1-MUSTAQIL ISH

1. Sirtga $M_0(x_0;y_0;z_0)$ nuqtada o‘tkazilgan urinma tekislik va normal tenglamalarini tuzing.

2. $z = f(x,y)$ funksiya berilgan tenglikni qanoatlantirishini ko‘rsating.

3. Murakkab funksiyaning ko‘rsatilgan hosilalarini toping.

4. Oshkormas ko‘rinishda berilgan $z=(x;y)$ funksiyaning birinchi tartibli xususiy hosilalarini toping.

5. Funksiyaning uchinchi tartibli differensialini toping.

6. Funksiyani ekstremumga tekshiring.

7. $z = f(x,y)$ funksiyaning D yopiq sohadagi eng katta va eng kichik qiymatlarini toping.

8. $z = f(x,y)$ funksiyalarning $\varphi(x,y)=0$ tenglama bilan bog‘langanlik shartidagi ekstremumlarini toping.

9. Eng katta va eng kichik qiymatlarni topishga oid amaliy masalalarni yeching.

10. x argument va $y = f(x)$ funksiyaning tajriba natijasida olingan qiymatlari jadvalda berilgan. x va y o‘zgaruvchilar orasidagi $y = ax^2+bx+c$ empirik funksiyani eng kichik kvadratlar usuli bilan toping. Tajriba nuqtalarini va empirik funksiyani to‘g‘ri burchakli dekart koordinatalar sistemasida tasvirlovchi chizmani chizig.

1-variant

1. $z = 2x^2-3y^2+4x-2y-10xy, \quad M_0(-1;1;3).$

2. $z=\ln(x^2+xy+y^2), \qquad (z'_x)^2-(z'_y)^2+z''_{xx}-z''_{yy}=0.$

3. $z=\ln(x^3+3y), \; x=utgv, \; y=\dfrac{v}{u^3}, \qquad \dfrac{\partial z}{\partial u}, \; \dfrac{\partial z}{\partial v}-?$

4. $x^3+2y^3+z^3-3xyz=2y.$

5. $z=x^3\cos y+y^3\sin x.$

6. $z=x^3+y^3-18xy+7.$

7. $z=5x^2-3xy+y^2+4, \quad D: x=-1, \; y=-1, \; x+y-1=0.$

8. $z=8-5x-4y, \quad x^2-y^2-9=0.$

9. Perimetri $2p$ ga teng uchburchak eng katta yuzaga ega bo‘lsa, uchburchakning tomonlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-0,8	0,4	0,3	-0,5	-2,0	-4,9

2-variant

1. $x^2 + y^2 - z^2 + 2x - 2xy - z = 0, \quad M_0(1;1;-2).$

2. $z = x^{y^2}, \qquad yz \cdot z''_{yy} - z \cdot z'_y - y \cdot (z'_y)^2 = 0.$

3. $u = \dfrac{yz}{x}, \; x = e^t, \; y = \ln t, \; z = t^2 - 1, \qquad \dfrac{du}{dt} - ?$

4. $xy^2 + yz^2 + zx^2 = 2xyz.$

5. $z = \cos(3x + e^{-y}).$

6. $z = \ln(x + y) - 2x^4 - 2y^4.$

7. $z = (x - y)(4 - x - y), \quad D: x = 0, x + 2y - 4 = 0, \; x - 2y - 4 = 0.$

8. $z = xy, \quad x^2 + y^2 - 1 = 0.$

9. Devorining qalinligi d ga va hajmi V ga teng ochiq quti (yashik) yasash uchun eng kam material sarflangan bo‘lsa, qutining tashqi o‘lchamlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-0,3	-2,4	-2,8	-1,8	-0,3	2,6

3-variant

1. $2x^2 - 3y^2 + xy + 3x - z - y = 0, \; M_0(1;-1;2).$

2. $z = xsh(x + y) + ych(x + y), \qquad z''_{xx} - 2z''_{xy} + z''_{yy} = 0.$

3. $z = arctg\,\dfrac{x+1}{y}, \; y = e^{(x+1)^2}, \qquad \dfrac{dz}{dx} - ?$

4. $z = x + arctg\,\dfrac{y}{z - x}.$

5. $z = e^{x+y} sh(x - y).$

6. $z = xy + \dfrac{1}{x} + \dfrac{8}{y}.$

7. $z = x^2 + 2xy - y^2 - 2x + 2y, \quad D: x = 0, \; y = 0, \; x - y + 2 = 0.$

8. $z = \dfrac{1}{\sqrt{x}} + \dfrac{2}{\sqrt{y}}, \quad x + 2y - 3 = 0.$

9. Tagi silindr ko‘rinishiga va tepasi konus shakliga ega chodirni tikish uchun eng kam material sarflangan bo‘lsa, chodirning o‘lchamlari nisbatini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-0,5	-1,5	-1,8	-0,8	1,6	4,5

4-variant

1. $x^2+y^2+z^2-4x+6z+8=0,\ M_0(2;1;-1).$

2. $z=\ln(x+e^{-y}),\qquad z'_x\cdot z''_{xy}-z'_y\cdot z''_{xx}=0.$

3. $z=x^y+y^x,\ x=u^2+v^2,\ y=v^2-u^2,\qquad \frac{\partial z}{\partial u},\ \frac{\partial z}{\partial v}-?$

4. $\frac{x}{z}=\ln\frac{x}{y}+yz^2.$ **5.** $z=\ln\cos(xy).$

6. $z=x\sqrt{y}-x^2-yx+6x+3.$

7. $z=xy(5-3x-15y),\quad D: x=0,\ y=0,\ 4x+y-8=0.$

8. $z=\frac{1}{\sqrt[3]{x}}+\frac{4}{\sqrt[3]{y}},\quad x+4y-5=0.$

9. Radiusi R ga teng aylanaga ichki chizilgan uchburchak eng katta yuzaga ega bo‘lsa, uning tomonlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-0,3	0,6	1,3	2,0	1,7	1,2

5-variant

1. $y^2+z^2-4x^2+2xy+3xz-6=0,\quad M_0(1;-2;2).$

2. $z=\frac{xy}{x-y},\qquad z''_{xx}+2z''_{xy}+z''_{yy}-\frac{2}{xy}\cdot z=0.$

3. $z=\frac{x}{y}+\frac{y}{x},\ x=u\sin v,\ y=v\cos u,\qquad \frac{\partial z}{\partial u},\ \frac{\partial z}{\partial v}-?$

4. $x^2-y^2-z^2=\cos z.$ **5.** $z=\frac{x}{y}+\frac{y}{x}.$

6. $z=\ln(x^2y)-x^2-9y^3.$

7. $z=x^3-3y^2-3xy,\quad D: x=0,\ x=2,\ y=0,\ y=1.$

8. $z=9-5x+3y,\quad x^2-y^2-16=0.$

9. Uchlari $x^2+3y^2=15$ ellipsning $A(\sqrt{3};-2)$, $B(-2\sqrt{3};1)$ va $C(x;y)$ nuqtalarida yotgan uchburchakning yuzasi eng katta bo‘lsa, $C(x;y)$ nuqtani toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	0,4	0,2	1,2	1,7	2,2	4,0

6-variant

1. $z = x^2 - y^2 - 2xy - x - 2y$, $M_0(-1;1;1)$.

2. $z = \dfrac{y}{\ln(x^2 - y^2)}$, $\dfrac{1}{x} \cdot z'_x + \dfrac{1}{y} z'_y - \dfrac{1}{y^2} \cdot z = 0$.

3. $z = \arcsin\dfrac{x}{y}$, $x = \sin t$, $y = \cos t$, $\dfrac{dz}{dt} - ?$

4. $x^2 + y^2 = e^{xz} + 2yz$.

5. $z = \dfrac{xy}{x + y}$.

6. $z = x^3 + 8y^3 - 6xy + 1$.

7. $z = x^2 + 2yx - 4x + 8y$, $D: x = 2,\ y = 0,\ 5x - 3y + 45 = 0$.

8. $z = 2\sqrt{x} - 3\sqrt{y}$, $4x - 6y + 1 = 0$.

9. Radiusi R ga teng sharga tashqi chizilgan konus eng kichik hajmga ega bo‘lsa, konusning o‘lchamlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	4,9	5,4	5,0	4,6	3,3	1,5

7-variant

1. $x^2 + y^2 - 3z^2 + xy + 2z = 0$, $M_0(1;0;1)$.

2. $z = xtg(x + y) + y^2 + xy$, $z''_{xx} - 2z''_{xy} + z''_{yy} = 0$.

3. $z = \dfrac{x^2 + xy}{1 + y}$, $y = x\cos x$, $\dfrac{dz}{dx} - ?$

4. $yz = x + y^2 tg\dfrac{x}{z}$.

5. $z = e^{4y}\ln(xy)$.

6. $z = y\sqrt{x} - y^2 - x + 6y$.

7. $z = xy(2 - 2x - y)$, $D: x = 0,\ x = 1,\ y = 0,\ y = 2$.

8. $z = 5 - \dfrac{1}{x} + \dfrac{1}{y^2}$, $x^2 - 4y - 5 = 0$.

9. Yon sirti S ga teng konus eng katta hajmga ega bo‘lsa, uning o‘lchamlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-0,5	-1,1	-0,3	0,4	2,0	4,8

8-variant

1. $z = 2x^2 + y^2 + 4xy - 5x - 10, \quad M_0(1;-7;8).$

2. $z = y\sqrt{\frac{y}{x}}, \qquad x^2 \cdot z''_{xx} - y^2 \cdot z''_{yy} = 0.$

3. $z = \sqrt{x-y} + \ln(x^2 + y), \; x = ve^u, \; y = ue^v, \qquad \frac{\partial z}{\partial u}, \; \frac{\partial z}{\partial v} - ?$

4. $yx = z\ln\frac{zx}{y}.$

5. $z = e^{x-y}ch(x+y).$

6. $z = 2xy + \frac{4}{x} + \frac{1}{y}.$

7. $z = 4x^2 + 9y^2 - 4x - 6y + 3, \quad D: x = 0, \; y = 0, \; x + y - 1 = 0.$

8. $z = 1 + \frac{2}{x} + \frac{3}{y}, \quad \frac{4}{x^2} + \frac{6}{y^2} - \frac{1}{10} = 0.$

9. $x + 3y - z = 0$ tekislikning $x^2 + y^2 = 10$ silindr bilan kesishish nuqtalari applikatalarining eng katta va eng kichik qiymatlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	1,0	1,5	1,1	0,2	-0,9	-2,9

9-variant

1. $x^2 + y^2 + z^2 - 6x + 4z - 4xz = 0, \quad M_0(1;2;-1).$

2. $z = \sqrt{x^2 + y^2}, \qquad z'_y \cdot z'_x + z \cdot z''_{xy} = 0.$

3. $z = \frac{\arcsin x}{y^2}, \; x = \frac{1}{5}u^5 + \frac{1}{7}v^7, \; y = \ln(uv), \qquad \frac{\partial z}{\partial u}, \; \frac{\partial z}{\partial v} - ?$

4. $x^2y - zy^2 = xe^{yz}.$

5. $z = \ln(x^y y^x)$.

6. $z = 3x^2y + y^3 - 18x - 30y.$

7. $z = 4 - 2x^2 - y^2, \quad D: y = 0, \; y = \sqrt{1 - x^2}.$

8. $z = 8 - 5x - 3y, \quad x^2 - y^2 - 16 = 0.$

9. $4x^2 + 36y^2 = 9$ ellipsning $4x + 9y - 25 = 0$ to‘g‘ri chiziqdan eng uzoq va eng yaqin joylashgan nuqtalarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-0,2	-0,4	0,7	0,7	2,6	4,5

10-variant

1. $x^2+y^2+z^2+6x+4y-8=0, \quad M_0(1;-1;2).$

2. $z=\frac{x}{x^2+y^2}, \quad z''_{xx}+z''_{yy}=0.$

3. $z=\frac{e^{xy}}{\sqrt{x+y}}, \; x=u\cos v, \; y=v\sin u, \quad \frac{\partial z}{\partial u}, \; \frac{\partial z}{\partial v}-?$

4. $x\ln y+y\ln z+z\ln x=4.$ **5.** $z=(x^2+y^2)\cdot e^{x+y}.$

6. $z=5x+y^3-3\ln(x^5y).$

7. $z=x^2+4xy-2y^2-6x-1, \quad D: x=0, \; y=0, \; x+y-3=0.$

8. $z=3\sqrt{x}+4\sqrt{y}, \; 3x+4y-28=0.$

9. Sirti S ga teng silindr shaklidagi usti ochiq idish eng ko‘p sig‘imga ega bo‘lsa, uning o‘lchamlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	1,4	1,8	1,7	0,8	-1,0	-3,0

11-variant

1. $y^2-2x^2-z^2-y+4z+13=0, \quad M_0(2;1;-1).$

2. $z=e^x(x\cos y-y\sin y), \quad z''_{xx}+z''_{yy}=0.$

3. $z=\frac{tg2x}{y^2}, \; x=arctg\sqrt{uv}, \; y=\frac{u}{v}, \quad \frac{\partial z}{\partial u}, \; \frac{\partial z}{\partial v}-?$

4. $zx=ye^{\frac{x}{z}}.$ **5.** $z=\ln\sin(xy).$

6. $z=xy+\frac{2}{x^2}+\frac{1}{2y}.$

7. $z=x^2y(4-x-y), \quad D: x=0, \; y=0, \; x+y-6=0.$

8. $z=4-\frac{3}{x}+\frac{1}{2y^2}, \; 3x+y-2=0.$

9. Perimetri $2p$ ga teng uchburchakni biror tomoni atrofida aylantirishdan hosil bo‘lgan jism eng katta hajmga ega bo‘lsa, uchburchakning tomonlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-0,1	-1,3	-1,2	-0,2	1,4	3,9

12-variant

1. $z = x^2 + y^2 - 4xy + 3y - 15, \quad M_0(3;-1;4).$

2. $z = \dfrac{x}{\cos(y^2 - x^2)}, \qquad \dfrac{1}{x} \cdot z'_x + \dfrac{1}{y} z'_y - \dfrac{1}{x^2} \cdot z = 0.$

3. $z = e^x \ln(x^2 + y^2), \; y = \dfrac{1}{2} x^2 + x, \qquad \dfrac{dz}{dx} - ?$

4. $\cos(xy + z) - \dfrac{xz}{y} = 0.$

5. $z = x^2 \cos y + y^3 \sin x.$

6. $z = 2x^3 + 2y^3 + x^2 y + y^2 x - 9x - 9y.$

7. $z = 4x^2 + y^2 + 4x + 2y + 6, \quad D: x = 0, \; y = 0, \; x + y + 2 = 0.$

8. $z = 5 + \dfrac{2}{x} + \dfrac{1}{y^2}, \quad x^2 + 2y - 3 = 0.$

9. Tekis metaldan (listdan) kesib olingan umumiy yuzasi S ga teng doira va to‘g‘ri to‘rtburchakdan silindr yasashda (bunda doiradan silindrning asosi va to‘g‘ri to‘rtburchakdan silindrning yon sirti yasaladi) eng kam payvand chokidan foydalanilgan bo‘lsa, silindrning o‘lchamlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	1,0	1,6	1,5	0,4	-1,3	-3,7

13-variant

1. $x^2 + y^2 + 2xz - z^2 + x - 2z - 2 = 0, \; M_0(1;1;1).$

2. $z = e^{xy} + e^{\frac{x}{y}}, \qquad x^2 \cdot z''_{xx} - y^2 z''_{yy} + x \cdot z'_x - y \cdot z'_y = 0.$

3. $z = \dfrac{x}{y^2} + 2y, \; x = u\sqrt{v}, \; y = v\cos u, \qquad \dfrac{\partial z}{\partial u}, \; \dfrac{\partial z}{\partial v} - ?$

4. $x + y^2 - z^3 = e^{-(x+y+z)}.$

5. $z = (x - y)\sin(x + y).$

6. $z = 4x + 3y - 2\ln(x^4 y^3).$

7. $z = x^3 + 8y^3 - 6xy + 1, \quad D: x = 0, \; x = 2, \; y = -1, \; y = 1.$

8. $z = x^2 y, \quad 2x + y - 1 = 0.$

9. $\dfrac{x^2}{4} + \dfrac{y^2}{9} + \dfrac{z^2}{25} = 1$ ellipsoidga ichki chizilgan to‘g‘ri burchakli parallelepiped eng katta hajmga ega bo‘lsa, uning o‘lchamlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-0,2	-1,2	-1,5	-1,4	0,3	2,0

