

## 5.5 THE FORCE OF GRAVITY AND WEIGHT

We are well aware that all objects are attracted to the Earth. The attractive force exerted by the Earth on an object is called the **force of gravity**  $\mathbf{F}_g$ . This force is directed toward the center of the Earth,<sup>3</sup> and its magnitude is called the **weight** of the object.

We saw in Section 2.6 that a freely falling object experiences an acceleration  $\mathbf{g}$  acting toward the center of the Earth. Applying Newton's second law  $\Sigma \mathbf{F} = m\mathbf{a}$  to a freely falling object of mass  $m$ , with  $\mathbf{a} = \mathbf{g}$  and  $\Sigma \mathbf{F} = \mathbf{F}_g$ , we obtain

$$\mathbf{F}_g = m\mathbf{g} \quad (5.6)$$

Thus, the weight of an object, being defined as the magnitude of  $\mathbf{F}_g$ , is  $mg$ . (You should not confuse the italicized symbol  $g$  for gravitational acceleration with the nonitalicized symbol  $g$  used as the abbreviation for "gram.")

Because it depends on  $g$ , weight varies with geographic location. Hence, weight, unlike mass, is not an inherent property of an object. Because  $g$  decreases with increasing distance from the center of the Earth, bodies weigh less at higher altitudes than at sea level. For example, a 1 000-kg palette of bricks used in the construction of the Empire State Building in New York City weighed about 1 N less by the time it was lifted from sidewalk level to the top of the building. As another example, suppose an object has a mass of 70.0 kg. Its weight in a location where  $g = 9.80 \text{ m/s}^2$  is  $F_g = mg = 686 \text{ N}$  (about 150 lb). At the top of a mountain, however, where  $g = 9.77 \text{ m/s}^2$ , its weight is only 684 N. Therefore, if you want to lose weight without going on a diet, climb a mountain or weigh yourself at 30 000 ft during an airplane flight!

Because weight  $= F_g = mg$ , we can compare the masses of two objects by measuring their weights on a spring scale. At a given location, the ratio of the weights of two objects equals the ratio of their masses.



The life-support unit strapped to the back of astronaut Edwin Aldrin weighed 300 lb on the Earth. During his training, a 50-lb mock-up was used. Although this effectively simulated the reduced weight the unit would have on the Moon, it did not correctly mimic the unchanging mass. It was just as difficult to accelerate the unit (perhaps by jumping or twisting suddenly) on the Moon as on the Earth.

Definition of weight

### QuickLab

Drop a pen and your textbook simultaneously from the same height and watch as they fall. How can they have the same acceleration when their weights are so different?

<sup>3</sup> This statement ignores the fact that the mass distribution of the Earth is not perfectly spherical.

**CONCEPTUAL EXAMPLE 5.2** How Much Do You Weigh in an Elevator?


You have most likely had the experience of standing in an elevator that accelerates upward as it moves toward a higher floor. In this case, you feel heavier. In fact, if you are standing on a bathroom scale at the time, the scale measures a force magnitude that is greater than your weight. Thus, you have tactile and measured evidence that leads you to believe you are heavier in this situation. *Are you heavier?*

**Solution** No, your weight is unchanged. To provide the acceleration upward, the floor or scale must exert on your feet an upward force that is greater in magnitude than your weight. It is this greater force that you feel, which you interpret as feeling heavier. The scale reads this upward force, not your weight, and so its reading increases.

**Quick Quiz 5.3**

A baseball of mass  $m$  is thrown upward with some initial speed. If air resistance is neglected, what forces are acting on the ball when it reaches (a) half its maximum height and (b) its maximum height?

**5.6** NEWTON'S THIRD LAW

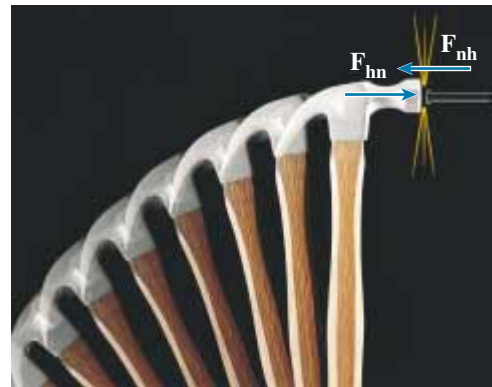
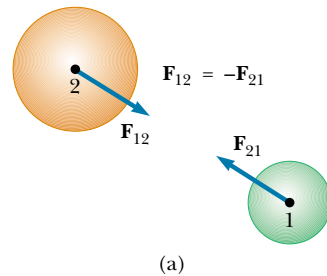
 If you press against a corner of this textbook with your fingertip, the book pushes back and makes a small dent in your skin. If you push harder, the book does the same and the dent in your skin gets a little larger. This simple experiment illustrates a general principle of critical importance known as **Newton's third law**:

