Optional Section

26.7 AN ATOMIC DESCRIPTION OF DIELECTRICS

In Section 26.5 we found that the potential difference ΔV_0 between the plates of a capacitor is reduced to $\Delta V_0/\kappa$ when a dielectric is introduced. Because the potential difference between the plates equals the product of the electric field and the separation d, the electric field is also reduced. Thus, if $\mathbf{E}_0$ is the electric field without the dielectric, the field in the presence of a dielectric is

$$\mathbf{E} = \frac{\mathbf{E}_0}{\kappa} \tag{26.21}$$

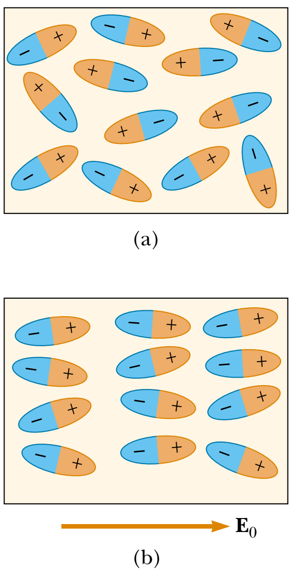

Figure 26.23 (a) Polar molecules are randomly oriented in the absence of an external electric field. (b) When an external field is applied, the molecules partially align with the field.

Let us first consider a dielectric made up of polar molecules placed in the electric field between the plates of a capacitor. The dipoles (that is, the polar molecules making up the dielectric) are randomly oriented in the absence of an electric field, as shown in Figure 26.23a. When an external field $\mathbf{E}_0$ due to charges on the capacitor plates is applied, a torque is exerted on the dipoles, causing them to partially align with the field, as shown in Figure 26.23b. We can now describe the dielectric as being polarized. The degree of alignment of the molecules with the electric field depends on temperature and on the magnitude of the field. In general, the alignment increases with decreasing temperature and with increasing electric field.

If the molecules of the dielectric are nonpolar, then the electric field due to the plates produces some charge separation and an *induced dipole moment.* These induced dipole moments tend to align with the external field, and the dielectric is polarized. Thus, we can polarize a dielectric with an external field regardless of whether the molecules are polar or nonpolar.

With these ideas in mind, consider a slab of dielectric material placed between the plates of a capacitor so that it is in a uniform electric field $\mathbf{E}_0$, as shown in Figure 26.24a. The electric field due to the plates is directed to the right and polarizes the dielectric. The net effect on the dielectric is the formation of an *induced* positive surface charge density σ_{ind} on the right face and an equal negative surface charge density $-\sigma_{\text{ind}}$ on the left face, as shown in Figure 26.24b. These induced surface charges on the dielectric give rise to an induced electric field $\mathbf{E}_{\text{ind}}$ in the direction opposite the external field $\mathbf{E}_0$. Therefore, the net electric field $\mathbf{E}$ in the

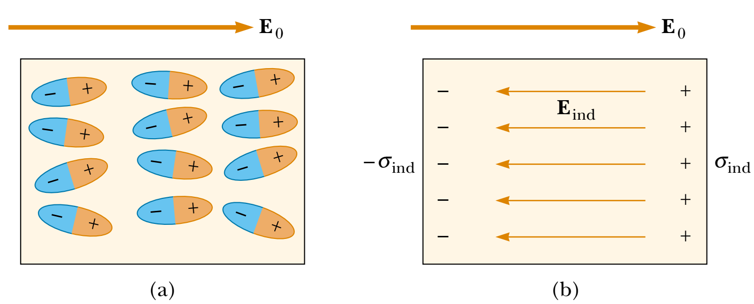

Figure 26.24 (a) When a dielectric is polarized, the dipole moments of the molecules in the dielectric are partially aligned with the external field $\mathbf{E}_0$. (b) This polarization causes an induced negative surface charge on one side of the dielectric and an equal induced positive surface charge on the opposite side. This separation of charge results in a reduction in the net electric field within the dielectric.

dielectric has a magnitude

$$E = E_0 - E_{\text{ind}} \tag{26.22}$$

In the parallel-plate capacitor shown in Figure 26.25, the external field E_0 is related to the charge density σ on the plates through the relationship $E_0 = \sigma/\epsilon_0$. The induced electric field in the dielectric is related to the induced charge density σ_{ind} through the relationship $E_{\text{ind}} = \sigma_{\text{ind}}/\epsilon_0$. Because $E = E_0/\kappa = \sigma/\kappa\epsilon_0$, substitution into Equation 26.22 gives

$$\frac{\sigma}{\kappa\epsilon_0} = \frac{\sigma}{\epsilon_0} - \frac{\sigma_{\text{ind}}}{\epsilon_0}$$

$$\sigma_{\text{ind}} = \left(\frac{\kappa - 1}{\kappa}\right)\sigma \tag{26.23}$$

Because $\kappa > 1$, this expression shows that the charge density σ_{ind} induced on the dielectric is less than the charge density σ on the plates. For instance, if $\kappa = 3$, we see that the induced charge density is two-thirds the charge density on the plates. If no dielectric is present, then $\kappa = 1$ and $\sigma_{\text{ind}} = 0$ as expected. However, if the dielectric is replaced by an electrical conductor, for which $E = 0$, then Equation 26.22 indicates that $E_0 = E_{\text{ind}}$; this corresponds to $\sigma_{\text{ind}} = \sigma$. That is, the surface charge induced on the conductor is equal in magnitude but opposite in sign to that on the plates, resulting in a net electric field of zero in the conductor.

