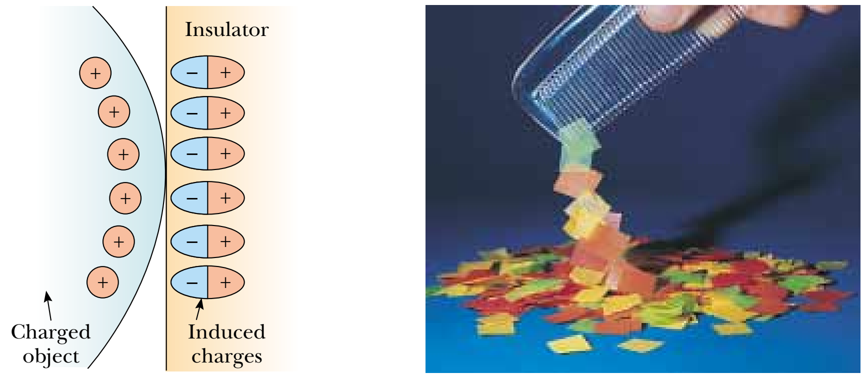


(a) (b)

Figure 23.4 (a) The charged object on the left induces charges on the surface of an insulator. (b) A charged comb attracts bits of paper because charges are displaced in the paper.

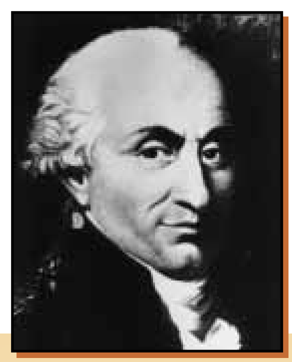
QuickLab

Tear some paper into very small pieces. Comb your hair and then bring the comb close to the paper pieces. Notice that they are accelerated toward the comb. How does the magnitude of the electric force compare with the magnitude of the gravitational force exerted on the paper? Keep watching and you might see a few pieces jump away from the comb. They don't just fall away; they are repelled. What causes this?

the negative charge in the rod that they move out of the sphere through the ground wire and into the Earth. If the wire to ground is then removed (Fig. 23.3d), the conducting sphere contains an excess of *induced* positive charge. When the rubber rod is removed from the vicinity of the sphere (Fig. 23.3e), this induced positive charge remains on the ungrounded sphere. Note that the charge remaining on the sphere is uniformly distributed over its surface because of the repulsive forces among the like charges. Also note that the rubber rod loses none of its negative charge during this process.

Charging an object by induction requires no contact with the body inducing the charge. This is in contrast to charging an object by rubbing (that is, by *conduction*), which does require contact between the two objects.

A process similar to induction in conductors takes place in insulators. In most neutral molecules, the center of positive charge coincides with the center of negative charge. However, in the presence of a charged object, these centers inside each molecule in an insulator may shift slightly, resulting in more positive charge on one side of the molecule than on the other. This realignment of charge within individual molecules produces an induced charge on the surface of the insulator, as shown in Figure 23.4. Knowing about induction in insulators, you should be able to explain why a comb that has been rubbed through hair attracts bits of electrically neutral paper and why a balloon that has been rubbed against your clothing is able to stick to an electrically neutral wall.



Charles Coulomb **(1736–1806)**
Coulomb's major contribution to science was in the field of electrostatics and magnetism. During his lifetime, he also investigated the strengths of materials and determined the forces that affect objects on beams, thereby contributing to the field of structural mechanics. In the field of ergonomics, his research provided a fundamental understanding of the ways in which people and animals can best do work.
(Photo courtesy of AIP Niels Bohr Library/E. Scott Barr Collection)

Quick Quiz 23.2

Object A is attracted to object B. If object B is known to be positively charged, what can we say about object A? (a) It is positively charged. (b) It is negatively charged. (c) It is electrically neutral. (d) Not enough information to answer.

23.3 COULOMB'S LAW

11.4 Charles Coulomb (1736–1806) measured the magnitudes of the electric forces between charged objects using the torsion balance, which he invented (Fig. 23.5).

