

Thus far our treatment of electrical phenomena has been confined to the study of charges at rest, or *electrostatics*. We now consider situations involving electric charges in motion. We use the term *electric current*, or simply *current*, to describe the rate of flow of charge through some region of space. Most practical applications of electricity deal with electric currents. For example, the battery in a flashlight supplies current to the filament of the bulb when the switch is turned on. A variety of home appliances operate on alternating current. In these common situations, the charges flow through a conductor, such as a copper wire. It also is possible for currents to exist outside a conductor. For instance, a beam of electrons in a television picture tube constitutes a current.

This chapter begins with the definitions of current and current density. A microscopic description of current is given, and some of the factors that contribute to the resistance to the flow of charge in conductors are discussed. A classical model is used to describe electrical conduction in metals, and some of the limitations of this model are cited.

## 27.1 ELECTRIC CURRENT

13.2

It is instructive to draw an analogy between water flow and current. In many localities it is common practice to install low-flow showerheads in homes as a water-conservation measure. We quantify the flow of water from these and similar devices by specifying the amount of water that emerges during a given time interval, which is often measured in liters per minute. On a grander scale, we can characterize a river current by describing the rate at which the water flows past a particular location. For example, the flow over the brink at Niagara Falls is maintained at rates between 1 400 m<sup>3</sup>/s and 2 800 m<sup>3</sup>/s.

Now consider a system of electric charges in motion. Whenever there is a net flow of charge through some region, a **current** is said to exist. To define current more precisely, suppose that the charges are moving perpendicular to a surface of area  $A$ , as shown in Figure 27.1. (This area could be the cross-sectional area of a wire, for example.) **The current is the rate at which charge flows through this surface.** If  $\Delta Q$  is the amount of charge that passes through this area in a time interval  $\Delta t$ , the **average current**  $I_{\text{av}}$  is equal to the charge that passes through  $A$  per unit time:

$$I_{\text{av}} = \frac{\Delta Q}{\Delta t} \quad (27.1)$$

If the rate at which charge flows varies in time, then the current varies in time; we define the **instantaneous current**  $I$  as the differential limit of average current:

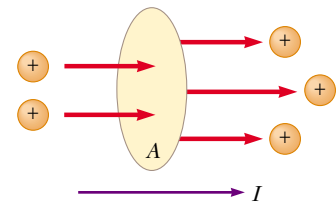
$$I \equiv \frac{dQ}{dt} \quad (27.2)$$

The SI unit of current is the **ampere** (A):

$$1 \text{ A} = \frac{1 \text{ C}}{1 \text{ s}} \quad (27.3)$$

That is, 1 A of current is equivalent to 1 C of charge passing through the surface area in 1 s.

The charges passing through the surface in Figure 27.1 can be positive or negative, or both. **It is conventional to assign to the current the same direction as the flow of positive charge.** In electrical conductors, such as copper or alu-



**Figure 27.1** Charges in motion through an area  $A$ . The time rate at which charge flows through the area is defined as the current  $I$ . The direction of the current is the direction in which positive charges flow when free to do so.

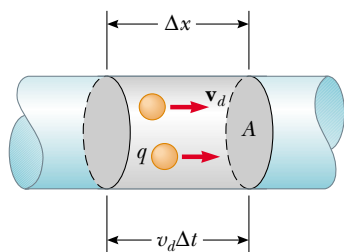
Electric current

The direction of the current

minimum, the current is due to the motion of negatively charged electrons. Therefore, when we speak of current in an ordinary conductor, **the direction of the current is opposite the direction of flow of electrons.** However, if we are considering a beam of positively charged protons in an accelerator, the current is in the direction of motion of the protons. In some cases—such as those involving gases and electrolytes, for instance—the current is the result of the flow of both positive and negative charges.

If the ends of a conducting wire are connected to form a loop, all points on the loop are at the same electric potential, and hence the electric field is zero within and at the surface of the conductor. Because the electric field is zero, there is no net transport of charge through the wire, and therefore there is no current. The current in the conductor is zero even if the conductor has an excess of charge on it. However, if the ends of the conducting wire are connected to a battery, all points on the loop are not at the same potential. The battery sets up a potential difference between the ends of the loop, creating an electric field within the wire. The electric field exerts forces on the conduction electrons in the wire, causing them to move around the loop and thus creating a current.

