MUSTAQIL UY ISHI

1.-3. Differensial tenglamaning umumiy yechimini toping.

4. Koshi masalasini yeching.

5.-6. Differensial tenglamaning umumiy yechimini toping.

7. Differensial tenglamani ixtiyoriy o‘zgarmasni variatsiyalash usuli bilan yeching.

8. $f_1(x)$, $f_2(x)$ berilgan. $y'' + 2y' = f_1(x) + f_2(x)$ differensial

1-variant

1. $(1+e^{-x})yy' = 1.$

2. $y^2 + x^2 y' = xyy'.$

3. $y' - \frac{y}{x} = x\sin x.$

4. $y'x + y = \frac{xy^2}{3}, \quad y(1) = 3.$

5. $(x\cos 2y + 1)dx - x^2 \sin 2y dy = 0.$

6. $y''' = \cos^2 x.$

7. $y'' + y = ctgx$.

8. $f_1(x) = e^{-2x}(3x+6), \; f_2(x) = \cos 2x + 2\sin 2x.$

9. $\begin{cases} y_1' = 3y_1 - y_2 + e^x, \\ y_2' = \; y_1 + y_2 + x. \end{cases}$

2-variant

1. $y' \ln y = e^{3x}.$

2. $xy^2 y' = x^3 + y^3.$

3. $y' - \frac{3y}{x} = e^x x^3.$

4. $y' + y = e^{\frac{x}{2}}\sqrt{y}, \quad y(0) = \frac{9}{4}.$

5. $e^{-y}dx + (1 - xe^{-y})dy = 0.$

6. $xy''' = 2.$

7. $y'' + 4y = tg2x.$

8. $f_1(x) = e-^{2x}(5x+4), \; f_2(x) = \cos x + 4\sin x.$

9. $\begin{cases} y_1' = 2y_1 - \; y_2 + \cos x, \\ y_2' = 3y_1 - 2y_2 + \sin x. \end{cases}$

3-variant

1. $\cos^3 y y' - \cos(2x-y) = (\cos 2x + y).$ **2.** $(4y+5x)dx + (5y+7x)dy = 0.$

3. $y' + 2y = e^{-x^2}.$ **4.** $y' - y = xy^2, \quad y(0) = 1.$

5. $(y + e^x \cos y)dx + (x - e^x \sin y)dy = 0.$

6. $(1+\sin x)y''' = y'' \cos x.$ **7.** $y'' + y = x\cos^2 x.$

8. $f_1(x) = 3x^2 + 2, \quad f_2(x) = e^{-2x}(\cos x + \sin x).$ **9.** $\begin{cases} y_1' = y_1 + \ \ y_2 + x, \\ y_2' = y_1 - 2y_2 + 2x. \end{cases}$

4-variant

1. $(e^x + 8)2y - ye^x dx = 0.$ **2.** $xy' = y\left(\ln\frac{y}{x} - 1\right).$

3. $y' - \frac{2y}{x+1} = (x+1)^2.$ **4.** $xy' + y = 2y^2 \ln x, \quad y(1) = \frac{1}{2}.$

5. $ye^x dx + (y + e^x)dy = 0.$

6. $xy'' + y' = \ln x.$ **7.** $y'' + y = tgx.$

8. $f_1(x) = 6x^2 + 1, \quad f_2(x) = e^{-2x}(2\cos x + \sin x).$ **9.** $\begin{cases} y_1' = -y_1 + y_2 + x, \\ y_2' = 3y_1 + y_2 + x^2. \end{cases}$

5-variant

1. $3^{x^2+y} dy + xdx = 0.$ **2.** $(2\sqrt{xy} - x)y' + y = 0.$

3. $y' + \frac{y}{x} = \frac{\ln x + 1}{x}.$ **4.** $y' + 2y = y^2 e^x, \quad y(0) = \frac{1}{2}.$

5. $(2x^3 - xy^2)dx + (2y^3 - x^2 y)dy = 0.$

6. $y'' tgx = y' + 1.$ **7.** $y'' + 4y = ctg 2x.$

8. $f_1(x) = e^{-2x}(2x-7), \quad f_2(x) = 2\cos 2x + 3\sin 2x.$ **9.** $\begin{cases} y_1' = y_1 - 3y_2 + e^{2x}, \\ y_2' = y_1 - \ \ y_2 + 2x. \end{cases}$

6-variant

1. $e^{-x^2} dy - x(1+y^2)dx = 0.$ **2.** $y' = \frac{y}{x} + \sin\frac{y}{x}.$

3. $y' - yctgx = \sin x.$ **4.** $3xy' + 5y = (4x-5)y^4, \quad y(1) = 1.$

5. $\frac{y}{x^2}dx - \frac{xy+1}{x}dy = 0.$

6. $y''' = x\sin x.$ **7.** $y'' + 2y' + y = xe^x$.