If two objects interact, the force  $\mathbf{F}_{12}$  exerted by object 1 on object 2 is equal in magnitude to and opposite in direction to the force  $\mathbf{F}_{21}$  exerted by object 2 on object 1:

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \quad (5.7)$$

This law, which is illustrated in Figure 5.6a, states that a force that affects the motion of an object must come from a second, *external*, object. The external object, in turn, is subject to an equal-magnitude but oppositely directed force exerted on it.

Newton's third law



**Figure 5.6** Newton's third law. (a) The force  $\mathbf{F}_{12}$  exerted by object 1 on object 2 is equal in magnitude to and opposite in direction to the force  $\mathbf{F}_{21}$  exerted by object 2 on object 1. (b) The force  $\mathbf{F}_{hn}$  exerted by the hammer on the nail is equal to and opposite the force  $\mathbf{F}_{nh}$  exerted by the nail on the hammer.

This is equivalent to stating that **a single isolated force cannot exist**. The force that object 1 exerts on object 2 is sometimes called the *action force*, while the force object 2 exerts on object 1 is called the *reaction force*. In reality, either force can be labeled the action or the reaction force. **The action force is equal in magnitude to the reaction force and opposite in direction. In all cases, the action and reaction forces act on different objects.** For example, the force acting on a freely falling projectile is  $\mathbf{F}_g = m\mathbf{g}$ , which is the force of gravity exerted by the Earth on the projectile. The reaction to this force is the force exerted by the projectile on the Earth,  $\mathbf{F}'_g = -\mathbf{F}_g$ . The reaction force  $\mathbf{F}'_g$  accelerates the Earth toward the projectile just as the action force  $\mathbf{F}_g$  accelerates the projectile toward the Earth. However, because the Earth has such a great mass, its acceleration due to this reaction force is negligibly small.

Another example of Newton's third law is shown in Figure 5.6b. The force exerted by the hammer on the nail (the action force  $\mathbf{F}_{\text{hn}}$ ) is equal in magnitude and opposite in direction to the force exerted by the nail on the hammer (the reaction force  $\mathbf{F}_{\text{nh}}$ ). It is this latter force that causes the hammer to stop its rapid forward motion when it strikes the nail.

You experience Newton's third law directly whenever you slam your fist against a wall or kick a football. You should be able to identify the action and reaction forces in these cases.



Compression of a football as the force exerted by a player's foot sets the ball in motion.

### Quick Quiz 5.4

A person steps from a boat toward a dock. Unfortunately, he forgot to tie the boat to the dock, and the boat scoots away as he steps from it. Analyze this situation in terms of Newton's third law.

The force of gravity  $\mathbf{F}_g$  was defined as the attractive force the Earth exerts on an object. If the object is a TV at rest on a table, as shown in Figure 5.7a, why does the TV not accelerate in the direction of  $\mathbf{F}_g$ ? The TV does not accelerate because the table holds it up. What is happening is that the table exerts on the TV an upward force  $\mathbf{n}$  called the **normal force**.<sup>4</sup> The normal force is a contact force that prevents the TV from falling through the table and can have any magnitude needed to balance the downward force  $\mathbf{F}_g$ , up to the point of breaking the table. If someone stacks books on the TV, the normal force exerted by the table on the TV increases. If someone lifts up on the TV, the normal force exerted by the table on the TV decreases. (The normal force becomes zero if the TV is raised off the table.)