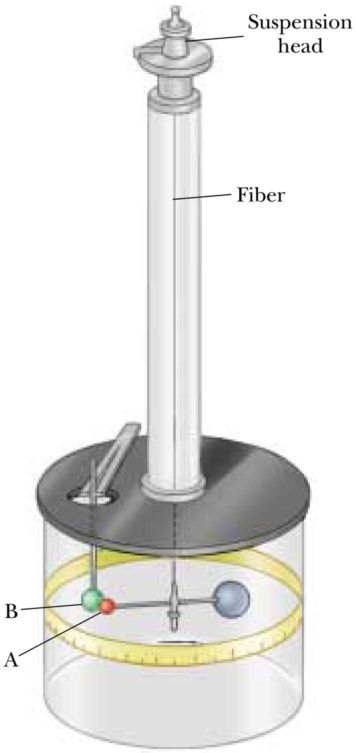


Figure 23.5 Coulomb's torsion balance, used to establish the inverse-square law for the electric force between two charges.

Coulomb confirmed that the electric force between two small charged spheres is proportional to the inverse square of their separation distance r—that is, $F_e \propto 1/r^2$. The operating principle of the torsion balance is the same as that of the apparatus used by Cavendish to measure the gravitational constant (see Section 14.2), with the electrically neutral spheres replaced by charged ones. The electric force between charged spheres A and B in Figure 23.5 causes the spheres to either attract or repel each other, and the resulting motion causes the suspended fiber to twist. Because the restoring torque of the twisted fiber is proportional to the angle through which the fiber rotates, a measurement of this angle provides a quantitative measure of the electric force of attraction or repulsion. Once the spheres are charged by rubbing, the electric force between them is very large compared with the gravitational attraction, and so the gravitational force can be neglected.

Coulomb's experiments showed that the **electric force** between two stationary charged particles

- is inversely proportional to the square of the separation r between the particles and directed along the line joining them;
- is proportional to the product of the charges q_1 and q_2 on the two particles;
- is attractive if the charges are of opposite sign and repulsive if the charges have the same sign.

From these observations, we can express **Coulomb's law** as an equation giving the magnitude of the electric force (sometimes called the *Coulomb force*) between two point charges:

$$F_e = k_e \frac{|q_1||q_2|}{r^2} \tag{23.1}$$

where k_e is a constant called the **Coulomb constant.** In his experiments, Coulomb was able to show that the value of the exponent of r was 2 to within an uncertainty of a few percent. Modern experiments have shown that the exponent is 2 to within an uncertainty of a few parts in 10^{16}.

The value of the Coulomb constant depends on the choice of units. The SI unit of charge is the **coulomb** (C). The Coulomb constant k_e in SI units has the value

Coulomb constant

$$k_e = 8.987\,5 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$$

This constant is also written in the form

$$k_e = \frac{1}{4\pi\epsilon_0}$$

where the constant ϵ_0 (lowercase Greek epsilon) is known as the *permittivity of free space* and has the value $8.854\,2 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2$.

The smallest unit of charge known in nature is the charge on an electron or proton,[1] which has an absolute value of

Charge on an electron or proton

$$|e| = 1.602\,19 \times 10^{-19}\text{ C}$$

Therefore, 1 C of charge is approximately equal to the charge of 6.24×10^{18} electrons or protons. This number is very small when compared with the number of

[1] No unit of charge smaller than e has been detected as a free charge; however, recent theories propose the existence of particles called *quarks* having charges $e/3$ and $2e/3$. Although there is considerable experimental evidence for such particles inside nuclear matter, *free* quarks have never been detected. We discuss other properties of quarks in Chapter 46 of the extended version of this text.