8. $f_1(x) = e^{-2x}(x^2+1), \quad f_2(x) = 3\cos 4x.$ **9.** $\begin{cases} y_1' = 2y_1 + y_2 + 1, \\ y_2' = -5y_1 - 2y_2 + x. \end{cases}$

7-variant

1. $e^{3y+x}dx = ydy.$ **2.** $x^3y' = y(y^2 + x^2).$

3. $y' + \frac{2y}{x} = \frac{1}{x^2}.$ **4.** $y' + 2xy = 2x^3y^2, \quad y(0) = \sqrt{2}.$

5. $(6xy^2 + 4x^3)dx + (6x^2y + y^3)dy = 0.$

6. $y'''tg4x = 4y''.$ **7.** $y'' - 4y' = e^{2x} - e^{-2x}.$

8. $f_1(x) = 3x^3 - 2x + 1, \quad f_2(x) = 2\cos 4x + 3\sin 4x.$ **9.** $\begin{cases} y_1' = y_1 + 4y_2, \\ y_2' = -y_1 + y_2 + e^{3x}. \end{cases}$

8-variant

1. $x + xy + y'(y + xy) = 0.$ **2.** $y' - \frac{y}{x} = tg\frac{y}{x}.$

3. $y' + \frac{y}{\cos^2 x} = \frac{\sin x}{\cos^3 x}.$ **4.** $y' + y = xy^2, \quad y(0) = 1.$

5. $\left(\frac{y}{x^2+y^2} + e^x\right)dx - \frac{xdy}{x^2+y^2} = 0.$

6. $xy''' - 2y'' = \frac{2}{x^2}.$ **7.** $y'' + 4y = \frac{1}{\sin 2x}.$

8. $f_1(x) = 3e^{-2x}, \quad f_2(x) = e^{-2x}(3\cos x + \sin x)$ **9.** $\begin{cases} y_1' = 3y_1 + y_2 + e^x, \\ y_2' = y_1 + 3y_2 - e^x. \end{cases}$

9-variant

1. $2yx^2dy=(1+x^2)dx.$ **2.** $xy'-y=(x+y)+\ln\left(\frac{x+y}{x}\right).$

3. $y'-\frac{y}{x}=x\cos x.$ **4.** $2(y'+y)=xy^2, \quad y(0)=2.$

5. $\left(\frac{2y}{x^3}+y\cos xy\right)dx+\left(\frac{1}{x^2}+x\cos xy\right)dy=0.$

6. $xy''=y'\ln\frac{y'}{x}.$ **7.** $y''+5y'+6y=\frac{1}{1+e^{2x}}$.

8. $f_1(x)=3x^2+2x+1, \quad f_2(x)=e^{-2x}(\cos x+3\sin x)$. **9.** $\begin{cases} y_1'=y_2-\cos x, \\ y_2'=2y_1+y_2. \end{cases}$

10-variant

1. $(xy^2+x)+y'(y-x^2y)=0.$ **2.** $xy'=y-xe^{\frac{y}{x}}.$

3. $y'+\frac{y}{1+x^2}=\frac{arctgx}{1+x^2}.$ **4.** $2(xy'+y)=y^2\ln x, \quad y(1)=2.$

5. $\left(xe^x+\frac{y}{x^2}\right)dx-\frac{1}{x}dy=0.$

6. $xy'''-y''=\frac{1}{x}.$ **7.** $y''-y=\frac{e^x}{e^x+1}.$.

8. $f_1(x)=x^2+3x, \quad f_2(x)=3\cos 2x+\sin 2x$. **9.** $\begin{cases} y_1'=4y_1-5y_2+4x+1, \\ y_2'=\ \ y_1-2y_2+x. \end{cases}$

11-variant

1. $xydy=(1-x^2)dx.$ **2.** $xy'=y\cos\left(\ln\frac{y}{x}\right).$

3. $y'-2xy=2x^3;$ **4.** $y'-ytgx=y^4\cos x, \quad y(0)=1.$

5. $\left(\frac{x}{\sqrt{x^2-y^2}}-1\right)dx-\frac{y}{\sqrt{x^2-y^2}}dy=0.$

6. $xy''' + y'' + x = 0$.

7. $y'' + 2y' + 2y = \frac{e^{-x}}{\cos x}$.

8. $f_1(x) = x^2 + 2, \quad f_2(x) = x\cos 2x$.

9. $\begin{cases} y_1' = -2y_1 - \ \ y_2 + \sin x, \\ y_2' = 4y_1 + 2y_2 + \cos x. \end{cases}$

12-variant

1. $y' + \sqrt{\frac{1-y^2}{1-x^2}} = 0.$

2. $(x^2 - 2xy)y' = xy - y^2.$

3. $x^2y' + xy + 1 = 0.$

4. $xyy' = y^2 + x, \quad y(1) = \sqrt{2}.$

5. $\frac{y}{x^2}dx - \frac{1}{x}dy = 0.$

6. $x^3y''' + x^2y'' = \sqrt{x}$.

7. $y'' + 4y' + 4y = \frac{e^{-x3}}{x^3}$.   .

8. $f_1(x) = e^{-2x}(x+1), \quad f_2(x) = e^{-2x}x\sin x$.

9. $\begin{cases} y_1' = \quad y_1 - y_2 - e^{-x}, \\ y_2' = -4y_1 + y_2 + xe^{-x}. \end{cases}$

13-variant

1. $\sin y\cos x dy = \cos y \sin x dx.$

2. $y' = \frac{y}{x} + \frac{x}{y}.$

3. $y' + \frac{y}{x+1} = x^2.$

4. $xy' - 2x^2\sqrt{y} = 4y, \quad y(1) = 1.$

5. $\left(x + \frac{y}{x^2+y^2}\right)dx + \left(y - \frac{x}{x^2+y^2}\right)dy = 0.$

6. $y'''ctg2x + 2y'' = 0$.