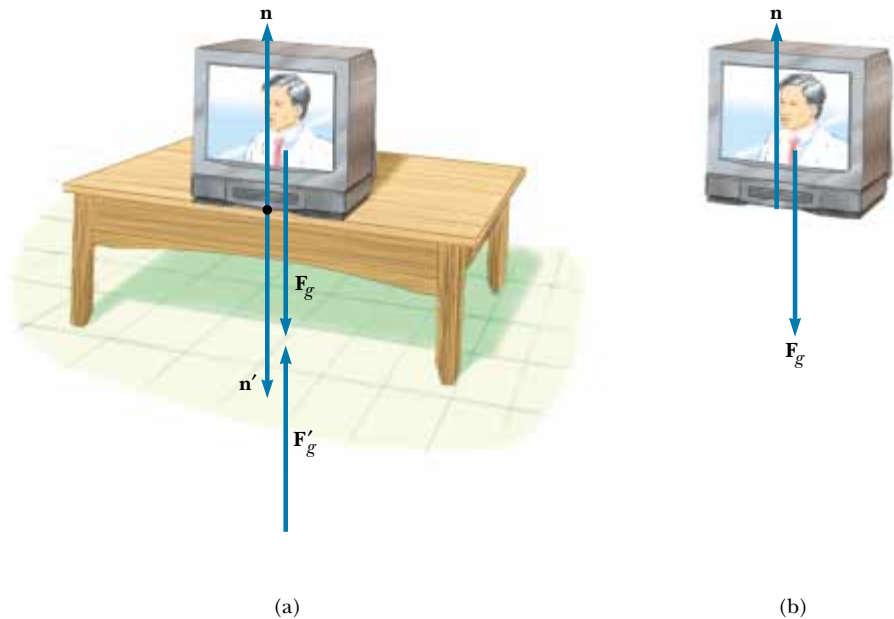
The two forces in an action–reaction pair **always act on different objects**. For the hammer-and-nail situation shown in Figure 5.6b, one force of the pair acts on the hammer and the other acts on the nail. For the unfortunate person stepping out of the boat in Quick Quiz 5.4, one force of the pair acts on the person, and the other acts on the boat.

For the TV in Figure 5.7, the force of gravity  $\mathbf{F}_g$  and the normal force  $\mathbf{n}$  are *not* an action–reaction pair because they act on the same body—the TV. The two reaction forces in this situation— $\mathbf{F}'_g$  and  $\mathbf{n}'$ —are exerted on objects other than the TV. Because the reaction to  $\mathbf{F}_g$  is the force  $\mathbf{F}'_g$  exerted by the TV on the Earth and the reaction to  $\mathbf{n}$  is the force  $\mathbf{n}'$  exerted by the TV on the table, we conclude that

$$\mathbf{F}_g = -\mathbf{F}'_g \quad \text{and} \quad \mathbf{n} = -\mathbf{n}'$$

<sup>4</sup> Normal in this context means *perpendicular*.

Definition of normal force



**Figure 5.7** When a TV is at rest on a table, the forces acting on the TV are the normal force  $\mathbf{n}$  and the force of gravity  $\mathbf{F}_g$ , as illustrated in part (b). The reaction to  $\mathbf{n}$  is the force  $\mathbf{n}'$  exerted by the TV on the table. The reaction to  $\mathbf{F}_g$  is the force  $\mathbf{F}'_g$  exerted by the TV on the Earth.

The forces  $\mathbf{n}$  and  $\mathbf{n}'$  have the same magnitude, which is the same as that of  $\mathbf{F}_g$  until the table breaks. From the second law, we see that, because the TV is in equilibrium ( $\mathbf{a} = 0$ ), it follows<sup>5</sup> that  $F_g = n = mg$ .

### Quick Quiz 5.5

If a fly collides with the windshield of a fast-moving bus, (a) which experiences the greater impact force: the fly or the bus, or is the same force experienced by both? (b) Which experiences the greater acceleration: the fly or the bus, or is the same acceleration experienced by both?

### CONCEPTUAL EXAMPLE 5.3 You Push Me and I'll Push You

A large man and a small boy stand facing each other on frictionless ice. They put their hands together and push against each other so that they move apart. (a) Who moves away with the higher speed?

**Solution** This situation is similar to what we saw in Quick Quiz 5.5. According to Newton's third law, the force exerted by the man on the boy and the force exerted by the boy on the man are an action–reaction pair, and so they must be equal in magnitude. (A bathroom scale placed between their hands would read the same, regardless of which way it faced.)

Therefore, the boy, having the lesser mass, experiences the greater acceleration. Both individuals accelerate for the same amount of time, but the greater acceleration of the boy over this time interval results in his moving away from the interaction with the higher speed.

(b) Who moves farther while their hands are in contact?

**Solution** Because the boy has the greater acceleration, he moves farther during the interval in which the hands are in contact.

