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MATEMATIKA

FANIDAN AMALIY MASHG‘ULOTLAR

Texnikumlarda ta’lim oluvchilar uchun o’quv qo’llanma

TOSHKENT

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Ushbu o'quv qo'llanma "TIQXMMI" Milliy tadqiqot universiteti Ilmiy Kengashining majlisida ko'rib chiqilgan va chop etishga tavsiya etilgan (2022 yil 15 noyabrdagi 418-a/f -buyruq, ro'yxatga olish raqami 418a/f - 087).

Ushbu kitob texnikumlarda matematika fanidan amaliy mash-g'ulotlarni bajarish bo'yicha o'quv qo'llanma bo'lib hisoblanadi. O'quv qo'llanma tekislikdagi analitik geometriya va differensial hisob elementlariga doir misol va masalalarni mustaqil yechish yo'llarini o'rgatishga bag'ishlangan.

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/Matematika fanidan amaliy mashg'ulotlar/

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KIRISH

Texnikumlarning o’quvchilari matematikadan masalalar yechishda ko’pincha bir qator qiyinchiliklarga duch keladilar. O’quvchilarga bu qiyinchiliklarni bartaraf etishda yordam berish, olingan nazariy bilimlarni matematika kursining barcha bo’limlariga doir masalalar yechishda tatbiq etishga o’rgatish mazkur qo’llanmaning asosiy vazifasidir.

Tajribada ko’rinadiki, ko’pchilik o’quvchilar masalalarni mustaqil yechishda ularni yechish usullari bo’yicha doimiy maslahatlarga muhtoj bo’ladilar, chunki masalani yechish yo’lini topish fan o’qituvchisining yoki shu sohada yozilgan qo’llanmaning yordamisiz o’quvchiga qiyinlik qiladi. Masalalar yechish bo’yicha ana shunday maslahatlarni o’quvchi mazkur kitobdan topishi mumkin.

Qo’llanma matematikaning tekislikdagi analitik geometriya va bir o’zgaruvchili funlsiyaning diffensial hisobi elementlariga bag’ishlangan bo’lib, mavzular o’rta umumiy ta’lim maktabida matematik bilimlarga ega bo’lgan o’quvchilarga tushinarli tilda bayon qilingan.

Tekislikdagi ikki nuqta orasidagi masofani topish va kesmani berilgan nisbatda bo’lishda koordinatalar usulidan foydalanilgan. To’g’ri chiziq va uning turli tenglamalarini tuzish masalalari, ikkita to’g’ri orasidagi burchak, to’g’ri chiziqlarning kesishish, parallellik va perpendikulyarlik shartlari o’rganilgan. Analitik geometriya usullari yordamida tekislikda nuqtalarning geometrik o’rnini topish, aylana, ellips, giperbola va parabolaning tenglamalarini tuzish hamda ikkinchi tartibli egri chiziqlarning asosiy elementlarini topishga doir masalalar o'rganilgan.

O'rta umumiy ta'lim maktablarida o'rganilmaydigan limitlar nazariyasi mavzusi o'quvchiga sodda va avvaldan tanish misollar yordamida tushuntirilgan. Ajoyib limitlarning qo'llanishlariga oid misollar berilgan. Limitlar mavzusining mantiqiy davomi – bu hosila mavzusidir. Qo'llanmada hosilani hisoblash qoidalari, uning geometrik, mexanik va fizik talqini, amaliy masalalarni yechishga tatbiqlari keng yoritilgan. Birinchi va ikkinchi tartibli hosilalar yordamida funksiyani to'la tekshirish, grafigini yasash, funksiyaning eng katta va eng kichik qiymatlarini topishga doir masalalar o'rganilgan. Misollar va masalalarni yechishda chizmalar va rasmlardan foydalanilgan.

Qo'llanmada yechilishlari bilan berilgan masalalardan tashqari mustaqil bajariladigan masalalar ham yetarlicha keltirilgan.

Yuqoridagilardan kelib chiqib, qo'llanmada yechilishlari bilan berilgan misollar va masalalar texnikumlarning o'quvchilariga matematikaga doir masalalarni yechish usullarini o'qituvchining yordamisisz, mustaqil o'rganishlari, ularni yechish usullari bo'yicha hosil qilingan malakalarni mustahkamlash va chuqurlashtirish imkonini beradi, deb hisoblsh mumkin.

Qo'llanma texnikumlarning oquvchilariga mo'ljallangan bo'lishiga qaramasdan undan texnikumlarda dars bera boshlagan o'qituvchilarga ham akademik guruhlarda ishlashda, uy vazifalari va nazorat topshiriqlar uchun misol va masalalar tanlashga foydali bo'lishi mimkin.

TEKISLIKDA ANALITIK GEOMETRIYA ELEMENTLARI

I - BOB

KOORDINATALAR USULI

Bu bobdagi masalalarni yechilishiga kirishayotgan o‘quvchi nuqtani χ ning koordinatalari bo‘yicha yasashni va berilgan nuqtaning koordinatalarini topishni bilishi kerak. “Masaladagi nuqtani toping” degan so‘z bu nuqtaning koordinatalarini topishni bildiradi.