7. $y'' + y = \frac{1}{\sin x}$.

8. $f_1(x) = e^{-2x}(3x+1), \quad f_2(x) = x^2\sin x$.

9. $\begin{cases} y_1' = 5y_1 + 4y_2 + e^x, \\ y_2' = 4y_1 + 5y_2 + 1. \end{cases}$

14-variant

1. $y' = 10^{x+y}.$

2. $(y + \sqrt{xy}) = xy'.$

3. $y' - \dfrac{2xy}{1+x^2} = 1 + x^2.$

4. $y' + x\sqrt[3]{y} = 3y, \quad y(0) = 1.$

5. $e^y dx + (\cos y + xe^y) dy = 0.$

6. $y'' = 1 - (y')^2.$

7. $y'' - 2y' + y = \dfrac{e^x}{x}.$

8. $f_1(x) = e^{-2x}(x-1), \ \ f_2(x) = e^{-2x}\sin x.$

9. $\begin{cases} y_1' = -2y_1 - \ \ y_2, \\ y_2' = 5y_1 + 2y_2 + x^2 + 1. \end{cases}$

15-variant

1. $\sqrt{1-x^2}dy - x\sqrt{1-y^2}dx = 0.$

2. $y \ln \dfrac{y}{x} dx - xdy = 0.$

3. $y' + \dfrac{y}{x} = \dfrac{\sin x}{x}.$

4.

$y' - ytgx = -\dfrac{2}{3} y^4 \sin x, \quad y(0) = 1.$

5. $\left(2x - 1 - \dfrac{y}{x^2}\right)dx - \left(2y - \dfrac{1}{x}\right)dy = 0.$

6. $yy'' - (y')^2 = y^4.$

7. $y'' - 2y' + y = \dfrac{e^x}{x^2}.$

8. $f_1(x) = e^x(x+1), \ \ f_2(x) = e^x x \sin x.$

9. $\begin{cases} y_1' = 5y_1 - 3y_2 + xe^{2x}, \\ y_2' = 3y_1 - \ \ y_2 + e^{2x}. \end{cases}$

16-variant

1. $(1+y)dx = (x-1)dy.$

2. $xyy' = y^2 + 2x^2.$

3. $y' + ytgx = \cos^2 x.$

4. $y' = \dfrac{x}{y} e^{2x} + y, \quad y(0) = 2.$

5. $\dfrac{ydx - xdy}{x^2 + y^2} = 0.$

6. $(y')^2 + 2yy'' = 0$.

7. $y'' + 2y' + y = \dfrac{1}{xe^x}$. .

8. $f_1(x) = e^{-2x}(3x+4), \;\; f_2(x) = e^{-2x}x\cos x$.

9. $\begin{cases} y_1' = 4y_1 - y_2, \\ y_2' = y_1 + 2y_2 + xe^x. \end{cases}$

17-variant

1. $\sqrt{4 - x^2}\,y' + xy^2 + x = 0.$

2. $xy + y^2 = (2x^2 + xy)y'.$

3. $(x^2 + 1)y' + 4xy = 3.$

4. $xy' + y = y^2 \ln x, \quad y(1) = 1.$

5. $(y^3 + \cos x)dx + (3xy^2 + e^y)dy = 0.$

6. $y'''x\ln x = y''$.

7. $y'' + 9y = \dfrac{1}{\sin 3x}$.

8. $f_1(x) = e^x(x^2+4), \;\; f_2(x) = e^x \sin x$.

9. $\begin{cases} y_1' = y_1 - 3y_2, \\ y_2' = y_1 + y_2 + e^x. \end{cases}$

18-variant

1. $x^2 dy - (2xy + 3y)dx = 0.$

2. $(y + 2x)dy - ydx = 0.$

3. $y = x(y' - x\cos x).$

4. $2(xy' + y) = xy^2, \quad y(1) = 1.$

5. $xy^2 dx + y(x^2 + y^2)dy = 0.$

6. $y''x - y' = x^2 e^x$.

7. $y'' - 4y' + 5y = \dfrac{e^{2x}}{\cos x}.$

8. $f_1(x) = e^x(x^2 - 2), \;\; f_2(x) = e^{-2x}x\cos x$.

9. $\begin{cases} y_1' = y_1 - 3y_2 + 1, \\ y_2' = -y_1 + y_2 + 2x. \end{cases}$

19-variant

1. $(1 + y^2)dx - \sqrt{x}dy = 0.$

2. $(2y^2 + 3x^2)xdy = (3y^3 + 6yx^2)dx.$

3. $y' + ytgx = \sin x.$

4. $3(xy' + y) = y^2 \ln x, \quad y(1) = 3.$

5. $(3y^3\cos 3x+7)dx+(3y^2\sin 3x-2y)dy=0.$

6. $y'''=e^{2x}+x.$

7. $y''+4y=\dfrac{1}{\cos 2x}.$.