14-variant

1. $x^2+y^2-z^2+6xy-z-6=0, \quad M_0(1;1;-2).$

2. $z=x\sin(x+y)+y\cos(x+y), \quad z''_{xx}-2z''_{xy}+z''_{yy}=0.$

3. $z=\arcsin\dfrac{x}{y}, \; y=\sqrt{x^2+1}, \quad \dfrac{dz}{dx}-?$

4. $xe^{xyz}+yx+zy=6.$ **5.** $z=e^{x+y}\cos(x-y).$

6. $z=xy+\dfrac{2}{x}+\dfrac{4}{y^2}.$

7. $z=3x^2+3y^2-2x-2y+2, \quad D: x=0, \; y=0, \; x+y-1=0.$

8. $z=6-4x-3y, \quad x^2+y^2-25=0.$

9. Diametri d ga teng sharga ichki chizilgan silindr eng kichik to‘la sirtga ega bo‘lsa, silindrning o‘lchamlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-1,6	-0,2	0,1	-0,7	-2,5	-5,5

15-variant

1. $4x^2-z^2+4xy-yz+3z-9=0, \quad M_0(-2;1;1).$

2. $z=arctg\dfrac{x}{y}, \quad z''_{xx}+z''_{yy}=0.$

3. $z=y^2tgx, \; x=e^t\sin t, \; y=e^t\cos t, \quad \dfrac{dz}{dt}-?$

4. $5z-\ln(x^2+y^2)=2yz.$ **5.** $z=\sin(e^x+2y).$

6. $z=6xy-x^2y-y^2x.$

7. $z=2x^3-xy^2+y^2, \quad D: x=0, x=1, \; y=0, \; y=6.$

8. $z=\dfrac{3}{\sqrt{x}}-\dfrac{1}{\sqrt{y}}, \quad 3x-y-8=0.$

9. Asosi a ga va balandligi H ga teng muntazam to‘ptburchakli piramida shaklidagi suv bilan to‘ldirilgan idishga kub (piramida va kub asoslarining markazlari bu asoslarga perpendikular to‘g‘ri chiziqda yotadi) tashlangan. Kubning idish ichidagi qismi idishdan eng ko‘p hajmdagi suv siqib chiqargan bo‘lsa, kubning qirrasini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-1,5	-2,8	-2,6	-1,6	0,4	3,1

16-variant

1. $z = y^2 - x^2 + 2xy - 3y + 5x - 4, \quad M_0(1;-1;2).$

2. $z = xe^{xy}, \qquad x^2 \cdot z''_{xx} - 2xy \cdot z''_{xy} + y^2 \cdot z''_{yy} = 0.$

3. $z = \dfrac{e^x + e^y}{x^2}, \;\; y = x\ln x, \qquad \dfrac{dz}{dx} - ?$

4. $y^2x^3 + yz^3 + x^2 = xyz.$

5. $z = e^{3x}\ln(xy).$

6. $z = 2x^2 + 3y^2 - 8\ln(x^2y^3).$

7. $z = x^2 + y^2, \quad D: x^2 + (y-1)^2 = 4.$

8. $z = 4 + \dfrac{2}{x} - \dfrac{3}{y}, \quad \dfrac{1}{x^2} - \dfrac{3}{2y^2} + \dfrac{1}{2} = 0.$

9. Radiusi R ga va balandligi H teng konusga ichki chizilgan to‘g‘ri burchakli parallelopiped eng katta hajmga ega bo‘lsa, parallelopipedning o‘lchamlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	1,3	1,9	1,8	0,7	-1,0	-3,4

17-variant

1. $x^2 + y^2 + xz - yz - 3xy - 2 = 0, \quad M_0(4;1;-1).$

2. $z = \cos(xy) + \cos\dfrac{x}{y}, \qquad x^2 \cdot z'''_{xx} - y^2 z''_{yy} + x \cdot z'_x - y \cdot z'_y = 0.$

3. $u = xz^3 + x^2y^2 + y^3z, \;\; x = t^{-2}, \;\; y = t^3, \;\; z = t^{-4}, \qquad \dfrac{du}{dt} - ?$

4. $2y^2x^3 + yz^3 + x^2z = 3.$

5. $z = \cos(x+y)\sin(x-y).$

6. $z = xy^2 + \dfrac{1}{x} + \dfrac{8}{y}.$

7. $z = x^2y(5 - 2x - 3y), \quad D: x = 0, \;\; y = 0, \;\; x + y + 2 = 0.$

8. $z = x^2 - 4y^2 + 12, \quad x + y + 3 = 0.$

9. Radiusi R ga teng shardan tayyorlangan materialdan eng katta hajmga ega silindr yasalgan bo‘lsa, silindrning o‘lchamlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-0,5	-1,5	-1,8	-1,7	0,1	1,7

18-variant

1. $2x^2+2y^2+z^2+8xz-z+6=0,\quad M_0(-2;1;1).$

2. $z=y^{\frac{y}{x}}\sin\frac{y}{x},\qquad x^2\cdot z'_x+xy\cdot z'_y-y\cdot z=0.$

3. $z=arctg\frac{x+1}{y},\ x=e^{2t},\ y=\ln(2t+1),\qquad \frac{dz}{dt}-?$

4. $z^3+xyz-xy^2=-x^3.$

5. $z=\ln sh(xy).$

6. $z=2x^3+2y^3+3x^2y+3y^2x-15x-15y.$

7. $z=x^3+y^3-6xy,\quad D: x=0,\ x=2,\ y=-1,\ y=2.$

8. $z=11+13x+5y,\quad x^2-y^2-144=0.$

9. O‘q kesimining perimetri $6a$ ga teng silindr eng katta hajmga ega bo‘lsa, uning o‘lchamlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-1,2	0,2	-0,3	-0,3	-2,1	-5,1

19-variant

1. $x^2-xy-8x+z^3-yz-8=0,\quad M_0(2;-3;2).$

2. $z=\arcsin(xy),\qquad \sqrt{1-x^2y^2}\,(z''_{xx}+z''_{yy})-(x^2+y^2)\cdot z'_x\cdot z'_y=0.$

3. $z=e^{\frac{x+y}{y}},\ y=\cos^4 x,\qquad \frac{dz}{dx}-?$

4. $\sqrt{x^2+y^2}+yx^3-3z=z^3$

5. $z=\ln ch(xy).$

6. $z=y\sqrt{x}-2y^2-x+14y.$

7. $z=(x+y)^2-2x+2y,\quad D: x=2,\ y=0,\ y-x-2=0.$

8. $z=\frac{5}{\sqrt[3]{x}}-\frac{1}{\sqrt[3]{y}},\quad 5x-y-12=0.$

9. Radiusi R ga teng yarim sharga ichki chizilgan to‘g‘ri burchakli parallelopiped eng katta hajmga ega bo‘lsa, parallelopipedning o‘lchamlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-1,3	-2,6	-2,4	-1,4	0,6	3,3

20-variant

1. $z = x^2 + y^2 - 2xy - x + 2y - 4, \quad M_0(-1;1;3).$

2. $z = y\ln(x^2 - y^2), \qquad y^2 \cdot z'_x + xy \cdot z'_y - x \cdot z = 0.$

3. $u = xy^3 + xz^3,\ x = t^2 + 1,\ y = t^3,\ z = \sin t, \qquad \frac{du}{dt} - ?$

4. $z^3 + 2x^2 + 3y = xyz.$ **5.** $z = (x + y)\cos(x - y).$

6. $z = 3xy + \frac{9}{x} + \frac{1}{y}.$

7. $z = 4x^2 - y^2 + 4xy - 8x, \quad D: x = 0,\ y = 2,\ 2x - y = 0.$

8. $z = 4 - \frac{1}{3x^2} + \frac{2}{y^2}, \quad x - 6y + 5 = 0.$

9. $M(x;y)$ nuqtadan $x = 0,\ y = 0,\ x - y + 1 = 0$ to‘g‘ri chiziqlargacha masofalar kvadratlarining yig‘indisi eng kichik bo‘lsa, bu nuqtani toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	5,2	5,7	5,3	4,9	3,6	1,8

21-variant

1. $x^2 + y^2 - 2z^2 + xy - 4z - 3xz - 4 = 0, \quad M_0(3;2;1).$

2. $z = x\sin y + y\cos x, \qquad z''_{xx} + z''_{yy} + z = 0.$

3. $z = e^{xy}\sqrt{y},\ x = \ln v,\ y = v\sin u, \qquad \frac{\partial z}{\partial u},\ \frac{\partial z}{\partial v} - ?$

4. $x^3 + y^2 + z = (x + y)arctgz.$ **5.** $z = (xy) \cdot e^{xy}.$

6. $z = 9x^3 + 2y^2 - \ln(xy).$

7. $z = xy^2(2 - x - y), \quad D: x = -3,\ y = 0,\ x + y + 1 = 0.$

8. $z = 8 - 4x + 3y, \quad x^2 + y^2 - 25 = 0.$

9. Perimetri p ga teng bo‘lgan tagi to‘g‘ri to‘rtburchak ko‘rinishiga va tepasi yarim aylana shakliga ega deraza romi orqali eng ko‘p yorug‘lik o‘tayotgan bol‘sa, pomning o‘lchamlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-0,3	-0,9	-0,1	0,6	2,2	5,0

22-variant

1. $z = x^2 + y^2 - 3xy + 3x - 2y - 5, \quad M_0(-1;2;-1).$

2. $z = tg(xy) + \dfrac{x}{y}, \quad x^2 \cdot z''_{xx} - y^2 \cdot z''_{yy} + x \cdot z'_x - y \cdot z'_y = 0.$

3. $z = \dfrac{xy - 2y^2}{\sqrt{1+y}}, \; y = xe^x, \quad \dfrac{dz}{dx} - ?$

4. $z^2 + 5 = z\ln(x + e^{-y}),$ **5.** $z = \cos(e^x + e^{-y}).$

6. $z = 2x^3 + 2y^3 - 6xy + 6.$

7. $z = 2x^2 + 3y^2 + 1, \quad D: y = \dfrac{3}{2}\sqrt{4 - x^2}.$

8. $z = 6xy + 5x - 5y, \quad x^2 + y^2 - 2 = 0.$

9. Sirti S ga teng to‘g‘ri burchakli ochiq hovuz eng katta sig‘imga ega bo‘lsa, uning o‘lchamlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	1,2	1,7	1,2	0,4	-0,7	-2,8

23-variant

1. $6xy - 2x^2 - xy^2 - z^2 + 3x = 0, \quad M_0(1;2;3).$

2. $z = \ln(x + e^{-y}), \quad z'_x - z''_{xy} - e^y z''_{yy} = 0.$

3. $z = \ln\dfrac{x}{y}, \; x = \sin\dfrac{u}{v}, \; y = \sqrt{\dfrac{u}{v}}, \quad \dfrac{\partial z}{\partial u}, \; \dfrac{\partial z}{\partial v} - ?$

4. $x^3 + y^3 + z^3 = 3xy + 3xz + 3yz.$ **5.** $z = \dfrac{x+y}{x-y}.$

6. $z = 3x^2 + y - 2\ln(x^3 y^4).$

7. $z = x^2 - 2xy - y^2 + 4x + 1, \quad D: x = -3, \; y = 0, \; x + y + 1 = 0.$

8. $z = 3 + \dfrac{1}{x} + \dfrac{1}{y}, \quad \dfrac{1}{x^2} + \dfrac{2}{y^2} - \dfrac{3}{8} = 0.$

9. Hajmi V ga teng konus eng kichik to‘la sirtga ega bo‘lsa, uning o‘lchamlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-0,5	-0,7	0,	0,4	2,3	4,2

24-variant

1. $x^2 - y^2 + z^2 - yz - 4yx - 8x = 0, \quad M_0(1;-2;-1).$

2. $z = \ln(xy) + \ln\dfrac{x}{y}, \qquad x^2 \cdot z'''_{xx} - y^2 z''_{yy} + x \cdot z'_x - y \cdot z'_y = 0.$

3. $z = xarctg(xy), \ x = e^t + 1, \ y = t^2 e^t, \qquad \dfrac{dz}{dt} - ?$

4. $x\sin y + (y+z)\sin x = z^3$ **5.** $z = \dfrac{x}{y}\ln(xy).$

6. $z = xy^2 + \dfrac{4}{x} + \dfrac{4}{y}.$

7. $z = 1 - x^2 - y^2, \quad D: (x-1)^2 + (y-1)^2 = 1.$

8. $z = 5 + \dfrac{3}{x^2} + \dfrac{1}{2y^2}, \quad 6x + y - 14 = 0.$

9. Uchlari $x^2 + 4y^2 = 4$ ellipsning $A\left(\sqrt{3};\dfrac{1}{2}\right)$, $B\left(1;\dfrac{\sqrt{3}}{2}\right)$ va $C(x;y)$ nuqtalarida yotgan uchburchakning yuzasi eng katta bo‘lsa, $C(x;y)$nuqtani toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	1,2	1,6	1,5	0,6	-1,2	-3,2

25-variant

1. $3x^2 - 4xy + 12xz - 3yz + z^2 + 15 = 0, \quad M_0(-1;-1;2).$

2. $z = y^x, \qquad x \cdot z'_x + z - y \cdot z''_{xy} = 0.$

3. $z = tg(xy), \ x = \ln(u^2 + v^2), \ y = \dfrac{v^2}{u^2}, \qquad \dfrac{\partial z}{\partial u}, \ \dfrac{\partial z}{\partial v} - ?$

4. $xe^y + ye^z + ze^x = x + y + z$ **5.** $z = e^{\sin(x-y)}.$

6. $z = 3x + y^4 - 6\ln x - 64\ln y.$

7. $z = xy(12 - 4x - 3y), \quad D: x = 0, \ y = 0, \ 4x + 3y - 8 = 0.$

8. $z = x^2 + y^2 - 4, \quad 4x + 3y - 12 = 0.$

9. Radiusi R ga va balandligi H teng konusga ichki chizilgan silindr eng katta hajmga ega bo‘lsa, silindrning o‘lchamlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-0,6	0,6	0,5	-0,3	-1,8	-4,7

26-variant

1. $z = x^2 + y^2 + 2xy - 2x - 3y - 8, \quad M_0(2;3;4).$

2. $z = (y - x)\sin y + \cos x, \qquad (x - y)z''_{xy} - z'_y + \sin y = 0.$

3. $z = tg\dfrac{x}{y}, \;\; x = \dfrac{2v}{u + v}, \;\; y = u^2 - 3v, \qquad \dfrac{\partial z}{\partial u}, \; \dfrac{\partial z}{\partial v} - ?$

4. $z^2 + x^3 = y\ln\dfrac{xz}{y}.$

5. $z = \sin(x + y)\cos(x - y).$

6. $z = x^3 + y^3 + x^2y + y^2x - 6x - 6y.$

7. $z = 3x^2 + 3y^2 - x - y - 2, \quad D: x = 5, \; y = 0, \; x - y - 1 = 0.$

8. $z = x + 2y, \quad x^2 + y^2 - 5 = 0.$

9. Radiusi R ga va balandligi H teng konusga ichki chizilgan to‘g‘ri burchakli parallelopiped eng katta hajmga ega. Parallelopiped asosining yuzasini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-0,2	-2,3	-2,7	-1,6	-0,2	2,7

27-variant

1. $x^2 - xy + xz + 3yz + 2z^2 + 2 = 0, \quad M_0(1;1;-1).$

2. $z = \ln(x^2 + y^2 + 2x + 1), \qquad z''_{xx} + z''_{yy} = 0.$

3. $z = \arccos\dfrac{2x}{y}, \;\; x = \sin t, \;\; y = \cos t, \qquad \dfrac{dz}{dt} - ?$

4. $xz^5 + zy^3 - x^3 = yx.$

5. $z = (x + y)\ln(xy).$

6. $z = 4xy + \dfrac{1}{x} + \dfrac{16}{y}.$

7. $z = x^2 - 2xy + 2y^2 - 4y, \quad D: x = 1, \; y = 1, \; x + 2y - 8 = 0.$

8. $z = 1 - 4x - 8y, \quad x^2 - 8y^2 - 8 = 0.$

9. Radiusi R ga teng shardan tayyorlangan materialdan eng katta hajmga ega silindr yasalgan bo‘lsa, silindrning balandligini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-0,3	-1,3	-1,6	-0,6	1,8	4,7

28-variant

1. $z = x^2 - y^2 + 6x + 3y - 2xy, \quad M_0(2;3;4).$

2. $z = tg\frac{x}{y}, \qquad z''_{xy} + \frac{x}{y} \cdot z''_{xx} + \frac{1}{y} \cdot z'_x = 0.$

