«Oliy matematika» kafedrasi

2-kurslar uchun

(3-SEMESTR)

1-namunaviy hisob ishi

TOSHKENT – 2020

1-tipik hisob

Nazariy savollar

1. Kompleks sonning algebraik ko’rinishi.
2. Algebraik ko’rinishdagi kompleks sonlar ustida amallar.
3. Kompleks sonning moduli va argumenti.
4. Kompleks sonning trigonometrik va ko’rsatkichli shakli.
5. Trigonometrik shakldagi sonni darajaga ko’tarish.
6. Trigonometrik sondan ildiz chiqarish.
7. Kompleks o’zgaruvchili funksiya va uning aniqlanish sohasi.
8. Kompleks o’zgaruvchili funksiyaning limiti va uzluksizligi.
9. Kompleks o’zgaruvchili funksiyani differensiallash.
10. Koshi-Riman sharti.
11. Kompleks o’zgaruvchili funksiyaning integrali va uni hisoblash.
12. Analitik funksiyalar.
13. Garmonik funksiyalar.
14. Koshining integral formulasi.
15. Kompleks hadli qatorlar.
16. Loran qatori.
17. Yakkalangan maxsus nuqtalar.
18. Chegirmalar.
19. Chegirmalar haqida Koshi teoremasi.
20. Laplas almashtirishi va uning xossalari.
21. Originallar sinfi, tasvirlar sinfi.
22. Operatsion hisobning asosiy teoremalari.
23. Differensial tenglamalarni va tenglamalar sistemasini operatsion hisob yordamida yechish.
24. Xususiy hosilalai differensial tenglamalar haqida tushuncha.
25. Ikkinchi tartibli chiziqli xususiy hosilalai differensial tenglamalar va ularning klassifikatsiyasi.
26. Cheksiz tor uchun Koshi masalasini yechish.
27. Chegaraviy va aralash masalalar

Namunaviy variantlar.

1-Misol. $z_1 = 2 - 3i$, $z_2 = 4 + 5i$, $z_3 = 3 - i$, $z_4 = -2 + 3i$ kompleks sonlar ustida$\frac{z_1 \cdot z_2}{z_3 + z_4}$, $z_1^2 + z_2^2 - 2z_3 z_4$ amallarni bajaring.

Yechish: Ko'paytirish va qo'shish amallarni bajaramiz:

$$z_1 \cdot z_2 = (2 - 3i)(4 + 5i) = 2 \cdot 4 + 2 \cdot 5i - 3i \cdot 4 - 3i \cdot 5i =$$

$$= 8 + 10i - 12i - 15i^2 = 8 + 15 - 2i = 23 - 2i;$$

$$z_3 + z_4 = (3 - i) + (-2 + 3i) = 3 - 2 + (-1 + 3)i = 1 + 2i.$$

Endi topilganlarning nisbatini topamiz:

$$\frac{z_1 \cdot z_2}{z_3 + z_4} = \frac{23 - 2i}{1 + 2i} = \frac{(23 - 2i)(1 - 2i)}{(1 + 2i)(1 - 2i)} = \frac{23 - 46i - 2i - 4}{1 - 4} =$$

$$= \frac{19 - 48i}{-3} = -\frac{19}{3} + 16i.$$

Xuddi shu singari

$$z_1^2 + z_2^2 - 2z_3 z_4 = (2 - 3i)^2 + (4 + 5i)^2 - 2(3 - i)(-2 + 3i) =$$

$$= 4 - 12i - 9 + 16 + 40i - 25 + 12 - 18i - 4i - 6 = -8 + 6i.$$

2-Misol. $z = 1 + i$ kompleks sonni trigonometrik shakldа yozing so'ngra z^4 darajaga ko'taring.

Yechish: $r = |z| = \sqrt{1^2 + 1^2} = \sqrt{2}$, $\operatorname{tg} \varphi = 1/1 = 1, \varphi = \arg z = \pi/4$. Shuning uchun $z = \sqrt{2}(\cos(\pi/4) + i \sin(\pi/4))$. Darajaga ko'tarish formulasiga ko'ra: $z^4 = \left(\sqrt{2}\right)^4 (\cos(4 \cdot \pi/4) + i \sin(4 \cdot \pi/4)) =$

$$= 4(\cos \pi + i \sin \pi) = -4.$$

3-Misol. $f(z)$ differensiallanuvchi funksiyaning $u(x, y) = 3x + 3y$ haqiqiy qismi beilgan. Shu $f(z)$ funksiyani toping. Bu yerda $z = x + iy$.

Yechish: Berilgan$u(x, y)$ funksiyaning x bo'yicha xususiy hosilasini topamiz: $\frac{\partial u}{\partial x} = 3$. Koshi-Riman shartlaridan biriga ko'ra $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$, u holda $\frac{\partial v}{\partial y} = 3$. Bu tenglikni y bo'yicha integrallab

$$v(x, y) = 3y + \varphi(x)$$

tenglikka ega bo'lamiz, bu yerda $\varphi(x)$ $-$ixtiyoriy funksiya.

