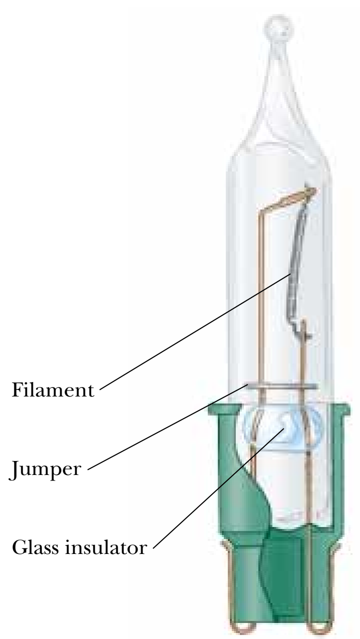
it failed, each bulb would represent a parallel circuit; in this circuit, the current would flow through the alternate connection, forming a short circuit, and the bulb would not glow.) When the filament breaks in one of these miniature lightbulbs, 120 V appears across the bulb because no current is present in the bulb and therefore no drop in potential occurs across the other bulbs. Inside the lightbulb, a small loop covered by an insulating material is wrapped around the filament leads. An arc burns the insulation and connects the filament leads when 120 V appears across the bulb—that is, when the filament fails. This "short" now completes the circuit through the bulb even though the filament is no longer active (Fig. 28.10).

Suppose that all the bulbs in a 50-bulb miniature-light string are operating. A 2.4-V potential drop occurs across each bulb because the bulbs are in series. The power input to this style of bulb is 0.34 W, so the total power supplied to the string is only 17 W. We calculate the filament resistance at the operating temperature to be $(2.4\text{ V})^2/(0.34\text{ W}) = 17\ \Omega$. When the bulb fails, the resistance across its terminals is reduced to zero because of the alternate jumper connection mentioned in the preceding paragraph. All the other bulbs not only stay on but glow more brightly because the total resistance of the string is reduced and consequently the current in each bulb increases.

Let us assume that the operating resistance of a bulb remains at 17 Ω even though its temperature rises as a result of the increased current. If one bulb fails, the potential drop across each of the remaining bulbs increases to 2.45 V, the current increases from 0.142 A to 0.145 A, and the power increases to 0.354 W. As more lights fail, the current keeps rising, the filament of each bulb operates at a higher temperature, and the lifetime of the bulb is reduced. It is therefore a good idea to check for failed (nonglowing) bulbs in such a series-wired string and replace them as soon as possible, in order to maximize the lifetimes of all the bulbs.



(a)

(b)

Figure 28.10 (a) Schematic diagram of a modern "miniature" holiday lightbulb, with a jumper connection to provide a current path if the filament breaks. (b) A Christmas-tree lightbulb.

28.3 KIRCHHOFF'S RULES

13.4 As we saw in the preceding section, we can analyze simple circuits using the expression $\Delta V = IR$ and the rules for series and parallel combinations of resistors. Very often, however, it is not possible to reduce a circuit to a single loop. The procedure for analyzing more complex circuits is greatly simplified if we use two principles called **Kirchhoff's rules:**

1. The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

$$\sum I_{\text{in}} = \sum I_{\text{out}} \tag{28.9}$$

2. The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{\text{closed loop}} \Delta V = 0 \qquad \textbf{(28.10)}$$

Gustav Kirchhoff (1824–1887) Kirchhoff, a professor at Heidelberg, Germany, and Robert Bunsen invented the spectroscope and founded the science of spectroscopy, which we shall study in Chapter 40. They discovered the elements cesium and rubidium and invented astronomical spectroscopy. Kirchhoff formulated another Kirchhoff's rule, namely, "a cool substance will absorb light of the same wavelengths that it emits when hot." *(AIP ESVA/W. F. Meggers Collection)*

Kirchhoff's first rule is a statement of conservation of electric charge. All current that enters a given point in a circuit must leave that point because charge cannot build up at a point. If we apply this rule to the junction shown in Figure 28.11a, we obtain

$$I_1 = I_2 + I_3$$

Figure 28.11b represents a mechanical analog of this situation, in which water flows through a branched pipe having no leaks. The flow rate into the pipe equals the total flow rate out of the two branches on the right.