3. $z = y^x, \ y = arctgx, \qquad \frac{dz}{dx} - ?$

4. $yz^2 = x^2y + z\ln(xy).$

5. $z = x^3 \sin y + y^2 \cos x.$

6. $z = x^3 + 3y^3 - 3\ln x - 48\ln y.$

7. $z = 2xy - 3x^2 - 2y^2 + 5, \quad D: x = -1, \ y = -1, \ x + y - 5 = 0.$

8. $z = 4 + 5x + 12y, \quad x^2 + y^2 - 169 = 0.$

9. Asosi a ga va uchidagi burchagi α ga teng uchburchak eng katta yuzaga ega bo‘lsa, uning qolgan ikki tomonini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-0,4	0,5	1,2	1,9	1,6	1,1

29-variant

1. $x^2 - 2y^2 - 2z^2 - xy - yz + 3 = 0, \ M_0(2;1;1).$

2. $z = xy + x\sin\frac{x}{y}, \qquad x \cdot z'_x + y \cdot z'_y - xy - z = 0.$

3. $u = x^2y^3z^4, \ x = \ln(t+1), \ y = t^2 + 1, \ z = t^3, \qquad \frac{du}{dt} - ?$

4. $e^{xyz} + xyz = x^2 + y.$

5. $z = e^{\cos(x-y)}.$

6. $z = x^3 + y^3 - 9xy + 6.$

7. $z = x^2 + 2xy - y^2 - 4x, \quad D: x = 0, \ y = 0, \ x + y + 2 = 0.$

8. $z = 3 + \frac{1}{x} + \frac{1}{2y^2}, \quad x - y - 2 = 0.$

9. Radiusi R ga teng sharga ichki chizilgan konus eng katta hajmga ega bol‘sa, konusning o‘lchamlarini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	-1,0	0,2	0,1	-0,7	-2,2	-5,1

30-variant

1. $x^3+y^3-z^2-2xyz-5xy-4y+2=0,\ M_0(2;1;-3).$

2. $z=x\ln(x+y)+ye^{x+y},\qquad z''_{xx}-2z''_{xy}+z''_{yy}=0.$

3. $z=arctg(xy),\ x=\ln(v^2-u^2),\ y=vu^2,\qquad \frac{\partial z}{\partial u},\ \frac{\partial z}{\partial v}-?$

4. $xz=e^{\frac{z}{y}}+x^3+y^3.$

5. $z=e^{x-y}\sin(x+y).$

6. $z=x^2y^2+\frac{1}{x}+\frac{4}{y}.$

7. $z=x^2+y^2,\quad D:3|x|+4|y|=12.$

8. $z=\frac{4}{x^2}-\frac{1}{2y^2},\quad x+y+1=0.$

9. Asosining radiusi R ga va balandligi H ga teng konus shaklidagi suv bilan to‘ldirilgan idishga kub (konus va kub asoslarining markazlari bu asoslarga perpendikular to‘g‘ri chiziqda yotadi) tashlangan. Kubning idish ichidagi qismi idishdan eng ko‘p hajmdagi suv siqib chiqargan bo‘lsa, kubning qirrasini toping.

10.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	0,7	0,5	1,5	2,0	2,5	4,3

B. NAMUNAVIY VARIANT YECHIMI

1. Sirtga $M_0(x_0;y_0;z_0)$ nuqtada o‘tkazilgan urinma tekislik va normal tenglamalarini tuzing.

1.30. $x^3+y^3-z^2-2xyz-5xy-4y+2=0,\ M_0(2;1;-3).$

$F(x,y,z)=x^3+y^3-z^2-2xyz-5xy-4y+2=0$ belgilash kiritamiz. U holda

$$F'_x(M_0)=3x_0^{\ 2}-2y_0z_0-5y_0=3\cdot 2^2-2\cdot 1\cdot(-3)-5\cdot 1=13,$$

$$F'_y(M_0)=3y_0^{\ 2}-2x_0z_0-5x_0-4=3\cdot 1^2-2\cdot 2\cdot(-3)-5\cdot 2-4=1,$$

$$F'_z(M_0)=-2z_0--2x_0y_0=-2\cdot(-3)-2\cdot 2\cdot 1=2.$$

Bu qiymatlarni

$$F'_x(x_0,y_0,z_0)(x-x_0)+F'_y(x_0,y_0,z_0)(y-y_0)+F'_z(x_0,y_0,z_0)(z-z_0)=0,$$

$$\frac{x-x_0}{F'_x(x_0,y_0,z_0)}=\frac{y-y_0}{F'_y(x_0,y_0,z_0)}=\frac{z-z_0}{F'_z(x_0,y_0,z_0)}$$

tenglamalarga qo‘yib, topamiz:

1) urinma tekislik tenglamasi

$$13\cdot(x-2)+1\cdot(y-1)+2\cdot(z+3)=0$$

yoki

$$13x+y+2z-21=0;$$

2) normal tenglamasi

$$\frac{x-2}{13}=\frac{y-1}{1}=\frac{z+3}{2}.$$

2. $z=f(x,y)$ funksiyaning berilgan tenglikni qanoatlantirishini ko‘rsating.

2.30. $z=x\ln(x+y)+ye^{x+y}$, $\quad z''_{xx}-2z''_{xy}+z''_{yy}=0.$

Funksiyaning birinchi tartibli xususiy hosilalarini topamiz:

$$z'_x=\ln(x+y)+x\cdot\frac{1}{x+y}+y\cdot e^{x+y}=\ln(x+y)+\frac{x}{x+y}+ye^{x+y},$$

$$z'_y=x\cdot\frac{1}{x+y}+1\cdot e^{x+y}+y\cdot e^{x+y}=\frac{x}{x+y}+(1+y)e^{x+y}.$$

Bundan

$$z''_{xx}=\frac{1}{x+y}+\frac{1}{x+y}-x\cdot\frac{1}{(x+y)^2}+y\cdot e^{x+y}=\frac{x+2y}{(x+y)^2}+ye^{x+y},$$

$$z''_{xy}=\frac{1}{x+y}-x\cdot\frac{1}{(x+y)^2}+1\cdot e^{x+y}+y\cdot e^{x+y}=\frac{y}{(x+y)^2}+(1+y)e^{x+y},$$

$$z''_{yy}=x\cdot\left(-\frac{1}{(x+y)^2}\right)+1\cdot e^{x+y}+(1+y)\cdot e^{x+y}=-\frac{x}{(x+y)^2}+(2+y)e^{x+y}.$$

z''_{xx}, z''_{xy}, z''_{yy} hosilalarni berilgan tenglamaga qo‘yamiz:

$$z''_{xx}-2z''_{xy}+z''_{yy}=\frac{x+2y}{x+y}+ye^{x+y}-2\cdot\left(\frac{y}{(x+y)^2}+(1+y)e^{x+y}\right)+$$

$$+\left(-\frac{x}{(x+y)^2}+(2+y)e^{x+y}\right)=\frac{x+2y-2y-x}{(x+y)^2}+e^{x+y}(y-2-2y+2+y)=0.$$

Demak, $z=x\ln(x+y)+ye^{x+y}$ funksiya $z''_{xx}-2z''_{xy}+z''_{yy}=0$ tenglikni qanoatlantiradi.

3. Murakkab funksiyaning ko‘rsatilgan hosilalarini toping.

3.30. $z = arctg(xy),\ x = \ln(v^2 - u^2),\ y = vu^2,\quad \frac{\partial z}{\partial u},\ \frac{\partial z}{\partial v} - ?$

Funksiyalarning xususiy hosilalarini topamiz:

$$\frac{\partial z}{\partial x} = \frac{1}{1+(xy)^2}(xy)'_x = \frac{y}{1+x^2y^2},\qquad \frac{\partial z}{\partial y} = \frac{1}{1+(xy)^2}(xy)'_y = \frac{x}{1+x^2y^2},$$

$$\frac{\partial x}{\partial u} = -\frac{2u}{v^2-u^2},\qquad \frac{\partial x}{\partial v} = \frac{2v}{v^2-u^2},\qquad \frac{\partial y}{\partial u} = 2uv,\qquad \frac{\partial y}{\partial v} = u^2.$$

U holda

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} = \frac{y}{1+x^2y^2}\cdot\left(-\frac{2u}{v^2-u^2}\right) + \frac{x}{1+x^2y^2}\cdot(2uv) =$$

$$= \frac{1}{1+x^2y^2}\cdot\left(-\frac{2u}{v^2-u^2}\cdot y + 2uv\cdot x\right)$$

yoki

$$\frac{\partial z}{\partial u} = \frac{2uv\cdot((v^2-u^2)\ln(v^2-u^2)-u^2)}{(v^2-u^2)\cdot(1+u^2v^2\ln^2(v^2-u^2))}.$$

Shu kabi

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v} = \frac{y}{1+x^2y^2}\cdot\left(\frac{2v}{v^2-u^2}\right) + \frac{x}{1+x^2y^2}\cdot(u^2) =$$

$$= \frac{1}{1+x^2y^2}\cdot\left(\frac{2v}{v^2-u^2}\cdot y + u^2\cdot x\right)$$

yoki

$$\frac{\partial z}{\partial v} = \frac{u^2\cdot(2v^2-(v^2-u^2)\ln(v^2-u^2))}{(v^2-u^2)\cdot(1+u^2v^2\ln^2(v^2-u^2))}.$$

4. Oshkormas ko‘rinishda berilgan $z=(x;y)$ funksiyaning birinchi tartibli xususiy hosilalarini toping.

4.30. $xz = e^{\frac{z}{y}} + x^3 + y^3.$

Misolning shartiga ko‘ra $F(x,y,z) = e^{\frac{z}{y}} + x^3 + y^3 - xz.$

Bundan

$$F'_x(x,y,z) = 3x^2 - z,\qquad F'_y(x,y,z) = e^{\frac{z}{y}}\left(-\frac{z}{y^2}\right) + 3y^2 = \frac{3y^4 - ze^{\frac{z}{y}}}{y^2},$$

$$F'_z(x,y,z) = e^{\frac{z}{y}}\left(\frac{1}{y}\right) - x = \frac{e^{\frac{z}{y}} - xy}{y}.$$

U holda

$$\frac{\partial z}{\partial x}=-\frac{F'_x(x,y,z)}{F'_z(x,y,z)}=\frac{(3x^2-z)y}{xy-e^{\frac{z}{y}}},\qquad \frac{\partial z}{\partial y}=-\frac{F'_y(x,y,z)}{F'_z(x,y,z)}=\frac{1}{y}\cdot\frac{3y^4-ze^{\frac{z}{y}}}{xy-e^{\frac{z}{y}}}.$$

5. Funksiyaning uchinchi tartibli differensialini toping.

5.30. $z=e^{x-y}\sin(x+y).$

Funksiyalarning birinchi tartibli xususiy hosilalarini topamiz:

$$z'_x=e^{x-y}\big(\sin(x+y)+\cos(x+y)\big),\qquad z'_y=e^{x-y}\big(\cos(x+y)-\sin(x+y)\big).$$

Bundan

$$z''_{x^2}=e^{x-y}(\sin(x+y)+\cos(x+y)+\cos(x+y)-\sin(x+y))=2e^{x-y}\cos(x+y),$$
$$z''_{xy}=e^{x-y}(-\sin(x+y)-\cos(x+y)+\cos(x+y)-\sin(x+y))=-2e^{x-y}\sin(x+y),$$
$$z''_{y^2}=e^{x-y}(-\cos(x+y)+\sin(x+y)-\sin(x+y)-\cos(x+y))=-2e^{x-y}\cos(x+y).$$

Funksiyalarning uchinchi tartibli xususiy hosilalarini topamiz:

$$z'''_{x^3}=2e^{x-y}\big(\cos(x+y)-\sin(x+y)\big),\qquad z'''_{x^2y}=-2e^{x-y}\big(\cos(x+y)+\sin(x+y)\big),$$
$$z'''_{y^2x}=-2e^{x-y}\big(\cos(x+y)-\sin(x+y)\big),\qquad z'''_{y^3}=2e^{x-y}\big(\cos(x+y)+\sin(x+y)\big).$$

Uchinchi tartibli xususiy hosilalarning topilgan qiymatlarini

$$d^3z=f'''_{x^3}(x,y)dx^3+3f'''_{x^2y}(x,y)dx^2dy+3f'''_{y^2x}(x,y)dxdy^2+f'''_{y^3}(x,y)dy^3$$

formulaga qo‘yib topamiz:

$$d^3z=(2e^{x-y}(\cos(x+y)-\sin(x+y))dx^3+3(-2e^{x-y}(\cos(x+y)+\sin(x+y))dx^2dy+$$
$$+3(-2e^{x-y}(\cos(x+y)-\sin(x+y))dxdy^2+(2e^{x-y}(\cos(x+y)+\sin(x+y))dy^3$$

yoki

$$d^3z=2e^{x-y}((\cos(x+y)-\sin(x+y))\cdot(dx^3-3dxdy^2)+$$
$$+((\cos(x+y)+\sin(x+y))\cdot(dy^3-3dx^2dy).$$

6. Funksiyani ekstremumga tekshiring.

6.30. $z=x^2y^2+\frac{1}{x}+\frac{4}{y}.$

Funksiyani ekstremumga belgilangan tartibda tekshiramiz.

1^o. Funksiyaning birinchi tartibli xususiy hosilalarini topamiz:

$$\frac{\partial z}{\partial x}=2xy^2-\frac{1}{x^2},\qquad \frac{\partial z}{\partial y}=2x^2y-\frac{4}{y^2}.$$

2^o. Statsionar nuqtalarni aniqlaymiz:

$$\begin{cases} 2x^3y^2 - 1 = 0, \\ x^2y^3 - 2 = 0. \end{cases}$$

Sistemani yechamiz: $P\left(\frac{1}{2};2\right)$.

3^o. Ikkinchi tartibli xususiy hosilalarni topamiz:

$$\frac{\partial^2 z}{\partial x^2} = 2y^2 + \frac{2}{x^3}, \quad \frac{\partial^2 z}{\partial x \partial y} = 4xy, \quad \frac{\partial^2 z}{\partial y^2} = 2x^2 + \frac{8}{y^3}.$$

4^o. $P\left(\frac{1}{2};2\right)$ statsionar nuqtada ikkinchi tartibli xususiy hosilalarni hisoblaymiz:

$$A = 2\cdot 2^2 + 2\cdot 2^3 = 24 > 0, \quad B = 4\cdot\frac{1}{2}\cdot 2 = 4, \quad C = 2\cdot\left(\frac{1}{2}\right)^2 + \frac{8}{2^3} = \frac{3}{2}.$$

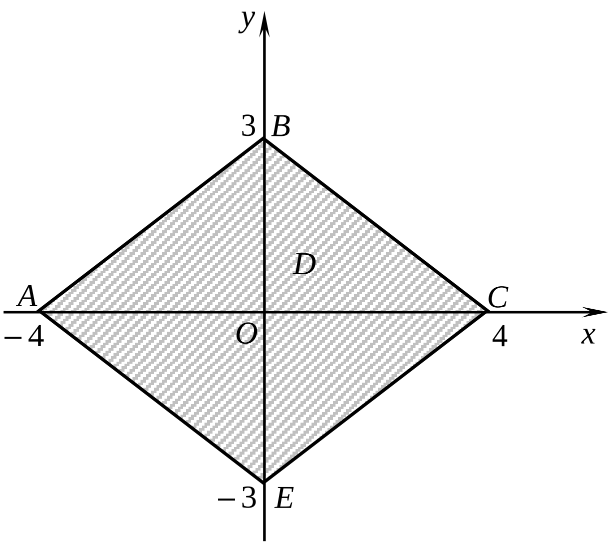
5^o. $P\left(\frac{1}{2};2\right)$ statsionar nuqtada $\Delta = AC - B^2 = 24\cdot\frac{3}{2} - 4^2 = 20 > 0$.

Demak, $P\left(\frac{1}{2};2\right)$ nuqta minimum nuqta va $z_{\min} = \left(\frac{1}{2}\right)^2 \cdot 2^2 + 1\cdot 2 + \frac{4}{2} = 5$. ◄

7. $z = f(x,y)$ funksiyaning D yopiq sohadagi eng katta va eng kichik qiymatlarini toping.

7.30. $z = x^2 + y^2, \quad D: 3|x| + 4|y| = 12.$

► D soha $ABCE$ rombdan iborat (5-shakl).



5-shakl.

1^o. Funksiyaning D sohada yotgan kritik nuqtalarini topamiz:

$$\begin{cases} \frac{\partial z}{\partial x} = 2x = 0, \\ \frac{\partial z}{\partial y} = 2y = 0. \end{cases}$$

Bundan $x = 0, y = 0$.

Demak, $P_0(0;0) = O(0;0)$, $z(P_0) = 0$.

2^o. Funksiyani soha chegarasida ekstremumga tekshiramiz. Soha chegarasi turli tenglamalar bilan aniqlanuvchi to‘rtta qismdan tashkil topgani sababli funksiyani har bir qismda ekstremumga alohida tekshiramiz.

