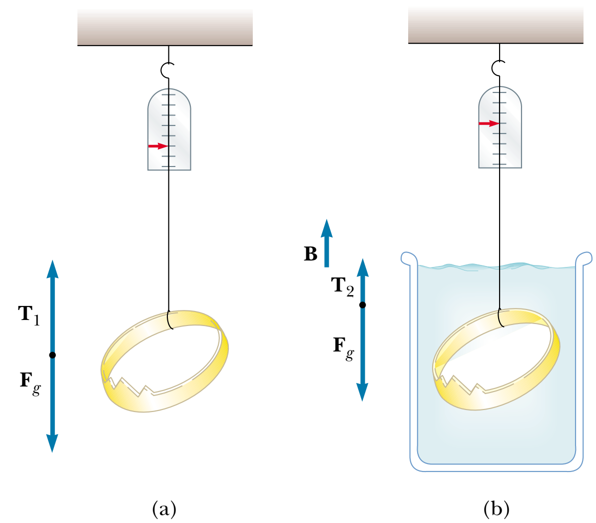


Figure 15.12 (a) When the crown is suspended in air, the scale reads its true weight $T_1 = F_g$ (the buoyancy of air is negligible). (b) When the crown is immersed in water, the buoyant force $\mathbf{B}$ reduces the scale reading to the apparent weight $T_2 = F_g - B$.

EXAMPLE 15.6 A Titanic Surprise

An iceberg floating in seawater, as shown in Figure 15.13a, is extremely dangerous because much of the ice is below the surface. This hidden ice can damage a ship that is still a considerable distance from the visible ice. What fraction of the iceberg lies below the water level?

Solution This problem corresponds to Case 2. The weight of the iceberg is $F_{gi} = \rho_i V_i g$, where $\rho_i = 917$ kg/m^3 and V_i is the volume of the whole iceberg. The magnitude of the upward buoyant force equals the weight of the displaced water: $B = \rho_w V_w g$, where V_w, the volume of the displaced water, is equal to the volume of the ice beneath the water (the shaded region in Fig. 15.13b) and ρ_w is the density of seawater, $\rho_w = 1\,030$ kg/m^3. Because $\rho_i V_i g = \rho_w V_w g$, the fraction of ice beneath the water's surface is

$$f = \frac{V_w}{V_i} = \frac{\rho_i}{\rho_w} = \frac{917 \text{ kg/m}^3}{1\,030 \text{ kg/m}^3} = 0.890 \quad \text{or} \quad \boxed{89.0\%}$$

(a)

(b)

Figure 15.13 (a) Much of the volume of this iceberg is beneath the water.
(b) A ship can be damaged even when it is not near the exposed ice.

15.5 FLUID DYNAMICS

Thus far, our study of fluids has been restricted to fluids at rest. We now turn our attention to fluids in motion. Instead of trying to study the motion of each particle of the fluid as a function of time, we describe the properties of a moving fluid at each point as a function of time.

Flow Characteristics

When fluid is in motion, its flow can be characterized as being one of two main types. The flow is said to be **steady,** or **laminar,** if each particle of the fluid follows a smooth path, such that the paths of different particles never cross each other, as shown in Figure 15.14. In steady flow, the velocity of the fluid at any point remains constant in time.

Above a certain critical speed, fluid flow becomes **turbulent;** turbulent flow is irregular flow characterized by small whirlpool-like regions, as shown in Figure 15.15.

The term **viscosity** is commonly used in the description of fluid flow to characterize the degree of internal friction in the fluid. This internal friction, or *viscous force,* is associated with the resistance that two adjacent layers of fluid have to moving relative to each other. Viscosity causes part of the kinetic energy of a fluid to be converted to internal energy. This mechanism is similar to the one by which an object sliding on a rough horizontal surface loses kinetic energy.

Because the motion of real fluids is very complex and not fully understood, we make some simplifying assumptions in our approach. In our model of an **ideal fluid,** we make the following four assumptions:

Properties of an ideal fluid

1. **The fluid is nonviscous.** In a nonviscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.
2. **The flow is steady.** In steady (laminar) flow, the velocity of the fluid at each point remains constant.

Figure 15.14 Laminar flow around an automobile in a test wind tunnel.

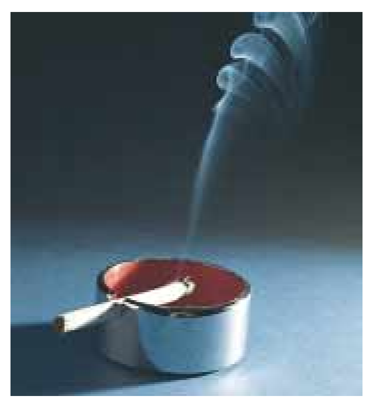


Figure 15.15 Hot gases from a cigarette made visible by smoke particles. The smoke first moves in laminar flow at the bottom and then in turbulent flow above.