1-§. Tekislikdagi ikki nuqta orasidagi masofa

Tekislikdan olingan $A(x_a; y_a)$ va $B(x_b; y_b)$ nuqtalar orasidagi masofa

$$d = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2} \tag{1.1}$$

formula bo‘yicha hisoblanadi, bu yerda d — bu nuqtalarni tutashtiruvchi kesmaning uzunligi.

Agar kesmaning uchlaridan biri koordinatalar boshi bilan ustma-ust tushsa, ikkinchisi esa $M(x_m; y_m)$ koordinatalarga ega bo‘lsa, (1.1) formula

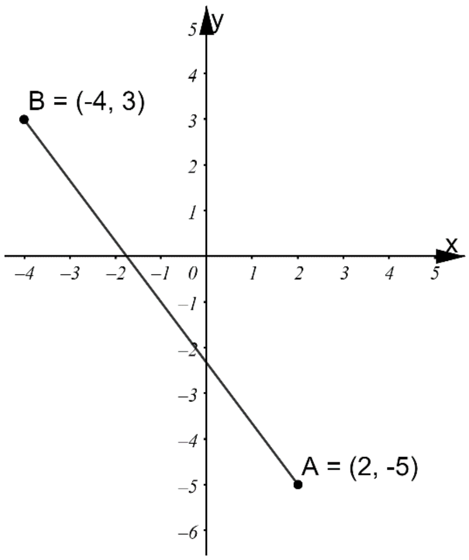
$$OM = \sqrt{x_m{}^2 + y_m{}^2} \tag{1.2}$$

ko‘rinishga keladi.

I. Ikki nuqta orasidagi masofani bu nuqtalarinng berilgan koordinatalari bo‘yicha hisoblash

1. $A(2; -5)$ va $B(-4; 3)$ nuqtalarni tutashtiruvchi kesmalarning uzunligini toping (1-rasm).

Yechilishi. Masala shartida $x_a = 2$, $x_b = -4$, $y_a = -5$, $y_b = 3$ berilgan. d ni topish kerak.



1-rasm

Yuqoridagi (1.1) formulani qo‘llab topamiz:

$$d = AB = \sqrt{\left(2-(-4)\right)^2 + \left(-5-3\right)^2} = 10$$

2. 1) $A(-1;2)$ va $B(2;6)$; 2) $M(2;-2)$ va $N(-4;1)$ nuqtalarni tutashtiruvchi kesmaning uzunligi nimaga teng?

3. Koordinatalar boshini 1) $A(3;-4)$; 2) $M(-5;-12)$ nuqtalar bilan tutashtiruvchi kesmaning uzunligini toping.

4. Uchlari 1) $A(4;0)$, $B(7;4)$ va $C(-4;6)$; 2) $A(6;7)$, $B(3;3)$ va $C(1;-5)$ nuqtalardan iborat bo‘lgan uchburchakning perimetrini hisoblang.

II. Berilgan uchta nuqtadan teng uzoqlashgan nuqtaning koordinatalarini hisoblash

5. $A(7;-1)$, $B(-2;2)$ va $C(-1;-5)$ nuqtalardan bir xil uzoqlashgan O_1 nuqtani toping.

Yechilishi. Masala shartidan $O_1A = O_1B = O_1C$ ekanligi kelib chiqadi. Izlanayotgan O_1 nuqta $(a;b)$ koordinatalarga ega bo‘lsin. (1.1) formula bo‘yicha topamiz:

$$O_1A = \sqrt{(a-7)^2 + (b+1)^2},$$

$$O_1B = \sqrt{(a+2)^2 + (b-2)^2},$$

$$O_1C = \sqrt{(a+1)^2 + (b+5)^2}.$$

Quyidagi tenglamalar sistemasini tuzamiz:

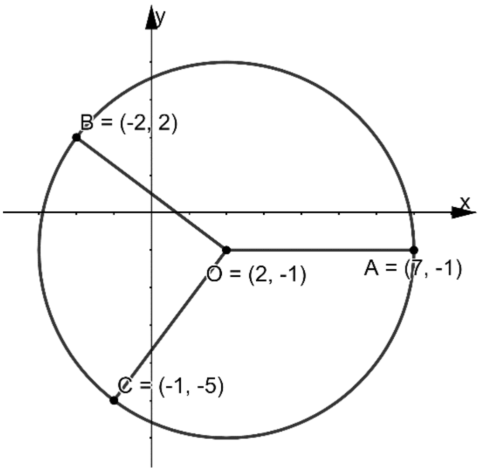
$$\begin{cases} \sqrt{(a-7)^2+(b+1)^2}=\sqrt{(a+2)^2+(b-2)^2}, \\ \sqrt{(a-7)^2+(b+1)^2}=\sqrt{(a+1)^2+(b+5)^2}. \end{cases}$$