Koshi-Riman shartining boshqasini qo'laymiz: $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$. Yuqoridagi tenglikdan $\frac{\partial v}{\partial x} = \varphi'(x)$. U holda $\frac{\partial u}{\partial y} = -\varphi'(x)$. Ammo masala shartiga ko'ra $\frac{\partial u}{\partial y} = 3$. Demak $-\varphi'(x) = 3$ yoki $\varphi(x) = -3x + C$. Bundan esa $f(z) = 3x + 3y + i(3y - 3x + C_1) = 3(x + iy) -$

$$= 3(-y + ix) + C_1 i = 3z - \frac{3(-y + ix)i}{i} + C = 3z - \frac{3(-yi - x)}{i} + C$$

$$= 3z + \frac{3z}{i} + C = 3z - 3zi + C = 3(1 - i)z + C.$$

4-Misol. $\int_{1-i}^{3+2i} 2zdz$ integralni hisoblang.

Yechish: Integral ostidagi funksiya analitik funksiya bo'lganligi uchun Nyuton_leybnis formulasini qo'llab

$$\int_{1-i}^{3+2i} 2zdz = z^2|_{1-i}^{3+2i} = (3 + 2i)^2 - (1 - i)^2 =$$

$$= 9 + 12i - 4 - (1 - 2i - 1) = 5 + 14i.$$

5-Misol. $f(z) = \frac{z}{z^2 - 4z + 3}$ funksiyaning chegirmalarini hisoblang.

Yechish: Funksiyani $f(z) = \frac{z}{(z-1)(z-3)}$ ko'rinishda yozib olamiz. Shuning uchun $z = 1$va$z = 3$ nuqtalar berilgan funksiyaning oddiy qutblari bo'ladi U holda

$$\underset{1}{\text{cheg}}\, f(z) = \lim_{z \to 1}(z - 1) \cdot \frac{z}{(z - 1)(z - 3)} = \lim_{z \to 1} \frac{z}{z - 3} = -\frac{1}{2},$$

$$\underset{3}{\text{cheg}}\, f(z) = \lim_{z \to 3}(z - 3) \cdot \frac{z}{(z - 1)(z - 3)} = \lim_{z \to 3} \frac{z}{z - 1} = \frac{3}{2}.$$

6-Misol. $f(t) = 2t - e^{3t} \cdot \cos t$ funksiyaning tasvirini toping.

Yechish: Tasvirlar jadvalidan har bir handing tasvirini topamiz:

$$L\{2t\} = 2\frac{1}{p^2} = \frac{2}{p^2};$$

$$L\{e^{3t} \cdot \cos t\} = \frac{p - 2}{(p - 2)^2 + 1}.$$

Shuning uchun

$$L\{2t - e^{3t} \cdot \cos t\} = L\{2t\} - L\{e^{3t} \cdot \cos t\} = \frac{2}{p^2} - \frac{p-2}{(p-2)^2 + 1}.$$

7-Misol. $\bar{f}(p) = \dfrac{p}{p^2 - 2p + 5}$ funksiyaning originalini toping.

Yechish: Kasrni originallari ma'lum bo'lgan kasrlarga yoyamiz:

$$\frac{p}{p^2 - 2p + 5} = \frac{p - 1 + 1}{(p-1)^2 + 4} = \frac{p-1}{(p-1)^2 + 4} + \frac{1}{(p-1)^2 + 4}.$$

Tasvirlar jadvalidan

$$\frac{p-1}{(p-1)^2 + 4} = L\{e^t \cos 2t\};\ \frac{1}{(p-1)^2 + 4} = \frac{1}{2} \cdot \frac{2}{(p-1)^2 + 4} = \frac{1}{2} L\{e^t \sin 2t\}$$

tengliklarni topamiz. Shuning uchun

$$\frac{p}{p^2 - 2p + 5} = L\left\{e^t \left(\cos 2t + \frac{1}{2} \sin 2t\right)\right\}.$$

8-Misol. $\dfrac{\partial^2 u}{\partial x^2} - 2\dfrac{\partial^2 u}{\partial x \partial y} + 2\dfrac{\partial^2 u}{\partial y^2} = 0$ tenglamani kanonik shaklga keltiring.