3. **The fluid is incompressible.** The density of an incompressible fluid is constant.
4. **The flow is irrotational.** In irrotational flow, the fluid has no angular momentum about any point. If a small paddle wheel placed anywhere in the fluid does not rotate about the wheel's center of mass, then the flow is irrotational.

15.6 STREAMLINES AND THE EQUATION OF CONTINUITY

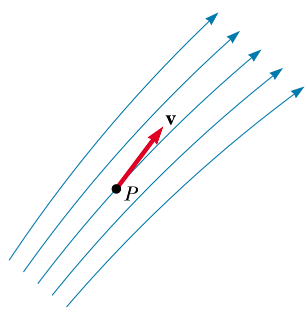


Figure 15.16 A particle in laminar flow follows a streamline, and at each point along its path the particle's velocity is tangent to the streamline.

The path taken by a fluid particle under steady flow is called a **streamline.** The velocity of the particle is always tangent to the streamline, as shown in Figure 15.16. A set of streamlines like the ones shown in Figure 15.16 form a *tube of flow.* Note that fluid particles cannot flow into or out of the sides of this tube; if they could, then the streamlines would cross each other.

Consider an ideal fluid flowing through a pipe of nonuniform size, as illustrated in Figure 15.17. The particles in the fluid move along streamlines in steady flow. In a time t, the fluid at the bottom end of the pipe moves a distance $\Delta x_1 = v_1 t$. If A_1 is the cross-sectional area in this region, then the mass of fluid contained in the left shaded region in Figure 15.17 is $m_1 = \rho A_1 \Delta x_1 = \rho A_1 v_1 t$, where ρ is the (nonchanging) density of the ideal fluid. Similarly, the fluid that moves through the upper end of the pipe in the time t has a mass $m_2 = \rho A_2 v_2 t$. However, because *mass is conserved* and because the flow is steady, the mass that crosses A_1 in a time t must equal the mass that crosses A_2 in the time t. That is, $m_1 = m_2$, or $\rho A_1 v_1 t = \rho A_2 v_2 t$; this means that

Equation of continuity

$$A_1 v_1 = A_2 v_2 = \text{constant} \tag{15.7}$$

This expression is called the **equation of continuity.** It states that

the product of the area and the fluid speed at all points along the pipe is a constant for an incompressible fluid.

This equation tells us that the speed is high where the tube is constricted (small A) and low where the tube is wide (large A). The product Av, which has the dimensions of volume per unit time, is called either the *volume flux* or the *flow rate.* The condition Av = constant is equivalent to the statement that the volume of fluid that enters one end of a tube in a given time interval equals the volume leaving the other end of the tube in the same time interval if no leaks are present.

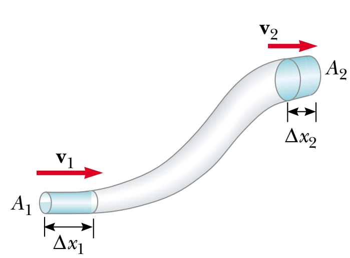


Figure 15.17 A fluid moving with steady flow through a pipe of varying cross-sectional area. The volume of fluid flowing through area A_1 in a time interval t must equal the volume flowing through area A_2 in the same time interval. Therefore, $A_1 v_1 = A_2 v_2$.

Figure 15.18

Quick Quiz 15.9

As water flows from a faucet, as shown in Figure 15.18, why does the stream of water become narrower as it descends?

EXAMPLE 15.7 Niagara Falls

Each second, 5 525 m^3 of water flows over the 670-m-wide cliff of the Horseshoe Falls portion of Niagara Falls. The water is approximately 2 m deep as it reaches the cliff. What is its speed at that instant?

Solution The cross-sectional area of the water as it reaches the edge of the cliff is $A = (670 \text{ m})(2 \text{ m}) = 1\,340 \text{ m}^2$. The flow rate of 5 525 m^3/s is equal to Av. This gives

$$v = \frac{5\,525 \text{ m}^3/\text{s}}{A} = \frac{5\,525 \text{ m}^3/\text{s}}{1\,340 \text{ m}^2} = \boxed{4 \text{ m/s}}$$

Note that we have kept only one significant figure because our value for the depth has only one significant figure.

Exercise A barrel floating along in the river plunges over the Falls. How far from the base of the cliff is the barrel when it reaches the water 49 m below?

Answer 13 m ≈ 10 m.

15.7 BERNOULLI'S EQUATION

When you press your thumb over the end of a garden hose so that the opening becomes a small slit, the water comes out at high speed, as shown in Figure 15.19. Is the water under greater pressure when it is inside the hose or when it is out in the air? You can answer this question by noting how hard you have to push your thumb against the water inside the end of the hose. The pressure inside the hose is definitely greater than atmospheric pressure.