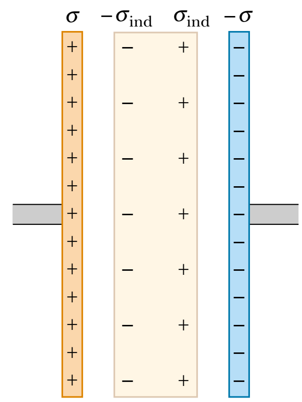

Figure 26.25 Induced charge on a dielectric placed between the plates of a charged capacitor. Note that the induced charge density on the dielectric is *less* than the charge density on the plates.

EXAMPLE 26.9 Effect of a Metallic Slab

A parallel-plate capacitor has a plate separation d and plate area A. An uncharged metallic slab of thickness a is inserted midway between the plates. (a) Find the capacitance of the device.

Solution We can solve this problem by noting that any charge that appears on one plate of the capacitor must induce a charge of equal magnitude but opposite sign on the near side of the slab, as shown in Figure 26.26a. Consequently, the net charge on the slab remains zero, and the electric field inside the slab is zero. Hence, the capacitor is equivalent to two capacitors in series, each having a plate separation $(d - a)/2$, as shown in Figure 26.26b.

Using the rule for adding two capacitors in series (Eq. 26.10), we obtain

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\dfrac{\epsilon_0 A}{(d-a)/2}} + \frac{1}{\dfrac{\epsilon_0 A}{(d-a)/2}}$$

$$C = \frac{\epsilon_0 A}{d - a}$$

Note that C approaches infinity as a approaches d. Why?

(b) Show that the capacitance is unaffected if the metallic slab is infinitesimally thin.

Solution In the result for part (a), we let $a \to 0$:

$$C = \lim_{a\to 0} \frac{\epsilon_0 A}{d - a} = \frac{\epsilon_0 A}{d}$$

which is the original capacitance.

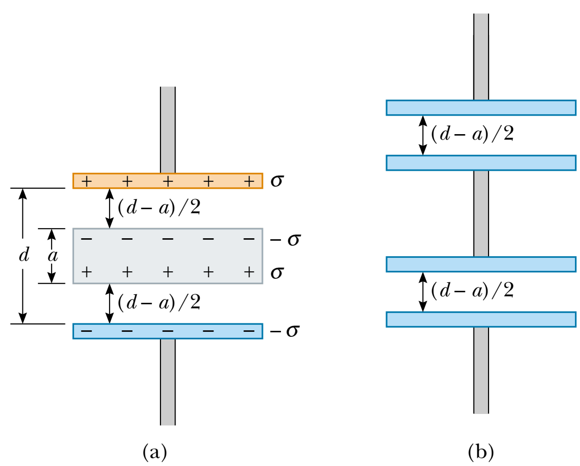

Figure 26.26 (a) A parallel-plate capacitor of plate separation d partially filled with a metallic slab of thickness a. (b) The equivalent circuit of the device in part (a) consists of two capacitors in series, each having a plate separation $(d - a)/2$.

(c) Show that the answer to part (a) does not depend on where the slab is inserted.

Solution Let us imagine that the slab in Figure 26.26a is moved upward so that the distance between the upper edge of the slab and the upper plate is b. Then, the distance between the lower edge of the slab and the lower plate is $d - b - a$. As in part (a), we find the total capacitance of the series combination:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\frac{\epsilon_0 A}{b}} + \frac{1}{\frac{\epsilon_0 A}{d-b-a}}$$

$$= \frac{b}{\epsilon_0 A} + \frac{d-b-a}{\epsilon_0 A} = \frac{d-a}{\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{d-a}$$

This is the same result as in part (a). It is independent of the value of b, so it does not matter where the slab is located.

EXAMPLE 26.10 A Partially Filled Capacitor

A parallel-plate capacitor with a plate separation d has a capacitance C_0 in the absence of a dielectric. What is the capacitance when a slab of dielectric material of dielectric constant κ and thickness $\frac{1}{3}d$ is inserted between the plates (Fig. 26.27a)?

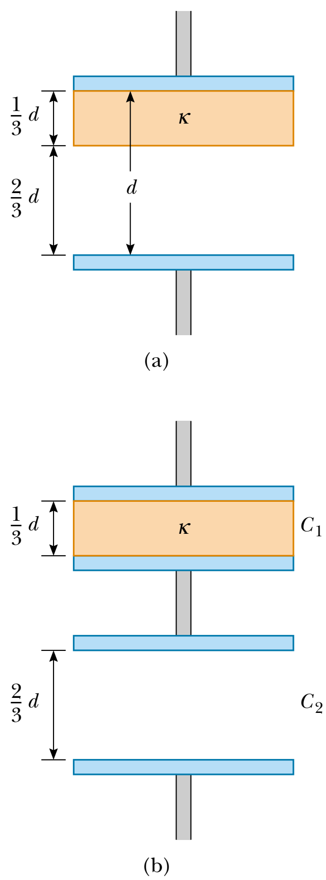

Figure 26.27 (a) A parallel-plate capacitor of plate separation d partially filled with a dielectric of thickness $d/3$. (b) The equivalent circuit of the capacitor consists of two capacitors connected in series.

