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Vertical nonlinear oscillations of viscoelastic systems with multiple degrees of freedom

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Vertical nonlinear oscillations of viscoelastic systems with multiple degrees of freedom

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Abstract. Vertical oscillations of three loads of different masses connected by nonlinear viscoelastic springs (suspensions) are considered in the paper. An account for rheological properties of the suspension, an integral model with the Koltunov-Rzhanitsyn relaxation kernel is used. Effective computational algorithms have been developed for solving problems based on the use of quadrature formulas. The effect of rheological properties of suspension on the mass displacement from the position of static equilibrium is investigated as well as the influence of nonlinear properties of the suspension on the mode of vibration and frequency.

1. Introduction

An account for several modes of vibration in nonlinear viscoelastic systems is much more complicated than in the systems with one degree of freedom, but the results of such an analysis are of undoubted interest. Note that one of the important applications of the effect of change in dissipative characteristics of mechanical systems is the problem of viscoelastic systems with several degrees of freedom.

2. Problem statement

Let us consider the vertical oscillations of three loads (Fig. 1) of masses m_1, m_2 and m_3 connected by nonlinear viscoelastic suspensions. Denote the displacements of masses m_1, m_2 and m_3 from the position of static equilibrium by x_1, x_2 and x_3, and the action force on the suspension mass - by F (z). Using the d'Alembert principle and considering the mass fictitious equilibrium (Fig. 2), to which the inertial forces and restoring forces are applied, we obtain [1, 2]:

$$m_1\ddot{x}_1 + F(x_1) - F(x_2 - x_1) = 0,$$

$$m_2\ddot{x}_2 + F(x_2 - x_1) - F(x_3 - x_2) = 0, \tag{1}$$

$$m_3\ddot{x}_3 + F(x_3 - x_2) = 0.$$

For function $F(z)$ the following expression is used [3-6]:

$$F(z) = k\left\{z(1 + \gamma z^2) - \int_0^t R(t,\tau)z(\tau)[1 + \gamma z^2(\tau)d\tau]\right\}, \qquad (2)$$

where k is the stiffness of suspension; γ is the coefficient of nonlinearity, depending on physical properties of the suspension material; R (t, τ) is the relaxation kernel.
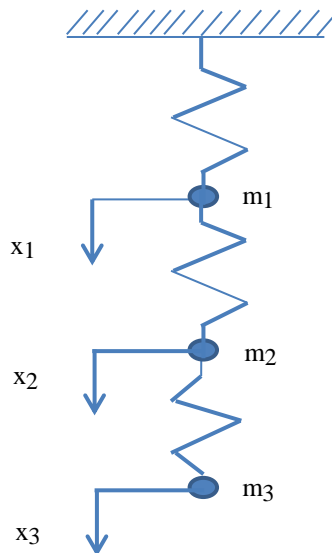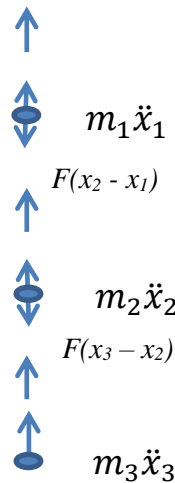

Figure 1. Vertical oscillations of three loads

Figure 2. The mass fictitious equilibrium

If k_1, k_2 and k_3 are the stiffness of the first, second and third suspension, then, with (2), the system (1) has the form[3,4]:

$$\ddot{x}_1 + \omega_1^2 x_1 = \omega_{21}^2(x_2 - x_1)[1 + \gamma_2(x_2 - x_1)^2] - \omega_1^2\gamma_1 x_1^2 +$$

$$+\omega_1^2 \int_0^t R_1(t,\tau)x_1(\tau)[1 + \gamma_1 x_1^2(\tau)]d\tau -$$

$$-\omega_{21}^2 \int_0^t R_2(t,\tau)[x_2(\tau) - x_1(\tau)]\{1 + \gamma_2[x_2(\tau) - x_1(\tau)]^2\}d\tau,$$