The relationship between fluid speed, pressure, and elevation was first derived in 1738 by the Swiss physicist Daniel Bernoulli. Consider the flow of an ideal fluid through a nonuniform pipe in a time t, as illustrated in Figure 15.20. Let us call the lower shaded part section 1 and the upper shaded part section 2. The force exerted by the fluid in section 1 has a magnitude P_1A_1. The work done by this force in a time t is $W_1 = F_1\Delta x_1 = P_1A_1\Delta x_1 = P_1V$, where V is the volume of section 1. In a similar manner, the work done by the fluid in section 2 in the same time t is $W_2 = -P_2A_2\Delta x_2 = -P_2V$. (The volume that passes through section 1 in a time t equals the volume that passes through section 2 in the same time.) This work is negative because the fluid force opposes the displacement. Thus, the net work done by these forces in the time t is

$$W = (P_1 - P_2)V$$

Daniel Bernoulli (1700–1782) Daniel Bernoulli, a Swiss physicist and mathematician, made important discoveries in fluid dynamics. Born into a family of mathematicians, he was the only member of the family to make a mark in physics.

Bernoulli's most famous work, *Hydrodynamica,* was published in 1738; it is both a theoretical and a practical study of equilibrium, pressure, and speed in fluids. He showed that as the speed of a fluid increases, its pressure decreases.

In *Hydrodynamica* Bernoulli also attempted the first explanation of the behavior of gases with changing pressure and temperature; this was the beginning of the kinetic theory of gases, a topic we study in Chapter 21. *(Corbis–Bettmann)*

Figure 15.19 The speed of water spraying from the end of a hose increases as the size of the opening is decreased with the thumb.

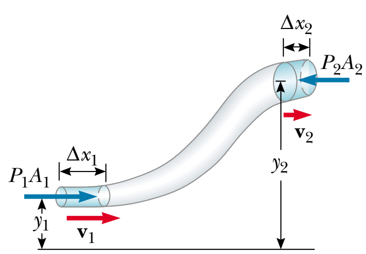


Figure 15.20 A fluid in laminar flow through a constricted pipe. The volume of the shaded section on the left is equal to the volume of the shaded section on the right.

QuickLab

Place two soda cans on their sides approximately 2 cm apart on a table. Align your mouth at table level and with the space between the cans. Blow a horizontal stream of air through this space. What do the cans do? Is this what you expected? Compare this with the force acting on a car parked close to the edge of a road when a big truck goes by. How does the outcome relate to Equation 15.9?

Part of this work goes into changing the kinetic energy of the fluid, and part goes into changing the gravitational potential energy. If m is the mass that enters one end and leaves the other in a time t, then the change in the kinetic energy of this mass is

$$\Delta K = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

The change in gravitational potential energy is

$$\Delta U = mgy_2 - mgy_1$$

We can apply Equation 8.13, $W = \Delta K + \Delta U$, to this volume of fluid to obtain

$$(P_1 - P_2)V = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + mgy_2 - mgy_1$$

If we divide each term by V and recall that $\rho = m/V$, this expression reduces to

$$P_1 - P_2 = \tfrac{1}{2}\rho v_2^2 - \tfrac{1}{2}\rho v_1^2 + \rho g y_2 - \rho g y_1$$

Rearranging terms, we obtain

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \qquad \textbf{(15.8)}$$

This is **Bernoulli's equation** as applied to an ideal fluid. It is often expressed as

Bernoulli's equation

$$\boxed{P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant}} \qquad \textbf{(15.9)}$$

This expression specifies that, in laminar flow, the sum of the pressure (P), kinetic energy per unit volume $(\frac{1}{2}\rho v^2)$, and gravitational potential energy per unit volume $(\rho g y)$ has the same value at all points along a streamline.

When the fluid is at rest, $v_1 = v_2 = 0$ and Equation 15.8 becomes

$$P_1 - P_2 = \rho g(y_2 - y_1) = \rho g h$$

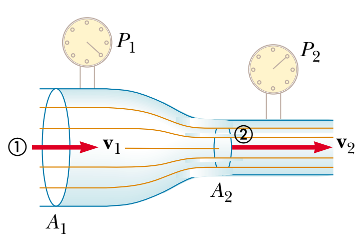
This is in agreement with Equation 15.4.

EXAMPLE 15.8 The Venturi Tube

The horizontal constricted pipe illustrated in Figure 15.21, known as a *Venturi tube,* can be used to measure the flow speed of an incompressible fluid. Let us determine the flow speed at point 2 if the pressure difference $P_1 - P_2$ is known.