Solution In Example 26.9, we found that we could insert a metallic slab between the plates of a capacitor and consider the combination as two capacitors in series. The resulting capacitance was independent of the location of the slab. Furthermore, if the thickness of the slab approaches zero, then the capacitance of the system approaches the capacitance when the slab is absent. From this, we conclude that we can insert an infinitesimally thin metallic slab anywhere between the plates of a capacitor without affecting the capacitance. Thus, let us imagine sliding an infinitesimally thin metallic slab along the bottom face of the dielectric shown in Figure 26.27a. We can then consider this system to be the series combination of the two capacitors shown in Figure 26.27b: one having a plate separation $d/3$ and filled with a dielectric, and the other having a plate separation $2d/3$ and air between its plates.

From Equations 26.15 and 26.3, the two capacitances are

$$C_1 = \frac{\kappa\epsilon_0 A}{d/3} \quad \text{and} \quad C_2 = \frac{\epsilon_0 A}{2d/3}$$

Using Equation 26.10 for two capacitors combined in series, we have

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d/3}{\kappa\epsilon_0 A} + \frac{2d/3}{\epsilon_0 A}$$

$$= \frac{d}{3\epsilon_0 A}\left(\frac{1}{\kappa} + 2\right) = \frac{d}{3\epsilon_0 A}\left(\frac{1+2\kappa}{\kappa}\right)$$

$$C = \left(\frac{3\kappa}{2\kappa+1}\right)\frac{\epsilon_0 A}{d}$$

Because the capacitance without the dielectric is $C_0 = \epsilon_0 A/d$, we see that

$$C = \left(\frac{3\kappa}{2\kappa+1}\right) C_0$$

SUMMARY

A **capacitor** consists of two conductors carrying charges of equal magnitude but opposite sign. The **capacitance** C of any capacitor is the ratio of the charge Q on either conductor to the potential difference ΔV between them:

$$C \equiv \frac{Q}{\Delta V} \tag{26.1}$$

This relationship can be used in situations in which any two of the three variables are known. It is important to remember that this ratio is constant for a given configuration of conductors because the capacitance depends only on the geometry of the conductors and not on an external source of charge or potential difference.

The SI unit of capacitance is coulombs per volt, or the **farad** (F), and 1 F = 1 C/V.

Capacitance expressions for various geometries are summarized in Table 26.2.

If two or more capacitors are connected in parallel, then the potential difference is the same across all of them. The equivalent capacitance of a parallel combination of capacitors is

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \tag{26.8}$$

If two or more capacitors are connected in series, the charge is the same on all of them, and the equivalent capacitance of the series combination is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \tag{26.10}$$

These two equations enable you to simplify many electric circuits by replacing multiple capacitors with a single equivalent capacitance.

Work is required to charge a capacitor because the charging process is equivalent to the transfer of charges from one conductor at a lower electric potential to another conductor at a higher potential. The work done in charging the capacitor to a charge Q equals the electric potential energy U stored in the capacitor, where

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}Q\,\Delta V = \tfrac{1}{2}C(\Delta V)^2 \tag{26.11}$$

TABLE 26.2 Capacitance and Geometry

Geometry	Capacitance	Equation
Isolated charged sphere of radius R (second charged conductor assumed at infinity)	$C = 4\pi\epsilon_0 R$	26.2
Parallel-plate capacitor of plate area A and plate separation d	$C = \epsilon_0 \frac{A}{d}$	26.3
Cylindrical capacitor of length ℓ and inner and outer radii a and b, respectively	$C = \frac{\ell}{2k_e \ln\left(\frac{b}{a}\right)}$	26.4
Spherical capacitor with inner and outer radii a and b, respectively	$C = \frac{ab}{k_e(b - a)}$	26.6

When a dielectric material is inserted between the plates of a capacitor, the capacitance increases by a dimensionless factor κ, called the **dielectric constant:**

$$C = \kappa C_0 \tag{26.14}$$

where C_0 is the capacitance in the absence of the dielectric. The increase in capacitance is due to a decrease in the magnitude of the electric field in the presence of the dielectric and to a corresponding decrease in the potential difference between the plates—if we assume that the charging battery is removed from the circuit before the dielectric is inserted. The decrease in the magnitude of **E** arises from an internal electric field produced by aligned dipoles in the dielectric. This internal field produced by the dipoles opposes the applied field due to the capacitor plates, and the result is a reduction in the net electric field.

The **electric dipole moment p** of an electric dipole has a magnitude

$$p \equiv 2aq \tag{26.16}$$

The direction of the electric dipole moment vector is from the negative charge toward the positive charge.