$$\ddot{x}_2 + \omega_2^2 x_2 = \omega_2^2 x_1[1 + \gamma_2(x_2 - x_1)^2] + \omega_{32}^2(x_3 - x_2)[1 + \gamma_3(x_3 - x_2)^2] - \quad (3)$$

$$-\omega_2^2\gamma_2 x_2(x_2 - x_1)^2 + \omega_2^2 \int_0^t R_2(t,\tau)[x_2(\tau) - x_1(\tau)] \cdot \{1 + \gamma_2[x_2(\tau) - x_1(\tau)]^2\}d\tau -$$

$$-\omega_{32}^2 \int_0^t R_3(t,\tau)[x_3(\tau) - x_2(\tau)] \cdot \{1 + \gamma_3[x_3(\tau) - x_2(\tau)]^2\}d\tau,$$

$$\ddot{x}_3 + \omega_3^2 x_3 = \omega_3^2 x_2[1 + \gamma_3(x_3 - x_2)^2] - \omega_3^2\gamma_3 x_3(x_3 - x_2)^2 +$$

$$+\omega_3^2 \int_0^t R_3(t,\tau)[x_3(\tau) - x_2(\tau)]\{1 + \gamma_3[x_3(\tau) - x_2(\tau)]^2\}d\tau,$$

where $\omega_i^2 = \dfrac{k_i}{m_i}$, $i = 1,2,3$; $\omega_{21}^2 = \dfrac{k_2}{m_1}$, $\omega_{32}^2 = \dfrac{k_3}{m_2}$.

Let the initial values of the mass displacements and their velocity be given as

$$x_i(0) = x_{i0}, \quad \dot{x}_i(0) = \vartheta_{i0}, \quad i = 1,2,3.$$

3. Methods
System (3) is solved by the methods based on the use of the quadrature formula [7-20]. Integrating system (3) twice over t on the interval [0; t] we have:

$$x_1(t) = x_{10} + \vartheta_{10}t + \int_0^t (t - s)\{\omega_{21}^2[x_2(s) - x_1(s)][1 + \gamma_2[x_2(s) - x_1(s)]^2] -$$

$$-\omega_1^2 x_1(s) - \omega_1^2\gamma_1 x_1^2(s)\}ds + \omega_1^2 \int_0^t \Gamma_1(t - s)x_1(s)[1 + \gamma_1 x_1^2(s)]ds -$$

$$-\omega_{21}^2 \int_0^t \Gamma_2(t - s)[x_2(s) - x_1(s)]\{1 + \gamma_2[x_2(s) - x_1(s)]^2\}ds,$$

$$x_2(t) = x_{20} + \vartheta_{20}t + \int_0^t (t - s)\{\omega_2^2 x_1(s)[1 + \gamma_2[x_2(s) - x_1(s)]^2] - \omega_2^2 x_2(s) +$$

$$+\omega_{32}^2[x_3(s) - x_2(s)]\{1 + \gamma_3[x_3(s) - x_2(s)]^2\} - \omega_2^2\gamma_2 x_2(s)[x_2(s) - x_1(s)]^2\}ds +$$

$$+\omega_2^2 \int_0^t \Gamma_2(t - s)[x_2(s) - x_1(s)]\{1 + \gamma_2[x_2(s) - x_1(s)]^2\}ds -$$

$$-\omega_{32}^2 \int_0^t \Gamma_3(t-s)[x_3(s) - x_2(s)]\{1 + \gamma_3[x_3(s) - x_2(s)]^2\}ds,$$

$$x_3(t) = x_{30} + \vartheta_{30}t + \int_0^t (t-s)\{\omega_2^2 x_2(s)[1 + \gamma_3[x_3(s) - x_2(s)]^2] - \omega_3^2 x_3(s) -$$

$$-\omega_3^2 \gamma_3 x_3(s)[x_3(s) - x_2(s)]^2\}ds + \omega_3^2 \int_0^t \Gamma_3(t-s)[x_3(s) - x_2(s)][1 + \gamma_3[x_3(s) - x_2(s)]^2]ds,$$

where

$$\Gamma_i(t-s) = \int_0^{t-s} (t-s-\tau)R_i(\tau)d\tau, \quad (i = 1,3); \quad R_i(t) = \varepsilon_i e^{-\beta_i t} \cdot t^{\alpha_i - 1}.$$