TABLE 23.1 Charge and Mass of the Electron, Proton, and Neutron

Particle	Charge (C)	Mass (kg)
Electron (e)	$-1.602\,191\,7 \times 10^{-19}$	$9.109\,5 \times 10^{-31}$
Proton (p)	$+1.602\,191\,7 \times 10^{-19}$	$1.672\,61 \times 10^{-27}$
Neutron (n)	0	$1.674\,92 \times 10^{-27}$

free electrons[2] in 1 cm^3 of copper, which is of the order of 10^{23}. Still, 1 C is a substantial amount of charge. In typical experiments in which a rubber or glass rod is charged by friction, a net charge of the order of 10^{-6} C is obtained. In other words, only a very small fraction of the total available charge is transferred between the rod and the rubbing material.

The charges and masses of the electron, proton, and neutron are given in Table 23.1.

EXAMPLE 23.1 The Hydrogen Atom

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately 5.3×10^{-11} m. Find the magnitudes of the electric force and the gravitational force between the two particles.

Solution From Coulomb's law, we find that the attractive electric force has the magnitude

$$F_e = k_e \frac{|e|^2}{r^2} = \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(1.60 \times 10^{-19}\ \text{C})^2}{(5.3 \times 10^{-11}\ \text{m})^2}$$

$$= \boxed{8.2 \times 10^{-8}\ \text{N}}$$

Using Newton's law of gravitation and Table 23.1 for the particle masses, we find that the gravitational force has the magnitude

$$F_g = G\frac{m_e m_p}{r^2}$$

$$= \left(6.7 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)$$

$$= \times \frac{(9.11 \times 10^{-31}\ \text{kg})(1.67 \times 10^{-27}\ \text{kg})}{(5.3 \times 10^{-11}\ \text{m})^2}$$

$$= \boxed{3.6 \times 10^{-47}\ \text{N}}$$

The ratio $F_e/F_g \approx 2 \times 10^{39}$. Thus, the gravitational force between charged atomic particles is negligible when compared with the electric force. Note the similarity of form of Newton's law of gravitation and Coulomb's law of electric forces. Other than magnitude, what is a fundamental difference between the two forces?

When dealing with Coulomb's law, you must remember that force is a vector quantity and must be treated accordingly. Thus, the law expressed in vector form for the electric force exerted by a charge q_1 on a second charge q_2, written $\mathbf{F}_{12}$, is

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2}\,\hat{\mathbf{r}} \qquad \textbf{(23.2)}$$

where $\hat{\mathbf{r}}$ is a unit vector directed from q_1 to q_2, as shown in Figure 23.6a. Because the electric force obeys Newton's third law, the electric force exerted by q_2 on q_1 is

[2] A metal atom, such as copper, contains one or more outer electrons, which are weakly bound to the nucleus. When many atoms combine to form a metal, the so-called *free electrons* are these outer electrons, which are not bound to any one atom. These electrons move about the metal in a manner similar to that of gas molecules moving in a container.

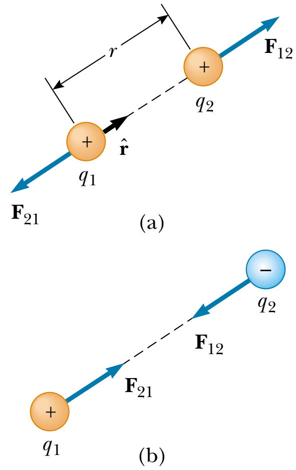


Figure 23.6 Two point charges separated by a distance r exert a force on each other that is given by Coulomb's law. The force $\mathbf{F}_{21}$ exerted by q_2 on q_1 is equal in magnitude and opposite in direction to the force $\mathbf{F}_{12}$ exerted by q_1 on q_2. (a) When the charges are of the same sign, the force is repulsive. (b) When the charges are of opposite signs, the force is attractive.

equal in magnitude to the force exerted by q_1 on q_2 and in the opposite direction; that is, $\mathbf{F}_{21} = -\mathbf{F}_{12}$. Finally, from Equation 23.2, we see that if q_1 and q_2 have the same sign, as in Figure 23.6a, the product q_1q_2 is positive and the force is repulsive. If q_1 and q_2 are of opposite sign, as shown in Figure 23.6b, the product q_1q_2 is negative and the force is attractive. Noting the sign of the product q_1q_2 is an easy way of determining the direction of forces acting on the charges.