Kirchhoff's second rule follows from the law of conservation of energy. Let us imagine moving a charge around the loop. When the charge returns to the starting point, the charge–circuit system must have the same energy as when the charge started from it. The sum of the increases in energy in some circuit elements must equal the sum of the decreases in energy in other elements. The potential energy decreases whenever the charge moves through a potential drop $-IR$ across a resistor or whenever it moves in the reverse direction through a source of emf. The potential energy increases whenever the charge passes through a battery from the negative terminal to the positive terminal. Kirchhoff's second rule applies only for circuits in which an electric potential is defined at each point; this criterion may not be satisfied if changing electromagnetic fields are present, as we shall see in Chapter 31.

QuickLab

Draw an arbitrarily shaped closed loop that does not cross over itself. Label five points on the loop *a*, *b*, *c*, *d*, and *e*, and assign a random number to each point. Now start at *a* and work your way around the loop, calculating the difference between each pair of adjacent numbers. Some of these differences will be positive, and some will be negative. Add the differences together, making sure you accurately keep track of the algebraic signs. What is the sum of the differences all the way around the loop?

In justifying our claim that Kirchhoff's second rule is a statement of conservation of energy, we imagined carrying a charge around a loop. When applying this rule, we imagine *traveling* around the loop and consider changes in *electric potential,* rather than the changes in *potential energy* described in the previous paragraph. You should note the following sign conventions when using the second rule:

- Because charges move from the high-potential end of a resistor to the low-potential end, if a resistor is traversed in the direction of the current, the change in potential ΔV across the resistor is $-IR$ (Fig. 28.12a).
- If a resistor is traversed in the direction *opposite* the current, the change in potential ΔV across the resistor is $+IR$ (Fig. 28.12b).
- If a source of emf (assumed to have zero internal resistance) is traversed in the direction of the emf (from $-$ to $+$), the change in potential ΔV is $+\mathcal{E}$ (Fig. 28.12c). The emf of the battery increases the electric potential as we move through it in this direction.
- If a source of emf (assumed to have zero internal resistance) is traversed in the direction opposite the emf (from $+$ to $-$), the change in potential ΔV is $-\mathcal{E}$ (Fig. 28.12d). In this case the emf of the battery reduces the electric potential as we move through it.

Limitations exist on the numbers of times you can usefully apply Kirchhoff's rules in analyzing a given circuit. You can use the junction rule as often as you need, so long as each time you write an equation you include in it a current that has not been used in a preceding junction-rule equation. In general, the number of times you can use the junction rule is one fewer than the number of junction

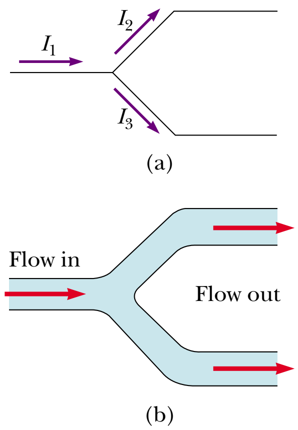


Figure 28.11 (a) Kirchhoff's junction rule. Conservation of charge requires that all current entering a junction must leave that junction. Therefore, $I_1 = I_2 + I_3$. (b) A mechanical analog of the junction rule: the amount of water flowing out of the branches on the right must equal the amount flowing into the single branch on the left.

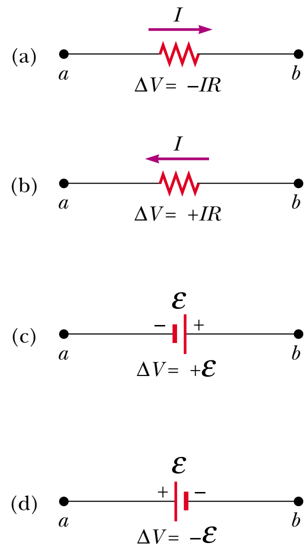


Figure 28.12 Rules for determining the potential changes across a resistor and a battery. (The battery is assumed to have no internal resistance.) Each circuit element is traversed from left to right.

points in the circuit. You can apply the loop rule as often as needed, so long as a new circuit element (resistor or battery) or a new current appears in each new equation. In general, **in order to solve a particular circuit problem, the number of independent equations you need to obtain from the two rules equals the number of unknown currents.**

Complex networks containing many loops and junctions generate great numbers of independent linear equations and a correspondingly great number of unknowns. Such situations can be handled formally through the use of matrix algebra. Computer programs can also be written to solve for the unknowns.

The following examples illustrate how to use Kirchhoff's rules. In all cases, it is assumed that the circuits have reached steady-state conditions—that is, the currents in the various branches are constant. Any capacitor **acts as an open circuit;** that is, the current in the branch containing the capacitor is zero under steady-state conditions.