1) AB to‘g‘ri chiziqda $-3x+4y=12$ yoki $y=\frac{12+3x}{4}$ va

$$z=x^2+\left(\frac{3x+12}{4}\right)^2\ (-4\le x\le 0).$$

U holda

$$z'_x=2x+2\left(\frac{3x+12}{4}\right)\cdot\frac{3}{4}=0 \text{dan } x=-\frac{36}{25}.\quad y=\frac{12+3x}{4}\text{dan } y=\frac{48}{25}.$$

Demak, $z\left(-\frac{36}{25},\frac{48}{25}\right)=\frac{144}{25}$.

AB to‘g‘ri chiziqning chetki nuqtalarida: $z(A)=z(-4,0)=16,\ z(B)=z(0,3)=9.$

2) BC to‘g‘ri chiziqda $3x+4y=12$ yoki $y=\frac{12-3x}{4}$.

Bundan $z=x^2+\left(\frac{12-3x}{4}\right)^2\ (0\le x\le 4).$

U holda $z'_x=2x+2\left(\frac{12-3x}{4}\right)\cdot\left(-\frac{3}{4}\right)=0\text{dan } x=\frac{36}{25}.\quad y=\frac{12-3x}{4}\text{dan } y=\frac{48}{25}.$

Demak, $z\left(\frac{36}{25},\frac{48}{25}\right)=\frac{144}{25}$.

BC to‘g‘ri chiziqning chetki nuqtalarida: $z(B)=9,\ z(C)=z(4,0)=16.$

3) CE to‘g‘ri chiziqda $3x-4y=12$ yoki $y=-\frac{12-3x}{4}$.

Bundan $z=x^2+\left(\frac{12-3x}{4}\right)^2\ (0\le x\le 4).$

U holda

$$z'_x=2x+2\left(\frac{12-3x}{4}\right)\cdot\left(-\frac{3}{4}\right)=0\text{dan } x=\frac{36}{25}.\quad y=-\frac{12-3x}{4}\text{dan } y=-\frac{48}{25}.$$

Demak, $z\left(\frac{36}{25},-\frac{48}{25}\right)=\frac{144}{25}$.

BC to‘g‘ri chiziqning chetki nuqtalarida: $z(C)=16,\ z(E)=z(0,-3)=9.$

4) EA to‘g‘ri chiziqda $-3x-4y=12$ yoki $y=-\frac{12+3x}{4}$.

Bundan $z=x^2+\left(\frac{12+3x}{4}\right)^2\ (-4\le x\le 0).$

U holda

$$z'_x = 2x + 2\left(\frac{12+3x}{4}\right)\cdot\left(\frac{3}{4}\right) = 0 \text{ dan } x = -\frac{36}{25}. \quad y = -\frac{12+3x}{4} \text{ dan } y = -\frac{48}{25}.$$

Demak, $z\left(-\frac{36}{25}, -\frac{48}{25}\right) = \frac{144}{25}$.

BC to‘g‘ri chiziqning chetki nuqtalarida: $z(E) = 9,\ z(A) = 16.$

3^o. Funksiyaning hisoblangan qiymatlarini taqqoslaymiz.

Demak,

$$z_{eng\ katta} = z(\pm 4{,}0) = 16 \quad \text{va} \quad Z_{eng\ kichik} = z(0{,}0) = 0.$$

8. $z = f(x,y)$ funksiyalarning $\varphi(x,y) = 0$ tenglama bilan bog‘langanlik shartidagi ekstremumlarini toping.

8.30. $z = \frac{4}{x^2} - \frac{1}{2y^2}, \quad x + y + 1 = 0.$

Funksiyani Lagranj ko‘paytuvchilari usulu bilan ekstremumga tekshiramiz.

1^o. Lagranj funksiyasini tuzamiz:

$$F(x,y,z) = f(x,y) + \lambda\varphi(x,y) = \frac{4}{x^2} - \frac{1}{2y^2} + \lambda(x + y + 1).$$

Bundan

$$F'_x = -\frac{8}{x^3} + \lambda, \quad F'_y = \frac{1}{y^3} + \lambda, \quad F'_\lambda = x + y + 1.$$

2^o. Shartli ekstremumning zaruruy shartiga ko‘ra

$$\begin{cases} -8 + \lambda x^3 = 0, \\ 1 + \lambda y^3 = 0, \\ x + y + 1 = 0. \end{cases}$$

Sistemani yechamiz: $x = -2,\ y = 1,\ \lambda = -1$. Demak, $P_0(-2;1)$ mumkin bo‘lgan shartli ekstremum nuqta.

3^o. Δ diterminantga qo‘yiladigan xususiy hosilalarni topamiz:

$$\varphi'_x = 1, \quad \varphi'_y = 1, \quad F''_{x^2} = \frac{24}{x^4}, \quad F''_{xy} = 0, \quad F''_{y^2} = -\frac{3}{y^4}.$$

Bundan

$$\varphi'_x(P_0) = 1, \quad \varphi'_y(P_0) = 1, \quad F''_{x^2}(P_0) = \frac{24}{(-2)^4} = \frac{3}{2}, \quad F''_{xy}(P_0) = 0, \quad F''_{y^2}(P_0) = -\frac{3}{1^4} = -3.$$

U holda

$$\Delta = -\begin{vmatrix} 0 & 1 & 1 \\ 1 & \frac{3}{2} & 0 \\ 1 & 0 & -3 \end{vmatrix} = -\frac{3}{2} < 0.$$

Demak, $P_0(-2;1)$ nuqtada funksiya shartli maksimumga ega:

$$z_{\max} = \frac{4}{(-2)^2} - \frac{1}{2\cdot 1^2} = \frac{1}{2}.$$

9. Eng katta va eng kichik qiymatlarnia topishga oid amaliy masalalarni yeching.

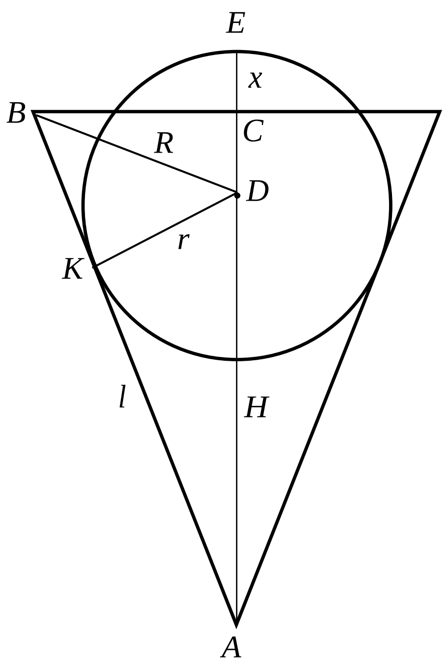
9.30. Asosining radiusi R ga va balandligi H ga teng konus shaklidagi idish suyuqlik bilan to‘ldirilgan. Idishga tashlangan sharning idish ichidagi qismi idishdan eng ko‘p miqdorda suyuqlik siqib chiqargan bo‘lsa, sharning radiusini toping.

Sharning idishdan tashqaridagi qismi, ya’ni shar sektorining balandligi $CE = x$ bo‘lsin (6-shakl). U holda bu sigmentining hajmi $V_{cek} = \frac{\pi}{3}\left(3x^2 r - x^3\right)$ ga teng bo‘ladi.

Sharning idish ichidagi qismining hajmini topamiz:

$$V = V_{sh} - V_{cek} = \frac{4}{3}\pi r^3 - \frac{\pi}{3}\left(3x^2 r - x^3\right) = \frac{\pi}{3}\left(4r^3 - 3rx^2 + x^3\right).$$

Sharning idishdan siqib chiqaradigan suyuqlik miqdori V hajmga bog‘liq bo‘ladi. Sharning idish ichidagi qismi idishdan eng ko‘p miqdorda suyuqlik siqib chiqaririshi uchun $4r^3 - 3rx^2 + x^3$ ifoda maksimumga erishishi kerak. Bunda shar bilan idishning o‘lchamlari uzviy bog‘lanishga ega bo‘ladi. Shu bog‘lanishni aniqlaymiz.



6-shakl

6-shakldan topamiz:

$$S_{\Delta ABC} = \frac{1}{2} BC\cdot AC = \frac{1}{2} RH, \quad S_{\Delta ABD} = \frac{1}{2} AB\cdot KD = \frac{1}{2} lr,$$

$$S_{\Delta DBC} = \frac{1}{2} BC\cdot DC = \frac{1}{2} R(ED - x) = \frac{1}{2} R(r - x).$$

Shu bilan birga $S_{\Delta ABC} = S_{\Delta ABD} + S_{\Delta DBC}$ yoki

$$\frac{1}{2} RH = \frac{1}{2} lr + \frac{1}{2} R(r - x).$$

Bundan $(l+R)r - Rx - RH = 0$.

Shunday qilib, sharning idish ichidagi qismi idishdan eng ko‘p miqdorda suyuqlik siqib chiqaririshini topish uchun $z(r,x) = 4r^3 - 3rx^2 + x^3$ funksiyaning $\varphi(r,x) = (l+R)r - Rx - RH = 0$ bog‘lanish tenglamasi bilan bog‘langanlik shartidagi maksimumini topish kerak bo‘ladi. Bu masalani Lagranj ko‘paytuvchilar usuli bilan yechamiz.

1^o. Lagranj funksiyasini tuzamiz:

$$F(r,x,z) = 4r^3 - 3rx^2 + x^3 + \lambda((l+R)r - Rx - RH).$$

Bundan

$$F'_r = 12r^2 - 3x^2 + \lambda(l+R), \quad F'_x = 3x^2 - 6rx - \lambda R, \quad F'_\lambda = (l+R)r - Rx - RH.$$

2^o. Shartli ekstremumning zaruruy shartiga ko‘ra

$$\begin{cases} 3(4r^2 - x^2) + \lambda l + \lambda R = 0, \\ 3(x^2 - 2rx) - \lambda R = 0, \\ (l+R)r - Rx - RH = 0 \end{cases} \Rightarrow \begin{cases} 6r(2r - x) + \lambda l = 0, \\ 3x(2r - x) + \lambda R = 0, \\ (l+R)r - Rx - RH = 0. \end{cases}$$

Sistemani yechib, r ni topamiz:

$$r = \frac{RH\sqrt{R^2 + H^2}}{(\sqrt{R^2 + H^2} - R)\cdot(\sqrt{R^2 + H^2} + 2R)}.$$

Demak, radiusning bu qiymatida idishga tashlangan shar idishdan eng ko‘p miqdorda suyuqlik siqib chiqaradi.

10. x argument va $y = f(x)$ funksiyaning tajriba natijasida olingan qiymatlari jadvalda berilgan. x va y o‘zgaruvchilar orasidagi $y = ax^2 + bx + c$ empirik funksiyani eng kichik kvadratlar usuli bilan toping. Tajriba nuqtalarini va empirik funksiyani to‘g‘ri chiziqli dekart koordinatalar sistemasida tasvirlovchi chizmani chizig.

10.30.

x_i	**0**	**1**	**2**	**3**	**4**	**5**
y_i	0,7	0,5	1,5	2,0	2,5	4,3

Empirik formulani $y = ax^2 + bx + c$ ko‘rinishda izlaymiz.

Bu funksiyaning a, b va c parametrlarini

$$\begin{cases} a\cdot\sum_{i=1}^{n} x_i^4 + b\cdot\sum_{i=1}^{n} x_i^3 + c\cdot\sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i^2 y_i, \\ a\cdot\sum_{i=1}^{n} x_i^3 + b\cdot\sum_{i=1}^{n} x_i + c\cdot\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i, \\ a\cdot\sum_{i=1}^{n} x_i^2 + b\cdot\sum_{i=1}^{n} x_i + c\cdot n = \sum_{i=1}^{n} y_i \end{cases}$$

tenglamalar sistemasidan topamiz.

Qulaylik uchun hisoblarni jadvalda bajaramiz:

i	x_i	x_i^2	x_i^3	x_i^4	y_i	$x_i y_i$	$x_i^2 y_i$
1	0	0	0	0	0,7	0	0
2	1	1	1	1	0,5	0,5	0,5
3	2	4	8	16	1,5	3,0	6,0
4	3	9	27	81	2,0	6,0	18,0
5	4	16	64	256	2,5	10,0	40,0
6	5	25	125	625	4,3	21,5	107,5
$\sum$	15	55	225	979	11,5	41	172

U holda sistema

$$\begin{cases} 979a + 225b + 55c = 172, \\ 225a + 55b + 15c = 41, \\ 55a + 15b + 6c = 11{,}5 \end{cases}$$

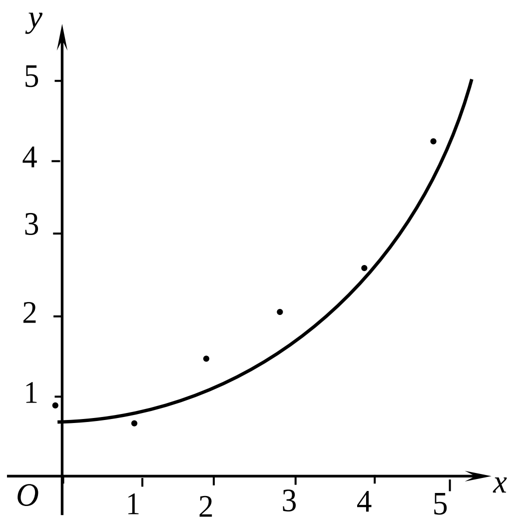
ko‘rinishga keladi.

Uni Kramer formulalari bilan yechamiz:

$$\Delta = \begin{vmatrix} 979 & 225 & 55 \\ 225 & 55 & 15 \\ 55 & 15 & 6 \end{vmatrix} = 3920,$$

$$\Delta_b = \begin{vmatrix} 979 & 172 & 55 \\ 225 & 41 & 15 \\ 55 & 11{,}5 & 6 \end{vmatrix} = -56,$$

$$\Delta_c = \begin{vmatrix} 979 & 225 & 172 \\ 225 & 55 & 41 \\ 55 & 15 & 11{,}5 \end{vmatrix} = 2520,$$



7-shakl.

$$a = \frac{560}{3920} = 0{,}14, \qquad b = -\frac{56}{3920} = -0{,}01, \qquad c = \frac{2520}{3920} = 0{,}64.$$

Demak, izlanayotgan funksiya

$$y = 0{,}1405x^2 - 0{,}01x + 0{,}64.$$

Tajriba nuqtalarini va empirik funksiyani to‘g‘ri burchakli dekart koordinatalar sistemasida tasvirlovchi chizmani chizamiz (7-shakl)

2-MUSTAQIL ISHI

1. Ikki karrali integralni hisoblang.
2. Berilgan chiziqlar bilan chegaralangan D tekis shakl yuzasini toping.
3. Uch karrali integrallarni hisoblang.
4. Berilgan sirtlar bilan chegaralangan jismning hajmini uch karrali integral bilan toping.
5. Birinchi tur egri chiziqli integralni hisoblang.
6. Ikkinchi tur egri chiziqli integrallarni hisoblang.
7. Birinchi tur sirt integralini hisoblang, bu yerda $\sigma -$ D tekislikning koordinata tekisliklari bilan ajratilgan qismi.
8. $u=u(x,y,z)$ funksiyaning M_1 nuqtadagi $\overrightarrow{M_1M_2}$ vektor yo‘nalishidagi hosilasini toping.
9. $\vec{a}$ vektor maydon oqimini D tekislik va koordinata tekisliklaridan hosil bo‘lgan piramidaning tashqi sirti bo‘yicha ikki usul bilan hisoblang: 1) oqim ta’rifidan foydalanib; 2) Ostrogradskiy-Gauss formulasi orqali.
10. $\vec{a}$ vektor maydon sirkulatsiyasini $Ax+By+Cz=D$ tekislikning koordinata tekisliklari bilan kesishishidan hosil bo‘lgan uchburchakning $\vec{n}=\{A;B;C\}$ vektorga nisbatan musbat yo‘nalishda aylanib konturi bo‘yicha ikki usul bilan hisoblang: 1) sirkulatsiya ta’rifidan foydalanib; 2) Stoks formulasi orqali.

1-variant

1. $\iint\limits_D y(1+x^2)dxdy,\ D:\ y=x^3,\ y=3x.$ **2.** $x=27-y^2,\ x=-6y.$

3. $\iiint\limits_V xy^2zdxdydz,\ V:\ -2\le x\le 1,\ 0\le y\le 2,\ 0\le z\le 3.$

4. $x\ge 0,\ y\ge 0,\ z\ge 0,\ 2x+y=2,\ z=y^2.$

5. $\int\limits_L ydl,\quad L:\ y^2=2x$ parabolaning $x^2=2y$ parabola kesgan yoyi.