Tenglamalarning chap va o‘ng tomonlarini kvadratga ko‘tarib, topamiz:

$$\begin{cases} (a-7)^2+(b+1)^2=(a+2)^2+(b-2)^2, \\ (a-7)^2+(b+1)^2=(a+1)^2+(b+5)^2. \end{cases}$$

Soddalashtirsak ushbu

$$\begin{cases} -3a+b+7=0, \\ -2a-b+3=0 \end{cases}$$



2-rasm

tenglamalar sistemasi hosil bo‘ladi, bu sistemani yechib, $a=2$ va $b=-1$ ni topamiz. $O_1(2;-1)$ nuqta bir nuqtada yotmaydigan berilgan uchta nuqtadan teng uzoqlashgan. Bu nuqta berilgan uchta nuqta orqali o‘tuvchi aylananing markazidir (2- rasm).

6. $A(10;7)$, $B(-4;7)$ va $C(12;-7)$ nuqtalardan teng uzoqlashgan O_1 nuqtaning koordinatalarini toping.

7. $A(-1;9)$, $B(-8;2)$, $C(9;9)$ nuqtalar orqali o‘tuvchi aylananing markazini va radiusining uzunligini toping.

III. Abssissalar (ordinatalar) o‘qida yotuvchi va berilgan nuqtadan berilgan masofada yotuvchi nuqtaning abssissasini (ordinatasini) hisoblash

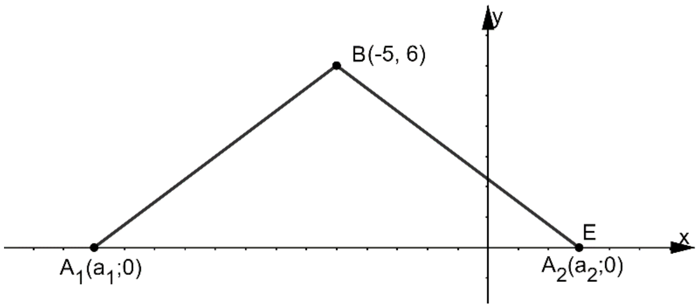
8. $B(-5;6)$ nuqtadan Ox o‘qda yotuvchi A nuqtagacha bo‘lgan masofa 10 ga teng. A nuqtani toping.

Yechilishi. Masala shartidan A nuqtaning ordinatasi 0 ga tengligi va $AB=10$ ekanligi kelib chiqadi. A nuqtaning abssissasini a bilan belgilab, $A(a;0)$ deb yozamiz. Ikki nuqta orasidagi masofani hisoblash formulasi bo‘yicha topamiz, ya’ni

$$AB=\sqrt{(a+5)^2+(0-6)^2}=\sqrt{(a+5)^2+36}.$$

Bundan $\sqrt{(a+5)^2+36}=10$ tenglamani hosil qilamiz. Ixchamlasak

$$a^2+10a-39=0$$



3-rasm

kvadrat tenglama hosil bo‘ladi. Bu tenglamaning ildizlari: $a_1=-13$ va $a_2=3$.

Natijada $A_1(-13;0)$ va $A_2(3;0)$ nuqtalarni hosil qilamiz.

Tekshirish:

$$A_1B=\sqrt{(-13+5)^2+(0-6)^2}=10,$$

$$A_2B=\sqrt{(3+5)^2+(0-6)^2}=10.$$

Har ikkala nuqta masala shartini qanoatlantiradi (3-rasm).

9. M nuqta Ox o‘qda yotadi. M nuqtadan $N(10;5)$ nuqtagacha bo‘lgan masofa 13 ga teng. M nuqtani toping.

10. B nuqta Oy o‘qda yotadi. B nuqtadan $A(3;-1)$ nuqtagacha bo‘lgan masofa 5 ga teng. B nuqtani toping.

IV. Abssissalar (ordinatalar) o‘qida yotuvchi va berilgan ikkita nuqtadan teng uzoqlashgan nuqtaning abssissasini (ordinatasini) hisoblash

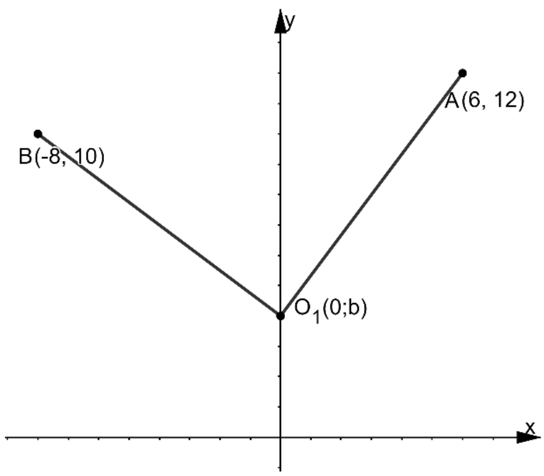
11. *Oy* o‘qda $A(6;12)$ va $B(-8;10)$ nuqtalardan teng uzoqlashgan nuqtani toping.