<sup>5</sup> Technically, we should write this equation in the component form  $F_{gy} = n_y = mg_y$ . This component notation is cumbersome, however, and so in situations in which a vector is parallel to a coordinate axis, we usually do not include the subscript for that axis because there is no other component.

## 5.7 SOME APPLICATIONS OF NEWTON'S LAWS

**4.6** In this section we apply Newton's laws to objects that are either in equilibrium ( $\mathbf{a} = 0$ ) or accelerating along a straight line under the action of constant external forces. We assume that the objects behave as particles so that we need not worry about rotational motion. We also neglect the effects of friction in those problems involving motion; this is equivalent to stating that the surfaces are *frictionless*. Finally, we usually neglect the mass of any ropes involved. In this approximation, the magnitude of the force exerted at any point along a rope is the same at all points along the rope. In problem statements, the synonymous terms *light*, *lightweight*, and *of negligible mass* are used to indicate that a mass is to be ignored when you work the problems.

**When we apply Newton's laws to an object, we are interested only in external forces that act on the object.** For example, in Figure 5.7 the only external forces acting on the TV are  $\mathbf{n}$  and  $\mathbf{F}_g$ . The reactions to these forces,  $\mathbf{n}'$  and  $\mathbf{F}'_g$ , act on the table and on the Earth, respectively, and therefore do not appear in Newton's second law applied to the TV.

When a rope attached to an object is pulling on the object, the rope exerts a force  $\mathbf{T}$  on the object, and the magnitude of that force is called the **tension** in the rope. Because it is the magnitude of a vector quantity, tension is a scalar quantity.

Consider a crate being pulled to the right on a frictionless, horizontal surface, as shown in Figure 5.8a. Suppose you are asked to find the acceleration of the crate and the force the floor exerts on it. First, note that the horizontal force being applied to the crate acts through the rope. Use the symbol  $\mathbf{T}$  to denote the force exerted by the rope on the crate. The magnitude of  $\mathbf{T}$  is equal to the tension in the rope. A dotted circle is drawn around the crate in Figure 5.8a to remind you that you are interested only in the forces acting on the crate. These are illustrated in Figure 5.8b. In addition to the force  $\mathbf{T}$ , this force diagram for the crate includes the force of gravity  $\mathbf{F}_g$  and the normal force  $\mathbf{n}$  exerted by the floor on the crate. Such a force diagram, referred to as a **free-body diagram**, shows all external forces acting on the object. The construction of a correct free-body diagram is an important step in applying Newton's laws. The *reactions* to the forces we have listed—namely, the force exerted by the crate on the rope, the force exerted by the crate on the Earth, and the force exerted by the crate on the floor—are *not* included in the free-body diagram because they act on *other* bodies and not on the crate.

We can now apply Newton's second law in component form to the crate. The only force acting in the  $x$  direction is  $\mathbf{T}$ . Applying  $\sum F_x = ma_x$  to the horizontal motion gives

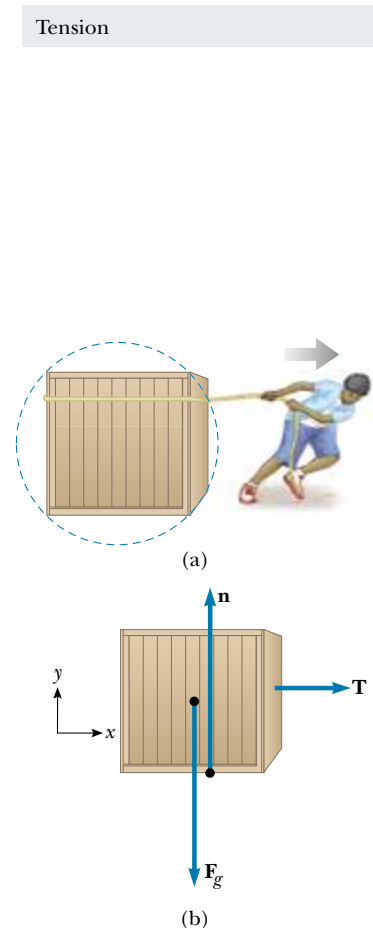
$$\sum F_x = T = ma_x \quad \text{or} \quad a_x = \frac{T}{m}$$

No acceleration occurs in the  $y$  direction. Applying  $\sum F_y = ma_y$  with  $a_y = 0$  yields

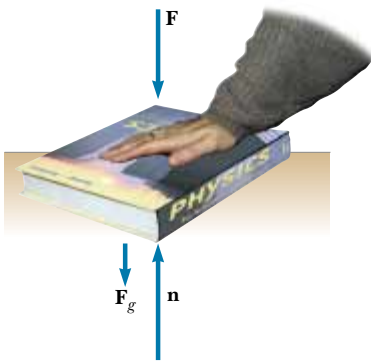
$$n + (-F_g) = 0 \quad \text{or} \quad n = F_g$$

That is, the normal force has the same magnitude as the force of gravity but is in the opposite direction.