Solution Because the pipe is horizontal, $y_1 = y_2$, and applying Equation 15.8 to points 1 and 2 gives

$$(1) \qquad P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$



(a)

(b)

Figure 15.21 (a) Pressure P_1 is greater than pressure P_2 because $v_1 < v_2$. This device can be used to measure the speed of fluid flow. (b) A Venturi tube.

From the equation of continuity, $A_1v_1 = A_2v_2$, we find that

$$(2) \qquad v_1 = \frac{A_2}{A_1}v_2$$

Substituting this expression into equation (1) gives

$$P_1 + \tfrac{1}{2}\rho\left(\frac{A_2}{A_1}\right)^2 v_2{}^2 = P_2 + \tfrac{1}{2}\rho v_2{}^2$$

$$\boxed{v_2 = A_1\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1{}^2 - A_2{}^2)}}}$$

We can use this result and the continuity equation to obtain an expression for v_1. Because $A_2 < A_1$, Equation (2) shows us that $v_2 > v_1$. This result, together with equation (1), indicates that $P_1 > P_2$. In other words, the pressure is reduced in the constricted part of the pipe. This result is somewhat analogous to the following situation: Consider a very crowded room in which people are squeezed together. As soon as a door is opened and people begin to exit, the squeezing (pressure) is least near the door, where the motion (flow) is greatest.

EXAMPLE 15.9 A Good Trick

It is possible to blow a dime off a table and into a tumbler. Place the dime about 2 cm from the edge of the table. Place the tumbler on the table horizontally with its open edge about 2 cm from the dime, as shown in Figure 15.22a. If you blow forcefully across the top of the dime, it will rise, be caught in the airstream, and end up in the tumbler. The mass of a dime is $m = 2.24$ g, and its surface area is $A = 2.50 \times 10^{-4}$ m^2. How hard are you blowing when the dime rises and travels into the tumbler?

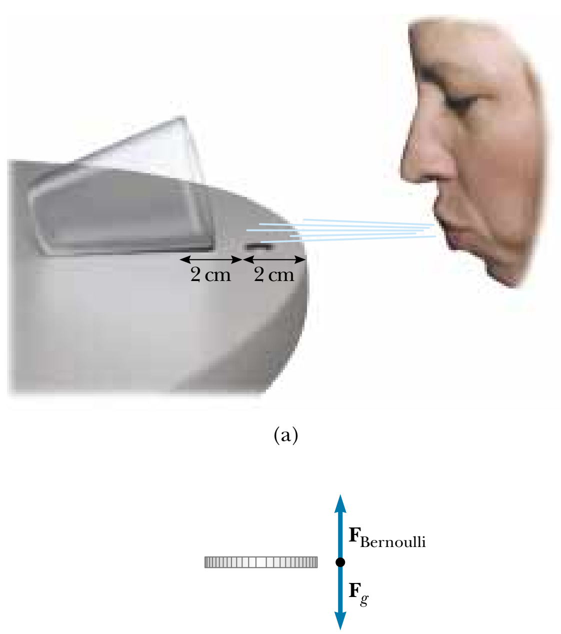


Figure 15.22

Solution Figure 15.22b indicates we must calculate the upward force acting on the dime. First, note that a thin stationary layer of air is present between the dime and the table. When you blow across the dime, it deflects most of the moving air from your breath across its top, so that the air above the dime has a greater speed than the air beneath it. This fact, together with Bernoulli's equation, demonstrates that the air moving across the top of the dime is at a lower pressure than the air beneath the dime. If we neglect the small thickness of the dime, we can apply Equation 15.8 to obtain

$$P_{\text{above}} + \tfrac{1}{2}\rho v_{\text{above}}^2 = P_{\text{beneath}} + \tfrac{1}{2}\rho v_{\text{beneath}}^2$$

Because the air beneath the dime is almost stationary, we can neglect the last term in this expression and write the difference as $P_{\text{beneath}} - P_{\text{above}} = \frac{1}{2}\rho v_{\text{above}}^2$. If we multiply this pressure difference by the surface area of the dime, we obtain the upward force acting on the dime. At the very least, this upward force must balance the gravitational force acting on the dime, and so, taking the density of air from Table 15.1, we can state that

$$F_g = mg = (P_{\text{beneath}} - P_{\text{above}})A = (\tfrac{1}{2}\rho v_{\text{above}}^2)A$$

$$v_{\text{above}} = \sqrt{\frac{2mg}{\rho A}} = \sqrt{\frac{2(2.24 \times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)}{(1.29\text{ kg/m}^3)(2.50 \times 10^{-4}\text{ m}^2)}}$$

$$v_{\text{above}} = 11.7\text{ m/s}$$

The air you blow must be moving faster than this if the upward force is to exceed the weight of the dime. Practice this trick a few times and then impress all your friends!