6. $\int\limits_L (xy-1)dx+x^2ydy,\ L:\ A(1;0)$ va $B(0;2)$ nuqtalarni tutashtiruvchi AB to‘g‘ri chiziq kesmasi.

7. $\iint\limits_\sigma zd\sigma,\ D:\ x+y+z=1.$

8. $u=\ln(1+x^2+y^2+z^2),\ M_1(1;1;1),\ M_2(5;-4;8).$

9. $\vec{a}=(3x+y)\vec{i}+(x+z)\vec{j}+y\vec{k},\ D:\ 2x+y+3z=6.$

10. $\vec{a}=(3x-y)\vec{i}+(2y+z)\vec{j}+(2z-x)\vec{k},\ \ 2x-3y+z=6.$

2-variant

1. $\iint\limits_D (xy-4x^3y^3)dxdy,\ D:\ x=1,\ y=x^2,\ y=-\sqrt{x}.$

2. $y=x^2,\ y=\frac{3}{4}x^2+1.$

3. $\iiint\limits_V (x^2+y^2+z^2)dxdydz,\ V:\ x^2+y^2+z^2=4,\ x\geq 0,\ y\geq 0,\ z\geq 0.$

4. $x^2+y^2=2y,\ z=\frac{13}{4}-x^2,\ z=0.$

5. $\int\limits_L x^2dl,\ \ L:\ x^2+y^2=R^2$ aylananing yuqori yoyi.

6. $\int\limits_L (xy-y)^2dx+xdy,\ \ L:\ y=x^2$ parabolaning $O(0;0)$ nuqtadan $B(1;1)$ nuqtagacha bo‘lgan yoyi.

7. $\iint\limits_\sigma (x+3y+2z)d\sigma,\ D:\ 2x+y+2z=2.$

8. $u=x^2+2y^2-4z^2-5,\ M_1(1;2;1),\ M_2(-3;-2;6).$

9. $\vec{a}=(x+y)\vec{i}+(y+z)\vec{j}+2(z+x)\vec{k},\ D:\ 3x-2y+2z=6.$

10. $\vec{a}=(x+2z)\vec{i}+(y-3z)\vec{j}+z\vec{k},\ \ 3x+2y+2z=6.$

3-variant

1. $\iint\limits_D \sqrt{1-x^2-y^2}dxdy,\ D:\ x^2+y^2=4.$

2. $y^2-2y+x^2=0,\ y^2-4y+x^2=0,\ y=x,\ x=0.$

3. $\iiint\limits_V 21xzdxdydz,\ V:\ y=x,\ y=0,\ x=2,\ z=xy,\ z=0.$

4. $z=3-7(x^2+y^2),\ z=3-14x.$

5. $\oint\limits_L (x^2+y^2)dl,\ \ L:\ x^2+y^2=4x$ aylana.

6. $\oint\limits_L (x^2y-x)dx+(y^2x-2y)dy,\ \ L:\ x=3\cos t,\ y=2\sin t$ ellipsning musbat yo‘nalishda aylanib o‘tishdagi yoyi.

7. $\iint\limits_\sigma (6x+4y+3z)d\sigma,\ D:\ x+2y+3z=6.$

8. $u=\ln(xy+yz+xz),\ M_1(-2;3;-1),\ M_2(2;1;-3).$

9. $\vec{a}=(x+y)\vec{i}+3y\vec{j}+(y-z)\vec{k},\ D:\ 2x-y-2z=-2.$

10. $\vec{a}=(x+z)\vec{i}+(x+3y)\vec{j}+y\vec{k},\ \ 2x+2y+z=2.$

4-variant

1. $\iint\limits_D y\sin xy dxdy,\ D:\ y=\frac{\pi}{2},\ y=\pi,\ x=1,\ x=2.$

2. $x=4-y^2,\ x-y+2=0.$

3. $\iiint\limits_V (xy-z^2)dxdydz,\ V:\ 0\le x\le 1,\ -1\le y\le 2,\ 0\le z\le 3.$

4. $z=8(x^2+y^2)+3,\ z=16x+3.$

5. $\oint\limits_L (x+y)dl,\ L:$ uchlari $A(1;0),\ B(0;1),\ O(0;0)$ nuqtalarda bo‘lgan uchburchak konturi.

6. $\int\limits_L xdy,\ \ L:\ y=\sin x$ sinusoidaning $O(\pi;0)$ nuqtadan $B(0;0)$ nuqtagacha bo‘lgan yoyi.

7. $\iint\limits_\sigma (4y-x+4z)d\sigma,\ D:\ x-2y+2z=2.$

8. $u=x^2y+y^2z+z^2x,\ M_1(1;-1;2),\ M_2(3;4;-1).$

9. $\vec{a}=3x\vec{i}+(y+z)\vec{j}+(x-z)\vec{k},\ D:\ x+3y+z=3.$

10. $\vec{a}=z\vec{i}+(x+y)\vec{j}+y\vec{k},\ \ 2x+y+2z=2.$

5-variant

1. $\iint\limits_D (6xy+24x^3y^3)dxdy,\ D:\ x=1,\ y=\sqrt{x},\ y=-x^2.$

2. $x=y^2,\ y^2=4-x.$

3. $\iiint\limits_V 5xyz^2dxdydz,\ V:\ -1\le x\le 0,\ 2\le y\le 3,\ 1\le z\le 2.$

4. $x\ge 0,\ z\ge 0,\ x+y=4,\ z=4\sqrt{y}.$

5. $\int\limits_L yxdl,\ L:\ y^2=6x$ parabolaning $x^2=6y$ parabola kesgan yoyi.

6. $\oint\limits_L ydx-xdy,\ L:\ r=R$ aylananing musbat yo‘nalishda aylanib o‘tishdagi yoyi.

7. $\iint\limits_\sigma (5x-8y-z)d\sigma,\ D:\ 2x-3y+z=6.$

8. $u=\frac{10}{1+x^2+y^2+z^2},\ M_1(-1;2;-2),\ M_2(2;0;1).$

9. $\vec{a}=(y+z)\vec{i}+(2x-z)\vec{j}+(y+3z)\vec{k},\ D:\ 2x+y+3z=6.$

10. $\vec{a}=(x+z)\vec{i}+2y\vec{j}+(x+y-z)\vec{k},\quad x+2y+z=2.$

6-variant

1. $\iint\limits_D x(y-1)dxdy,\ D:\ y=5x,\ y=x,\ x=3.$

2. $x=8-y^2,\ x=-2y.$

3. $\iiint\limits_V (3x^2+y^2)dxdydz,\ V:\ z=10y,\ x+y=1,\ x=0,\ y=0,\ z=0.$

4. $x^2+y^2=4x,\ z=10-y^2,\ z=0.$

5. $\int\limits_L y^2 dl,\ L:\ x=3(t-\sin t),\ y=3(1-\cos t)$ sikloidaning bir arkasi.

6. $\int\limits_L \cos z dx-\sin x dz,\ L:\ A(2;0;-2)$ va $B(-2;0;2)$ nuqtalarni tutashtiruvchi AB to‘g‘ri chiziq kesmasi.

7. $\iint\limits_\sigma (2x+3y+2z)d\sigma,\ D:\ x+3y+z=3.$

8. $u=x-2y+e^x,\ M_1(-4;-5;0),\ M_2(2;3;4).$

9. $\vec{a}=(x+y+z)\vec{i}+2z\vec{j}+(y-7z)\vec{k},\ D:\ 2x+3y+z=6.$

10. $\vec{a}=x\vec{i}+(y-2z)\vec{j}+(2x-y+2z)\vec{k},\quad x+2y+2z=2.$

7-variant

1. $\iint\limits_D \dfrac{dxdy}{1+x^2+y^2},\ D:\ x^2+y^2=9.$

2. $y=\dfrac{3}{x},\ y=8e^x,\ y=3,\ y=8.$

3. $\iiint\limits_V (x-y-z)dxdydz,\ V:\ 0\le x\le 3,\ 0\le y\le 1,\ -2\le z\le 1.$

4. $z=-2(x^2+y^2)-1,\ z=4y-1.$

5. $\oint\limits_L xydl,\ L:$ tomonlari $x=1,\ x=-1,\ y=1,\ y=-1$ bo‘lgan kvadrat konturi.

6. $\int\limits_L \dfrac{ydx+xdy}{x^2+y^2},\ L:\ A(1;2)$ va $B(3;6)$ nuqtalarni tutashtiruvchi AB to‘g‘ri chiziq kesmasi.

7. $\iint\limits_\sigma (5x-y+5z)d\sigma,\ D:\ 3x+2y+z=6.$

8. $u=\sqrt{1+x^2+y^2+z^2},\ M_1(1;1;1),\ M_2(3;2;1).$

9. $\vec{a}=(x+y-z)\vec{i}-2y\vec{j}+(x+2z)\vec{k},\ D:\ x+2y+z=2.$

10. $\vec{a}=(2y-z)\vec{i}+(x+y)\vec{j}+x\vec{k},\ \ x+2y+2z=4.$

8-variant

1. $\iint\limits_D y\cos xy dxdy,\ D:\ y=\frac{\pi}{2},\ y=\pi,\ x=1,\ x=2.$

2. $x^2-2x+y^2=0,\ x^2-6x+y^2=0,\ y=0,\ y=\frac{x}{\sqrt{3}}.$

3. $\iiint\limits_V (y^2+z)dxdydz,\ V:\ z=x+y,\ x+y=1,\ x=0,\ y=0,\ z=0.$

4. $x^2+y^2=9,\ z=5-x-y,\ z\geq 0.$

5. $\oint\limits_L \sqrt{x^2+y^2}dl,\ \ L:\ x^2+y^2=2y$ aylana.

6. $\int\limits_L (x^2+y)dx+(x+y^2)dy,\ \ L:\ ABC$ siniq chiziq, $A(2;0), B(5;3),\ C(5;0).$

7. $\iint\limits_\sigma (7x+y+2z)d\sigma,\ D:\ 3x-2y+2z=6.$

8. $u=5xy^3z^2,\ M_1(2;1;-1),\ M_2(4;-3;0).$

9. $\vec{a}=(3x-1)\vec{i}+(y-x+z)\vec{j}+4z\vec{k},\ D:\ 2x-y-2z=2.$

10. $\vec{a}=(x+z)\vec{i}+z\vec{j}+(2x-y)\vec{k},\ \ 3x+2y+z=2.$

9-variant

1. $\iint\limits_D ye^{\frac{xy}{2}}dxdy,\ D:\ y=\ln 2,\ y=\ln 3,\ x=2,\ x=4.$

2. $x=5-y^2,\ x=-4y.$

3. $\iiint\limits_V y^2dxdydz,\ V:\ z=2(3x+y),\ x+y=1,\ x=0,\ y=0,\ z=0.$

4. $z\geq 0,\ x^2+y^2=4,\ z=x^2+y^2.$

5. $\int\limits_L (x+y)dl,\ \ L:\ r^2=\cos 2\varphi\ \left(-\frac{\pi}{4}\leq\varphi\leq\frac{\pi}{4}\right)$ Bernulli limniskatasining bo‘lagi.

6. $\int\limits_L 4x\sin^2 ydx+y\cos 2xdy,\ L:\ A(0;0)$ va $B(3;6)$ nuqtalarni tutashtiruvchi AB to‘g‘ri chiziq kesmasi.

7. $\iint\limits_\sigma (3y-x-z)d\sigma,\ D:\ x-y+z=2.$

8. $u=\frac{x}{y}+\frac{y}{z}-\frac{z}{x},\ M_1(-1;1;1),\ M_2(2;3;4).$

9. $\vec{a}=(y+z)\vec{i}+(x+6y)\vec{j}+y\vec{k},\ D:\ x+2y+2z=2.$

10. $\vec{a}=(y+2z)\vec{i}+(x+2z)\vec{j}+(x-2y)\vec{k},\ \ 2x+y+2z=2.$

10-variant

1. $\iint\limits_D y^2(1+2x)dxdy,\ D:\ y=2-x^2,\ x=0.$

2. $x=y^2,\ x=\dfrac{3}{4}y^2+1.$

3. $\iiint\limits_V (2x-y^2-z)dxdydz,\ V:\ 1\le x\le 5,\ 0\le y\le 2,\ -1\le z\le 0.$

4. $z\ge 0,\ y^2=2-x,\ z=3x.$

5. $\int\limits_L (4\sqrt[3]{x}-3\sqrt{y})dl,\ L:\ A(-1;0)$ va $B(0;1)$ nuqtalarni tutashtiruvchi to‘g‘ri chiziq kesmasi.

6. $\int\limits_L \dfrac{x^2dy-y^2dx}{3\sqrt[3]{x^5}+\sqrt[3]{y^5}},\quad L:\ x=2\cos^3 t,\ y=2\sin^3 t$ astroidaning $A(2;0)$ nuqtadan $B(0;2)$ nuqtagacha bo‘lgan yoyi.

7. $\iint\limits_\sigma (2+y-7x+9z)d\sigma,\ D:\ 2x-y-2z=-2.$

8. $u=\ln\left(1+x^3+y^3+z\right),\ M_1(1;3;0),\ M_2(-4;1;3).$

9. $\vec{a}=(2x-z)\vec{i}+(y-x)\vec{j}+(x+2z)\vec{k},\ D:\ x-y+z=2.$

10. $\vec{a}=(y-z)\vec{i}+(2x+y)\vec{j}+z\vec{k},\ \ 2x+y+z=2.$

11-variant

1. $\iint\limits_D xy^2dxdy,\ D:\ y=x,\ y=0,\ x=1.$

2. $y=\dfrac{\sqrt{x}}{2},\ y=\dfrac{1}{2x},\ x=\dfrac{y}{2}.$

3. $\iiint\limits_V x^2yzdxdydz,\ V:\ -1\le x\le 2,\ 0\le y\le 3,\ 2\le z\le 3.$

4. $x\ge 0,\ z\ge 0,\ x+y=2,\ z=y^2.$

5. $\int\limits_L (x^2+y^2)dl,\ L:\ r=2$ aylananing birinchi choragi.

6. $\int\limits_L xydx+(y-x)dy,\ L:\ y=x^3$ kubik parabolaning $O(0;0)$ nuqtadan $B(1;1)$ nuqtagacha bo‘lgan yoyi.

7. $\iint\limits_\sigma (2x+3y+z)d\sigma,\ D:\ 2x+2y+z=2.$

8. $u=\ln({}^2+y^2+z^2),\ M_1(-1;2;1),\ M_2(3;1;-1).$

9. $\vec{a}=(y-z)\vec{i}+(2x+y)\vec{j}+z\vec{k},\ D:\ 2x+y+z=2.$

10. $\vec{a}=(2z-x)\vec{i}+(x-y)\vec{j}+(3x+z)\vec{k},\quad x+y+2z=2.$

12-variant

1. $\iint\limits_D e^y dxdy,\ D:\ y=\ln x,\ y=0,\ x=e.$

2. $y=\sqrt{2-x^2},\ y=x^2.$

3. $\iiint\limits_V (1+2x^3)dxdydz,\ V:\ y=4x,\ y=0,\ x=1,\ z=\sqrt{xy},\ z=0.$

4. $x^2+y^2=4x,\ z=12-y^2,\ z=0.$

5. $\int\limits_L ydl,\ L:\ y=x^2$ parabolaning $A(2;4)$ va $B(1;1)$ nuqtalar orasidagi yoyi.

6. $\int\limits_L ydx-xdy,\ L:\ x=a\cos^3 t,\ y=a\sin^3 t\ \left(0\le t\le \frac{\pi}{2}\right)$ astroida yoyi.

7. $\iint\limits_\sigma (2x+3y+z)d\sigma,\ D:\ 2x+3y+z=6.$

8. $u=x^3+xy^2-6xyz,\ M_1(1;3;-5),\ M_2(4;2;-2).$

9. $\vec{a}=x\vec{i}+(x+z)\vec{j}+(y+z)\vec{k},\ D:\ 3x+3y+z=3.$

10. $\vec{a}=(y+z)\vec{i}+x\vec{j}+(x+2y)\vec{k},\quad 2x+3y+2z=6.$

13-variant

1. $\iint\limits_D ye^{2xy}dxdy,\ D:\ y=\ln 3,\ y=\ln 4,\ x=\frac{1}{2},\ x=1.$

2. $y^2-6y+x^2=0,\ y^2-8y+x^2=0,\ y=x,\ x=0.$

3. $\iiint\limits_V (4+8x^3)dxdydz,\ V:\ y=x,\ y=0,\ x=1,\ z=\sqrt{xy},\ z=0.$

4. $y\ge 0,\ z\ge 0,\ x=4,\ y=2x,\ z=x^2.$

5. $\oint\limits_L (x-y)dl,\quad L:\ x^2+y^2=2ax$ aylana.