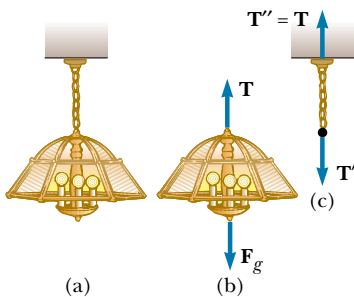
If  $\mathbf{T}$  is a constant force, then the acceleration  $a_x = T/m$  also is constant. Hence, the constant-acceleration equations of kinematics from Chapter 2 can be used to obtain the crate's displacement  $\Delta x$  and velocity  $v_x$  as functions of time. Be-



**Figure 5.8** (a) A crate being pulled to the right on a frictionless surface. (b) The free-body diagram representing the external forces acting on the crate.



**Figure 5.9** When one object pushes downward on another object with a force  $\mathbf{F}$ , the normal force  $\mathbf{n}$  is greater than the force of gravity:  $n = F_g + F$ .



**Figure 5.10** (a) A lamp suspended from a ceiling by a chain of negligible mass. (b) The forces acting on the lamp are the force of gravity  $\mathbf{F}_g$  and the force exerted by the chain  $\mathbf{T}$ . (c) The forces acting on the chain are the force exerted by the lamp  $\mathbf{T}'$  and the force exerted by the ceiling  $\mathbf{T}''$ .

cause  $a_x = T/m = \text{constant}$ , Equations 2.8 and 2.11 can be written as

$$v_{xf} = v_{xi} + \left(\frac{T}{m}\right)t$$

$$\Delta x = v_{xi}t + \frac{1}{2}\left(\frac{T}{m}\right)t^2$$

In the situation just described, the magnitude of the normal force  $\mathbf{n}$  is equal to the magnitude of  $\mathbf{F}_g$ , but this is not always the case. For example, suppose a book is lying on a table and you push down on the book with a force  $\mathbf{F}$ , as shown in Figure 5.9. Because the book is at rest and therefore not accelerating,  $\Sigma F_y = 0$ , which gives  $n - F_g - F = 0$ , or  $n = F_g + F$ . Other examples in which  $n \neq F_g$  are presented later.

Consider a lamp suspended from a light chain fastened to the ceiling, as in Figure 5.10a. The free-body diagram for the lamp (Figure 5.10b) shows that the forces acting on the lamp are the downward force of gravity  $\mathbf{F}_g$  and the upward force  $\mathbf{T}$  exerted by the chain. If we apply the second law to the lamp, noting that  $\mathbf{a} = 0$ , we see that because there are no forces in the  $x$  direction,  $\Sigma F_x = 0$  provides no helpful information. The condition  $\Sigma F_y = ma_y = 0$  gives

$$\Sigma F_y = T - F_g = 0 \quad \text{or} \quad T = F_g$$

Again, note that  $\mathbf{T}$  and  $\mathbf{F}_g$  are *not* an action–reaction pair because they act on the same object—the lamp. The reaction force to  $\mathbf{T}$  is  $\mathbf{T}'$ , the downward force exerted by the lamp on the chain, as shown in Figure 5.10c. The ceiling exerts on the chain a force  $\mathbf{T}''$  that is equal in magnitude to the magnitude of  $\mathbf{T}'$  and points in the opposite direction.

## Problem-Solving Hints

### Applying Newton's Laws

The following procedure is recommended when dealing with problems involving Newton's laws:

- Draw a simple, neat diagram of the system.
- Isolate the object whose motion is being analyzed. Draw a free-body diagram for this object. For systems containing more than one object, draw *separate* free-body diagrams for each object. *Do not* include in the free-body diagram forces exerted by the object on its surroundings. Establish convenient coordinate axes for each object and find the components of the forces along these axes.
- Apply Newton's second law,  $\Sigma \mathbf{F} = m\mathbf{a}$ , in component form. Check your dimensions to make sure that all terms have units of force.
- Solve the component equations for the unknowns. Remember that you must have as many independent equations as you have unknowns to obtain a complete solution.
- Make sure your results are consistent with the free-body diagram. Also check the predictions of your solutions for extreme values of the variables. By doing so, you can often detect errors in your results.