The torque acting on an electric dipole in a uniform electric field **E** is

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E} \tag{26.18}$$

The potential energy of an electric dipole in a uniform external electric field **E** is

$$U = -\mathbf{p} \cdot \mathbf{E} \tag{26.20}$$

Problem-Solving Hints

Capacitors

- Be careful with units. When you calculate capacitance in farads, make sure that distances are expressed in meters and that you use the SI value of ϵ_0. When checking consistency of units, remember that the unit for electric fields can be either N/C or V/m.
- When two or more capacitors are connected in parallel, the potential difference across each is the same. The charge on each capacitor is proportional to its capacitance; hence, the capacitances can be added directly to give the equivalent capacitance of the parallel combination. The equivalent capacitance is always larger than the individual capacitances.
- When two or more capacitors are connected in series, they carry the same charge, and the sum of the potential differences equals the total potential difference applied to the combination. The sum of the reciprocals of the capacitances equals the reciprocal of the equivalent capacitance, which is always less than the capacitance of the smallest individual capacitor.
- A dielectric increases the capacitance of a capacitor by a factor κ (the dielectric constant) over its capacitance when air is between the plates.
- For problems in which a battery is being connected or disconnected, note whether modifications to the capacitor are made while it is connected to the battery or after it has been disconnected. If the capacitor remains connected to the battery, the voltage across the capacitor remains unchanged (equal to the battery voltage), and the charge is proportional to the capaci-

tance, although it may be modified (for instance, by the insertion of a dielectric). If you disconnect the capacitor from the battery before making any modifications to the capacitor, then its charge remains fixed. In this case, as you vary the capacitance, the voltage across the plates changes according to the expression $\Delta V = Q/C$.

Questions

1. If you were asked to design a capacitor in a situation for which small size and large capacitance were required, what factors would be important in your design?
2. The plates of a capacitor are connected to a battery. What happens to the charge on the plates if the connecting wires are removed from the battery? What happens to the charge if the wires are removed from the battery and connected to each other?
3. A farad is a very large unit of capacitance. Calculate the length of one side of a square, air-filled capacitor that has a plate separation of 1 m. Assume that it has a capacitance of 1 F.
4. A pair of capacitors are connected in parallel, while an identical pair are connected in series. Which pair would be more dangerous to handle after being connected to the same voltage source? Explain.
5. If you are given three different capacitors C_1, C_2, C_3, how many different combinations of capacitance can you produce?
6. What advantage might there be in using two identical capacitors in parallel connected in series with another identical parallel pair rather than a single capacitor?
7. Is it always possible to reduce a combination of capacitors to one equivalent capacitor with the rules we have developed? Explain.
8. Because the net charge in a capacitor is always zero, what does a capacitor store?
9. Because the charges on the plates of a parallel-plate capacitor are of opposite sign, they attract each other. Hence, it would take positive work to increase the plate separation. What happens to the external work done in this process?
10. Explain why the work needed to move a charge Q through a potential difference ΔV is $W = Q\Delta V$, whereas the energy stored in a charged capacitor is $U = \frac{1}{2}Q\Delta V$. Where does the $\frac{1}{2}$ factor come from?
11. If the potential difference across a capacitor is doubled, by what factor does the stored energy change?
12. Why is it dangerous to touch the terminals of a high-voltage capacitor even after the applied voltage has been turned off? What can be done to make the capacitor safe to handle after the voltage source has been removed?
13. Describe how you can increase the maximum operating voltage of a parallel-plate capacitor for a fixed plate separation.
14. An air-filled capacitor is charged, disconnected from the power supply, and, finally, connected to a voltmeter. Explain how and why the voltage reading changes when a dielectric is inserted between the plates of the capacitor.
15. Using the polar molecule description of a dielectric, explain how a dielectric affects the electric field inside a capacitor.
16. Explain why a dielectric increases the maximum operating voltage of a capacitor even though the physical size of the capacitor does not change.
17. What is the difference between dielectric strength and the dielectric constant?
18. Explain why a water molecule is permanently polarized. What type of molecule has no permanent polarization?
19. If a dielectric-filled capacitor is heated, how does its capacitance change? (Neglect thermal expansion and assume that the dipole orientations are temperature dependent.)

Problems

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
☐ = paired numerical/symbolic problems

Section 26.1 Definition of Capacitance

1. (a) How much charge is on each plate of a 4.00-μF capacitor when it is connected to a 12.0-V battery? (b) If this same capacitor is connected to a 1.50-V battery, what charge is stored?
2. Two conductors having net charges of $+10.0\ \mu$C and $-10.0\ \mu$C have a potential difference of 10.0 V. Determine (a) the capacitance of the system and (b) the potential difference between the two conductors if the charges on each are increased to $+100\ \mu$C and $-100\ \mu$C.

Section 26.2 Calculating Capacitance

3. An isolated charged conducting sphere of radius 12.0 cm creates an electric field of 4.90×10^4 N/C at a distance 21.0 cm from its center. (a) What is its surface charge density? (b) What is its capacitance?

4. (a) If a drop of liquid has capacitance 1.00 pF, what is its radius? (b) If another drop has radius 2.00 mm, what is its capacitance? (c) What is the charge on the smaller drop if its potential is 100 V?

5. Two conducting spheres with diameters of 0.400 m and 1.00 m are separated by a distance that is large compared with the diameters. The spheres are connected by a thin wire and are charged to 7.00 μC. (a) How is this total charge shared between the spheres? (Neglect any charge on the wire.) (b) What is the potential of the system of spheres when the reference potential is taken to be $V = 0$ at $r = \infty$?

6. Regarding the Earth and a cloud layer 800 m above the Earth as the "plates" of a capacitor, calculate the capacitance if the cloud layer has an area of 1.00 km^2. Assume that the air between the cloud and the ground is pure and dry. Assume that charge builds up on the cloud and on the ground until a uniform electric field with a magnitude of 3.00×10^6 N/C throughout the space between them makes the air break down and conduct electricity as a lightning bolt. What is the maximum charge the cloud can hold?