6. $\oint\limits_L (x+y)dx+(x-y)dy,\ L:\ x=2\cos t,\ y=3\sin t$ ellipsning musbat yo‘nalishda aylanib o‘tishdagi yoyi.

7. $\iint\limits_\sigma (3y-2x-2z)d\sigma,\ D:\ 2x-y-2z=-2.$

8. $u = e^{xy+z^2}$, $M_1(-5;0;2)$, $M_2(2;4;-3)$.

9. $\vec{a} = (2y - z)\vec{i} + (x + 2y)\vec{j} + y\vec{k}$, $D:\ x + 3y + 2z = 6$.

10. $\vec{a} = (x + z)\vec{i} + (z - x)\vec{j} + (x + 2y + z)\vec{k}$, $x + y + z = 2$.

14-variant

1. $\iint\limits_D \frac{xydxdy}{x^2 + y^2}$, $D:\ x^2 + y^2 = 9$.

2. $y = \frac{2}{x}$, $y = 7e^x$, $y = 2$, $y = 7$.

3. $\iiint\limits_V xyz^2 dxdydz$, $V:\ 0 \le x \le 2,\ -1 \le y \le 0,\ 0 \le z \le 4$.

4. $z = 32(x^2 + y^2) + 3$, $z = 3 - 64x$.

5. $\oint\limits_L \sqrt{z^2 + y^2}dl$, $L:\ z^2 + y^2 = 4$ aylana.

6. $\oint\limits_L y\cos xdx + \sin xdy$, $L:$ uchlari $A(1;0)$, $B(0;2)$, $C(2;0)$ nuqtalarda bo‘lgan ABC uchburchakning musbat yo‘nalishda aylanib o‘tishdagi konturi.

7. $\iint\limits_\sigma (6x + y + 4z)d\sigma$, $D:\ 3x + 3y + z = 3$.

8. $u = xe^y + ye^x - z^2$, $M_1(3;0;2)$, $M_2(4;1;3)$.

9. $\vec{a} = (2y - z)\vec{i} + (x + y)\vec{j} + x\vec{k}$, $D:\ x + 2y + 2z = 4$.

10. $\vec{a} = (x + y - z)\vec{i} - 2y\vec{j} + (x + 2z)\vec{k}$, $x + 2y + z = 2$.

15-variant

1. $\iint\limits_D (y + x^2)dxdy$, $D:\ y = x^2,\ x = y^2$.

2. $y = x^2 + 2$, $y = -3x$.

3. $\iiint\limits_V (x^2 + y^2 + z^2)dxdydz$, $V:\ 0 \le x \le 3,\ -1 \le y \le 2,\ 0 \le z \le 2$.

4. $z \ge 0$, $y + z = 2$, $x^2 + y^2 = 4$.

5. $\int\limits_L (x^2 + y^2 + z^2)dl$, $L:\ x = 4\cos t,\ y = 4\sin t,\ z = 3t$ vint chizig‘ining birinchi o‘rami.

6. $\oint\limits_L (x^2 - y)dx$, $L:\ x = 0,\ y = 0,\ x = 1,\ y = 2$ to‘g‘ri chiziqlardan tuzilgan to‘g‘ri to‘rtburchakning musbat yo‘nalishda aylanib o‘tishdagi konturi.

7. $\iint\limits_\sigma (3x + 10y - z)d\sigma$, $D:\ x + 3y + 2z = 6$.

8. $u = ze^{x^2+y^2+z^2}$, $M_1(0;0;0)$, $M_2(3;-4;2)$.

9. $\vec{a} = x\vec{i} + (y-2z)\vec{j} + (2x-y+2z)\vec{k}$, $D:\ x+2y+2z=2$.

10. $\vec{a} = (2x-z)\vec{i} + (y-x)\vec{j} + (x+2z)\vec{k}$, $x-y+z=2$.

16-variant

1. $\iint\limits_D xy^3dxdy$, $D:\ y^2 = 1-x,\ x \ge 0$.

2. $x^2 = 3y$, $y^2 = 3x$.

3. $\iiint\limits_V (x+2y)dxdydz$, $V:\ z = x^2+3y^2$, $y=x$, $x=1$, $y=0$, $z=0$.

4. $z \ge 0$, $y=2$, $y=x$, $z=x^2$.

5. $\int\limits_L ydl$, $L:\ x=\cos^3 t$, $y=\sin^3 t$ astroidaning $A(1;0)$ va $B(0;1)$ nuqtalar orasidagi yoyi.

6. $\int\limits_L (xy-y^2)dx + xdy$, $L: y=2x^2$ parabolaning $O(0;0)$ nuqtadan $B(1;2)$ nuqtagacha bo‘lgan yoyi.

7. $\iint\limits_\sigma (4x-y+z)d\sigma$, $D:\ x-y+z=2$.

8. $u = \dfrac{x}{y} - \dfrac{y}{z} - \dfrac{x}{z}$, $M_1(2;2;2)$, $M_2(-3;4;1)$.

9. $\vec{a} = (x+z)\vec{i} + (z-x)\vec{j} + (x+2y+z)\vec{k}$, $D:\ x+y+z=2$.

10. $\vec{a} = (2y-z)\vec{i} + (x+2y)\vec{j} + y\vec{k}$, $x+2y+2z=2$.

17-variant

1. $\iint\limits_D \dfrac{dxdy}{\sqrt{1+x^2+y^2}}$, $D:\ x^2+y^2=3$.

2. $x = y^2+1$, $y+x=3$.

3. $\iiint\limits_V 2xy^2z^2dxdydz$, $V:\ 0 \le x \le 3$, $-2 \le y \le 0$, $1 \le z \le 2$.

4. $z \ge 0$, $z=x$, $x=\sqrt{4-y^2}$.

5. $\int\limits_L \dfrac{dl}{x-y}$, $L:\ A(0;4)$ va $B(4;0)$ nuqtalarni tutashtiruvchi to‘g‘ri chiziq kesmasi.

6. $\oint\limits_L xdy$, $L:\ x^2+y^2=R^2$ aylananing musbat yo‘nalishda aylanib o‘tishdagi yoyi.

7. $\iint\limits_{\sigma}(2x-3y+z)d\sigma,\ D:\ x+2y+z=2.$

8. $u=e^{zy},\ M_1(3;1;4),\ M_2(1;-1;-1).$

9. $\vec{a}=(y+z)\vec{i}+x\vec{j}+(y-2z)\vec{k},\ D:\ 2x+2y+z=2.$

10. $\vec{a}=x\vec{i}+(x+z)\vec{j}+(y+z)\vec{k},\ \ 3x+3y+z=3.$

18-variant

1. $\iint\limits_{D}(y^2+x^2)dxdy,\ D: x=1,\ x=y^2.$

2. $y=\dfrac{8}{x^2+4},\ x^2=4y.$

3. $\iiint\limits_{V}(x+2y+3z^2)dxdydz,\ V:\ -1\le x\le 2,\ 0\le y\le 1,\ 1\le z\le 2.$

4. $y\ge 0,\ z\ge 0,\ y+x=2,\ z=x^2.$

5. $\oint\limits_{L}\sqrt{x^2+y^2}dl,\quad L:\ x^2+y^2=2x$ aylana.

6. $\int\limits_{L}xye^x dx+(x-1)e^x dy,\ L:\ A(0;2)$ va $B(1;2)$ nuqtalarni tutashtiruvchi AB to‘g‘ri chiziq qismi.

7. $\iint\limits_{\sigma}(x+2y+3z)d\sigma,\ D:\ x+y+z=2.$

8. $u=3xy^2+z^2-xyz,\ M_1(1;1;2),\ M_2(3;-1;4).$

9. $\vec{a}=(2z-x)\vec{i}+(x-y)\vec{j}+(3x+z)\vec{k},\ D:\ x+y+2z=2.$

10. $\vec{a}=(x+y)\vec{i}+3y\vec{j}+(y-z)\vec{k},\ \ 2x-y-2z=-2.$

19-variant

1. $\iint\limits_{D}(x^3-2y)dxdy,\ D: y=x^2-1,\ x\ge 0,\ y\le 0.$

2. $xy=1,\ x^2=y,\ y=2,\ x=0.$

3. $\iiint\limits_{V}\sqrt{x^2+y^2+z^2}dxdydz,\ V:\ x^2+y^2+z^2=9,\ x\ge 0,\ y\ge 0,\ z\ge 0.$

4. $z=2-18(x^2+y^2),\ z=2-36y.$

5. $\int\limits_{L}\dfrac{dl}{\sqrt{x^2+y^2}},\quad L:\ r=2(1+\cos\varphi)\ \left(0\le\varphi\le\dfrac{\pi}{2}\right)$ kardioida.

6. $\int\limits_{L}2xydx-x^2dy,\ L:\ x=2y^2$ parabolaning $O(0;0)$ nuqtadan $B(2;1)$ nuqtagacha bo‘lgan yoyi.

7. $\iint\limits_{\sigma}(2x+15y+z)d\sigma,\ D:\ x+2y+2z=2.$

8. $u=e^{x-yz},\ M_1(1;0;3),\ M_2(2;-4;5).$

9. $\vec{a}=(x+2z)\vec{i}+(y-3z)\vec{j}+z\vec{k},\ D:\ 3x+2y+2z=6.$

10. $\vec{a}=(x+y+z)\vec{i}+2z\vec{j}+(y-7z)\vec{k},\ \ 2x+3y+z=6.$

20-variant

1. $\iint\limits_{D}xy^2dxdy,\ D:\ y=x^2,\ y=2x.$

2. $y=3\sqrt{x},\ y=\dfrac{3}{x},\ x=\dfrac{y}{12}.$

3. $\iiint\limits_{V}(1+2z)dxdydz,\ V:\ y=4x,\ y=0,\ x=1,\ z=\sqrt{xy},\ z=0.$

4. $x^2+y^2+4x=0,\ z=8-y^2,\ z=0.$

5. $\int\limits_{L}\dfrac{z^2dl}{x^2+y^2},\ L:\ x=2\cos t,\ y=2\sin t,\ z=2t$ vint chizig‘ining birinchi o‘rami.

6. $\int\limits_{L}(x^2+y^2)dx+xydy,\ L:\ y=e^x$ chiziqning $A(0;1)$ nuqtadan $B(1;e)$ nuqtagacha bo‘lgan yoyi.

7. $\iint\limits_{\sigma}(6x-y+8z)d\sigma,\ D:\ x+y+2z=2.$

8. $u=(x^2+y^2+z^2)^3,\ M_1(1;2;-1),\ M_2(0;-1;3).$

9. $\vec{a}=(y+2z)\vec{i}+(x+2z)\vec{j}+(x-2y)\vec{k},\ D:\ 2x+y+2z=2.$

10. $\vec{a}=(y+z)\vec{i}+(x+6y)\vec{j}+y\vec{k},\ \ x+2y+2z=2.$

21-variant

1. $\iint\limits_{D}x(2x+y)dxdy,\ D:\ y=1-x^2,\ y\geq 0.$

2. $y=\dfrac{2}{x},\ y=5e^x,\ y=2,\ y=5.$

3. $\iiint\limits_{V}(x^2+2y^2-z)dxdydz,\ V:\ 0\leq x\leq 1,\ 0\leq y\leq 3,\ -1\leq z\leq 2.$

4. $z\geq 0,\ z=y^2,\ x^2+y^2=9.$

5. $\int\limits_{L}ydl,\ L:\ y^2=2x$ parabolaning $A(0;0)$ va $B(1;\sqrt{2})$ nuqtalar orasidagi yoyi.

6. $\int\limits_{L}2y\sin 2xdx-\cos 2xdy,\ L:\ A\left(\dfrac{\pi}{4};2\right)$ va $B\left(\dfrac{\pi}{6};1\right)$ nuqtalarni tutashtiruvchi AB to‘g‘ri chiziq kesmasi.

7. $\iint\limits_{\sigma}(5x+y-z)d\sigma,\ D:\ x+2y+2z=2.$

8. $u=5x^2yz-xy^2z+yz^2,\ M_1(1;1;1),\ M_2(9;-3;-9).$

9. $\vec{a}=(x+z)\vec{i}+z\vec{j}+(2x-y)\vec{k},\ D:\ 3x+2y+z=6.$

10. $\vec{a}=(3x-1)\vec{i}+(y-x+z)\vec{j}+4z\vec{k},\ \ 2x-y-2z=-2.$

22-variant

1. $\iint\limits_{D}\frac{dxdy}{\sqrt{x^2+y^2}},\ D:\ x^2+y^2=4.$

2. $x^2-2x+y^2=0,\ x^2-6x+y^2=0,\ y=0,\ y=x.$

3. $\iiint\limits_{V}x^3yzdxdydz,\ V:\ -1\le x\le 2,\ 1\le y\le 3,\ 0\le z\le 1.$

4. $z=4-x,\ x^2+y^2=4x.$

5. $\int\limits_{L}\frac{dl}{\sqrt{8-x^2-y}}dl,\ \ L:\ A(0;0)$ va $B(2;2)$ nuqtalarni tutashtiruvchi to‘g‘ri chiziq kesmasi.

6. $\int\limits_{L}y^2dx+x^2dy,\ \ \ L:\ x=5\cos t,\ y=2\sin t$ ellipsning musbat yo‘nalishda aylanib o‘tishdagi yuqori yoyi.

7. $\iint\limits_{\sigma}(3x-2y+6z)d\sigma,\ D:\ 2x+y+2z=2.$

8. $u=(x-y)^z,\ M_1(1;5;0),\ M_2(3;7;-2).$

9. $\vec{a}=4x\vec{i}+(x-y-z)\vec{j}+(3y+2z)\vec{k},\ D:\ 2x+y+z=4.$

10. $\vec{a}=(2y+z)\vec{i}+(x-y)\vec{j}-2z\vec{k},\ \ x-y+z=2.$

23-variant

1. $\iint\limits_{D}e^{x^2+y^2}\sqrt{x^2+y^2}dxdy,\ D:\ x^2+y^2=9.$

2. $x=y^2,\ x=\sqrt{2-y^2}.$

3. $\iiint\limits_{V}3(2y+3x)dxdydz,\ V:\ y=x,\ x=0,\ x=1,\ z=x^2+y^2,\ z=0.$

4. $z=0,\ x^2+y^2=4y,\ z=4-x^2.$

5. $\int\limits_{L}\frac{dl}{x^2+y^2+z^2},\ \ L:\ x=\cos t,\ y=\sin t,\ z=t$ vint chizig‘ining birinchi o‘rami.

6. $\int\limits_{L}2xydx-x^2dy+zdz,\ L:\ O(0;0;0)$ va $B(2;1;-1)$ nuqtalarni tutashtiruvchi OB to‘g‘ri chiziq kesmasi.

7. $\iint\limits_{\sigma}(2x+5y+10z)d\sigma,\ D:\ 2x+y+3z=6.$

8. $u=\dfrac{x}{x^2+y^2+z^2},\ M_1(1;2;2),\ M_2(-3;2;-1).$

9. $\vec{a}=(x+z)\vec{i}+2y\vec{j}+(x+y-z)\vec{k},\ D:\ x+2y+z=2.$

10. $\vec{a}=(x+y)\vec{i}+(y+z)\vec{j}+2(x+z)\vec{k},\ 3x-2y+2z=6.$

24-variant

1. $\iint\limits_{D}(x+1)y^2dxdy,\ D:\ y=3x^2,\ y=3.$

2. $x=\sqrt{4-y^2},\ y=\sqrt{3x}.$

3. $\iiint\limits_{V}(x+y+z)dxdydz,\ V:\ x+y+z=1,\ x\ge 0,\ y\ge 0,\ z\ge 0.$

4. $z=24(x^2+y^2),\ z=48x.$

5. $\int\limits_{L}(x^2+y^2)^2dl,\ L:\ x=3\cos t,\ y=3\sin t$ aylana.

6. $\int\limits_{L}(2a-y)dx+xdy,\quad L:\ x=a(t-\sin t),\ y=a(1-\cos t)\ (0\le t\le 2\pi)$ sikloidaning birinchi arkasi.