**EXAMPLE 5.4** A Traffic Light at Rest

A traffic light weighing 125 N hangs from a cable tied to two other cables fastened to a support. The upper cables make angles of  $37.0^\circ$  and  $53.0^\circ$  with the horizontal. Find the tension in the three cables.

**Solution** Figure 5.11a shows the type of drawing we might make of this situation. We then construct two free-body diagrams—one for the traffic light, shown in Figure 5.11b, and one for the knot that holds the three cables together, as seen in Figure 5.11c. This knot is a convenient object to choose because all the forces we are interested in act through it. Because the acceleration of the system is zero, we know that the net force on the light and the net force on the knot are both zero.

In Figure 5.11b the force  $\mathbf{T}_3$  exerted by the vertical cable supports the light, and so  $T_3 = F_g = 125 \text{ N}$ . Next, we choose the coordinate axes shown in Figure 5.11c and resolve the forces acting on the knot into their components:

Force	x Component	y Component
$\mathbf{T}_1$	$-T_1 \cos 37.0^\circ$	$T_1 \sin 37.0^\circ$
$\mathbf{T}_2$	$T_2 \cos 53.0^\circ$	$T_2 \sin 53.0^\circ$
$\mathbf{T}_3$	0	$-125 \text{ N}$

Knowing that the knot is in equilibrium ( $\mathbf{a} = 0$ ) allows us to write

$$(1) \quad \sum F_x = -T_1 \cos 37.0^\circ + T_2 \cos 53.0^\circ = 0$$

$$(2) \quad \sum F_y = T_1 \sin 37.0^\circ + T_2 \sin 53.0^\circ + (-125 \text{ N}) = 0$$

From (1) we see that the horizontal components of  $\mathbf{T}_1$  and  $\mathbf{T}_2$  must be equal in magnitude, and from (2) we see that the sum of the vertical components of  $\mathbf{T}_1$  and  $\mathbf{T}_2$  must balance the weight of the light. We solve (1) for  $T_2$  in terms of  $T_1$  to obtain

$$T_2 = T_1 \left( \frac{\cos 37.0^\circ}{\cos 53.0^\circ} \right) = 1.33 T_1$$

This value for  $T_2$  is substituted into (2) to yield

$$T_1 \sin 37.0^\circ + (1.33 T_1)(\sin 53.0^\circ) - 125 \text{ N} = 0$$

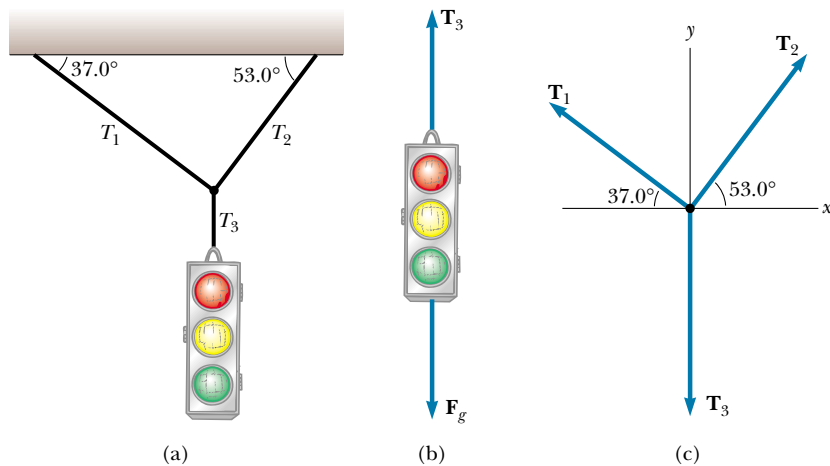
$$T_1 = 75.1 \text{ N}$$

$$T_2 = 1.33 T_1 = 99.9 \text{ N}$$

This problem is important because it combines what we have learned about vectors with the new topic of forces. The general approach taken here is very powerful, and we will repeat it many times.

**Exercise** In what situation does  $T_1 = T_2$ ?

**Answer** When the two cables attached to the support make equal angles with the horizontal.



**Figure 5.11** (a) A traffic light suspended by cables. (b) Free-body diagram for the traffic light. (c) Free-body diagram for the knot where the three cables are joined.