Yechish: Bu yerda $a = 1, b = -1, c = 2, b^2 - ac = -1 < 0$, ya'ni tenglama elliptik turdagi tenglama ekan. Uning xarakteristik tenglamasi

$(dy)^2 + 2dxdy + 2(dx)^2 = 0$yoki${y'}^2 + 2y' + 2 = 0$ ko'rinishda bo'ladi. Bundan esa $y' = -1 \pm i$ va $y = (-1 \pm i)x$ ya'ni $y + x - ix = C_1$ va $y + x + ix = C_2$ ikkita mavhum xarakteristik oilalarga ega bo'lamiz. Shuning uchun yangi o'zgaruvchilarni $\xi = x + y$va$\eta = x$ tengliklar orqali kiritamiz. Ularning xususiy hosilalari

$$\frac{\partial \xi}{\partial x} = 1, \frac{\partial \xi}{\partial y} = 1, \frac{\partial \eta}{\partial x} = 1, \frac{\partial \eta}{\partial y} = 0$$

bo'ladi.U holda

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \xi} \cdot \frac{\partial \xi}{\partial x} + \frac{\partial u}{\partial \eta} \cdot \frac{\partial \eta}{\partial x} = \frac{\partial u}{\partial \xi} + \frac{\partial u}{\partial \eta};$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \xi} \cdot \frac{\partial \xi}{\partial y} + \frac{\partial u}{\partial \eta} \cdot \frac{\partial \eta}{\partial y} = \frac{\partial u}{\partial \xi} + \frac{\partial u}{\partial \eta} = \frac{\partial u}{\partial \xi};$$

$$\frac{\partial^2 u}{\partial x^2} = \left(\frac{\partial^2 u}{\partial \xi^2}\cdot\frac{\partial \xi}{\partial x} + \frac{\partial^2 u}{\partial \xi \partial \eta}\cdot\frac{\partial \eta}{\partial x}\right) + \left(\frac{\partial^2 u}{\partial \eta \partial \xi}\cdot\frac{\partial \xi}{\partial x} + \frac{\partial^2 u}{\partial \eta^2}\cdot\frac{\partial \eta}{\partial x}\right) =$$

$$= \frac{\partial^2 u}{\partial \xi^2} + 2\frac{\partial^2 u}{\partial \xi \partial \eta} + \frac{\partial^2 u}{\partial \eta^2};$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial \xi^2}\cdot\frac{\partial \xi}{\partial x} + \frac{\partial^2 u}{\partial \xi \partial \eta}\cdot\frac{\partial \eta}{\partial x} = \frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \xi \partial \eta};$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial \xi^2}\cdot\frac{\partial \xi}{\partial y} + \frac{\partial^2 u}{\partial \xi \partial \eta}\cdot\frac{\partial \eta}{\partial y} = \frac{\partial^2 u}{\partial \xi^2}.$$

Topilgan xususiy hosilalarni berilgan differensial tenglamaga qo’yib

$$\frac{\partial^2 u}{\partial \xi^2} + 2\frac{\partial^2 u}{\partial \xi \partial \eta} + \frac{\partial^2 u}{\partial \eta^2} - 2\frac{\partial^2 u}{\partial \xi^2} - 2\frac{\partial^2 u}{\partial \xi \partial \eta} + 2\frac{\partial^2 u}{\partial \xi^2} = 0$$

yoki

$$\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2} = 0.$$

Hisoblash uchun vazifalar

1.Berilgan kompleks sonlar ustida ko'rsatilgan amallarni bajaring.

2. Berilgan kompleks sonni trigonometrik shaklda yozing so'ngra ko'rsatilgan darajaga ko'taring.

3. $f(z)$ differensiallanuvchi funksiyaning haqiqiy yoki mavhum qismi berilgan.Ana shu $f(z)$ funksiyani toping. Bu yerda $z = x + iy$.

4. Berilgan integralni hisoblang.

5. Berilgan funksiyaning chegirmalarini hisoblang.

6. Berilgan funksiyaning tasvirini toping.

7. Berilgan funksiyaning originalini toping.

8. Ikkinchi tartibli xususiy hosilalai differensial tenglamani kanonik shaklga keltiring.

VARIANTLAR

1-Variant

1. $z_1 = 1 - i, z_2 = 3 + 5i, z_3 = 3 - 2i, z_4 = 2 - 3i.$
   $(z_1 + z_2)(z_3 - z_4); \frac{z_1 z_4}{z_2 - z_3}$
2. $z = \sqrt{3} + i.\quad z^3 - ?$
3. $u(x,y) = -2x + 2y$
4. $\int_{1+2i}^{-1-i} 2z dz$
5. $f(z) = \frac{z+3}{z^2 - 3z + 2}$
6. $f(t) = t^2 - 6e^{4t} \cdot \cos 2t$
7. $\bar{f}(p) = \frac{p-3}{p^2 - 4p + 5}$
8. $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + 10\frac{\partial^2 u}{\partial y^2} = 0$

2-variant

1. $z_1 = 2 - i, z_2 = 1 + 5i, z_3 = 2 - i, z_4 = -2 + i.$
   $(z_1 + z_2)(z_3 - z_4); \frac{z_1 + z_4}{z_2 z_3}$
2. $z = 1 + i\sqrt{3}.\quad z^3 - ?$
3. $v(x,y) = 7x - 7y$
4. $\int_{1-2i}^{3+i} (2z - 1) dz$
5. $f(z) = \frac{2z+5}{z^2 - 7z + 10}$
6. $f(t) = 5t - e^{-2t} \cdot \cos t$
7. $\bar{f}(p) = \frac{p+3}{p^2 - 2p + 10}$
8. $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} - 15\frac{\partial^2 u}{\partial y^2} = 0$