Quick Quiz 23.3

Object A has a charge of $+2\ \mu$C, and object B has a charge of $+6\ \mu$C. Which statement is true?

(a) $\mathbf{F}_{AB} = -3\mathbf{F}_{BA}$. (b) $\mathbf{F}_{AB} = -\mathbf{F}_{BA}$. (c) $3\mathbf{F}_{AB} = -\mathbf{F}_{BA}$.

When more than two charges are present, the force between any pair of them is given by Equation 23.2. Therefore, the resultant force on any one of them equals the vector sum of the forces exerted by the various individual charges. For example, if four charges are present, then the resultant force exerted by particles 2, 3, and 4 on particle 1 is

$$\mathbf{F}_1 = \mathbf{F}_{21} + \mathbf{F}_{31} + \mathbf{F}_{41}$$

EXAMPLE 23.2 Find the Resultant Force

Consider three point charges located at the corners of a right triangle as shown in Figure 23.7, where $q_1 = q_3 = 5.0\ \mu$C, $q_2 = -2.0\ \mu$C, and $a = 0.10$ m. Find the resultant force exerted on q_3.

Solution First, note the direction of the individual forces exerted by q_1 and q_2 on q_3. The force F_{23} exerted by q_2 on q_3 is attractive because q_2 and q_3 have opposite signs. The force F_{13} exerted by q_1 on q_3 is repulsive because both charges are positive.

The magnitude of $\mathbf{F}_{23}$ is

$$\begin{aligned} F_{23} &= k_e \frac{|q_2||q_3|}{a^2} \\ &= \left(8.99 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(2.0 \times 10^{-6}\ \text{C})(5.0 \times 10^{-6}\ \text{C})}{(0.10\ \text{m})^2} \\ &= 9.0\ \text{N} \end{aligned}$$

Note that because q_3 and q_2 have opposite signs, $\mathbf{F}_{23}$ is to the left, as shown in Figure 23.7.

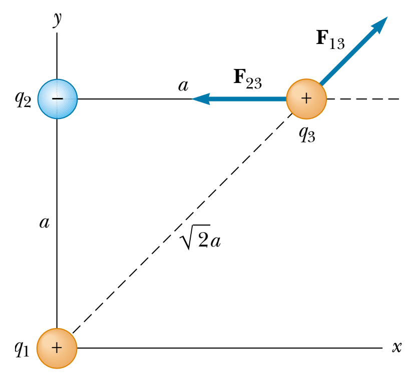


Figure 23.7 The force exerted by q_1 on q_3 is $\mathbf{F}_{13}$. The force exerted by q_2 on q_3 is $\mathbf{F}_{23}$. The resultant force $\mathbf{F}_3$ exerted on q_3 is the vector sum $\mathbf{F}_{13} + \mathbf{F}_{23}$.

The magnitude of the force exerted by q_1 on q_3 is

$$F_{13} = k_e \frac{|q_1||q_3|}{(\sqrt{2}a)^2}$$

$$= \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(5.0 \times 10^{-6}\ \text{C})(5.0 \times 10^{-6}\ \text{C})}{2(0.10\ \text{m})^2}$$

$$= 11\ \text{N}$$

The force $\mathbf{F}_{13}$ is repulsive and makes an angle of 45° with the x axis. Therefore, the x and y components of $\mathbf{F}_{13}$ are equal, with magnitude given by $F_{13} \cos 45° = 7.9$ N.

The force $\mathbf{F}_{23}$ is in the negative x direction. Hence, the x and y components of the resultant force acting on q_3 are

$$F_{3x} = F_{13x} + F_{23} = 7.9\ \text{N} - 9.0\ \text{N} = -1.1\ \text{N}$$

$$F_{3y} = F_{13y} = 7.9\ \text{N}$$

We can also express the resultant force acting on q_3 in unit-vector form as

$$\mathbf{F}_3 = \boxed{(-1.1\mathbf{i} + 7.9\mathbf{j})\ \text{N}}$$

Exercise Find the magnitude and direction of the resultant force $\mathbf{F}_3$.