8. $f_1(x)=x^3+2x-1,\quad f_2(x)=x(\sin 3x+\cos 3x).$

9. $\begin{cases} y_1'=4y_1+\ \ y_2-e^{3x}, \\ y_2'=-y_1+2y_2. \end{cases}$

20-variant

1. $1+(1+y')e^y=0.$

2. $y^2=x(x+y)y'.$

3. $xy'-2y+x^2=0.$

4. $yx'+x=-yx^2,\quad x(1)=2.$

5. $(3x^2+2y)dx+(2x-3)dy=0.$

6. $y''(3+2y)=(2y')^2.$

7. $y''+3y'+2y=\dfrac{1}{e^x+1}.$

8. $f_1(x)=x^2-4,\quad f_2(x)=e^x(\sin x+\cos x).$

9. $\begin{cases} y_1'=\ \ 2y_1+\ \ y_2+x, \\ y_2'=-5y_1-2y_2+x^2. \end{cases}$

21-variant

1. $(4x+2xy^2)dx-(3y-3x^2y)dy=0.$

2. $(x^2-3y^2)dx+2xydy=0.$

3. $y'\sqrt{1-x^2}+y=\arcsin x.$

4. $y'-y+y^2\cos x=0,\quad y(0)=2.$

5. $(3x^2y+y^3)dx+(x^3+3xy^2)dy=0.$

6. $y'''+y''tgx=0.$

7. $y''+4y'+4y=e^{-2x}\ln x.$.

8. $f_1(x)=e^{-2x}(x+2),\quad f_2(x)=e^{-2x}\sin x.$

9. $\begin{cases} y_1'=2y_1+\ \ y_2-\cos 3x, \\ y_2'=-y_1+4y_2+\sin 3x. \end{cases}$

22-variant

1. $\sin y y'-y\cos x=2\cos x.$

2. $(y^2-2xy)dx-x^2dy=0.$

3. $y'\sin x-y\cos x=1.$

4. $xy^2y'=x^2+y^3,\quad y(1)=\sqrt[3]{3}.$

5. $3x^2e^ydx+(x^3e^y-1)dy=0.$

6. $y''(1+y)=(y')^2+y'$.

7. $y''-2y=xe^{-x}$.

8. $f_1(x)=e^x(2x+6),\quad f_2(x)=e^x(\sin x+4\cos x)$.

9. $\begin{cases} y_1'=2y_1-5y_2, \\ y_2'=\ y_1-2y_2+e^{2x}. \end{cases}$

23-variant

1. $y'=(2y+1)tgx.$

2. $ydx-xdy=\sqrt{x^2+y^2}dy.$

3. $(1-x)(y'+y)=e^{-x}.$

4. $xy'-2\sqrt{x^3}y=y,\quad y(2)=8.$

5. $(3x^2y+\sin x)dx+(x^3-\cos y)dy=0.$

6. $y''(1+y)=(5y')^2$.

7. $y''-y=e^{2x}\sin(e^x).$

8. $f_1(x)=e^{-2x}(3x-2),\quad f_2(x)=3\cos 3x$.

9. $\begin{cases} y_1'=2y_1+4y_2+\cos x, \\ y_2'=3y_1-2y_2+\sin x. \end{cases}$

24-variant

1. $\sqrt{3+y^2}dx-ydy=x^2ydy.$

2. $xy'=4\sqrt{2x^2+y^2}+y.$

3. $x(y'-y)=e^x.$

4. $xy'+y=xy^2,\quad y(1)=1.$

5. $(e^{x+y}+3x^2)dx+(e^{xy}+4y^3)dy=0.$

6. $(1+x^2)y''+1+(y')^2=0$.

7. $y''-y=e^{2x}\cos(e^x).$

8. $f_1(x)=e^x(4x-3),\quad f_2(x)=2\sin 2x+3\cos 2x$.

9. $\begin{cases} y_1'=2y_1+3y_2+e^x, \\ y_2'=\ y_1-2y_2+2xe^x. \end{cases}$

25-variant

1. $x(4+e^y)dx-e^ydy=0.$

2. $\left(xye^{\frac{x}{y}}+y^2\right)=x^2e^{\frac{x}{y}}y'.$

3. $y'+\dfrac{x}{1-x^2}=\dfrac{1}{1-x^2}.$

4. $y'-y=\dfrac{x}{y}e^x;\quad y(0)=4.$

5. $\frac{2x}{y^3}dx+\frac{y^2-3x^2}{y^4}dy=0.$

6. $yy''-2yy'\ln y=(y')^2.$

7. $y''-4y'+4y=\frac{e^{2x}}{x^3}.$

8. $f_1(x)=x^2+6x+4,\quad f_2(x)=e^x x\sin 3x.$

9. $\begin{cases} y_1'=-2y_1-\ y_2+e^{-x}, \\ y_2'=\ 3y_1+2y_2-e^{-x}. \end{cases}$

26-variant

1. $2x+2xy^2+\sqrt{1+x^2}\,y'=0.$

2. $x\ln\frac{x}{y}dy-ydx=0.$

3. $y'+\frac{y}{x}=\frac{\sin x}{x}.$

4. $xdx=\left(\frac{x^2}{y}-y^3\right)dy,\quad x(1)=\sqrt{2}.$

5. $\left(\frac{\sin 2x}{y}+x\right)dx+\left(y-\frac{\sin^2 x}{y^2}\right)dy=0.$

6. $y'''(x-1)-y''=0.$

7. $y''+2y'=\frac{1}{\cos 3x}.$

8. $f_1(x)=x^2-5x+1,\quad f_2(x)=e^x x\cos 3x.$

9. $\begin{cases} y_1'=\ y_1+\ y_2-\cos x, \\ y_2'=3y_1-\ y_2+\sin x+\cos x. \end{cases}$