WEB 7. An air-filled capacitor consists of two parallel plates, each with an area of 7.60 cm^2, separated by a distance of 1.80 mm. If a 20.0-V potential difference is applied to these plates, calculate (a) the electric field between the plates, (b) the surface charge density, (c) the capacitance, and (d) the charge on each plate.

8. A 1-megabit computer memory chip contains many 60.0-fF capacitors. Each capacitor has a plate area of 21.0×10^{-12} m^2. Determine the plate separation of such a capacitor (assume a parallel-plate configuration). The characteristic atomic diameter is 10^{-10} m = 0.100 nm. Express the plate separation in nanometers.

9. When a potential difference of 150 V is applied to the plates of a parallel-plate capacitor, the plates carry a surface charge density of 30.0 nC/cm^2. What is the spacing between the plates?

10. A variable air capacitor used in tuning circuits is made of N semicircular plates each of radius R and positioned a distance d from each other. As shown in Figure P26.10, a second identical set of plates is enmeshed with its plates halfway between those of the first set. The second set can rotate as a unit. Determine the capacitance as a function of the angle of rotation θ, where $\theta = 0$ corresponds to the maximum capacitance.

WEB 11. A 50.0-m length of coaxial cable has an inner conductor that has a diameter of 2.58 mm and carries a charge of 8.10 μC. The surrounding conductor has an inner diameter of 7.27 mm and a charge of -8.10 μC. (a) What is the capacitance of this cable? (b) What is the potential difference between the two conductors? Assume the region between the conductors is air.

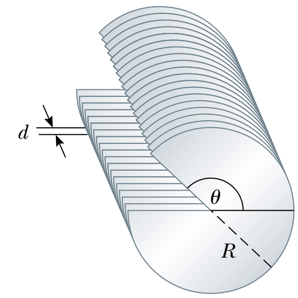

Figure P26.10

12. A 20.0-μF spherical capacitor is composed of two metallic spheres, one having a radius twice as large as the other. If the region between the spheres is a vacuum, determine the volume of this region.

13. A small object with a mass of 350 mg carries a charge of 30.0 nC and is suspended by a thread between the vertical plates of a parallel-plate capacitor. The plates are separated by 4.00 cm. If the thread makes an angle of 15.0° with the vertical, what is the potential difference between the plates?

14. A small object of mass m carries a charge q and is suspended by a thread between the vertical plates of a parallel-plate capacitor. The plate separation is d. If the thread makes an angle θ with the vertical, what is the potential difference between the plates?

15. An air-filled spherical capacitor is constructed with inner and outer shell radii of 7.00 and 14.0 cm, respectively. (a) Calculate the capacitance of the device. (b) What potential difference between the spheres results in a charge of 4.00 μC on the capacitor?

16. Find the capacitance of the Earth. (*Hint:* The outer conductor of the "spherical capacitor" may be considered as a conducting sphere at infinity where V approaches zero.)

Section 26.3 Combinations of Capacitors

17. Two capacitors $C_1 = 5.00$ μF and $C_2 = 12.0$ μF are connected in parallel, and the resulting combination is connected to a 9.00-V battery. (a) What is the value of the equivalent capacitance of the combination? What are (b) the potential difference across each capacitor and (c) the charge stored on each capacitor?

18. The two capacitors of Problem 17 are now connected in series and to a 9.00-V battery. Find (a) the value of the equivalent capacitance of the combination, (b) the voltage across each capacitor, and (c) the charge on each capacitor.

19. Two capacitors when connected in parallel give an equivalent capacitance of 9.00 pF and an equivalent ca-

pacitance of 2.00 pF when connected in series. What is the capacitance of each capacitor?

20. Two capacitors when connected in parallel give an equivalent capacitance of C_p and an equivalent capacitance of C_s when connected in series. What is the capacitance of each capacitor?

WEB 21. Four capacitors are connected as shown in Figure P26.21. (a) Find the equivalent capacitance between points *a* and *b*. (b) Calculate the charge on each capacitor if $\Delta V_{ab} = 15.0$ V.

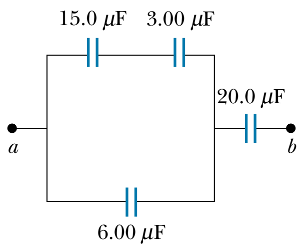

Figure P26.21

22. Evaluate the equivalent capacitance of the configuration shown in Figure P26.22. All the capacitors are identical, and each has capacitance *C*.

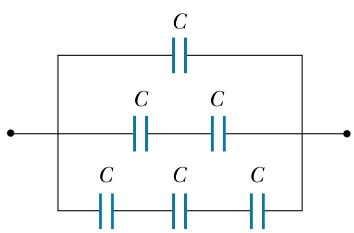

Figure P26.22

23. Consider the circuit shown in Figure P26.23, where $C_1 = 6.00\ \mu$F, $C_2 = 3.00\ \mu$F, and $\Delta V = 20.0$ V. Capacitor C_1 is first charged by the closing of switch S_1. Switch S_1 is then opened, and the charged capacitor is connected to the uncharged capacitor by the closing of S_2. Calculate the initial charge acquired by C_1 and the final charge on each.