Yechilishi. *Oy* o‘qda yotuvchi izlanayotgan nuqtaning koordinatalari $O_1(0;b)$ (*Oy* o‘qda yotuvchi nuqtaning abssissasi 0 ga teng) bo‘lsin.

Masala shartidan $O_1A = O_1B$ ekanligi kelib chiqadi. (1.1) formula bo’yicha topamiz:

$$O_1A = \sqrt{(0-6)^2 + (b-12)^2} = \sqrt{36 + (b-12)^2},$$

$$O_1B = \sqrt{(0+8)^2 + (b-10)^2} = \sqrt{64 + (b-10)^2}.$$



4-rasm

Ushbu tenglamaga egamiz:

$$\sqrt{36 + (b-12)^2} = \sqrt{64 + (b-10)^2}$$

yoki $36 + (b-12)^2 = 64 + (b-10)^2$.

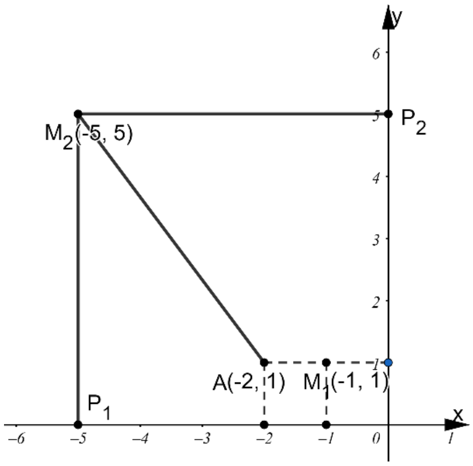
Soddalashtirishlardan so‘ng, $b - 4 = 0$, ya’ni $b = 4$ ni topamiz. Izlanayotgan nuqta $O_1(0;4)$ ekan (4-rasm).

12. *Oy* o‘qda 1) $A(-4;0)$ va $B(-3;-7)$; 2) $A(-3;-1)$ va $B(6;2)$ nuqtalardan teng uzoqlashgan nuqtaning koordinatalarini hisoblang.

13. *Ox* o‘qda 1) $A(5;13)$ va $B(-12;-4)$; 2) $A(0;6)$ va $B(2;-4)$ nuqtalardan teng uzoqlashgan nuqtani toping.

V. Koordinata o‘qlaridan va berilgan nuqtadan teng uzoqlashgan nuqtaning koordinatalarini hisoblash

14. Koordinata o‘qlaridan va $A(-2;1)$ nuqtadan teng uzoqlashgan M nuqtani toping.



5-rasm

Yechilishi. Izlanayotgan M nuqta $A(-2;1)$ nuqta kabi ikkinchi koordinata choragida yotadi, chunki u A va P_1 nuqtalardan teng uzoqlashgan (5-rasm), M nuqtadan koordinata o‘qlarigacha bo‘lgan masofalar teng, demak, uning koordinatalari $(-a;a)$ bo‘ladi, bu yerda $a>0$.

Masala shartidan $MA = MP_1 = MP_2$, $MP_1 = a$; $MP_2 = |-a|$; ya’ni $|-a| = a$ ekanligi kelib chiqadi:

$$MA = \sqrt{(-a+2)^2 + (a-1)^2}\,.$$

Ushbu tenglamani tuzamiz:

$$\sqrt{(-a+2)^2 + (a-1)^2} = a\,.$$

Kvadratga ko‘tarib, soddalashtirgandan so‘ng, $a^2 + 6a + 5 = 0$ kvadrat tenglamaga ega bo‘lamiz. Tenglamani yechib, $a_1 = 1$, $a_2 = 5$ ildizlarni topamiz.

$M_1(-1;1)$ va $M_2(-5;5)$ nuqtalarni hosil qilamiz, bu nuqtalar masala shartini qanoatlantiradi.

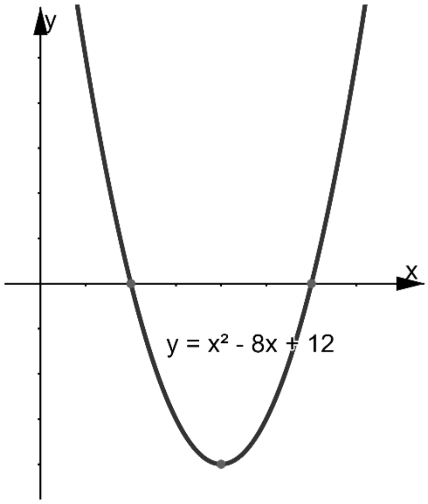
15. Koordinata o‘qlaridan va 1) $A(-8;-1)$; 2) $A(4;2)$ nuqtadan teng uzoqlashgan M nuqtaning koordinatalarini hisoblang.