27-variant

1. $y(1+\ln y)+xy'=0.$

2. $3y\sin\frac{3x}{y}+\left(y-3x\sin\frac{3x}{y}\right)y'=0.$

3. $y'-\frac{y}{x\ln x}=x\ln x.$

4. $y'-xy=-y^3e^{-x^2};\quad y(0)=\frac{\sqrt{2}}{2}.$

5. $\frac{(x-y)dx+(x+y)dy}{x^2+y^2}=0.$

6. $2xy'''y''=y''^2-1.$

7. $y''+y=\frac{2}{\sin^2 x}.$

8. $f_1(x)=e^x(3x-2),\quad f_2(x)=x^2\sin 2x.$

9. $\begin{cases} y_1'=\ 4y_1+y_2+36x, \\ y_2'=-2y_1+y_2+2e^x. \end{cases}$

28-variant

1. $y'\sqrt{1-x^2}-\cos^2 y=0.$

2. $y=x(y'-\sqrt[x]{e^y}).$

3. $y'-\dfrac{y}{x+2}=x^2+2x.$

4. $y'x+y=-xy^2;\quad y(1)=2.$

5. $\left(\dfrac{x}{\sqrt{x^2+y^2}}-\dfrac{y}{x^2}\right)dx+\left(\dfrac{y}{\sqrt{x^2+y^2}}+\dfrac{1}{x}\right)dy=0.$

6. $xy'''+y''=\dfrac{1}{\sqrt{x^2}}.$

7. $y''+\pi^2 y=\dfrac{\pi^2}{\sin \pi x}.$

8. $f_1(x)=e^{-2x}(5x+4),\quad f_2(x)=x\cos 2x.$

9. $\begin{cases} y_1'=2y_1-\ y_2, \\ y_2'=\ y_1+4y_2+xe^x. \end{cases}$

29-variant

1. $(1+e^x)ydy-e^y dx=0.$

2. $(y^2-x^2)dy=2xydx.$

3. $y'+y\cos x=\dfrac{1}{2}\sin 2x.$

4. $xyy'-x=y^2,\quad y(1)=\sqrt{2}.$

5. $\left(\dfrac{1}{x-y}+3x^2y^7\right)dx+\left(7x^3y^6-\dfrac{1}{x-y}\right)dy=0.$

6. $xy''-y=2x^2e^x.$

7. $y''+y=\dfrac{1}{\sin x}.$

8. $f_1(x)=x^2-5x+1,\quad f_2(x)=e^x x\cos 3x.$

9.

$\begin{cases} y_1'=-y_2+\cos x, \\ y_2'=3y_1-4y_2+4\cos x-\sin x. \end{cases}$

30-variant

1. $x\sqrt{4+y^2}dx+y\sqrt{3+x^2}dy=0.$

2. $(3xy+x^2)y'-3y^2=0.$

3. $xy' + y + xe^{-x^2} = 0$.

4. $y' - ytgx + y^2\cos x = 0,\ \ y(0) = \frac{1}{2}$

.

5. $\frac{2x(1-e^y)}{(1+x^2)^2}dx + \frac{e^y}{1+x^2}dy = 0$.

6. $2xy''y' = (y')^2 - 4$.

7. $y'' + 9y = \frac{1}{\sin 3x}$.

8. $f_1(x) = 6e^x(\cos x + \sin x),\ \ f_2(x) = e^{-2x}(5x-2)$.

9. $\begin{cases} y_1' = \ \ y_1 + y_2 + \sin x, \\ y_2' = 3y_1 - y_2 - \cos x. \end{cases}$

NAMUNAVIY VARIANT YECHIMI

1. Differensial tenglamaning umumiy yechimini toping.

1.30. $x\sqrt{4+y^2}dx + y\sqrt{3+x^2}dy = 0$.

O‘zgaruvchilari ajraladigan differensial tenglama berilgan. Uning har ikkala tomonini $\sqrt{4+y^2}\cdot\sqrt{3+x^2} \neq 0$ ga bo‘lib, o‘zgaruvchilarni ajratamiz:

$$\frac{xdx}{\sqrt{3+x^2}} + \frac{ydy}{\sqrt{4+y^2}} = 0.$$

Bu tenglikni integrallaymiz:

$$\sqrt{3+x^2} + \sqrt{4+y^2} = C.$$

Bundan

$$\sqrt{4+y^2} = C - \sqrt{3+x^2}$$

yoki

$$y = \sqrt{(C-\sqrt{3+x^2})^2 - 4}.$$

2. Differensial tenglamaning umumiy yechimini toping.

2.30. $(3xy + x^2)y' - 3y^2 = 0$.

Berilgan tenglamani

$$y' = \frac{3y^2}{3xy + x^2}$$

ko‘rinishga keltiramiz. Bu ifodada

$$f(x, y) = \frac{3y^2}{3xy + x^2}$$

bir jinsli funksiya. Demak, berilgan tenglama bir jinsli tenglama.