**CONCEPTUAL EXAMPLE 5.5** Forces Between Cars in a Train

In a train, the cars are connected by *couplers*, which are under tension as the locomotive pulls the train. As you move down the train from locomotive to caboose, does the tension in the couplers increase, decrease, or stay the same as the train speeds up? When the engineer applies the brakes, the couplers are under compression. How does this compression force vary from locomotive to caboose? (Assume that only the brakes on the wheels of the engine are applied.)

**Solution** As the train speeds up, the tension decreases from the front of the train to the back. The coupler between

the locomotive and the first car must apply enough force to accelerate all of the remaining cars. As you move back along the train, each coupler is accelerating less mass behind it. The last coupler has to accelerate only the caboose, and so it is under the least tension.

When the brakes are applied, the force again decreases from front to back. The coupler connecting the locomotive to the first car must apply a large force to slow down all the remaining cars. The final coupler must apply a force large enough to slow down only the caboose.

**EXAMPLE 5.6** Crate on a Frictionless Incline

A crate of mass  $m$  is placed on a frictionless inclined plane of angle  $\theta$ . (a) Determine the acceleration of the crate after it is released.

**Solution** Because we know the forces acting on the crate, we can use Newton's second law to determine its acceleration. (In other words, we have classified the problem; this gives us a hint as to the approach to take.) We make a sketch as in Figure 5.12a and then construct the free-body diagram for the crate, as shown in Figure 5.12b. The only forces acting on the crate are the normal force  $\mathbf{n}$  exerted by the inclined plane, which acts perpendicular to the plane, and the force of gravity  $\mathbf{F}_g = m\mathbf{g}$ , which acts vertically downward. For problems involving inclined planes, it is convenient to choose the coordinate axes with  $x$  downward along the incline and  $y$  perpendicular to it, as shown in Figure 5.12b. (It is possible to solve the problem with "standard" horizontal and vertical axes. You may want to try this, just for practice.) Then, we re-

place the force of gravity by a component of magnitude  $mg \sin \theta$  along the positive  $x$  axis and by one of magnitude  $mg \cos \theta$  along the negative  $y$  axis.

Now we apply Newton's second law in component form, noting that  $a_y = 0$ :

$$(1) \quad \sum F_x = mg \sin \theta = ma_x$$

$$(2) \quad \sum F_y = n - mg \cos \theta = 0$$

Solving (1) for  $a_x$ , we see that the acceleration along the incline is caused by the component of  $\mathbf{F}_g$  directed down the incline:

$$(3) \quad a_x = g \sin \theta$$

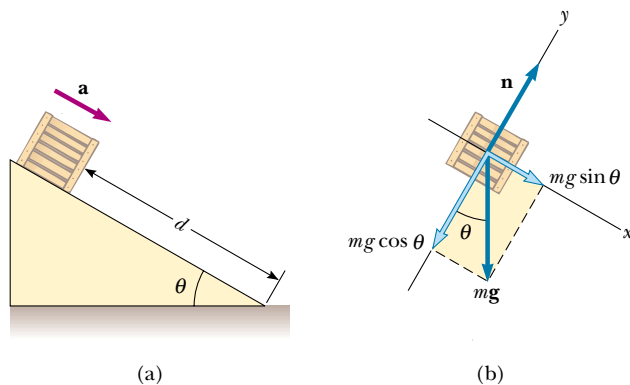
Note that this acceleration component is independent of the mass of the crate! It depends only on the angle of inclination and on  $g$ .

From (2) we conclude that the component of  $\mathbf{F}_g$  perpendicular to the incline is balanced by the normal force; that is,  $n = mg \cos \theta$ . This is one example of a situation in which the normal force is *not* equal in magnitude to the weight of the object.

**Special Cases** Looking over our results, we see that in the extreme case of  $\theta = 90^\circ$ ,  $a_x = g$  and  $n = 0$ . This condition corresponds to the crate's being in free fall. When  $\theta = 0$ ,  $a_x = 0$  and  $n = mg$  (its maximum value); in this case, the crate is sitting on a horizontal surface.

(b) Suppose the crate is released from rest at the top of the incline, and the distance from the front edge of the crate to the bottom is  $d$ . How long does it take the front edge to reach the bottom, and what is its speed just as it gets there?