3-Variant

1. $z_1 = 8 - 3i, z_2 = -5 + 4i, z_3 = 7 - 2i, z_4 = 2 - 5i.$
   $(z_1 + z_2)(z_3 - z_4); \frac{z_1 z_4}{z_2 - z_3}$
2. $z = 3\sqrt{2} + 3\sqrt{2}i.\quad z^4 - ?$
3. $u(x,y) = x^2 + 2x - y^2 - 1$
4. $\int_{5+7i}^{1-5i} (4z^3 - z) dz$
5. $f(z) = \frac{5z-2}{z^2 - 6z + 8}$
6. $f(t) = t^2 \cdot e^{-6t} - e^{4t} \cdot \cos 2t$
7. $\bar{f}(p) = \frac{p-3}{p^2 - 6p + 13}$
8. $\frac{\partial^2 u}{\partial x^2} - 10\frac{\partial^2 u}{\partial x \partial y} + 29\frac{\partial^2 u}{\partial y^2} = 0$

4-variant

1. $z_1 = 2 - 3i, z_2 = 1 - 4i,$
   $z_3 = 7 - 5i, z_4 = 2 + 7i.$
   $(z_1 + z_2)(z_3 - z_4); \frac{z_1 + z_4}{z_2 z_3}$
2. $z = 3\sqrt{3} - 3i.\quad z^5 - ?$
3. $v(x,y) = 2y(x + 1)$
4. $\int_{5+7i}^{3-4i} 6z^2 dz$
5. $f(z) = \frac{4z+5}{z^2 - 2z - 24}$
6. $f(t) = \mathrm{ch}5t - e^{2t} \cdot \sin 3t$
7. $\bar{f}(p) = \frac{3}{p^2 - 4p + 20}$
8. $\frac{\partial^2 u}{\partial x^2} + 8\frac{\partial^2 u}{\partial x \partial y} + 15\frac{\partial^2 u}{\partial y^2} = 0$

5-Variant

1. $z_1 = 9 - 3i, z_2 = -2 + 3i,$
   $z_3 = 7 - 2i, z_4 = 3 - 8i.$
   $(z_1 + z_2)(z_3 - z_4); \frac{z_1 z_4}{z_2 - z_3}$
2. $z = -3\sqrt{3} + 3i. \quad z^2 - ?$
3. $u(x, y) = 2x^2 - 4x - 2y^2 + 1$
4. $\int_{1-8i}^{7-2i} (z + 3)dz$
5. $f(z) = \frac{2z + 3}{z^2 - 4z - 3}$
6. $f(t) = t^3 - 6 \operatorname{sh} 2t$
7. $\bar{f}(p) = \frac{2p - 7}{p^2 - 6p + 13}$
8. $\frac{\partial^2 u}{\partial x^2} - 6\frac{\partial^2 u}{\partial x \partial y} + 10\frac{\partial^2 u}{\partial y^2} = 0$

6-variant

1. $z_1 = 9 - 2i, z_2 = 6 + 5i,$
   $z_3 = 5 - 8i, z_4 = -7 + 3i.$
   $(z_1 + z_2)(z_3 - z_4); \frac{z_1 + z_4}{z_2 z_3}$
2. $z = -2 + 2\sqrt{3}i. \quad z^4 - ?$
3. $v(x, y) = 4y(x - 1)$
4. $\int_{7-2i}^{6-5i} (2z^3 + z^2 - z + 1)dz$
5. $f(z) = \frac{2z^2 + 1}{z^2 - 7z + 12}$
6. $f(t) = 7t^2 e^{-2t} + 5\cos 2t$
7. $\bar{f}(p) = \frac{p + 3}{p^2 - 10}$
8. $\frac{\partial^2 u}{\partial x^2} + 6\frac{\partial^2 u}{\partial x \partial y} + 10\frac{\partial^2 u}{\partial y^2} = 0$

7-Variant

1. $z_1 = 11 - i, z_2 = 9 + 5i,$
   $z_3 = 3 - 12i, z_4 = 2 - 5i.$
   $(z_1 + z_2)(z_3 - z_4); \frac{z_1 z_4}{z_2 - z_3}$
2. $z = -4\sqrt{3} + 4i. \quad z^2 - ?$
3. $u(x, y) = x^2 - 5x - y^2$
4. $\int_{3-8i}^{7-i} (z^2 + 3z - 1)dz$
5. $f(z) = \frac{2z + 11i}{z^2 + 6z + 5}$
6. $f(t) = t^3 - 6 \operatorname{sh} 2t$
7. $\bar{f}(p) = \frac{2p - 7}{p^2 + 16}$
8. $\frac{\partial^2 u}{\partial x^2} + 8\frac{\partial^2 u}{\partial x \partial y} + 16\frac{\partial^2 u}{\partial y^2} = 0$