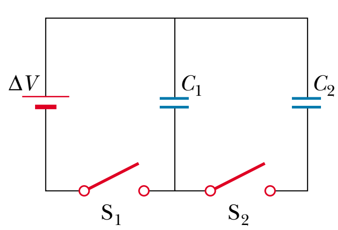

Figure P26.23

24. According to its design specification, the timer circuit delaying the closing of an elevator door is to have a capacitance of 32.0 μF between two points *A* and *B*. (a) When one circuit is being constructed, the inexpensive capacitor installed between these two points is found to have capacitance 34.8 μF. To meet the specification, one additional capacitor can be placed between the two points. Should it be in series or in parallel with the 34.8-μF capacitor? What should be its capacitance? (b) The next circuit comes down the assembly line with capacitance 29.8 μF between *A* and *B*. What additional capacitor should be installed in series or in parallel in that circuit, to meet the specification?

25. The circuit in Figure P26.25 consists of two identical parallel metallic plates connected by identical metallic springs to a 100-V battery. With the switch open, the plates are uncharged, are separated by a distance $d = 8.00$ mm, and have a capacitance $C = 2.00\ \mu$F. When the switch is closed, the distance between the plates decreases by a factor of 0.500. (a) How much charge collects on each plate and (b) what is the spring constant for each spring? (*Hint:* Use the result of Problem 35.)

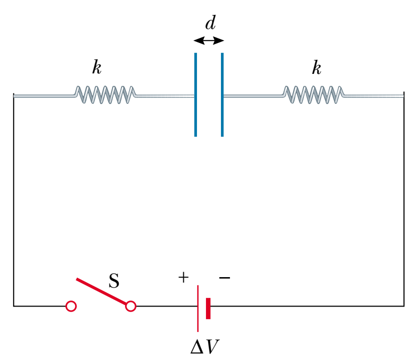

Figure P26.25

26. Figure P26.26 shows six concentric conducting spheres, A, B, C, D, E, and F having radii *R*, 2*R*, 3*R*, 4*R*, 5*R*, and 6*R*, respectively. Spheres B and C are connected by a conducting wire, as are spheres D and E. Determine the equivalent capacitance of this system.

27. A group of identical capacitors is connected first in series and then in parallel. The combined capacitance in parallel is 100 times larger than for the series connection. How many capacitors are in the group?

28. Find the equivalent capacitance between points *a* and *b* for the group of capacitors connected as shown in Figure P26.28 if $C_1 = 5.00\ \mu$F, $C_2 = 10.0\ \mu$F, and $C_3 = 2.00\ \mu$F.

29. For the network described in the previous problem if the potential difference between points *a* and *b* is 60.0 V, what charge is stored on C_3?

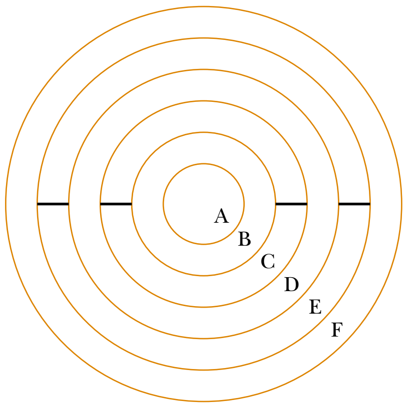

Figure P26.26

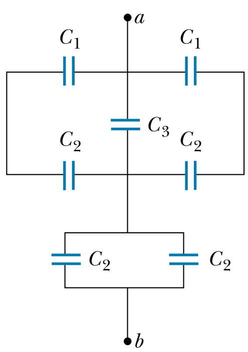

Figure P26.28 Problems 28 and 29.

30. Find the equivalent capacitance between points a and b in the combination of capacitors shown in Figure P26.30.

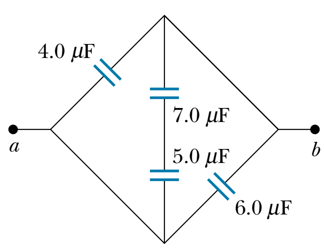

Figure P26.30

Section 26.4 Energy Stored in a Charged Capacitor

31. (a) A 3.00-μF capacitor is connected to a 12.0-V battery. How much energy is stored in the capacitor? (b) If the capacitor had been connected to a 6.00-V battery, how much energy would have been stored?

32. Two capacitors $C_1 = 25.0\ \mu$F and $C_2 = 5.00\ \mu$F are connected in parallel and charged with a 100-V power supply. (a) Draw a circuit diagram and calculate the total energy stored in the two capacitors. (b) What potential difference would be required across the same two capacitors connected in series so that the combination stores the same energy as in part (a)? Draw a circuit diagram of this circuit.

33. A parallel-plate capacitor is charged and then disconnected from a battery. By what fraction does the stored energy change (increase or decrease) when the plate separation is doubled?

34. A uniform electric field $E = 3\ 000$ V/m exists within a certain region. What volume of space contains an energy equal to 1.00×10^{-7} J? Express your answer in cubic meters and in liters.

WEB **35.** A parallel-plate capacitor has a charge Q and plates of area A. Show that the force exerted on each plate by the other is $F = Q^2/2\epsilon_0 A$. (*Hint:* Let $C = \epsilon_0 A/x$ for an arbitrary plate separation x; then require that the work done in separating the two charged plates be $W = \int F\,dx$.)

36. Plate a of a parallel-plate, air-filled capacitor is connected to a spring having force constant k, and plate b is fixed. They rest on a table top as shown (top view) in Figure P26.36. If a charge $+Q$ is placed on plate a and a charge $-Q$ is placed on plate b, by how much does the spring expand?