VI. Abssissalar (ordinatalar) o‘qidan va berilgan nuqtadan bir xil masofaga uzoqlashgan nuqtaning koordinatalarini hisoblash

16. Ordinatalar o‘qigacha va $A(8;6)$ nuqtagacha bo‘lgan masofasi 5 ga teng bo‘lgan M nuqtani toping.

Yechilishi. Masala shartidan $MA=5$ va M nuqtaning abssissasi 5 ga tengligi kelib chiqadi. M nuqtaning ordinatasi b ga teng bo‘lsin, u holda $M(5;b)$ (6-rasm) formula bo‘yicha quyidagiga egamiz:

$$MA=\sqrt{(5-8)^2+(b-6)^2}\,.$$



6-rasm

Ushbu tenglamani tuzamiz:

$$\sqrt{(5-8)^2+(b-6)^2}=5.$$

Uni soddalashtirib, $b^2-12b+20=0$ kvadrat tenglamaga ega bo’lamiz. Bu tengnlamaning ildizlari $b_1=2$ va $b_2=10$. Demak, masala shartini qanoatlantiruvchi ikkita nuqta bor ekan: $M_1(5;2)$ va $M_2(5;10)$.

17. Abssissalar o‘qigacha va $A(1;2)$ nuqtagacha bo‘lgan masofasi 10 ga teng bo‘lgan M nuqtaning koordinatalarini hisoblang.

18. Ordinatalar o‘qigacha va $M(1;3)$ nuqtagacha bo‘lgan masofasi 13 ga teng bo‘lgan A nuqtani toping.

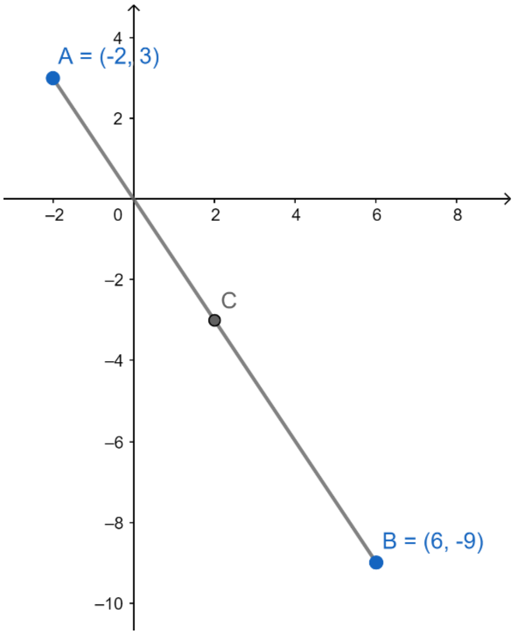
2-§. Kesmani berilgan nisbatda bo‘lish

Kesmani berilgan nisbatda bo‘luvchi nuqtaning koordinatalari ushbu formulalar bo‘yicha hisoblanadi:

$$x_C=\frac{x_A+\lambda x_B}{1+\lambda},\qquad y_C=\frac{y_A+\lambda y_B}{1+\lambda},\tag{1.3}$$

bu yerda (x_A, y_A) va (x_B, y_B) nuqtalar berilgan AB kesma uchlarining koordinatalaridir; $\lambda=\dfrac{AC}{BC}$ esa AB kesma $(x_C; y_C)$ nuqta bilan bo‘linadigan nisbat.

Agar C nuqta AB kesmani teng ikkiga bo‘lsa, u holda $\lambda = 1$ bo‘lib, (1.3) formulalar quyidagi ko‘rinishda bo‘ladi:



7-rasm

$$x_C = \frac{x_A + x_B}{2}, \quad y_C = \frac{y_A + y_B}{2}. \quad (1.4)$$

Masalalarda λ odatda kesmalar uzunliklarining nisbati kabi berilishini, shu sababli nisbatning oldingi va keyingi hadlari kesmalar uzunliklarini berilgan o‘lchov birligida ifodalamasligini nazarda tutish kerak. Masalan, $AC = 12$ sm; $CB = 16$ sm:

$$\lambda = \frac{AC}{BC} = \frac{12 \text{ sm}}{16 \text{ sm}} = \frac{3}{4}.$$

I. Kesma o‘rtasining koordinatalarini uning uchlarining koordinatalari bo‘yicha hisoblash.

19. $A(-2;3)$ va $B(6;-9)$ nuqtalar AB kesmaning uchlari. AB kesmaning o‘rtasi bo‘lgan C nuqtani toping**.**

Yechilishi. Masala shartida quyidagilar berilgan: $x_A = -2$, $x_B = 6$, $y_A = 3$ va $y_B = -9$. $C(x_C; y_C)$ nuqtani topish kerak:

$$x_C = \frac{-2+6}{2} = 2, \quad y_C = \frac{3+(-9)}{2} = -3.$$

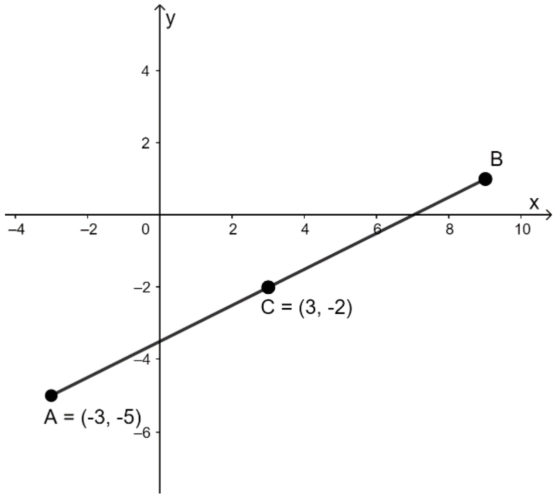
AB kesmaning o‘rtasi $C(-2;3)$ nuqta bo‘ladi (7-rasm).