7. $\iint\limits_{\sigma}(3x+2y+2z)d\sigma,\ D:\ 3x+2y+2z=6.$

8. $u=x^3y+y^3z-3z^2,\ M_1(0;-2;-1),\ M_2(12;-5;0).$

9. $\vec{a}=(x+z)\vec{i}+(x+3y)\vec{j}+y\vec{k},\ D:\ 2x+2y+z=4.$

10. $\vec{a}=(y+z)\vec{i}+(2x-z)\vec{j}+(y+3z)\vec{k},\ 2x+y+3z=6.$

25-variant

1. $\iint\limits_{D}\dfrac{y^2}{x^2}dxdy,\ D:\ y=x,\ xy=1,\ y=2.$

2. $2y=\sqrt{x},\ x+y=5.$

3. $\iiint\limits_{V}x^2y^2z^3dxdydz,\ V:\ -1\le x\le 3,\ 0\le y\le 2,\ 1\le z\le 2.$

4. $x^2+y^2=3z,\ x+y=6.$

5. $\int\limits_{L}(4\sqrt[3]{x}-3\sqrt[3]{y})dl,\ L:\ x=\cos^3 t,\ y=\sin^3 t$ astroidaning $A(1;0)$ va $B(0;1)$ nuqtalar orasidagi yoyi.

6. $\int\limits_{L}\sin y dx+\sin x dy,\ L:\ A(0;\pi)$ va $B(\pi;0)$ nuqtalarni tutashtiruvchi

AB to‘g‘ri chiziq kesmasi.

7. $\iint\limits_{\sigma}(x+2y+3z)d\sigma,\ D:\ 2x-y+z=2.$

8. $u=3xy^2z^3,\ M_1(-3;-2;1),\ M_2(0;1;-3).$

9. $\vec{a}=4z\vec{i}+(x-y-z)\vec{j}+(3y+z)\vec{k},\ D:\ x-2y+2z=2.$

10. $\vec{a}=(2z-x)\vec{i}+(x+2y)\vec{j}+3z\vec{k},\quad x+4y+2z=8.$

26-variant

1. $\iint\limits_{D}x^2(1+3y)dxdy,\ D:\ x=0,\ y^2=2-x.$

2. $y+2x=0,\ x^2=3-y.$

3. $\iiint\limits_{V}(x^2+y^2+z^2)dxdydz,\ V:\ 0\le x\le 1,\ -2\le y\le 1,\ 1\le z\le 3.$

4. $x^2+y^2=2x,\ z=\dfrac{13}{4}-y^2,\ z=0.$

5. $\int\limits_{L}x\ d,\quad L:\ x=\cos^3 t,\ y=\sin^3 t$ astroidaning $A(1;0)$ va $B(0;1)$ nuqtalar orasidagi yoyi.

6. $\int\limits_{L}(xy-2)dx+y^2xdy,\ L:\ A(2;1)$ va $B(1;2)$ nuqtalarni tutashtiruvchi AB to‘g‘ri chiziq kesmasi.

7. $\iint\limits_{\sigma}(3x-y+2z)d\sigma,\ D:\ x+2y+z=4.$

8. $u=xe^{x^2+y^2+z^2},\ M_1(0;0;0),\ M_2(2;-4;3).$

9. $\vec{a}=(x+y)\vec{i}+(x+z)\vec{j}+2(y+z)\vec{k},\ D:\ 2x-3y+2z=6.$

10. $\vec{a}=(x+y)\vec{i}+(x+3z)\vec{j}+z\vec{k},\quad 2x+y+2z=2.$

27-variant

1. $\iint\limits_{D}(x+y^2)dxdy,\ D:\ y=x^2,\ x=y^2.$

2. $xy=2,\ x=5e^y,\ x=2,\ x=5.$

3. $\iiint\limits_{V}8x^2yz^2dxdydz,\ V:\ -2\le x\le 1,\ 0\le y\le 2,\ -1\le z\le 3.$

4. $z=10-x^2,\ z=0,\ x^2+y^2=4y.$

5. $\oint\limits_{L}(x+y)dl,\quad L:\ x^2+y^2=2ay$ aylana.

6. $\int_L ydx$, $L:\ y=\cos x$ cosinusoidaning $O(\pi;-1)$ nuqtadan $B(0;1)$ nuqtagacha bo‘lgan yoyi.

7. $\iint_\sigma (3x-2y+z)d\sigma,\ D:\ 2x+y+z=4.$

8. $u=3yx^2+z^2-xyz,\ M_1(1;1;2),\ M_2(-1;3;4).$

9. $\vec{a}=(x+y+z)\vec{i}+2z\vec{j}+(x-7z)\vec{k},\ D:\ 3x+2y+z=6.$

10. $\vec{a}=y\vec{i}+(x-2z)\vec{j}+(2y-x+2z)\vec{k},\ \ 2x+y+2z=2.$

28-variant

1. $\iint_D \frac{dxdy}{\sqrt{1+x^2+y^2}},\ D:\ x^2+y^2=8.$

2. $y=\frac{2}{x^2+1},\ x^2=y.$

3. $\iiint_V (2+3y^3)dxdydz,\ V:\ x=4y,\ x=0,\ y=1,\ z=\sqrt{xy},\ z=0.$

4. $z\ge 0,\ y^2+x^2=4,\ z=x^2.$

5. $\oint_L \sqrt{x^2+y^2}dl$, $L:\ x^2+y^2=4x$ aylana.

6. $\oint_L (x-y)dx+(x+y)dy$, $L:\ x=3\cos t,\ y=2\sin t$ ellipsning musbat yo‘nalishda aylanib o‘tishdagi yoyi.

7. $\iint_\sigma (x+6y+4z)d\sigma,\ D:\ 2x+2y+z=2.$

8. $u=x^2y+xz^2+zy^2,\ M_1(1;1;1),\ M_2(-1;0;2).$

9. $\vec{a}=(2x-z)\vec{i}+(x+y)\vec{j}+y\vec{k},\ D:\ 2x+y+2z=4.$

10. $\vec{a}=(2x-z)\vec{i}+(z-y)\vec{j}+(x+3z)\vec{k},\ \ 2x+y+z=2.$

29-variant

1. $\iint_D \frac{xydxdy}{x^2+y^2},\ D:\ x^2+y^2=16.$

2. $x^2+y^2=4,\ x^2=3y.$

3. $\iiint_V (x^2+2y+z^2)dxdydz,\ V:\ 1\le x\le 2,\ 0\le y\le 2,\ -1\le z\le 2.$

4. $z=4-y,\ x^2+y^2=4y.$

5. $\int_L \frac{dl}{y-x}$, $L:\ A(1;3)$ va $B(3;1)$ nuqtalarni tutashtiruvchi to‘g‘ri chiziq kesmasi.

6. $\oint\limits_L ydx$, $L:\ x^2+y^2=16$ aylananing musbat yo‘nalishda aylanib o‘tishdagi yoyi.

7. $\iint\limits_\sigma (4x+y+2z)d\sigma,\ D:\ x+y+z=1.$

8. $u=\frac{1}{2}x^2y^2z^2,\ M_1(1;-1;0),\ M_2(2;-1;2).$

9. $\vec{a}=(2x+z)\vec{i}+(y-2z)\vec{j}+x\vec{k},\ D:\ 2x+2y+3z=6.$

10. $\vec{a}=(x+z)\vec{i}+y\vec{j}+(y+2x)\vec{k},\ \ 3x+2y+2z=6.$

30-variant

1. $\iint\limits_D (x^2+3y)dxdy,\ D:\ x+y=1,\ y=x^2-1,\ x\geq 0.$

2. $y^2=4x,\ x^2=4y.$

3. $\iiint\limits_V (3x^2+2y+z)dxdydz,\ V:\ 0\leq x\leq 1,\ 0\leq y\leq 1,\ -1\leq z\leq 3.$

4. $x=1,\ y=2x,\ y\geq 0,\ z=y^2,\ z\geq 0.$

5. $\int\limits_L \sqrt{2y}dl,\ \ L:\ x=2(t-\sin t),\ y=2(1-\cos t)$ sikloidaning bir arkasi.

6. $\int\limits_L y^2dx+x^2dy,\ L:\ x=a\cos t,\ y=b\sin t$ ellipsning soat strelkasi yo‘nalishida aylanib o‘tishdagi yoyi.

7. $\iint\limits_\sigma (4x-y+4z)d\sigma,\ D:\ 2x+2y+z=4.$

8. $u=\ln(1+x+y^2),\ M_1(1;1;1),\ M_2(3;-5;4).$

9. $\vec{a}=(2z-x)\vec{i}+(x+2y)\vec{j}+3z\vec{k},\ D:\ x+4y+2z=8.$

10. $\vec{a}=z\vec{i}+(x+y)\vec{j}+y\vec{k},\ \ 2x+y+2z=2.$

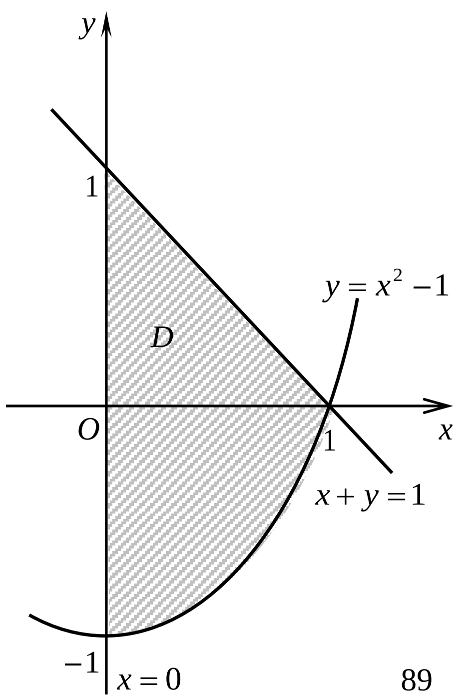
NAMUNAVIY VARIANT YECHIMI

1. Ikki karrali integralni hisoblang.

1.30. $\iint\limits_D (x^2+3y)dxdy,\ D:\ x+y=1,\ y=x^2-1,\ x\geq 0.$

D integrallash sohasi 18 - shaklda keltirilgan.

Agar ichki integrallash y bo‘yicha va tashqi integrallash x bo‘yicha bajarilsa berilgan ikki karrali integral bitta takroriy integral bilan ifodalanadi. Integralni hisoblaymiz:



18-shakl.

$$\iint_D (x^2+3y)dxdy = \int_0^1 dx \int_{x^2-1}^{1-x} (x^2+3y)dy = \int_0^1 \left(x^2 y + \frac{3}{2}y^2 \right)\Bigg|_{x^2-1}^{1-x} dx =$$

$$= \int_0^1 \left(x^2 - x^3 - x^4 + x^2 + \frac{3}{2}(1-2x+x^2-x^4+2x^2-1) \right) dx =$$

$$= \frac{1}{2}\int_0^1 (4x^2-2x^3-2x^4+9x^2-3x^4-6x)dx = = \frac{1}{2}\int_0^1 (13x^2-2x^3-5x^4-6x)dx =$$

$$\frac{1}{2}\left(\frac{13}{3}x^3 - \frac{1}{2}x^4 - x^5 - 3x^2 \right)\Bigg|_0^1 = -\frac{1}{12}.$$

2. Berilgan chiziqlar bilan chegaralangan D tekis shakl yuzasini toping.

2.30. $y^2 = 4x,\ x^2 = 4y.$

Tekis shakl quyidan $y = \frac{1}{4}x^2$ parabola bilan yuqoridan $y^2 = 4x$ parabola bilan chegaralangan (19-shakl).

Bundan

$$S = \iint_D dxdy = \int_0^4 dx \int_{\frac{1}{4}x^2}^{2\sqrt{x}} dy = \int_0^4 \left(2\sqrt{x} - \frac{1}{4}x^2 \right) dx = \left(\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{12}x^3 \right)\Bigg|_0^4 = \frac{16}{3}.$$

3. Uch karrali integrallarni hisoblang.

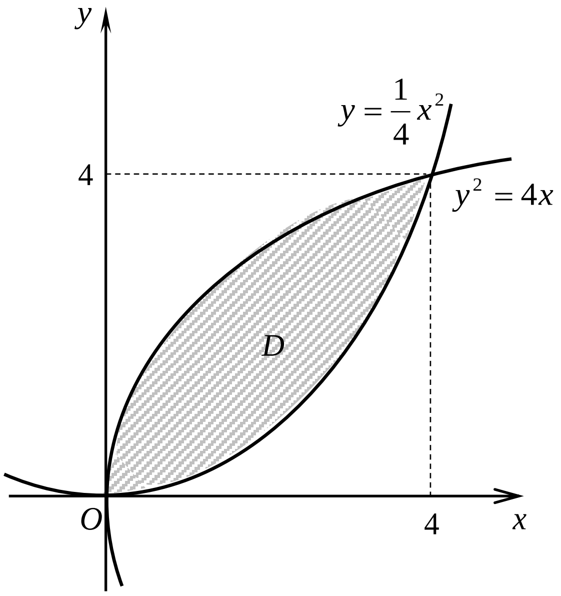
3.30. $\iiint_V (3x^2+2y+z)dxdydz,\ V:\ 0 \le x \le 1,\ 0 \le y \le 1,\ -1 \le z \le 3.$

Berilgan to‘g‘ri burchakli parllelopiped uchun topamiz:

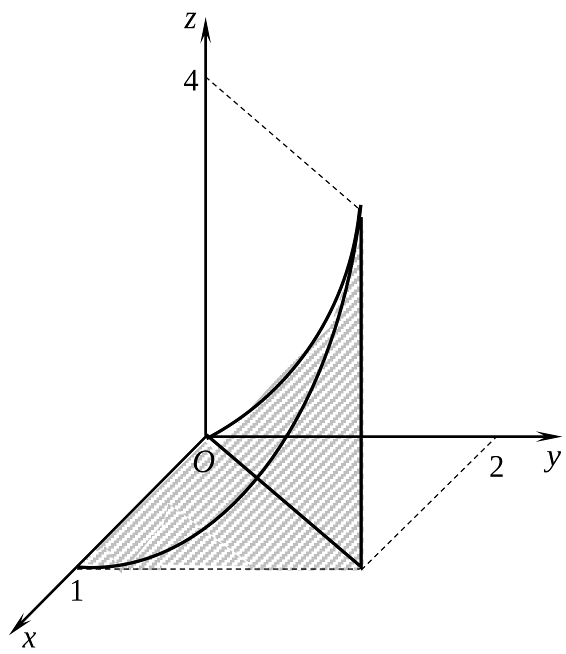
$$\iiint_V (3x^2+2y+z)dxdydz = \int_0^1 dx \int_0^1 dy \int_{-1}^3 (3x^2+2y+z)dz =$$

$$= \int_0^1 dx \int_0^1 \left((3x^2+2y)z + \frac{z^2}{2} \right)\Bigg|_{-1}^3 dy = 4\int_0^1 dx \int_0^1 (3x^2+2y+1)dy =$$

$$=4\int_0^1((3x^2+1)y+y^2)\Big|_0^1dx=4\int_0^1(3x^2+2)dx=4(x^3+2x)\Big|_0^1=12.$$



19-shakl.



20-shakl.

4. Berilgan sirtlar bilan chegaralangan jismning hajmini uch karrali integral bilan toping.

4.30. $x=1,\ \ y=2x,\ \ y\geq 0,\ \ z=y^2,\ \ z\geq 0.$

Berilgan jism (20- shakl) hajmini hisoblaymiz:

$$V=\iiint_V dxdydz=\int_0^1dx\int_0^{2x}dy\int_0^{y^2}dz=\int_0^1dx\int_0^{2x}z\Big|_0^{y^2}dy=\int_0^1dx\int_0^{2x}y^2dy=\int_0^1\frac{y^3}{3}\Bigg|_0^{2x}dy=\frac{8}{3}\int_0^1x^3dx=\frac{2}{3}x^4\Big|_0^1=\frac{2}{3}.$$

5. Birinchi tur egri chiziqli integralni hisoblang.

5.30. $\int_L\sqrt{2y}dl$, $\ L:\ x=2(t-\sin t),\ \ y=2(1-\cos t)$ sikloidaning bir arkasi.

Sikloidaning parametrik tenglamasidan topamiz:

$$x_t'=2(1-\cos t),\ \ y_t'=2\sin t,$$

$$dl=\sqrt{4(1-\cos t)^2+4\sin^2 t}dt=2\sqrt{2}\sqrt{1-\cos t}dt.$$

U holda

$$\int_L\sqrt{2y}dl=\int_0^{2\pi}\sqrt{2\cdot 2(1-\cos t)}2\sqrt{2}\sqrt{1-\cos t}dt=$$

$$=4\sqrt{2}\int_0^{2\pi}(1-\cos t)dt=4\sqrt{2}(t-\sin t)\Big|_0^{2\pi}=8\pi\sqrt{2}.$$

6. Ikkinchi tur egri chiziqli integrallarni hisoblang.

6.30. $\int_L y^2dx + x^2dy$, $L:\ x = a\cos t,\ \ y = b\sin t$ ellipsning soat strelkasi yo‘nalishida aylanib o‘tishdagi yuqori yoyi.

Ellipsning parametrik tenglamasiga ko‘ra $dx = -a\sin tdt,\ dy = b\cos tdt.$ Bunda soat strelkasi yo‘nalishida t parametr π dan 0 gacha o‘zgaradi.