Tenglamada $y = ux,\ y' = u'x + x$ o‘rniga qo‘yish bajaramiz:

$$u'x + u = \frac{3x^2u^2}{3x^2u + x^2} \quad \text{yoki} \quad u'x + u = \frac{3u^2}{3u + 1}.$$

Bundan

$$u'x = \frac{3u^2 - 3u^2 - u}{3u + 1} \quad \text{yoki} \quad u'x = -\frac{u}{3u + 1}.$$

O‘zgaruvchilarni ajratamiz:

$$\frac{3u + 1}{u} du = -\frac{dx}{x}.$$

Tenglamani integrallaymiz:

$$\int \frac{3u + 1}{u} du = \ln C - \int \frac{dx}{x} \quad \text{yoki} \quad \ln|u| + 3u = \ln C - \ln|x|.$$

Bundan $3u = \ln \frac{C}{xu}$. $u = \frac{y}{x}$ o‘rniga qo‘yish bajaramiz:

$$3\frac{y}{x} = \ln \frac{C}{y} \quad \text{yoki} \quad y = Ce^{-\frac{3y}{x}}.$$

3. Differensial tenglamaning umumiy yechimini toping.

3.30. $xy' + y + xe^{-x^2} = 0$.

Tenglamani

$$y' + \frac{y}{x} = -e^{-x^2}$$

ko‘rinishiga keltiramiz. Bu tenglama chiziqli tenglama.

Bunda

$$P(x)=\frac{1}{x},\quad Q(x)=-e^{-x^2}.$$

$y=uv,\quad y'=u'v+v'u$ o‘rniga qo‘yish bajaramiz:

$$u'v+u\left(v'+\frac{v}{x}\right)=-e^{-x^2}$$

Bu tenglamada v funksiyani tanlaymiz va

$$\begin{cases} v'+\dfrac{v}{x}=0, \\ u'v=-e^{-x^2} \end{cases}$$

tenglamalar sistemasini hosil qilamiz.

Birinchi tenglamani integrallaymiz:

$$\frac{dv}{v}=-\frac{dx}{x} \quad \text{yoki} \quad \int\frac{dv}{v}=-\int\frac{dx}{x}.$$

Bundan

$$\ln|v|=-\ln|x|+\ln C \quad \text{yoki } C=1 \text{ da } v=\frac{1}{x}.$$

$v$ni sistemaning ikkinchi tenglamasiga qo‘yamiz:

$$u'\frac{1}{x}=-e^{-x^2}.$$

U holda

$$u'=-xe^{-x^2} \quad \text{yoki} \quad u=\frac{1}{2}e^{-x^2}+C.$$

Demak, tenglamaning umumiy yechimi

$$y=uv==\frac{e^{-x^2}}{2x}+\frac{C}{x}.$$

4. Koshi masalasini yeching.

4.30. $y' - ytgx + y^2\cos x = 0,\ \ y(0) = \frac{1}{2}.$

Tenglamani $y' - ytgx = -y^2\cos x$ ko‘rinishda yozamiz. Bu tenglama Bernulli tenglamasi. Bunda $n = 2$.

$z = y^{1-2} = y^{-1}$ belgilash kiritamiz va chiziqli

$$z' + ztgx = \cos x$$

tenglamani hosil qilamiz.

$z = uv,\quad z' = u'v + v'u$ o‘rniga qo‘yish bajaramiz:

$$u'v + u(v' + vtgx) = \cos x.$$

$u,\ v$ funksiyalarni topish uchun

$$\begin{cases} v' + vtgx = 0, \\ u'v = \cos x \end{cases}$$

sistemani tuzamiz.

Sistemaning birinchi tenglamasidan $v = \cos x$ xususiy yechimni topamiz va uni sistemaning ikkinchi tenglamasiga qo‘yamiz:

$$u'\cos x = \cos x \quad \text{yoki} \quad u' = 1.$$

Bundan

$$u = x + C.$$

Berilgan tenglamaning umumiy yechimini topamiz:

$$z = uv, \qquad z = (x + C)\cos x.$$

Bundan

$$y^{-1} = (x + C)\cos x \quad \text{yoki} \quad y = \frac{1}{(x + C)\cos x}.$$

Tenglamaning xususiy yechimni topish uchun ixtiyoriy o‘zgarmasning qiymatini boshlang‘ich shartdan topamiz:

$$\frac{1}{2} = \frac{1}{C} \quad \text{yoki} \quad C = 2.$$

Demak, tenglamaning izlanayotgan xususiy yechimi

$$y = \frac{1}{(x+2)\cos x}.$$ ◘

5. Differensial tenglamaning umumiy yechimini toping.

5.30. $\frac{2x(1-e^y)}{(1+x^2)^2}dx + \frac{e^y}{1+x^2}dy = 0.$

◘ Tenglamada $M(x,y) = \frac{2x(1-e^y)}{(1+x^2)^2}$, $N(x,y) = \frac{e^y}{1+x^2}$.

Bundan

$$\frac{\partial M}{\partial y} = -\frac{2xe^y}{(1+x^2)^2}, \quad \frac{\partial N}{\partial x} = -\frac{2xe^y}{(1+x^2)^2}, \text{ ya'ni } \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

Demak, tenglama to‘liq differensialli.