In the latter system, replacing the integrals with the quadrature formulas of the trapezoid, to determine the load displacement from the position of static equilibrium $x_{1i} = x_1(t_i)$, $x_{2i} = x_2(t_i)$ и $x_{3i} = x_3(t_i)$ ($i = 1,2,3,\dots$), we have the following recurrent ratio:

$$x_{1n} = x_{10} + \vartheta_{10}t_n + \sum_{i=0}^{n-1} A_i(t_n - t_i)\{\omega_{21}^2(x_{2i} - x_{1i})[1 + \gamma_2(x_{2i} - x_{1i})^2] - \omega_1^2 x_{1i} - \omega_1^2 \gamma_1 x_{1i}^2\}$$
$$+$$

$$+\omega_1^2 \sum_{i=0}^{n-1} A_i\Gamma_1(t_n - t_i)x_{1i}(1 + \gamma_1 x_{1i}^2) - \omega_{21}^2 \sum_{i=0}^{n-1} A_i\Gamma_2(t_n - t_i)(x_{2i} - x_{1i})[1 + \gamma_2(x_{2i} - x_{1i})^2],$$

$$x_{2n} = x_{20} + \vartheta_{20}t_n + \sum_{i=0}^{n-1} A_i(t_n - t_i)\{\omega_2^2 x_{1i}[1 + \gamma_2(x_{2i} - x_{1i})^2] - \omega_2^2 x_{2i} +$$

$$+\omega_{32}^2(x_{3i} - x_{2i})[1 + \gamma_3(x_{3i} - x_{2i})^2] - \omega_2^2 \gamma_2 x_{2i}(x_{2i} - x_{1i})^2\} +$$

$$+\omega_2^2 \sum_{i=0}^{n-1} A_i\Gamma_2(t_n - t_i)(x_{2i} - x_{1i})[1 + \gamma_2(x_{2i} - x_{1i})^2] -$$

$$-\omega_{32}^2 \sum_{i=0}^{n-1} A_i\Gamma_3(t_n - t_i)(x_{3i} - x_{2i})[1 + \gamma_3(x_{3i} - x_{2i})^2],$$

$$x_{3n} = x_{30} + \vartheta_{30}t_n + \sum_{i=0}^{n-1} A_i(t_n - t_i)\{\omega_3^2 x_{2i}[1 + \gamma_3(x_{3i} - x_{2i})^2] - \omega_3^2 x_{3i} -$$

$$-\omega_3^2 \gamma_3 x_{3i}(x_{3i} - x_{2i})^2\} + \omega_3^2 \sum_{i=0}^{n-1} A_i\Gamma_3(t_n - t_i)(x_{3i} - x_{2i})[1 + \gamma_3(x_{3i} - x_{2i})^2],$$

where $A_0 = A_n = \frac{\Delta t}{2}$; $A_j = \Delta t, j = \overline{1, n-1}$.