U holda

$$\int_L y^2dx + x^2dy = \int_\pi^0 (-b^2\sin^2 ta\cos t + a^2\cos^2 tb\sin t)dt =$$

$$= \int_\pi^0 b^2a(1-\cos^2 t)d(\cos t) + \int_\pi^0 a^2b(1-\sin^2 t)d(\sin t) =$$

$$= b^2a\left(\cos t - \frac{1}{3}\cos^3 t\right)\Bigg|_\pi^0 + a^2b\left(\sin t - \frac{1}{3}\sin^3 t\right)\Bigg|_\pi^0 = \frac{4}{3}ab^2.$$

7. Birinchi tur sirt integralini hisoblang, bu yerda $\sigma -\ D$ tekislikning koordinata tekisliklari bilan ajratilgan qismi.

7.30. $\iint_\sigma (4x - y + 4z)d\sigma,\ D:\ 2x + 2y + z = 4.$

Tekislik tenglamasidan topamiz:

$$z = 4 - 2x - 2y,\ z'_x = -2,\ z'_y = -2.$$

U holda $d\sigma = \sqrt{1 + z_x'^2 + z_y'^2}dxdy = 3dxdy.$

Sirt integralini σ_{xy} soha bo‘yicha ikki karrali integralni hisoblashga keltiramiz, bu yerda $\sigma_{xy} - \sigma$ sirtning Oxy tekislikdagi proeksiyasi bo‘lgan AOB uchburchak (21-shakl).

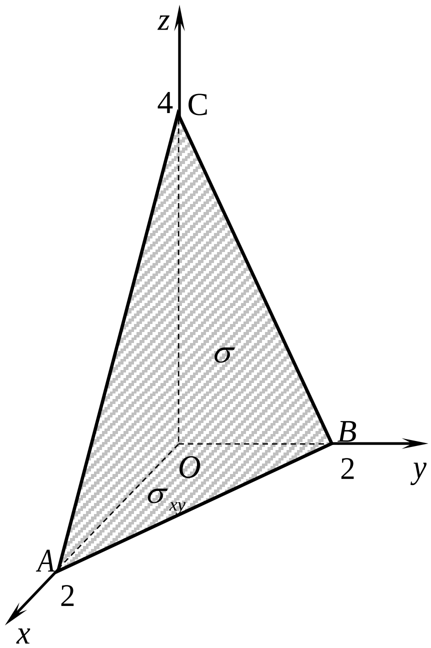
$$\iint_\sigma (4x - y + 4z)d\sigma = \iint_\sigma (4x - y + 16 - 8x - 8y)3dxdy =$$

$$= 3\int_0^2 dx\int_0^{2-x}(16 - 4x - 9y)dy = 3\int_0^2\left(\left(16-4x\right)y - \frac{9}{2}y^2\right)\Bigg|_0^{2-x}dx =$$

$$= 3\int_0^2 (2-x)\left((16-4x) - \frac{9(2-x)}{2}\right)dx =$$

$$= \frac{3}{2}\int_0^2 (2-x)(x+14)dx =$$

$$= \frac{3}{2}\int_0^2\left(28 - 12x - x^2\right)dx = \frac{3}{2}\left(28x - 6x^2 - \frac{x^3}{3}\right)\Bigg|_0^2 = 44.$$



21-shakl.

8. $u = u(x,y,z)$ funksiyaning M_1 nuqtadagi $\overrightarrow{M_1M_2}$ vektor yo‘nalishidagi hosilasini toping.

8.30. $u=\ln(1+x+y^2+z^2)$, $M_1(1;1;1)$, $M_2(3;-5;4)$.

$\overrightarrow{M_1M_2}$ vektor yo‘nalishidagi $\vec{l}$ birlik vektorning yo‘naltiruvchi kosinuslarini topamiz:

$$\overrightarrow{M_1M_2}=\{2;-6;3\},\quad \vec{l}^{\,0}=\frac{\overrightarrow{M_1M_2}}{\left|\overrightarrow{M_1M_2}\right|}=\frac{2\vec{i}-6\vec{j}+3\vec{k}}{7}=\frac{2}{7}\vec{i}-\frac{6}{7}\vec{j}+\frac{3}{7}\vec{k},$$

$$\cos\alpha=\frac{2}{7},\ \cos\beta=-\frac{6}{7},\ \cos\gamma=\frac{3}{7}.$$

$u=\ln(1+x+y^2+z^2)$ funksiya xususiy hosilalarining $M_1(1;1;1)$ nuqtadagi qiymatlarini topamiz:

$$\left.\frac{\partial u}{\partial x}\right|_{M_0}=\left.\frac{1}{1+x+y^2+z^2}\right|_{M_0}=\frac{1}{4},\quad \left.\frac{\partial u}{\partial y}\right|_{M_0}=\left.\frac{2y}{1+x+y^2+z^2}\right|_{M_0}=\frac{1}{2},$$

$$\left.\frac{\partial u}{\partial z}\right|_{M_0}=\left.\frac{2z}{1+x+y^2+z^2}\right|_{M_0}=\frac{1}{2}.$$

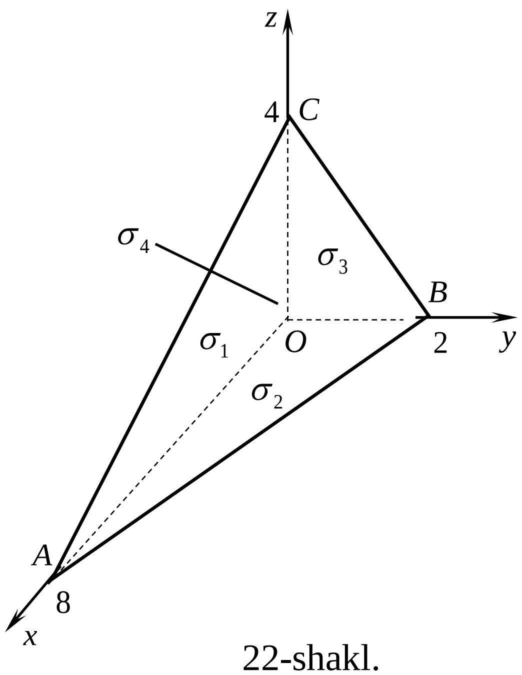
U holda

$$\frac{\partial u}{\partial l}=\frac{1}{4}\cdot\frac{2}{7}+\frac{1}{2}\cdot\left(-\frac{6}{7}\right)+\frac{1}{2}\cdot\frac{3}{7}=-\frac{1}{7}.$$

9. $\vec{a}$ vektor maydon oqimini D tekislik va koordinata tekisliklaridan hosil bo‘lgan piramidaning tashqi sirti bo‘yicha ikki usul bilan hisoblang:
1) oqim ta’rifidan foydalanib;
2)Ostrogradskiy-Gauss formulasi orqali.

9.30. $\vec{a}=(2z-x)\vec{i}+(x+2y)\vec{j}+3z\vec{k}$,
D: $x+4y+2z=8$.

1) Vektor maydon oqimini $\Pi=\iint\limits_{\sigma}\vec{a}\vec{n}^{0}d\sigma$ formula bilan piramidaning (22-shakl) har bir tomoni (to‘rtta uchburchak) orqali hisoblaymiz:



22-shakl.

ΔAOC da $y=0$, $\vec{n}^{0}=-\vec{j}$, $x+2z=8$.

$$\Pi_1 = -\iint_{\sigma} x d\sigma = -\iint_{\varsigma_1} x dx dz = -\int_0^4 dz \int_0^{2(4-z)} x dx = -\frac{1}{2}\int_0^4 x^2 \Big|_0^{2(4-x)} dz =$$

$$= -2\int_0^4 \left(16 - 8z + z^2\right) dz = -2\left(16z - 4z^2 + \frac{z^3}{3}\right)\Bigg|_0^4 = -\frac{128}{3}.$$

ΔAOB da $z = 0,\ \vec{n}^0 = -\vec{k},\ x + 4y = 8.$

$$\Pi_2 = \iint_{\sigma} 0 d\sigma = 0.$$

ΔBOC da $x = 0,\ \vec{n}^0 = -\vec{i},\ z + 2y = 4.$

$$\Pi_3 = -\iint_{\sigma} 2z d\sigma = -\iint_{\sigma_3} 2z dy dz = -\int_0^2 dy \int_0^{2(2-y)} 2z dz = -\int_0^2 z^2 \Big|_0^{2(2-y)} dy =$$

$$= -4\int_0^2 (4 - 4y + y^2) dy = -4\left(4y - 2y^2 + \frac{y^3}{3}\right)\Bigg|_0^2 = -\frac{32}{3}.$$

ΔABC da $\quad \vec{n}^0 = \dfrac{\vec{i} + 4\vec{j} + 2\vec{k}}{\sqrt{21}},\ z = \dfrac{8 - x - 4y}{2},\ z'_x = -\dfrac{1}{2},\ z'_y = -2,$

$$d\sigma = \sqrt{1 + z_x'^2 + z_y'^2}\, dx dy = \sqrt{1 + \frac{1}{4} + 4}\, dx dy = \frac{\sqrt{21}}{2} dx dy,$$

$$\vec{a}\vec{n}^0 = \frac{1}{\sqrt{21}}(2z - x + 4(x + 2y) + 2\cdot 3z) = \frac{8z + 3x + 8y}{\sqrt{21}}.$$

$$\Pi_4 = -\frac{1}{\sqrt{21}}\iint_{\sigma}(3x + 8y + 8z) d\sigma = \frac{1}{\sqrt{21}}\cdot\frac{\sqrt{21}}{2}\iint_{\sigma_4}(3x + 8y + 32 - 4x - 16y) dx dy =$$

$$= \frac{1}{2}\iint_{D_4}(32 - x - 8y) dx dy = \frac{1}{2}\int_0^2 dy \int_0^{4(2-y)}(32 - x - 8y) dx =$$

$$= \frac{1}{2}\int_0^2\left((32 - 8y)x - \frac{x^2}{2}\right)\Bigg|_0^{4(2-y)} dy = \frac{1}{2}\cdot 8\int_0^4((16 - 4y)(2 - y) - (2 - y)^2) dy =$$

$$= 4\int_0^2 (2 - y)(16 - 4y - 2 + y) dy = 4\int_0^2 (2 - y)(14 - 3y) dy =$$

$$= 4\int_0^2 (28 - 20y + 3y^2) dy = 4(28y - 10y^2 + y^3)\Big|_0^2 = 96.$$

Demak,

$$\Pi = \Pi_1 + \Pi_2 + \Pi_3 + \Pi_4 = -\frac{128}{3} + 0 - \frac{32}{3} + 96 = \frac{128}{3}.$$

2) Vektor maydon oqimini Ostrogradskiy-Gauss formulasi orqali hisoblaymiz.

$$\Pi=\iiint_V\left(\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}\right)dxdydz=\iiint_V(-1+2+3)dxdydz=$$

$$=4\int_0^2 dy\int_0^{4(2-y)}dx\int_0^{\frac{8-x-4y}{2}}dz=4\int_0^2 dy\int_0^{4(2-y)}z\Big|_0^{\frac{8-x-4y}{2}}dx=2\int_0^2 dy\int_0^{4(2-y)}(8-4y-x)dx=$$

$$=2\int_0^2\left((8-4y)x-\frac{x^2}{2}\right)\Bigg|_0^{4(2-y)}dy=16\int_0^2(2-y)(4-2y-2+y)dy=$$

$$=16\int_0^2(4-4y+y^2)dy=16\left(4y-2y^2+\frac{y^3}{3}\right)\Bigg|_0^2=\frac{128}{3}.$$

10. $\vec{a}$ vektor maydon sirkulatsiyasini tekislikning koordinata tekisliklari bilan kesishishidan hosil bo‘lgan uchburchakning $\bar{n}=\{A;B;C\}$ vektorga nisbatan musbat yo‘nalishda aylanish konturi bo‘yicha ikki usul bilan hisoblang: 1) sirkulatsiya ta’rifidan foydalanib; 2) Stoks formulasi orqali.

10.30. $\vec{a}=z\vec{i}+(x+y)\vec{j}+y\vec{k},\ \ 2x+y+2z=2.$

1) Sirkulatsiyani $ABCA$ kontur (23-shakl) bo‘yicha topamiz:

$$Ц=\oint_L\vec{a}d\vec{r}=\int_{AB}\vec{a}d\vec{r}+\int_{BC}\vec{a}d\vec{r}+\int_{CA}\vec{a}d\vec{r}.$$

AB kesmada $z=0,\ dz=0,\ 2x+y=2,\ y=2(1-x),\ dy=-2dx.$ U holda

$$\vec{a}=(x+y)\vec{j}+y\vec{k},\ d\vec{r}=dx\vec{i}+dy\vec{j},\ \vec{a}d\vec{r}=(x+y)dy.$$

Bundan

$$Ц_1=\int_{AB}\vec{a}d\vec{r}=\int_{AB}(x+y)dy=-2\int_1^0(x+2-2x)dx=-2\int_1^0(2-x)dx=-2\left(2x-\frac{x^2}{2}\right)\Bigg|_1^0=3.$$

BC kesmada $x=0,\ dx=0,\ 2z+y=2,\ z=\dfrac{2-y}{2},\ dz=-\dfrac{1}{2}dy.$

U holda $\vec{a}=z\vec{i}+y\vec{j}+y\vec{k},\ d\vec{r}=dy\vec{j}+dz\vec{k},\ \vec{a}d\vec{r}=ydy+ydz.$

Bundan

$$Ц_2=\int_{BC}\vec{a}d\vec{r}=\int_{BC}ydy+ydz=\int_2^0\left(y-\frac{1}{2}y\right)dy=\frac{1}{2}\frac{y^2}{2}\Bigg|_2^0=-1.$$

CA kesmada $y=0,\ dy=0,\ x+z=1,\ z=1-x,\ dz=-dx.$

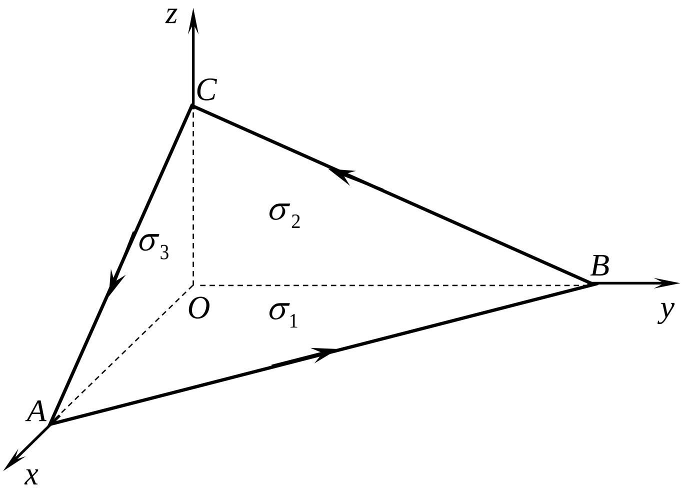
U holda $\vec{a}=z\vec{i}+x\vec{j},\ d\vec{r}=dx\vec{i}+dz\vec{k},\ \vec{a}d\vec{r}=zdx.$

Bundan

$$Ц_3=\int_{CA}\vec{a}d\vec{r}=\int_{CA}zdx=\int_0^1(1-x)dx=\left(x-\frac{x^2}{2}\right)\Bigg|_0^1=\frac{1}{2}.$$

Demak,

$$Ц = Ц_1 + Ц_2 + Ц_3 = 3 - 1 + \frac{1}{2} = \frac{5}{2}.$$



23-shakl.

2) Sirkulyatsiyani Stoks formulasidan foydalanib topamiz:

$\vec{a} = z\vec{i} + (x+y)\vec{j} + y\vec{k}$ dan

$P = z,\ Q = x + y,\ R = y.$

Bundan

$$\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} = 1, \qquad \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} = 1,$$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 1.$$

U holda

$$Ц = \iint_{\sigma} rot\vec{a}d\vec{\sigma} = \iint_{\sigma} dydz + dzdx + dxdy = \iint_{D_1} dydz + \iint_{D_2} dzdx + \iint_{D_3} dxdy =$$

$$= \int_0^1 dz \int_0^{2(1-z)} dy + \int_0^1 dx \int_0^{1-x} dz + \int_0^1 dx \int_0^{2(1-x)} dy = \int_0^1 (2-2z)dz + \int_0^1 (1-x)dx + \int_0^1 (2-2x)dx =$$

$$= (2z - z2)\Big|_0^1 + \left(x - \frac{x^2}{2} \right)\Bigg|_0^1 + (2x - x^2)_0^1 = 1 + \frac{1}{2} + 1 = \frac{5}{2}.$$