8-variant

1. $z_1 = 1 - 8i, z_2 = 7 + 5i,$
   $z_3 = 2 - 3i, z_4 = 9 + 2i.$
   $(z_1 + z_2)(z_3 - z_4); \frac{z_1 + z_4}{z_2 z_3}$
2. $z = -3\sqrt{2} + 3\sqrt{2}i. \quad z^4 - ?$
3. $v(x, y) = y(2x - 5)$
4. $\int_{2-i}^{9-2i} (3z^2 + 2z - 3)dz$
5. $f(z) = \frac{-z^2 + i}{z^2 + 7z + 12}$
6. $f(t) = 7t^2 e^{-2t} + 5\cos 2t$
7. $\bar{f}(p) = \frac{p + 8}{p^2 - 16}$
8. $\frac{\partial^2 u}{\partial x^2} + 8\frac{\partial^2 u}{\partial x \partial y} + 20\frac{\partial^2 u}{\partial y^2} = 0$

9-Variant

1. $z_1 = -3i, z_2 = -9 + 8i,$
   $z_3 = 12 - 7i, z_4 = 3 - 11i.$
   $(z_1 + z_2)(z_3 - z_4); \frac{z_1 z_4}{z_2 - z_3}$
2. $z = -3\sqrt{2} - 3\sqrt{2}i.\ \ z^4 - ?$
3. $u(x, y) = x^2 + 3x - y^2 - 8$
4. $\int_{7-8i}^{8-2i} (z+3)^2 dz$
5. $f(z) = \frac{2z + 3i}{z^2 - 12z + 11}$
6. $f(t) = 5t^3 - \operatorname{sh} 7t$
7. $\bar{f}(p) = \frac{2p - 7}{p^2 + 25}$
8. $\frac{\partial^2 u}{\partial x^2} - 5\frac{\partial^2 u}{\partial x \partial y} + 25\frac{\partial^2 u}{\partial y^2} = 0$

10-variant

1. $z_1 = 7 - 9i, z_2 = -6 + 11i,$
   $z_3 = 5 + 3i, z_4 = 7 - 9i.$
   $(z_1 + z_2)(z_3 - z_4); \frac{z_1 + z_4}{z_2 z_3}$
2. $z = -5 + 5\sqrt{3}i.\ \ z^5 - ?$
3. $v(x, y) = y(2x + 3)$
4. $\int_{9-2i}^{6-3i} (8z^3 + 3z^2 - 2z + 1) dz$
5. $f(z) = \frac{2z^2 + i}{z^2 + 10z - 11}$
6. $f(t) = t^3 e^{-12t} - 7\cos 3t$
7. $\bar{f}(p) = \frac{p + 8}{p^2 - 16}$
8. $\frac{\partial^2 u}{\partial x^2} + 10\frac{\partial^2 u}{\partial x \partial y} + 29\frac{\partial^2 u}{\partial y^2} = 0$

11-Variant

1. $z_1 = 1 - 9i, z_2 = 7 + 8i,$
   $z_3 = -2 + 2i, z_4 = 6 - 5i.$
   $(z_1 + z_2)(z_3 - z_4); \frac{z_1 z_4}{z_2 - z_3}$
2. $z = -4\sqrt{3} - 4i.\ \ z^2 - ?$
3. $u(x, y) = y^2 + 2x - x^2 + 7$
4. $\int_{3-9i}^{7-2i} (3z^2 + 4z - 5) dz$
5. $f(z) = \frac{2zi + 6}{z^2 + 2z - 35}$
6. $f(t) = t^3 - 6\operatorname{sh} 2t$
7. $\bar{f}(p) = \frac{2p - 7}{p^2 + 16}$
8. $\frac{\partial^2 u}{\partial x^2} + 8\frac{\partial^2 u}{\partial x \partial y} + 16\frac{\partial^2 u}{\partial y^2} = 0$

12-variant

1. $z_1 = 7 - 8i, z_2 = -1 + 3i,$
   $z_3 = 2 - 9i, z_4 = 7 - 4i.$
   $(z_1 + z_2)(z_3 - z_4); \frac{z_1 + z_4}{z_2 z_3}$
2. $z = -3\sqrt{2} - 3\sqrt{2}i.\ \ z^4 - ?$
3. $v(x, y) = 2y(x + 1)$
4. $\int_{2-5i}^{1-2i} (6z^2 + 2z + 3) dz$
5. $f(z) = \frac{-z^2 + 5i}{z^2 + 7z + 12}$
6. $f(t) = 7t^2 e^{-2t} + 5\cos 2t$
7. $\bar{f}(p) = \frac{p + 8}{p^2 - 16}$
8. $\frac{\partial^2 u}{\partial x^2} + 8\frac{\partial^2 u}{\partial x \partial y} + 20\frac{\partial^2 u}{\partial y^2} = 0$