$\frac{\partial u}{\partial x} = M(x,y) = \frac{2x(1-e^y)}{(1+x^2)^2}$ tenglikni x bo‘yicha integrallaymiz :

$$u = (1-e^y)\left(-\frac{1}{1+x^2}\right) + \varphi(y) \quad \text{yoki} \quad \varphi(y) = u + \frac{1-e^y}{1+x^2}.$$

Bundan

$$\varphi'(y) = \frac{\partial u}{\partial y} - \frac{e^y}{1+x^2}.$$

U holda

$$\frac{\partial u}{\partial y} = N(x,y) = \frac{e^y}{1+x^2}$$

ekanidan

$$\varphi'(y) = 0 \quad \text{yoki} \quad \varphi(y) = \overline{C}.$$

Demak,

$$u = \overline{C} + \frac{e^y - 1}{1+x^2} \quad \text{yoki} \quad \frac{e^y - 1}{1+x^2} = C.$$ ◘

6. Differensial tenglamaning umumiy yechimini toping.

6.30. $2xy''y' = (y')^2 - 4.$

$y' = p(x),\ \ y'' = p'(x)$ o‘rniga qo‘yish bajaramiz:

$$2xpp' = p^2 - 4.$$

Bu tenglamada o‘zgaruvchilarni ajratamiz:

$$2xp\frac{dp}{dx} = p^2 - 4 \quad \text{yoki} \quad \frac{2pdp}{p^2 - 4} = \frac{dx}{x}.$$

Integrallaymiz:

$$\ln|p^2 - 4| = \ln C_1 + \ln x.$$

Bundan

$$p = \sqrt{C_1 x + 4}.$$

y o‘zgaruvchiga qaytamiz:

$$y' = \sqrt{C_1 x + 4}.$$

Bundan

$$y = \int \sqrt{C_1 x + 4}\,dx + C_2 \quad \text{yoki} \quad y = \frac{2}{3C_1}(C_1 x + 4)^{\frac{3}{2}} + C_2.$$

7. Tenglamani ixtiyoriy o‘zgarmalarni variatsiyalash usuli bilan yeching.

7.30. $y'' + 9y = \dfrac{1}{\sin 3x}.$

$k^2 + 9 = 0$ xarakteristik tenglama $k_{1,2} = \pm 3i$ ildizlarga ega. U holda mos bir jinsli tenglamaning umumiy yechimi $y_1 = C_1\cos 3x + C_2\sin 3x$ ko‘rinishda bo‘ladi.

Berilgan tenglamaning xususiy yechimini

$$\bar{y} = C_1(x)\cos 3x + C_2(x)\sin 3x$$

ko‘rinishda izlaymiz.

$C_1(x)$ va $C_2(x)$ funksiyalarni topish uchun

$$\begin{cases} C_1'(x)\cos 3x + C_2'(x)\sin 3x = 0, \\ -3C_1'(x)\sin 3x + 3C_2'(x)\cos 3x = \dfrac{1}{\sin 3x} \end{cases}$$

sistemani tuzamiz va yechamiz:

$$C_1'(x) = -\frac{1}{3}, \ C_2'(x) = \frac{1}{3} ctg 3x.$$

Bundan

$$C_1(x) = -\frac{1}{3}x, \ C_2(x) = \frac{1}{9}\ln|\sin 3x|.$$

Demak, berilgan tenglamaning xususiy yechimini

$$\bar{y} = -\frac{1}{3}x\cos 3x + \frac{1}{9}\ln|\sin 3x|\sin 3x$$

va umumiy yechimi

$$y = C_1\cos 3x + C_2\sin 3x - \frac{1}{3}x\cos 3x + \frac{1}{9}\ln|\sin 3x|\sin 3x$$

yoki

$$y = \left(C_1 - \frac{1}{3}x\right)\cos 3x + \left(C_2 + \frac{1}{9}\ln|\sin 3x|\right)\sin 3x.$$ ◐

8. $f_1(x)$, $f_2(x)$ berilgan. $y'' + 2y' = f_1(x) + f_2(x)$ differensial tenglamaning umumiy yechimini toping.

8.30. $f_1(x) = 6e^x(\cos x + \sin x)$, $f_2(x) = e^{-2x}(5x-2)$.

◐ $k^2 + 2k = 0$ xarakteristik tenglama $k_1 = 0$, $k_2 = -2$ ildizlarga ega. Mos bir jinsli tenglamaning umumiy yechimi $y = C_1 + C_2e^{-2x}$ ga teng.

Tenglamaning o‘ng tomoni ikkita $f_1(x) = 6e^x(\cos x + \sin x)$ va $f_2(x) = e^{-2x}(5x-2)$ funksiyalarning yig‘indisidan iborat. Shu sababli ikkita bir jinsli bo‘lmagan

$$y'' + 2y' = 6e^x(\cos x + \sin x) \quad \text{va} \quad y'' + 2y' = e^{-2x}(5x-2)$$

tenglamalarni yechamiz.

Birinchi tenglamaning o‘ng tomoni $f(x)=e^{\alpha x}(P_n(x)\cos\beta x+Q_m(x)\sin\beta x)$

ko‘rinishda berilgan. Bunda $\alpha=1,\ \beta=1,\ P_0(x)=6,\ Q_0(x)=6$, $\alpha\pm i\beta=1\pm i$ xarakteristik tenglamaning ildizi emas.