4. Results and discussion

To conduct a computational experiment, a computer program was developed, in which the results obtained are reflected in the form of graphs. In calculations, the following initial data were used: : $x_{10} = x_{20} = 0$; $x_{30} = 1$; $\vartheta_{10} = \vartheta_{20} = \vartheta_{30} = 0$; $\omega_1^2 = \omega_2^2 = \omega_3^2 = \omega_{21}^2 = \omega_{32}^2 = 1$; $\alpha_1 = \alpha_2 = \alpha_3 = 0.25$; $\beta_1 = \beta_2 = \beta_3 = 0.05$; $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.05$; $\gamma_1 = \gamma_2 = \gamma_3 = 0.462$. Figures 3, 4, 5 show nonlinear ($\gamma_i = 0.462$) oscillations of loads of masses m_1, m_2 and m_3 from the position of static equilibrium, where $\varepsilon_i = 0$ (a solid line), $\varepsilon_i = 0.01$ (a dashed line) and $\varepsilon_i = 0.05$ (a dotted line). The graph shows that an account for rheological property of a suspension leads to a decrease in load amplitude from the position of static equilibrium. A decrease in the load oscillation frequency leads to a phase shift. Over time, the viscoelastic properties of the suspension significantly affect the amplitudes and frequencies.
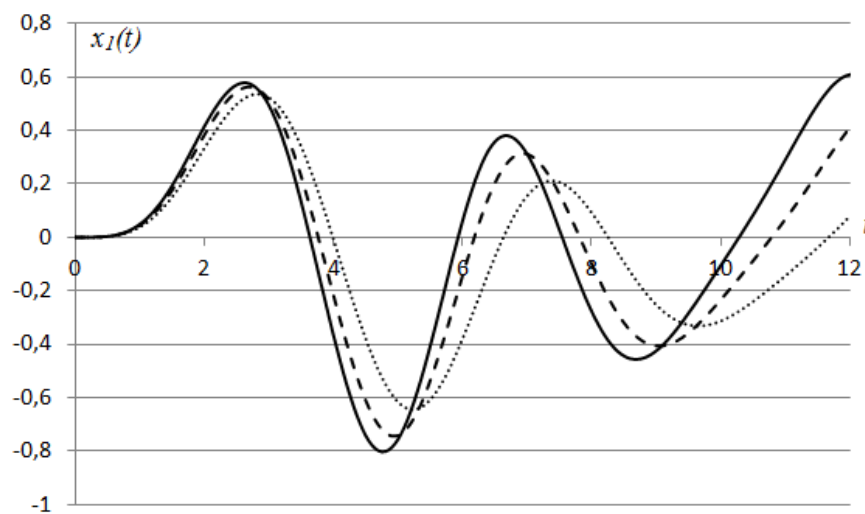

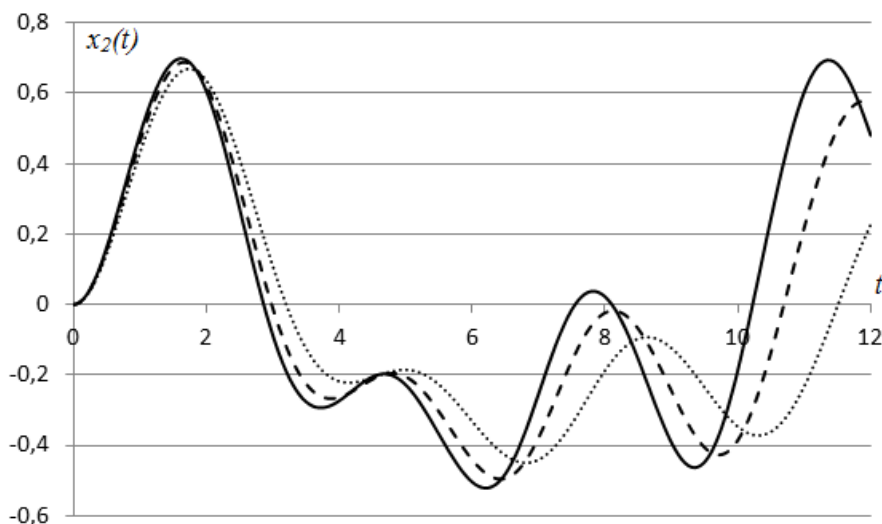**Figure 3.** Modes of nonlinear oscillations of the loads of mass m_1.