13-Variant

1. $z_1 = 7 - 3i, z_2 = 9 - 8i,$
   $z_3 = 13 - 5i, z_4 = 3 - 11i.$
   $(z_1 + z_2)(z_3 - z_4); \frac{iz_1 z_4}{z_2 - iz_3}$
2. $z = -5\sqrt{3} - 5i.\ \ z^4 -?$
3. $u(x, y) = x^2 - x - y^2 + 11$
4. $\int_{7-i}^{8-2i} (2zi + 3)^2 dz$
5. $f(z) = \frac{iz + 8}{z^2 + 11z + 18}$
6. $f(t) = 5te^{-2t} - e^{6t} \sin(-3t)$
7. $\bar{f}(p) = \frac{9}{(p-5)^6}$
8. $\frac{\partial^2 u}{\partial x^2} - 14 \frac{\partial^2 u}{\partial x \partial y} + 49 \frac{\partial^2 u}{\partial y^2} = 0$

14-variant

1. $z_1 = 12 - 5i, z_2 = 6 - 12i,$
   $z_3 = 8 + 3i, z_4 = 7 - 5i.$
   $(z_1 + z_2)(z_3 - z_4); \frac{z_1 + 2iz_4}{-iz_2 z_3}$
2. $z = 5\sqrt{3} - 5i.\ \ z^5 -?$
3. $v(x, y) = y(2x - 1)$
4. $\int_{1-2i}^{1-i} (5iz + 2)^3 dz$
5. $f(z) = \frac{z^2 - 5i}{z^2 - 10z + 18}$
6. $f(t) = t^3 e^{-3t} - \cos 4t$
7. $\bar{f}(p) = \frac{13}{(p-8)^8}$
8. $\frac{\partial^2 u}{\partial x^2} + 14 \frac{\partial^2 u}{\partial x \partial y} + 49 \frac{\partial^2 u}{\partial y^2} = 0$

15-Variant

1. $z_1 = 10 - 9i, z_2 = 1 + 8i,$
   $z_3 = -2 + i, z_4 = -5 - 5i.$
   $(z_1 + z_2)(z_3 - z_4); \frac{z_1 z_4}{iz_2 - z_3}$
2. $z = 7\sqrt{3} - 7i.\ \ z^2 -?$
3. $u(x, y) = y^2 + 2x - x^2 + 7$
4. $\int_{3-9i}^{3-2i} (3z^2 + 8z - 5) dz$
5. $f(z) = \frac{2zi + 6}{z^2 + 2z - 35}$
6. $f(t) = t^3 - 6 \operatorname{ch} 2t$
7. $\bar{f}(p) = \frac{2p - 7}{p^2 + 25}$
8. $\frac{\partial^2 u}{\partial x^2} - 16 \frac{\partial^2 u}{\partial x \partial y} + 68 \frac{\partial^2 u}{\partial y^2} = 0$

16-variant

1. $z_1 = -3 - 8i, z_2 = -1 - 7i,$
   $z_3 = 2 - 5i, z_4 = 7 + 6i.$
   $(z_1 + z_2)(z_3 - z_4); \frac{z_1 + 2z_4}{z_2 z_3}$
2. $z = 7\sqrt{2} + 7\sqrt{2}i.\ \ z^4 -?$
3. $v(x, y) = 2y(x + 1)$
4. $\int_{2-5i}^{1-2i} (6z^2 + 2z + 3) dz$
5. $f(z) = \frac{-z^2 + 5i}{z^2 + 7z + 12}$
6. $f(t) = 7t^2 e^{-3t} + 5 \cos 12t$
7. $\bar{f}(p) = \frac{p + 8}{p^2 - 25}$
8. $\frac{\partial^2 u}{\partial x^2} + 16 \frac{\partial^2 u}{\partial x \partial y} + 68 \frac{\partial^2 u}{\partial y^2} = 0$

17-Variant

1. $z_1 = 7 - 3i, z_2 = 3 - 8i,$
   $z_3 = 3 - 5i, z_4 = 3 - i.$
   $(z_1 + 2iz_2)(z_3 - z_4); \frac{iz_1 z_4}{z_2 - iz_3}$
2. $z = 8 + 8\sqrt{3}i.\ \ z^4 - ?$
3. $u(x, y) = x^2 + 5x - y^2 + 12$
4. $\int_{2-3i}^{1-2i} (zi + 3)^3 dz$
5. $f(z) = \frac{iz + 8}{(z - 2i)(z + 2i)}$
6. $f(t) = t^2 e^{3t} - e^t \cos(-4t)$
7. $\bar{f}(p) = \frac{5}{(2p - 8)^6}$
8. $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + 5\frac{\partial^2 u}{\partial y^2} = 0$