Problem-Solving Hints

Kirchhoff's Rules

- Draw a circuit diagram, and label all the known and unknown quantities. You must assign a *direction* to the current in each branch of the circuit. Do not be alarmed if you guess the direction of a current incorrectly; your result will be negative, but *its magnitude will be correct.* Although the assignment of current directions is arbitrary, you must adhere rigorously to the assigned directions when applying Kirchhoff's rules.
- Apply the junction rule to any junctions in the circuit that provide new relationships among the various currents.

- Apply the loop rule to as many loops in the circuit as are needed to solve for the unknowns. To apply this rule, you must correctly identify the change in potential as you imagine crossing each element in traversing the closed loop (either clockwise or counterclockwise). Watch out for errors in sign!
- Solve the equations simultaneously for the unknown quantities.

EXAMPLE 28.7 A Single-Loop Circuit

A single-loop circuit contains two resistors and two batteries, as shown in Figure 28.13. (Neglect the internal resistances of the batteries.) (a) Find the current in the circuit.

Solution We do not need Kirchhoff's rules to analyze this simple circuit, but let us use them anyway just to see how they are applied. There are no junctions in this single-loop circuit; thus, the current is the same in all elements. Let us assume that the current is clockwise, as shown in Figure 28.13. Traversing the circuit in the clockwise direction, starting at *a*, we see that $a \rightarrow b$ represents a potential change of $+\mathcal{E}_1$, $b \rightarrow c$ represents a potential change of $-IR_1$, $c \rightarrow d$ represents a potential change of $-\mathcal{E}_2$, and $d \rightarrow a$ represents a potential change of $-IR_2$. Applying Kirchhoff's loop rule gives

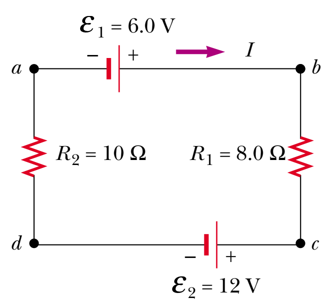


Figure 28.13 A series circuit containing two batteries and two resistors, where the polarities of the batteries are in opposition.

$$\sum \Delta V = 0$$

$$\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$$

Solving for I and using the values given in Figure 28.13, we obtain

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{6.0\text{ V} - 12\text{ V}}{8.0\ \Omega + 10\ \Omega} = \boxed{-0.33\text{ A}}$$

The negative sign for I indicates that the direction of the current is opposite the assumed direction.

(b) What power is delivered to each resistor? What power is delivered by the 12-V battery?

Solution

$$\mathcal{P}_1 = I^2R_1 = (0.33\text{ A})^2(8.0\ \Omega) = \boxed{0.87\text{ W}}$$

$$\mathcal{P}_2 = I^2R_2 = (0.33\text{ A})^2(10\ \Omega) = \boxed{1.1\text{ W}}$$

Hence, the total power delivered to the resistors is $\mathcal{P}_1 + \mathcal{P}_2 = 2.0$ W.

The 12-V battery delivers power $I\mathcal{E}_2 = 4.0$ W. Half of this power is delivered to the two resistors, as we just calculated. The other half is delivered to the 6-V battery, which is being charged by the 12-V battery. If we had included the internal resistances of the batteries in our analysis, some of the power would appear as internal energy in the batteries; as a result, we would have found that less power was being delivered to the 6-V battery.

EXAMPLE 28.8 Applying Kirchhoff's Rules

Find the currents I_1, I_2, and I_3 in the circuit shown in Figure 28.14.

Solution Notice that we cannot reduce this circuit to a simpler form by means of the rules of adding resistances in series and in parallel. We must use Kirchhoff's rules to analyze this circuit. We arbitrarily choose the directions of the currents as labeled in Figure 28.14. Applying Kirchhoff's junction rule to junction *c* gives

$$(1) \qquad I_1 + I_2 = I_3$$

We now have one equation with three unknowns—I_1, I_2, and I_3. There are three loops in the circuit—*abcda, befcb,* and *aefda*. We therefore need only two loop equations to determine the unknown currents. (The third loop equation would give no new information.) Applying Kirchhoff's loop rule to loops *abcda* and *befcb* and traversing these loops clockwise, we obtain the expressions

$$(2) \qquad abcda \qquad 10\text{ V} - (6\ \Omega)I_1 - (2\ \Omega)I_3 = 0$$

$$(3) \qquad befcb \qquad -14\text{ V} + (6\ \Omega)I_1 - 10\text{ V} - (4\ \Omega)I_2 = 0$$

Note that in loop *befcb* we obtain a positive value when traversing the 6-Ω resistor because our direction of travel is opposite the assumed direction of I_1.