20. 1) $A(5;-4)$ va $B(-1;2)$; 2) $A(6;-3)$ va $(-2;-7)$ nuqtalarni tutashtiruvchi kesmaning o‘rtasi bo‘lgan C nuqtaning koordinatalarini hisoblang**.**

II. Kesma uchining koordinatalarini uning o'rtasi va ikkinchi uchining koordinatalari bo'yicha hisoblash

21. Kesmaning uchi $A(-3;-5)$ nuqtadan, uning o'rtasi esa $C(3;-2)$ nuqtadan iborat. Kesmaning ikkinchi uchi B nuqtani toping.

Yechilishi. Masala shartida quyidagilar berilgan: $x_A = -3$, $y_A = -5$, $x_C = 3$ va $y_C = -2$.



8-rasm

Bu qiymatlarni (1.4) formulaga qo'yib topamiz :

$$3 = \frac{-3 + x_B}{3}, \quad -2 = \frac{-5 + y_B}{2}.$$

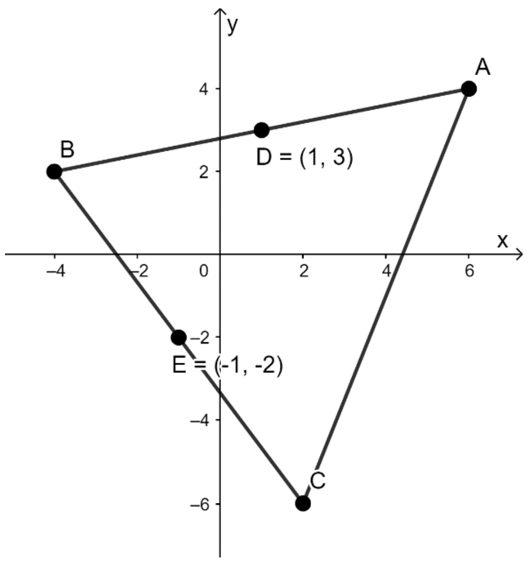
Birinchi tenglamani x_B ga, ikkinchisini y_B ga nisbatan yechib, $x_B = 9$ va $y_B = 1$ ni topamiz, ya'ni izlanayotgan nuqta $B(9;1)$ bo'ladi (8-rasm).

22. Kesmaning uchi $A(8;-5)$ nuqtadan, uning o'rtasi $C(5;-2)$ nuqtadan iborat. Kesmaning ikkinchi uchi B nuqtani toping.

23. Kesmaning uchi $B(-3;-2)$ va uning o'rtasi $C(-2;3)$ bo'yicha kesmaning ikkinchi uchi A nuqtani toping.

III. Uchburchak uchlarining koordinatalarini uning tomonlari o'rtalarining koordinatalari bo'yicha topish

24. Uchburchak tomonlarining o'rtalari $D(1;3)$, $E(-1;-2)$ va



9-rasm

$F(4;-1)$ nuqtalardan iborat. Uchburchakning uchlarini toping.

Yechilishi. Berilgan A, B va C nuqtalar uchburchakning uchlari, D nuqta AB tomonning, E nuqta BC tomonning va F nuqta AC tomonning o‘rtasi bo‘lsin (9-rasm). A, B va C nuqtalarning koordinatalarini topish talab etiladi.

Uchburchakning uchlarini mos ravishda $A(x_A;y_A)$, $B(x_B;y_B)$ va $C(x_C;y_C)$ bilan belgilab va D, E hamda F nuqtalarning koordinatalarini bilgan holda (1.4) formulalar bo‘yicha ushbu sistemalarni hosil qilamiz:

$$\begin{cases} 1=\dfrac{x_A+x_B}{2}, \\ -1=\dfrac{x_B+x_C}{2}, \\ 4=\dfrac{x_A+x_C}{2} \end{cases} \quad \text{va} \quad \begin{cases} 3=\dfrac{y_A+y_B}{2}, \\ -2=\dfrac{y_B+y_C}{2}, \\ -1=\dfrac{y_A+y_C}{2}. \end{cases}$$

Tenglamalarni butun ko‘rinishga keltiramiz:

$$\begin{cases} x_A+x_B=2, \\ x_B+x_C=-2, \\ x_A+x_C=8, \end{cases} \qquad \begin{cases} y_A+y_B=6, \\ y_B+y_C=-4, \\ y_A+y_C=-2. \end{cases}$$