18-variant

1. $z_1 = 2 - 5i, z_2 = 6 - 7i,$
   $z_3 = 8 + 3i, z_4 = 4 - 5i.$
   $(z_1 + z_2)(z_3 - 2iz_4); \frac{z_1 + 2iz_4}{-iz_2 z_3}$
2. $z = 8\sqrt{3} + 8i.\ \ z^3 - ?$
3. $v(x, y) = y(2x + 5)$
4. $\int_{3-2i}^{1-2i} (5z + 2i)^2 dz$
5. $f(z) = \frac{z^2 - 5i}{(z + 3i)(z - 2i)}$
6. $f(t) = t^3 e^{-7t} - 8\cos 4t$
7. $\bar{f}(p) = \frac{-4}{(3p - 27)^8}$
8. $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + 17\frac{\partial^2 u}{\partial y^2} = 0$

19-Variant

1. $z_1 = 1 - 9i, z_2 = -1 + 8i,$
   $z_3 = -2 + 3i, z_4 = -5 - 6i.$
   $(z_1 + z_2)(z_3 - iz_4); \frac{z_1 z_4}{z_2 - iz_3}$
2. $z = 6 - 6\sqrt{3}i.\ \ z^2 - ?$
3. $u(x, y) = x^2 - 7x - y^2 - 12$
4. $\int_{3-i}^{1-2i} (3z - 5)^2 dz$
5. $f(z) = \frac{2z + 6i}{z^2 + 12z + 35}$
6. $f(t) = e^{-3t} - 5\cos 2t$
7. $\bar{f}(p) = \frac{p - 7}{p^2 + 8p + 25}$
8. $\frac{\partial^2 u}{\partial x^2} - 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} = 0$

20-variant

1. $z_1 = -3 + 5i, z_2 = 1 - 7i,$
   $z_3 = 2 - 7i, z_4 = 3 + 8i.$
   $(z_1 + iz_2)(z_3 - z_4); \frac{z_1 + 2iz_4}{z_2 z_3}$
2. $z = -6\sqrt{2} - 6\sqrt{2}i.\ \ z^3 - ?$
3. $v(x, y) = y(2x - 7)$
4. $\int_{2-i}^{1-2i} (2z + 3)^3 dz$
5. $f(z) = \frac{-z^2 + 5i}{z^2 + 13z + 30}$
6. $f(t) = 7t^3 e^{4t} + 5\,\mathrm{sh}\,6t$
7. $\bar{f}(p) = \frac{p + 8}{p^2 - 8p + 25}$
8. $\frac{\partial^2 u}{\partial x^2} + 16\frac{\partial^2 u}{\partial x \partial y} + 15\frac{\partial^2 u}{\partial y^2} = 0$

21-Variant

1. $z_1 = 5 - 3i, z_2 = 3 - 5i,$
   $z_3 = 4 - 5i, z_4 = 3 - 5i.$
   $(z_1 + 2iz_2)(z_3 - iz_4); \frac{iz_1 z_4}{z_2 - iz_3}$
2. $z = -8 + 8\sqrt{3}i.\ \ z^4 -?$
3. $u(x,y) = x^2 - 4x - y^2 + 1$
4. $\int_{2-3i}^{1-2i} (-zi + 3)^2 dz$
5. $f(z) = \frac{iz + 1}{(z - 2i)(z + 5i)}$
6. $f(t) = t^2 e^t - e^{-5t} \cos 4t$
7. $\bar{f}(p) = \frac{5}{(2p - 12)^6}$
8. $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$

22-variant

1. $z_1 = 2 - i, z_2 = 6 + i,$
   $z_3 = 8 + i, z_4 = 4 - i.$
   $(z_1 + iz_2)(z_3 - 2iz_4); \frac{z_1 + 2iz_4}{-iz_2 z_3}$
2. $z = 8\sqrt{3} - 8i.\ \ z^3 -?$
3. $v(x,y) = 2(x - 2)y$
4. $\int_{3-2i}^{1-2i} (z - 5i)^2 dz$
5. $f(z) = \frac{z^2 - 3i}{(z + 3i)(z - i)}$
6. $f(t) = t^3 e^{2t} - 5 \cos 4t$
7. $\bar{f}(p) = \frac{7}{(3p - 15)^8}$
8. $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$

23-Variant

1. $z_1 = 1 - 6i, z_2 = -1 + 8i,$
   $z_3 = -2 + 3i, z_4 = -5 - 6i.$
   $(z_1 + 2z_2)(z_3 - iz_4); \frac{z_1 z_4}{z_2 - iz_3}$
2. $z = -6 - 6\sqrt{3}i.\ \ z^2 -?$
3. $u(x,y) = 3x^2 - 3y^2 - 5$
4. $\int_{2-i}^{1-2i} (3z - 1)^2 dz$
5. $f(z) = \frac{2z + 6i}{z^2 - 6z + 10}$
6. $f(t) = 6e^{-3t} - 2 \cos 5t$
7. $\bar{f}(p) = \frac{2p - 7}{p^2 + 6p + 25}$
8. $\frac{\partial^2 u}{\partial x^2} - 4\frac{\partial^2 u}{\partial x \partial y} - 6\frac{\partial^2 u}{\partial y^2} = 0$