U holda tenglamaning xususiy yechimini

$$\bar{y}_1=e^x(A\cos x+B\sin x)$$

ko‘rinishda izlaymiz.

$\bar{y}_1'=e^x((A+B)\cos x+(B-A)\sin x),\quad \bar{y}_1''=e^x(2B\cos x-2A\sin x)$ larni berilgan

tenglamaga qo‘yamiz:

$$e^x(2B\cos x-2A\sin x)+2e^x((A+B)\cos x+(B-A)\sin x)=6e^x(\cos x+\sin x).$$

Chap va o‘ng tomondagi $\cos x$ va $\sin x$ lar oldidagi koeffitsiyentlarni tenglab, topamiz: $A=-\frac{3}{5},\ B=\frac{9}{5}$.

Demak, birinchi tenglamaning xususiy yechimi

$$\bar{y}_1=\frac{3}{5}e^x(3\sin x-\cos x).$$

Ikkinchi tenglamaning o‘ng tomoni $f(x)=e^{\alpha x}P_n(x)$ ko‘rinishda berilgan.

Bunda $\alpha=-2,\ P_1(x)=5x-2.$ $\alpha=-2$ xarakteristik tenglamaning bir karrali ildizi.

Tenglamaning xususiy yechimini

$$\bar{y}_2=xe^{-2x}(Cx+D)$$

ko‘rinishda izlaymiz.

$$\bar{y}_2'=e^{-2x}(2Cx^2+(2C-2D)x+D),$$

$$\bar{y}_2''=e^{-2x}(4Cx^2+(4D-8C)x+2C-4D)$$

larni berilgan tenglamaga qo‘yamiz:

$$e^{-2x}(4Cx^2+(4D-8C)x+2C-4D)+2e^{-2x}(-2Cx^2+(2C-2D)x+D)=e^{-2x}(Cx+D)$$

Bundan $C=-\frac{5}{4},\ D=-\frac{1}{4}.$

Demak, ikkinchi tenglamaning xususiy yechimi

$$\bar{y}_2=-\frac{1}{4}e^{-2x}x(5x+1).$$

Shunday qilib, berilgan tenglamaning umumiy yechimi

$$y=C_1+C_2e^{-2x}+\frac{3}{5}e^x(3\sin x-\cos x)-\frac{1}{4}e^{-2x}x(5x+1).$$

9. Differensial tenglamalar sistemasining umumiy yechimini toping.

9.30. $\begin{cases} y_1'=\ y_1+y_2+\sin x, \\ y_2'=3y_1-y_2-\cos x. \end{cases}$

1) Sistemaga mos bir jinsli tenglamani tuzamiz:

$$\begin{cases} y_1'=\ y_1+y_2, \\ y_2'=3y_1-y_2. \end{cases}$$

Sistemaning xarakteristik tenglamasini tuzamiz va yechamiz:

$$\begin{vmatrix} 1-\lambda & 1 \\ 3 & -1-\lambda \end{vmatrix}=0,\quad \lambda_1=-2,\ \lambda_2=2.$$

$\lambda_1=-2$ da $3\alpha_{11}+\alpha_{21}=0$ tenglikdan $\alpha_{21}=-3\alpha_{11}$ yoki $\alpha_{11}=1$ desak, $\alpha_{21}=-3$ kelib chiqadi.

$\lambda_2=2$ da shu kabi topamiz: $\alpha_{12}=1,\ \alpha_{22}=1.$

U holda bir jinsli sistemaning yechimi

$$\begin{cases} y_1=\ C_1e^{-2x}+C_2e^{2x}, \\ y_2=-3C_1e^{-2x}+C_2e^{2x} \end{cases}$$

bo‘ladi.

Berilgan sistemaning xususiy yechimini

$$\begin{cases} \bar y_1 = A_1 \cos x + B_1 \sin x, \\ \bar y_2 = A_2 \cos x + B_2 \sin x \end{cases}$$

ko‘rinishda izlaymiz. Bundan

$$\begin{cases} \bar y_1' = -A_1 \sin x + B_1 \cos x, \\ \bar y_2' = -A_2 \sin x + B_2 \cos x. \end{cases}$$

$\bar y_1,\ \bar y_2,\ \bar y_1,\ \bar y_2'$ larni berilgan sistemaga qo‘yamiz $\cos x$ va $\sin x$ lar oldidagi koeffitsiyentlarni tenglab, topamiz:

$$A_1 = 0,\ B_1 = -\frac{1}{5},\ A_2 = -\frac{1}{5},\ B_2 = -\frac{4}{5}.$$

Demak, berilgan sistemaning xususiy yechimi va umumiy yechimi:

$$\begin{cases} \bar y_1 = -\dfrac{1}{5}\sin x, \\ \bar y_2 = -\dfrac{1}{5}\cos x - \dfrac{4}{5}\sin x \end{cases}$$

$$\begin{cases} y_1 = C_1 e^{-2x} + C_2 e^{2x} - \dfrac{1}{5}\sin x, \\ y_2 = -3C_1 e^{-2x} + C_2 e^{2x} - \dfrac{1}{5}\cos x - \dfrac{4}{5}\sin x. \end{cases}$$