Answer 8.0 N at an angle of 98° with the x axis.

EXAMPLE 23.3 Where Is the Resultant Force Zero?

Three point charges lie along the x axis as shown in Figure 23.8. The positive charge $q_1 = 15.0\ \mu$C is at $x = 2.00$ m, the positive charge $q_2 = 6.00\ \mu$C is at the origin, and the resultant force acting on q_3 is zero. What is the x coordinate of q_3?

Solution Because q_3 is negative and q_1 and q_2 are positive, the forces $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$ are both attractive, as indicated in Figure 23.8. From Coulomb's law, $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$ have magnitudes

$$F_{13} = k_e \frac{|q_1||q_3|}{(2.00 - x)^2} \qquad F_{23} = k_e \frac{|q_2||q_3|}{x^2}$$

For the resultant force on q_3 to be zero, $\mathbf{F}_{23}$ must be equal in magnitude and opposite in direction to $\mathbf{F}_{13}$, or

$$k_e \frac{|q_2||q_3|}{x^2} = k_e \frac{|q_1||q_3|}{(2.00 - x)^2}$$

Noting that k_e and q_3 are common to both sides and so can be dropped, we solve for x and find that

$$(2.00 - x)^2 |q_2| = x^2 |q_1|$$

$$(4.00 - 4.00x + x^2)(6.00 \times 10^{-6}\ \text{C}) = x^2(15.0 \times 10^{-6}\ \text{C})$$

Solving this quadratic equation for x, we find that $\boxed{x = 0.775\ \text{m.}}$ Why is the negative root not acceptable?

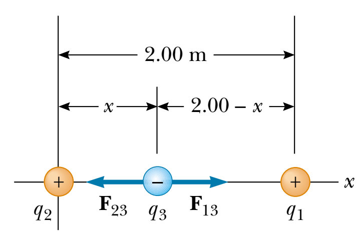


Figure 23.8 Three point charges are placed along the x axis. If the net force acting on q_3 is zero, then the force $\mathbf{F}_{13}$ exerted by q_1 on q_3 must be equal in magnitude and opposite in direction to the force $\mathbf{F}_{23}$ exerted by q_2 on q_3.

EXAMPLE 23.4 Find the Charge on the Spheres

Two identical small charged spheres, each having a mass of 3.0×10^{-2} kg, hang in equilibrium as shown in Figure 23.9a. The length of each string is 0.15 m, and the angle θ is 5.0°. Find the magnitude of the charge on each sphere.

Solution From the right triangle shown in Figure 23.9a, we see that $\sin \theta = a/L$. Therefore,

$$a = L \sin \theta = (0.15\ \text{m}) \sin 5.0° = 0.013\ \text{m}$$

The separation of the spheres is $2a = 0.026$ m.

The forces acting on the left sphere are shown in Figure 23.9b. Because the sphere is in equilibrium, the forces in the horizontal and vertical directions must separately add up to zero:

$$(1) \qquad \sum F_x = T\sin\theta - F_e = 0$$

$$(2) \qquad \sum F_y = T\cos\theta - mg = 0$$

From Equation (2), we see that $T = mg/\cos\theta$; thus, T can be eliminated from Equation (1) if we make this substitution. This gives a value for the magnitude of the electric force F_e:

$$(3) \qquad F_e = mg\tan\theta$$
$$= (3.0 \times 10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)\tan 5.0°$$
$$= 2.6 \times 10^{-2}\ \text{N}$$