24-variant

1. $z_1 = -3 + i, z_2 = 1 - 3i,$
   $z_3 = 7 - 2i, z_4 = 3 + 11i.$
   $(z_1 + iz_2)(z_3 - z_4); \frac{z_1 + 2iz_4}{z_2 z_3}$
2. $z = 6\sqrt{2} + 6\sqrt{2}i.\ \ z^3 -?$
3. $v(x,y) = 2xy$
4. $\int_{2-3i}^{3-2i} (2z + 1)^3 dz$
5. $f(z) = \frac{-z^2 + 5i}{z^2 + 8z + 17}$
6. $f(t) = 7t^4 e^{3t} - 2\, ch\, 3t$
7. $\bar{f}(p) = \frac{2p + 7}{p^2 - 6p + 25}$
8. $\frac{\partial^2 u}{\partial x^2} + 5\frac{\partial^2 u}{\partial x \partial y} + 6\frac{\partial^2 u}{\partial y^2} = 0$

25-Variant

1. $z_1 = 9 - 3i, z_2 = 3 - 5i,$
   $z_3 = 4 - 5i, z_4 = 3 - 8i.$
   $(z_1 + 2iz_2)(z_3 - iz_4); \frac{iz_1z_4}{z_2 - iz_3}$
2. $z = -8 + 8\sqrt{3}i.\ \ z^4 - ?$
3. $u(x, y) = 5x^2 + 6x - 5y^2 - 1$
4. $\int_{2-5i}^{1-2i} (-zi + 5)^2 dz$
5. $f(z) = \frac{iz + 5}{(z - 3i)(z + 5i)}$
6. $f(t) = t^3 - 6\,\mathrm{sh}\,2t$
7. $\bar{f}(p) = \frac{2p - 7}{p^2 + 16}$
8. $\frac{\partial^2 u}{\partial x^2} - 8\frac{\partial^2 u}{\partial x \partial y} + 20\frac{\partial^2 u}{\partial y^2} = 0$

26-variant

1. $z_1 = 2 - i, z_2 = 6 + 5i,$
   $z_3 = 8 + i, z_4 = 4 - i.$
   $(z_1 + iz_2)(z_3 - 2iz_4); \frac{z_1 + 2iz_4}{-iz_2z_3}$
2. $z = 8\sqrt{3} - 8i.\ \ z^3 - ?$
3. $v(x, y) = 2(5x + 3)y$
4. $\int_{1-2i}^{1-2i} (2z - i)^2 dz$
5. $f(z) = \frac{4z^2 - 3i}{(z + i)(z - i)}$
6. $f(t) = 7t^2 e^{-2t} + 5\cos 2t$
7. $\bar{f}(p) = \frac{p + 8}{p^2 - 16}$
8. $\frac{\partial^2 u}{\partial x^2} + 8\frac{\partial^2 u}{\partial x \partial y} + 20\frac{\partial^2 u}{\partial y^2} = 0$

27-Variant

1. $z_1 = 1 - 6i, z_2 = -1 + 8i,$
   $z_3 = -2 + 3i, z_4 = -5 + 8i.$
   $(z_1 + 2z_2)(z_3 - iz_4); \frac{z_1z_4}{z_2 - iz_3}$
2. $z = -6\sqrt{2} - 6\sqrt{2}i.\ \ z^2 - ?$
3. $u(x, y) = 3x^2 - 3y^2 - 5$
4. $\int_{2-3i}^{1-6i} (2z - 3)^2 dz$
5. $f(z) = \frac{2z + 6i}{z^2 - 14z + 53}$
6. $f(t) = t^2 \cdot e^{-6t} - e^{4t} \cdot \cos 2t$
7. $\bar{f}(p) = \frac{p - 3}{p^2 - 6p + 13}$
8. $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial^2 u}{\partial x \partial y} + 73\frac{\partial^2 u}{\partial y^2} = 0$

28-variant

1. $z_1 = -3 + 5i, z_2 = 2 - 3i,$
   $z_3 = 7 - 2i, z_4 = 3 + 7i.$
   $(z_1 + iz_2)(z_3 - z_4); \frac{z_1 + 2iz_4}{z_2z_3}$
2. $z = 10\sqrt{2} + 10\sqrt{2}i.\ \ z^3 - ?$
3. $v(x, y) = 2xy$
4. $\int_{2-3i}^{4-2i} (-z + 1)^3 dz$
5. $f(z) = \frac{-z^2 + 5i}{z^2 + 8z + 12}$
6. $f(t) = \mathrm{ch}5t - e^{2t} \cdot \sin 3t$
7. $\bar{f}(p) = \frac{3}{p^2 - 4p + 20}$
8. $\frac{\partial^2 u}{\partial x^2} + 16\frac{\partial^2 u}{\partial x \partial y} + 73\frac{\partial^2 u}{\partial y^2} = 0$