**Solution** Because  $a_x = g \sin \theta$  is constant, we can apply Equation 2.11,  $x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$ , to analyze the crate's motion.



**Figure 5.12** (a) A crate of mass  $m$  sliding down a frictionless incline. (b) The free-body diagram for the crate. Note that its acceleration along the incline is  $g \sin \theta$ .



With the displacement  $x_f - x_i = d$  and  $v_{xi} = 0$ , we obtain

$$d = \frac{1}{2}a_x t^2$$

$$(4) \quad t = \sqrt{\frac{2d}{a_x}} = \sqrt{\frac{2d}{g \sin \theta}}$$

Using Equation 2.12,  $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$ , with  $v_{xi} = 0$ , we find that

$$v_{xf}^2 = 2a_x d$$

$$(5) \quad v_{xf} = \sqrt{2a_x d} = \sqrt{2gd \sin \theta}$$

We see from equations (4) and (5) that the time  $t$  needed to reach the bottom and the speed  $v_{xf}$ , like acceleration, are independent of the crate's mass. This suggests a simple method you can use to measure  $g$ , using an inclined air track: Measure the angle of inclination, some distance traveled by a cart along the incline, and the time needed to travel that distance. The value of  $g$  can then be calculated from (4).

### EXAMPLE 5.7 One Block Pushes Another

Two blocks of masses  $m_1$  and  $m_2$  are placed in contact with each other on a frictionless horizontal surface. A constant horizontal force  $\mathbf{F}$  is applied to the block of mass  $m_1$ . (a) Determine the magnitude of the acceleration of the two-block system.

**Solution** Common sense tells us that both blocks must experience the same acceleration because they remain in contact with each other. Just as in the preceding example, we make a labeled sketch and free-body diagrams, which are shown in Figure 5.13. In Figure 5.13a the dashed line indicates that we treat the two blocks together as a system. Because  $\mathbf{F}$  is the only external horizontal force acting on the system (the two blocks), we have

$$\sum F_x(\text{system}) = F = (m_1 + m_2)a_x$$

$$(1) \quad a_x = \frac{F}{m_1 + m_2}$$

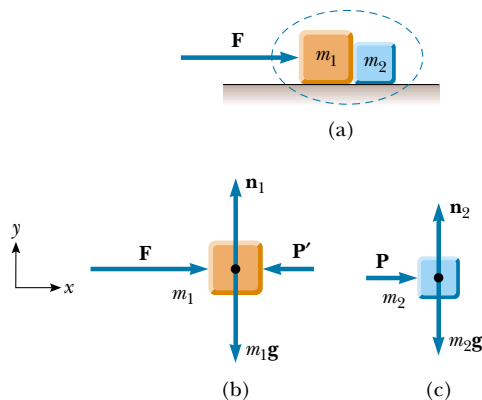


Figure 5.13

Treating the two blocks together as a system simplifies the solution but does not provide information about internal forces.

(b) Determine the magnitude of the contact force between the two blocks.

**Solution** To solve this part of the problem, we must treat each block separately with its own free-body diagram, as in Figures 5.13b and 5.13c. We denote the contact force by  $\mathbf{P}$ . From Figure 5.13c, we see that the only horizontal force acting on block 2 is the contact force  $\mathbf{P}$  (the force exerted by block 1 on block 2), which is directed to the right. Applying Newton's second law to block 2 gives

$$(2) \quad \sum F_x = P = m_2 a_x$$

Substituting into (2) the value of  $a_x$  given by (1), we obtain

$$(3) \quad P = m_2 a_x = \left( \frac{m_2}{m_1 + m_2} \right) F$$

From this result, we see that the contact force  $\mathbf{P}$  exerted by block 1 on block 2 is *less* than the applied force  $\mathbf{F}$ . This is consistent with the fact that the force required to accelerate block 2 alone must be less than the force required to produce the same acceleration for the two-block system.

It is instructive to check this expression for  $P$  by considering the forces acting on block 1, shown in Figure 5.13b. The horizontal forces acting on this block are the applied force  $\mathbf{F}$  to the right and the contact force  $\mathbf{P}'$  to the left (the force exerted by block 2 on block 1). From Newton's third law,  $\mathbf{P}'$  is the reaction to  $\mathbf{P}$ , so that  $|\mathbf{P}'| = |\mathbf{P}|$ . Applying Newton's second law to block 1 produces

$$(4) \quad \sum F_x = F - P' = F - P = m_1 a_x$$