From Coulomb's law (Eq. 23.1), the magnitude of the electric force is

$$F_e = k_e \frac{|q|^2}{r^2}$$

where $r = 2a = 0.026$ m and $|q|$ is the magnitude of the charge on each sphere. (Note that the term $|q|^2$ arises here because the charge is the same on both spheres.) This equation can be solved for $|q|^2$ to give

$$|q|^2 = \frac{F_e r^2}{k_e} = \frac{(2.6 \times 10^{-2}\ \text{N})(0.026\ \text{m})^2}{8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}$$

$$|q| = \boxed{4.4 \times 10^{-8}\ \text{C}}$$

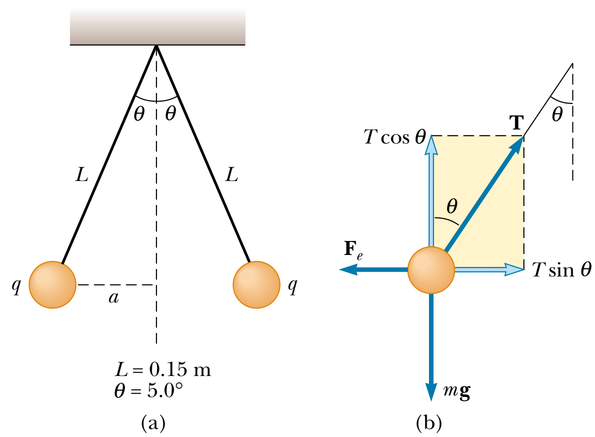


Figure 23.9 (a) Two identical spheres, each carrying the same charge q, suspended in equilibrium. (b) The free-body diagram for the sphere on the left.

Exercise If the charge on the spheres were negative, how many electrons would have to be added to them to yield a net charge of -4.4×10^{-8} C?

Answer 2.7×10^{11} electrons.

QuickLab

For this experiment you need two 20-cm strips of transparent tape (mass of each $\approx$ 65 mg). Fold about 1 cm of tape over at one end of each strip to create a handle. Press both pieces of tape side by side onto a table top, rubbing your finger back and forth across the strips. Quickly pull the strips off the surface so that they become charged. Hold the tape handles together and the strips will repel each other, forming an inverted "V" shape. Measure the angle between the pieces, and estimate the excess charge on each strip. Assume that the charges act as if they were located at the center of mass of each strip.

23.4 THE ELECTRIC FIELD

11.5 Two field forces have been introduced into our discussions so far—the gravitational force and the electric force. As pointed out earlier, field forces can act through space, producing an effect even when no physical contact between the objects occurs. The gravitational field $\mathbf{g}$ at a point in space was defined in Section 14.6 to be equal to the gravitational force $\mathbf{F}_g$ acting on a test particle of mass m divided by that mass: $\mathbf{g} \equiv \mathbf{F}_g/m$. A similar approach to electric forces was developed by Michael Faraday and is of such practical value that we shall devote much attention to it in the next several chapters. In this approach, an **electric field** is said to exist in the region of space around a charged object. When another charged object enters this electric field, an electric force acts on it. As an example, consider Figure 23.10, which shows a small positive test charge q_0 placed near a second object carrying a much greater positive charge Q. We define the strength (in other words, the magnitude) of the electric field at the location of the test charge to be the electric force *per unit charge,* or to be more specific

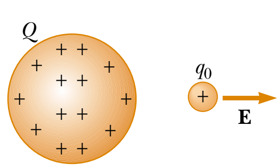


Figure 23.10 A small positive test charge q_0 placed near an object carrying a much larger positive charge Q experiences an electric field $\mathbf{E}$ directed as shown.

the electric field E at a point in space is defined as the electric force $\mathbf{F}_e$ acting on a positive test charge q_0 placed at that point divided by the magnitude of the test charge:

$$\mathbf{E} \equiv \frac{\mathbf{F}_e}{q_0} \qquad (23.3)$$

Definition of electric field

Note that **E** is the field produced by some charge *external* to the test charge—it is not the field produced by the test charge itself. Also, note that the existence of an electric field is a property of its source. For example, every electron comes with its own electric field.

The vector **E** has the SI units of newtons per coulomb (N/C), and, as Figure 23.10 shows, its direction is the direction of the force a positive test charge experiences when placed in the field. We say that **an electric field exists at a point if a test charge at rest at that point experiences an electric force.** Once the magnitude and direction of the electric field are known at some point, the electric force exerted on *any* charged particle placed at that point can be calculated from

This dramatic photograph captures a lightning bolt striking a tree near some rural homes.