Tenglamalar sistemasini yechib (hisoblashlarni mustaqil bajarishni tavsiya etamiz), quyidagi yechimlarni olamiz:

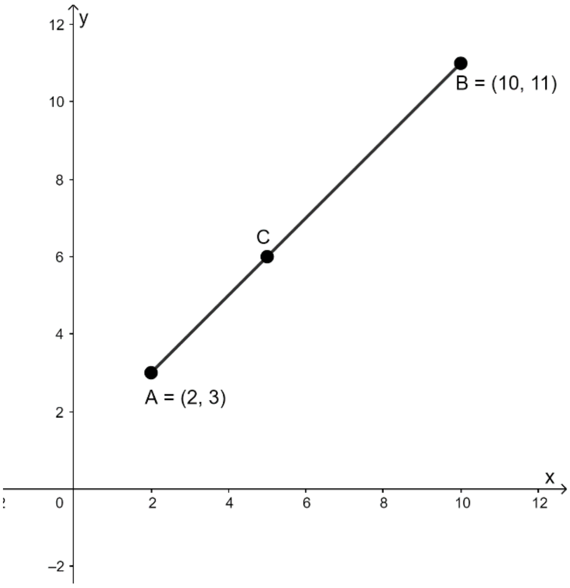
$$x_A=6, \quad x_B=-4, \quad x_C=2,$$
$$y_A=4, \quad y_B=2, \quad y_C=-6.$$

Uchburchakning uchlari $A(6;4)$, $B(-4;2)$ va $C(2;-6)$ nuqtalardan iborat.

25. Uchburchak tomonlari o‘rtalarining koordinatalari berilgan: $(2;1)$, $(0;-4)$ va $(-4;-1)$. Uchburchakning uchlarini toping.

IV. Uchlarining koordinatalari bilan berilgan kesmani berilgan nisbatda bo‘luvchi nuqtalarning koordinatalarini hisoblash

26. C nuqta AB kesmani (A dan B ga qarab) $3:5$ kabi nisbatda bo‘ladi. Kesmaning uchlari $A(2;3)$ va $B(10;11)$ nuqtalardan iborat. C nuqtani toping.



10-rasm

Yechilishi. Masala shartida quyidagilar berilgan: $x_A = 2$, $x_B = 10$; $y_A = 3$, $y_B = 11$; $\lambda = \dfrac{AC}{CB} = \dfrac{3}{5}$. $C(x_C; y_C)$ ni topish kerak (10-rasm).

(1.3) formulalar bo‘yicha topamiz:

$$x_C = \frac{2 + \frac{3}{5}\cdot 10}{1 + \frac{3}{5}} = 5, \quad y_C = \frac{3 + \frac{3}{5}\cdot 11}{1 + \frac{3}{5}} = 6. \quad \text{Demak, } C(5;6).$$

Tekshirish:

$$AC = 3\sqrt{2}, \quad CB = 5\sqrt{2}, \quad \lambda = \frac{AC}{CB} = \frac{3\sqrt{2}}{5\sqrt{2}} = \frac{3}{5}.$$

***Izoh**. Agar masala shartida kesmaning berilgan nisbatda bo‘linishi A dan B ga qarab bajarilishi ko‘rsatilmaganda edi, u holda masala ikkita yechimga ega bo‘lar edi. Ikkinchi yechim kesmani B dan A ga qarab bo‘lish bilan topilar edi.*

27. Uchlari $A(-3;-2)$ va $B(9;6)$ bo‘lgan kesma C nuqta bilan (B dan A ga qarab) $1:3$ kabi nisbatda bo‘linadi. C nuqtani toping.

Izoh. *Birinchi nuqta deb* B *olinadi, shu sababli (1.3) formulalar bu masala uchun ushbu ko‘rinishda bo‘ladi:*

$$x_C = \frac{x_B + \lambda x_A}{1+\lambda}, \quad y_C = \frac{y_B + \lambda y_A}{1+\lambda}.$$

28. Uchlari $A(3;-2)$ va $B(10;-9)$ bo‘lgan kesma C nuqta bilan 2:5 kabi nisbatda bo‘linadi. C nuqtani toping.

29. Uchlari $A(-11;1)$ va $B(9;11)$ bo‘lgan kesma 2:3:5 kabi nisbatda (A dan B ga qarab) bo‘lingan. Bo‘lish nuqtalarini toping.

Yechilishi. A dan B ga qarab bo‘linish nuqtalarini C va D bilan belgilaymiz. Quyidagilar masala shartida berilgan: $x_A = -11,\ x_B = 9,$ $y_A = 1,\ y_B = 11$ va $AC:CB:BD = 2:3:5$. $C(x_C; y_C)$ va $D(x_D; y_D)$ nuqtalarni topish kerak. C nuqta AB kesmani

$$\lambda = \frac{AC}{CB} = \frac{2}{3+5} = \frac{2}{8} = \frac{1}{4}$$

nisbatda bo‘ladi.