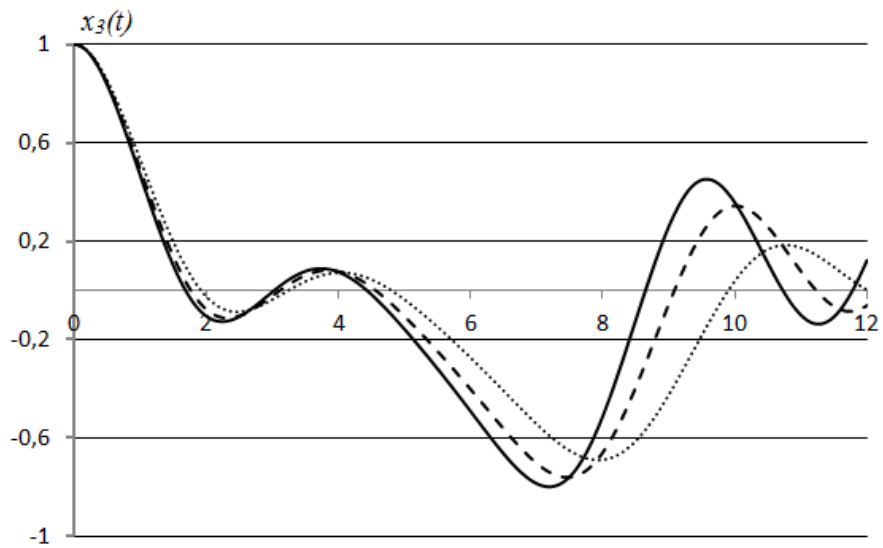**Figure 4.** Modes of nonlinear oscillations of the loads of mass m_2.

Figure 5. Modes of nonlinear oscillations of the loads of mass m_3

The effect of nonlinear suspension properties on the load displacement from the position of static equilibrium is investigated. Fig. 6 shows the influence of nonlinear properties of suspension on the mode of vibration of load of mass m_1. Here $\gamma_1 = \gamma_2 = \gamma_3 = 0$ (a solid line) and $\gamma_1 = \gamma_2 = \gamma_3 = 0.462$ (a dashed line); $\gamma_1 = \gamma_2 = \gamma_3 = 0.645$ (a dotted line). The graph shows that, with an increase in nonlinear property of suspension, the frequency that comes to the phase shift increases. The effect of nonlinearity on the amplitude of the mass oscillations is insignificant.
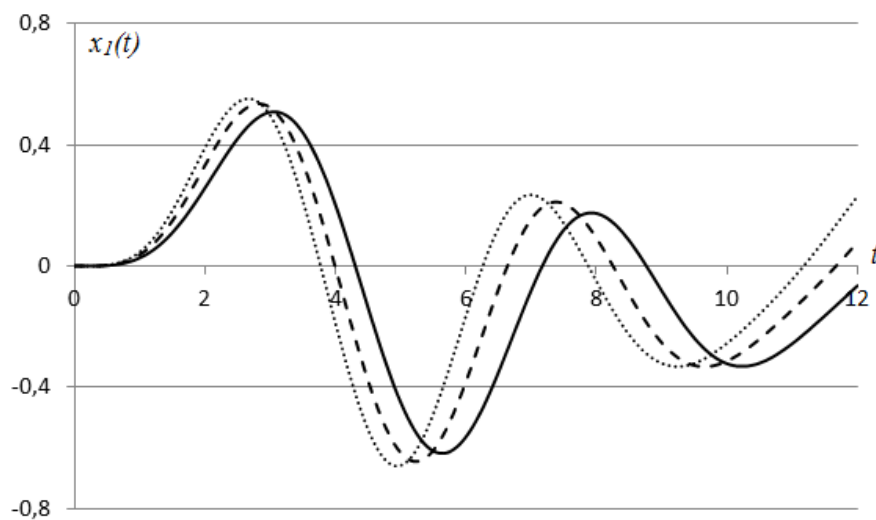

Figure 6. The influence of nonlinear properties of suspension on the mode of vibration of load of mass m_1