It is common to refer to a moving charge (positive or negative) as a mobile **charge carrier.** For example, the mobile charge carriers in a metal are electrons.



**Figure 27.2** A section of a uniform conductor of cross-sectional area  $A$ . The mobile charge carriers move with a speed  $v_d$ , and the distance they travel in a time  $\Delta t$  is  $\Delta x = v_d \Delta t$ . The number of carriers in the section of length  $\Delta x$  is  $nA v_d \Delta t$ , where  $n$  is the number of carriers per unit volume.

Average current in a conductor

### Microscopic Model of Current

We can relate current to the motion of the charge carriers by describing a microscopic model of conduction in a metal. Consider the current in a conductor of cross-sectional area  $A$  (Fig. 27.2). The volume of a section of the conductor of length  $\Delta x$  (the gray region shown in Fig. 27.2) is  $A \Delta x$ . If  $n$  represents the number of mobile charge carriers per unit volume (in other words, the charge carrier density), the number of carriers in the gray section is  $nA \Delta x$ . Therefore, the charge  $\Delta Q$  in this section is

$$\Delta Q = \text{number of carriers in section} \times \text{charge per carrier} = (nA \Delta x)q$$

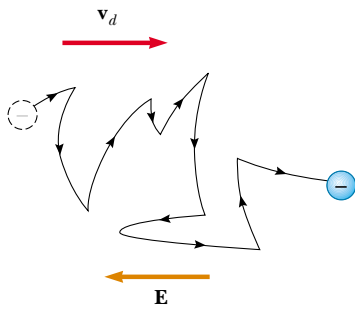
where  $q$  is the charge on each carrier. If the carriers move with a speed  $v_d$ , the distance they move in a time  $\Delta t$  is  $\Delta x = v_d \Delta t$ . Therefore, we can write  $\Delta Q$  in the form

$$\Delta Q = (nA v_d \Delta t)q$$

If we divide both sides of this equation by  $\Delta t$ , we see that the average current in the conductor is

$$I_{\text{av}} = \frac{\Delta Q}{\Delta t} = nq v_d A \quad (27.4)$$

The speed of the charge carriers  $v_d$  is an average speed called the **drift speed.** To understand the meaning of drift speed, consider a conductor in which the charge carriers are free electrons. If the conductor is isolated—that is, the potential difference across it is zero—then these electrons undergo random motion that is analogous to the motion of gas molecules. As we discussed earlier, when a potential difference is applied across the conductor (for example, by means of a battery), an electric field is set up in the conductor; this field exerts an electric force on the electrons, producing a current. However, the electrons do not move in straight lines along the conductor. Instead, they collide repeatedly with the metal atoms, and their resultant motion is complicated and zigzag (Fig. 27.3). Despite the collisions, the electrons move slowly along the conductor (in a direction opposite that of  $\mathbf{E}$ ) at the drift velocity  $\mathbf{v}_d$ .

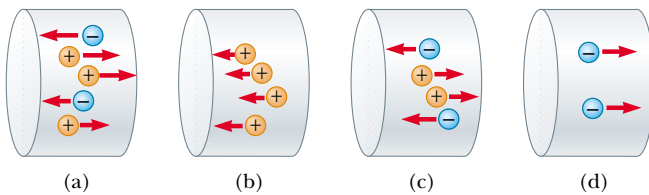


**Figure 27.3** A schematic representation of the zigzag motion of an electron in a conductor. The changes in direction are the result of collisions between the electron and atoms in the conductor. Note that the net motion of the electron is opposite the direction of the electric field. Each section of the zigzag path is a parabolic segment.

We can think of the atom–electron collisions in a conductor as an effective internal friction (or drag force) similar to that experienced by the molecules of a liquid flowing through a pipe stuffed with steel wool. The energy transferred from the electrons to the metal atoms during collision causes an increase in the vibrational energy of the atoms and a corresponding increase in the temperature of the conductor.

### Quick Quiz 27.1

Consider positive and negative charges moving horizontally through the four regions shown in Figure 27.4. Rank the current in these four regions, from lowest to highest.



**Figure 27.4**

### EXAMPLE 27.1 Drift Speed in a Copper Wire

The 12-gauge copper wire in a typical residential building has a cross-sectional area of  $3.31 \times 10^{-6} \text{ m}^2$ . If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that each copper atom contributes one free electron to the current. The density of copper is  $8.95 \text{ g/cm}^3$ .