Expressions (1), (2), and (3) represent three independent equations with three unknowns. Substituting Equation (1) into Equation (2) gives

$$10\text{ V} - (6\ \Omega)I_1 - (2\ \Omega)\ (I_1 + I_2) = 0$$

$$(4) \qquad 10\text{ V} = (8\ \Omega)I_1 + (2\ \Omega)I_2$$

Dividing each term in Equation (3) by 2 and rearranging gives

$$(5) \qquad -12\text{ V} = -(3\ \Omega)I_1 + (2\ \Omega)I_2$$

Subtracting Equation (5) from Equation (4) eliminates I_2, giving

$$22\text{ V} = (11\ \Omega)I_1$$

$$I_1 = \boxed{2\text{ A}}$$

Using this value of I_1 in Equation (5) gives a value for I_2:

$$(2\ \Omega)I_2 = (3\ \Omega)I_1 - 12\text{ V} = (3\ \Omega)\ (2\text{ A}) - 12\text{ V} = -6\text{ V}$$

$$I_2 = \boxed{-3\text{ A}}$$

Finally,

$$I_3 = I_1 + I_2 = \boxed{-1\text{ A}}$$

The fact that I_2 and I_3 are both negative indicates only that the currents are opposite the direction we chose for them. However, the numerical values are correct. What would have happened had we left the current directions as labeled in Figure 28.14 but traversed the loops in the opposite direction?

Exercise Find the potential difference between points *b* and *c*.

Answer 2 V.

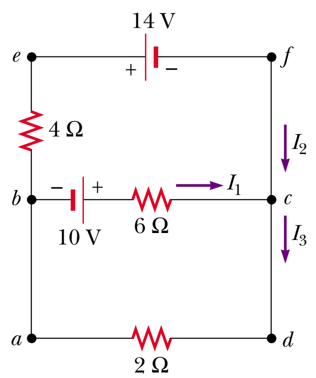


Figure 28.14 A circuit containing three loops.

EXAMPLE 28.9 A Multiloop Circuit

(a) Under steady-state conditions, find the unknown currents I_1, I_2, and I_3 in the multiloop circuit shown in Figure 28.15.

Solution First note that because the capacitor represents an open circuit, there is no current between *g* and *b* along path *ghab* under steady-state conditions. Therefore, when the charges associated with I_1 reach point *g*, they all go through the 8.00-V battery to point *b*; hence, $I_{gb} = I_1$. Labeling the currents as shown in Figure 28.15 and applying Equation 28.9 to junction *c*, we obtain

$$(1) \quad I_1 + I_2 = I_3$$

Equation 28.10 applied to loops *defcd* and *cfgbc*, traversed clockwise, gives

$$(2) \quad defcd \qquad 4.00\text{ V} - (3.00\ \Omega)I_2 - (5.00\ \Omega)I_3 = 0$$

$$(3) \quad cfgbc \qquad (3.00\ \Omega)I_2 - (5.00\ \Omega)I_1 + 8.00\text{ V} = 0$$

From Equation (1) we see that $I_1 = I_3 - I_2$, which, when substituted into Equation (3), gives

$$(4) \quad (8.00\ \Omega)I_2 - (5.00\ \Omega)I_3 + 8.00\text{ V} = 0$$

Subtracting Equation (4) from Equation (2), we eliminate I_3 and find that

$$I_2 = -\frac{4.00\text{ V}}{11.0\ \Omega} = \boxed{-0.364\text{ A}}$$

Because our value for I_2 is negative, we conclude that the direction of I_2 is from *c* to *f* through the 3.00-Ω resistor. Despite

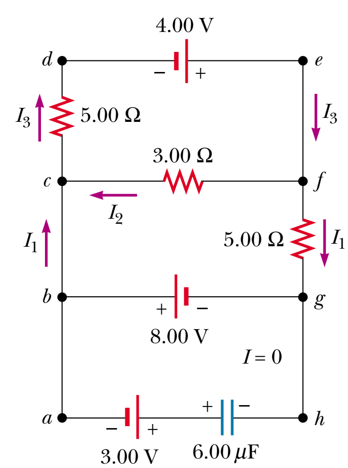


Figure 28.15 A multiloop circuit. Kirchhoff's loop rule can be applied to *any* closed loop, including the one containing the capacitor.