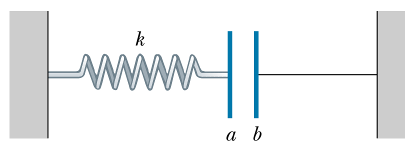

Figure P26.36

37. Review Problem. A certain storm cloud has a potential difference of 1.00×10^8 V relative to a tree. If, during a lightning storm, 50.0 C of charge is transferred through this potential difference and 1.00% of the energy is absorbed by the tree, how much water (sap in the tree) initially at 30.0°C can be boiled away? Water has a specific heat of 4 186 J/kg · °C, a boiling point of 100°C, and a heat of vaporization of 2.26×10^6 J/kg.

38. Show that the energy associated with a conducting sphere of radius R and charge Q surrounded by a vacuum is $U = k_e Q^2/2R$.

39. Einstein said that energy is associated with mass according to the famous relationship $E = mc^2$. Estimate the radius of an electron, assuming that its charge is distributed uniformly over the surface of a sphere of radius R and that the mass–energy of the electron is equal to the total energy stored in the resulting nonzero electric field between R and infinity. (See Problem 38. Experimentally, an electron nevertheless appears to be a point particle. The electric field close to the electron must be described by quantum electrodynamics, rather than the classical electrodynamics that we study.)

Section 26.5 Capacitors with Dielectrics

40. Find the capacitance of a parallel-plate capacitor that uses Bakelite as a dielectric, if each of the plates has an area of 5.00 cm^2 and the plate separation is 2.00 mm.

41. Determine (a) the capacitance and (b) the maximum voltage that can be applied to a Teflon-filled parallel-plate capacitor having a plate area of 1.75 cm^2 and plate separation of 0.040 0 mm.

42. (a) How much charge can be placed on a capacitor with air between the plates before it breaks down, if the area of each of the plates is 5.00 cm^2? (b) Find the maximum charge if polystyrene is used between the plates instead of air.

43. A commercial capacitor is constructed as shown in Figure 26.15a. This particular capacitor is rolled from two strips of aluminum separated by two strips of paraffin-coated paper. Each strip of foil and paper is 7.00 cm wide. The foil is 0.004 00 mm thick, and the paper is 0.025 0 mm thick and has a dielectric constant of 3.70. What length should the strips be if a capacitance of 9.50×10^{-8} F is desired? (Use the parallel-plate formula.)

44. The supermarket sells rolls of aluminum foil, plastic wrap, and waxed paper. Describe a capacitor made from supermarket materials. Compute order-of-magnitude estimates for its capacitance and its breakdown voltage.

45. A capacitor that has air between its plates is connected across a potential difference of 12.0 V and stores 48.0 μC of charge. It is then disconnected from the source while still charged. (a) Find the capacitance of the capacitor. (b) A piece of Teflon is inserted between the plates. Find its new capacitance. (c) Find the voltage and charge now on the capacitor.

46. A parallel-plate capacitor in air has a plate separation of 1.50 cm and a plate area of 25.0 cm^2. The plates are charged to a potential difference of 250 V and disconnected from the source. The capacitor is then immersed in distilled water. Determine (a) the charge on the plates before and after immersion, (b) the capacitance and voltage after immersion, and (c) the change in energy of the capacitor. Neglect the conductance of the liquid.

47. A conducting spherical shell has inner radius a and outer radius c. The space between these two surfaces is filled with a dielectric for which the dielectric constant is κ_1 between a and b, and κ_2 between b and c (Fig. P26.47). Determine the capacitance of this system.

48. A wafer of titanium dioxide ($\kappa = 173$) has an area of 1.00 cm^2 and a thickness of 0.100 mm. Aluminum is evaporated on the parallel faces to form a parallel-plate capacitor. (a) Calculate the capacitance. (b) When the capacitor is charged with a 12.0-V battery, what is the magnitude of charge delivered to each plate? (c) For the situation in part (b), what are the free and induced surface charge densities? (d) What is the magnitude E of the electric field?

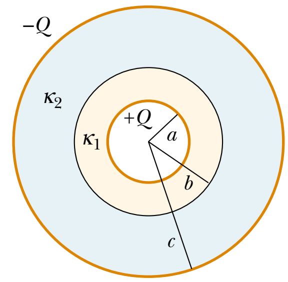

Figure P26.47

49. Each capacitor in the combination shown in Figure P26.49 has a breakdown voltage of 15.0 V. What is the breakdown voltage of the combination?

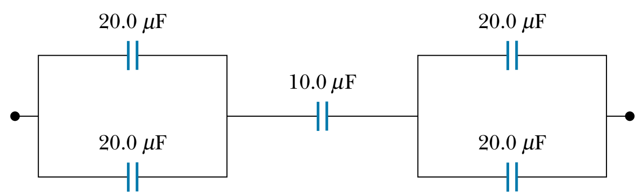

Figure P26.49

(Optional)
Section 26.6 Electric Dipole in an Electric Field

50. A small rigid object carries positive and negative 3.50-nC charges. It is oriented so that the positive charge is at the point (− 1.20 mm, 1.10 mm) and the negative charge is at the point (1.40 mm, − 1.30 mm). (a) Find the electric dipole moment of the object. The object is placed in an electric field $\mathbf{E} = (7\,800\mathbf{i} - 4\,900\mathbf{j})$ N/C. (b) Find the torque acting on the object. (c) Find the potential energy of the object in this orientation. (d) If the orientation of the object can change, find the difference between its maximum and its minimum potential energies.