**Solution** From the periodic table of the elements in Appendix C, we find that the molar mass of copper is 63.5 g/mol. Recall that 1 mol of any substance contains Avogadro's number of atoms ( $6.02 \times 10^{23}$ ). Knowing the density of copper, we can calculate the volume occupied by 63.5 g (= 1 mol) of copper:

$$V = \frac{m}{\rho} = \frac{63.5 \text{ g}}{8.95 \text{ g/cm}^3} = 7.09 \text{ cm}^3$$

Because each copper atom contributes one free electron to the current, we have

$$\begin{aligned} n &= \frac{6.02 \times 10^{23} \text{ electrons}}{7.09 \text{ cm}^3} (1.00 \times 10^6 \text{ cm}^3/\text{m}^3) \\ &= 8.49 \times 10^{28} \text{ electrons/m}^3 \end{aligned}$$

From Equation 27.4, we find that the drift speed is

$$v_d = \frac{I}{nqA}$$

where  $q$  is the absolute value of the charge on each electron. Thus,


$$\begin{aligned} v_d &= \frac{I}{nqA} \\ &= \frac{10.0 \text{ C/s}}{(8.49 \times 10^{28} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})(3.31 \times 10^{-6} \text{ m}^2)} \\ &= 2.22 \times 10^{-4} \text{ m/s} \end{aligned}$$

**Exercise** If a copper wire carries a current of 80.0 mA, how many electrons flow past a given cross-section of the wire in 10.0 min?

**Answer**  $3.0 \times 10^{20}$  electrons.

Example 27.1 shows that typical drift speeds are very low. For instance, electrons traveling with a speed of  $2.46 \times 10^{-4}$  m/s would take about 68 min to travel 1 m! In view of this, you might wonder why a light turns on almost instantaneously when a switch is thrown. In a conductor, the electric field that drives the free electrons travels through the conductor with a speed close to that of light. Thus, when you flip on a light switch, the message for the electrons to start moving through the wire (the electric field) reaches them at a speed on the order of  $10^8$  m/s.

## 27.2 RESISTANCE AND OHM'S LAW

 In Chapter 24 we found that no electric field can exist inside a conductor. However, this statement is true *only* if the conductor is in static equilibrium. The purpose of this section is to describe what happens when the charges in the conductor are allowed to move.

Charges moving in a conductor produce a current under the action of an electric field, which is maintained by the connection of a battery across the conductor. An electric field can exist in the conductor because the charges in this situation are in motion—that is, this is a *nonelectrostatic* situation.

Consider a conductor of cross-sectional area  $A$  carrying a current  $I$ . The **current density**  $J$  in the conductor is defined as the current per unit area. Because the current  $I = nqv_dA$ , the current density is

$$J \equiv \frac{I}{A} = nqv_d \quad (27.5)$$

where  $J$  has SI units of A/m<sup>2</sup>. This expression is valid only if the current density is uniform and only if the surface of cross-sectional area  $A$  is perpendicular to the direction of the current. In general, the current density is a vector quantity:

$$\mathbf{J} = nq\mathbf{v}_d \quad (27.6)$$

From this equation, we see that current density, like current, is in the direction of charge motion for positive charge carriers and opposite the direction of motion for negative charge carriers.

**A current density  $\mathbf{J}$  and an electric field  $\mathbf{E}$  are established in a conductor whenever a potential difference is maintained across the conductor.** If the potential difference is constant, then the current also is constant. In some materials, the current density is proportional to the electric field:

$$\mathbf{J} = \sigma\mathbf{E} \quad (27.7)$$

where the constant of proportionality  $\sigma$  is called the **conductivity** of the conductor.<sup>1</sup> Materials that obey Equation 27.7 are said to follow **Ohm's law**, named after Georg Simon Ohm (1787–1854). More specifically, Ohm's law states that

for many materials (including most metals), the ratio of the current density to the electric field is a constant  $\sigma$  that is independent of the electric field producing the current.

Materials that obey Ohm's law and hence demonstrate this simple relationship between  $\mathbf{E}$  and  $\mathbf{J}$  are said to be *ohmic*. Experimentally, it is found that not all materials have this property, however, and materials that do not obey Ohm's law are said to

<sup>1</sup> Do not confuse conductivity  $\sigma$  with surface charge density, for which the same symbol is used.

Current density

Ohm's law