How do rheological parameters affect the mode of viscoelastic mass oscillation? The change in the parameter α (Fig. 7) and the parameter β (Fig. 8) by the mode of oscillations is studied. The graph shows that a small change in these parameters considerably affects the change in oscillation frequency. The dependence of the parameter α and the frequency is proportional; the dependence of the parameter β and the frequency is inversely proportional. This is explained by the fact that with an increase in parameter α, the suspension material becomes more viscous, and with an increase in parameter β, less viscous (Sharipov et al., 2019).
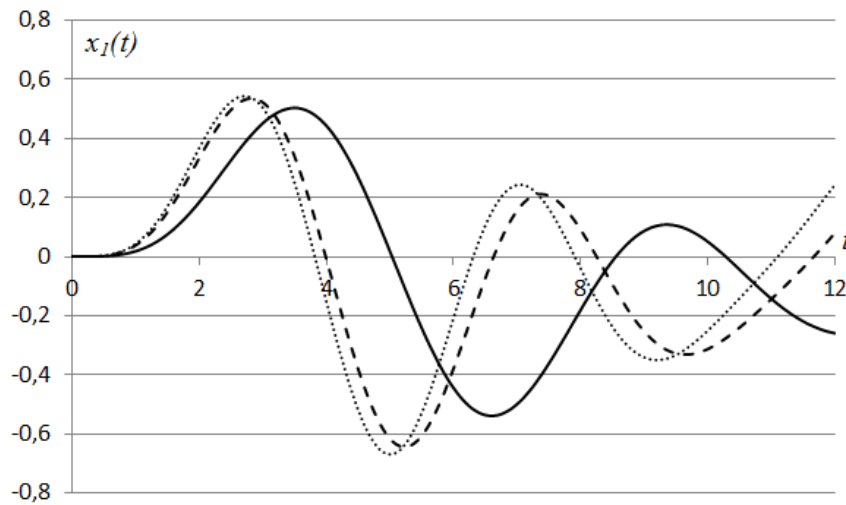
Figure 7. The effect of the parameter α on the mode of vibration of load of mass m_1. Solid line (α = 0.1), dashed line (α = 0.25), dotted line (α = 0.4). $\gamma_i = 0.462$, $\varepsilon_i = 0.05$.
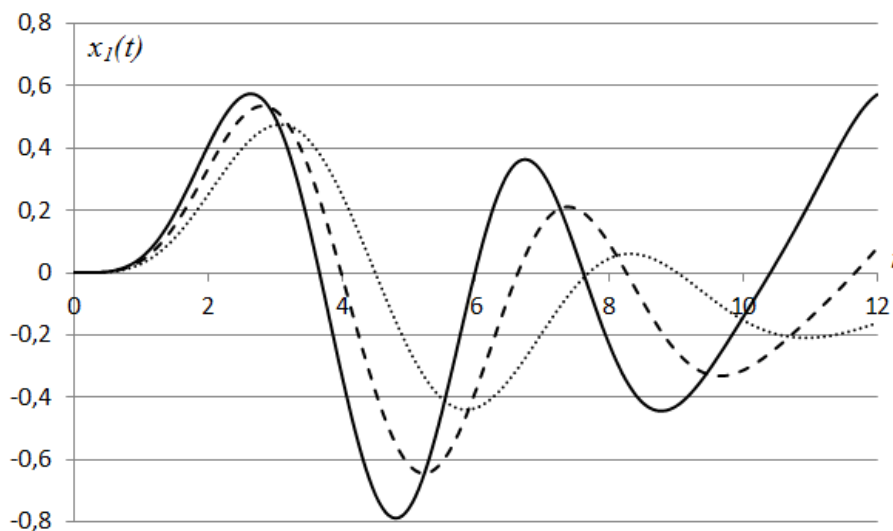

Figure 8. The effect of the parameter ε on the mode of vibration of load of mass m_1. Solid line (ε = 0.005), dashed line (ε = 0.05), dotted line (ε = 0.1).

Conclusions

Effective computational algorithms have been developed for solving problems based on the use of quadrature formulas. The effect of rheological properties of suspension on the mass displacement from the position of static equilibrium is investigated as well as the influence of nonlinear properties of the suspension on the mode of vibration and frequency.

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