51. A small object with electric dipole moment $\mathbf{p}$ is placed in a nonuniform electric field $\mathbf{E} = E(x)\,\mathbf{i}$. That is, the field is in the x direction, and its magnitude depends on the coordinate x. Let θ represent the angle between the dipole moment and the x direction. (a) Prove that the dipole experiences a net force $F = p(dE/dx)\cos\theta$ in the direction toward which the field increases. (b) Consider the field created by a spherical balloon centered at the origin. The balloon has a radius of 15.0 cm and carries a charge of 2.00 μC. Evaluate dE/dx at the point (16 cm, 0, 0). Assume that a water droplet at this point has an induced dipole moment of $(6.30\mathbf{i})$ nC·m. Find the force on it.

(Optional)
Section 26.7 An Atomic Description of Dielectrics

52. A detector of radiation called a Geiger–Muller counter consists of a closed, hollow, conducting cylinder with a fine wire along its axis. Suppose that the internal diameter of the cylinder is 2.50 cm and that the wire along the axis has a diameter of 0.200 mm. If the dielectric strength of the gas between the central wire and the cylinder is 1.20×10^6 V/m, calculate the maximum voltage that can be applied between the wire and the cylinder before breakdown occurs in the gas.

53. The general form of Gauss's law describes how a charge creates an electric field in a material, as well as in a vacuum. It is

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon}$$

where $\epsilon = \kappa\epsilon_0$ is the permittivity of the material. (a) A sheet with charge Q uniformly distributed over its area A is surrounded by a dielectric. Show that the sheet creates a uniform electric field with magnitude $E = Q/2A\epsilon$ at nearby points. (b) Two large sheets of area A carrying opposite charges of equal magnitude Q are a small distance d apart. Show that they create a uniform electric field of magnitude $E = Q/A\epsilon$ between them. (c) Assume that the negative plate is at zero potential. Show that the positive plate is at a potential $Qd/A\epsilon$. (d) Show that the capacitance of the pair of plates is $A\epsilon/d = \kappa A\epsilon_0/d$.

ADDITIONAL PROBLEMS

54. For the system of capacitors shown in Figure P26.54, find (a) the equivalent capacitance of the system, (b) the potential difference across each capacitor, (c) the charge on each capacitor, and (d) the total energy stored by the group.

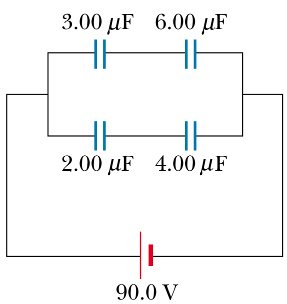

Figure P26.54

55. Consider two *long*, parallel, and oppositely charged wires of radius d with their centers separated by a distance D. Assuming the charge is distributed uniformly on the surface of each wire, show that the capacitance per unit length of this pair of wires is

$$\frac{C}{\ell} = \frac{\pi\epsilon_0}{\ln\left(\frac{D-d}{d}\right)}$$

56. A 2.00-nF parallel-plate capacitor is charged to an initial potential difference $\Delta V_i = 100$ V and then isolated. The dielectric material between the plates is mica ($\kappa = 5.00$). (a) How much work is required to withdraw the mica sheet? (b) What is the potential difference of the capacitor after the mica is withdrawn?

WEB 57. A parallel-plate capacitor is constructed using a dielectric material whose dielectric constant is 3.00 and whose dielectric strength is 2.00×10^8 V/m. The desired capacitance is 0.250 μF, and the capacitor must withstand a maximum potential difference of 4 000 V. Find the minimum area of the capacitor plates.

58. A parallel-plate capacitor is constructed using three dielectric materials, as shown in Figure P26.58. You may assume that $\ell \gg d$. (a) Find an expression for the capacitance of the device in terms of the plate area A and d, κ_1, κ_2, and κ_3. (b) Calculate the capacitance using the values $A = 1.00$ cm^2, $d = 2.00$ mm, $\kappa_1 = 4.90$, $\kappa_2 = 5.60$, and $\kappa_3 = 2.10$.

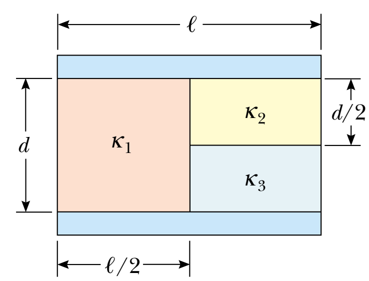

Figure P26.58

59. A conducting slab of thickness d and area A is inserted into the space between the plates of a parallel-plate capacitor with spacing s and surface area A, as shown in Figure P26.59. The slab is not necessarily halfway between the capacitor plates. What is the capacitance of the system?

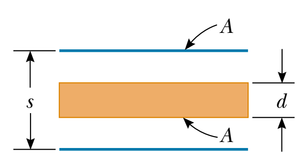

Figure P26.59

60. (a) Two spheres have radii a and b and their centers are a distance d apart. Show that the capacitance of this system is

$$C \approx \frac{4\pi\epsilon_0}{\frac{1}{a} + \frac{1}{b} - \frac{2}{d}}$$

provided that d is large compared with a and b. (*Hint:* Because the spheres are far apart, assume that the