(1.3) formulalarga asosan topamiz:

$$x_C = \frac{-11 + \frac{1}{4}\cdot 9}{1+\frac{1}{4}} = -7, \quad y_C = \frac{1 + \frac{1}{4}\cdot 11}{1+\frac{1}{4}} = 3; \quad C(-7;3).$$

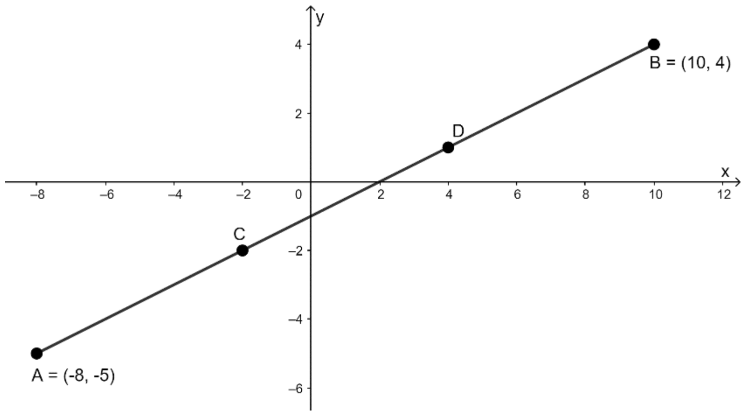
D nuqta AB kesmaning o‘rtasi. Uni (1.4) formulalarni qo‘llab topamiz:

$$x_D = \frac{-11+9}{2} = -1, \qquad y_D = \frac{1+11}{2} = 6, \qquad D(-1;6).$$

30. Uchlari $A(-5;-2)$ va $B(4;2,5)$ bo‘lgan kesma A dan B ga qarab 3:4:2 kabi nisbatda bo‘lingan. Bo‘linish nuqtalarini toping.

V. Kesmaning bo‘linish nuqtalari koordinatalarini uning uchlarini koordinatalari va bu kesma bo‘lingan bo‘laklar soni bo‘yicha topish

31. Kesmaning uchlari $A(-8;-5)$ va $B(10;4)$ nuqtalardan iborat. Bu kesmani uchta teng bo‘lakka bo‘luvchi C va D nuqtalarni toping.



11-rasm

Yechilishi. Masalaning shartida quyidagilar berilgan: $x_A=-8$, $x_B=10$, $y_A=-5$, $y_B=4$ va $n=3$. $C(x_D;y_D)$ va $D(x_D;y_D)$ nuqtalarni hisoblash kerak. Dastlab AB kesmani $\lambda=\frac{1}{2}$ nisbatda bo‘luvchi C nuqtani topamiz. Bo‘lishni A dan B ga qarab boshlaymiz. (1.3) formulaga ko‘ra,

$$x_C=\frac{-8+\frac{1}{2}\cdot 10}{1+\frac{1}{2}}=-2,\quad y_C=\frac{-5+\frac{1}{2}\cdot 4}{1+\frac{1}{2}}=-2;\quad C(-2;-2).$$

CD kesmani bo‘lish 1:1 nisbatda bo‘lganligi uchun (1.4) formulalardan foydalanamiz:

$$x_D=\frac{-2+10}{2}=4,\quad y_D=\frac{-2+4}{2}=1;\quad D(4;1).$$

Bo‘linish nuqtalari: $C(-2;-2)$ va $D(4;1)$ (11-rasm).

Izoh. *D nuqtani AB kesmani 2:1 kabi nisbatda bo‘lib ham topish mumkin edi, u holda yana (1.3) formulalarni qo‘llash kerak bo‘lar edi.*

32. AB kesmani uchta teng bo‘lakka bo‘luvchi nuqtalarni toping. Kesmaning uchlari $A(3;-3)$ va $B(-3;9)$.

33. Kesmaning uchlari $A(5;-6)$va $B(-5;9)$ nuqtalardan iborat. Bu kesmani beshta teng bo‘lakka bo‘luvchi nuqtalarning koordinatalarini toping.

Yechilishi. A dan B ga qarab ketma-ket bo‘lish nuqtalarini $C(x_C;y_C), D(x_D;y_D)$, $E(x_E;y_E)$ va $F(x_F;y_F)$ bilan belgilaymiz. Masala shartida quyidagilar berilgan: $x_A=5$, $x_B=-5$, $y_A=-6$, $y_B=9$, $n=5$.

AB kesmani $\lambda=\frac{1}{4}$ nisbatda bo‘luvchi C nuqtani (1.3) formulalar bo‘yicha topamiz:

$$x_C=\frac{5+\frac{1}{4}\cdot(-5)}{1+\frac{1}{4}}=3,\quad y_C=\frac{-6+\frac{1}{4}\cdot 9}{1+\frac{1}{4}}=-3;\quad C(3;-3).$$

AB kesmani $\lambda=\frac{2}{3}$ nisbatda bo‘luvchi D nuqtani topamiz:

$$x_D=\frac{5+\frac{2}{3}\cdot(-5)}{1+\frac{2}{3}}=1,\quad y_D=\frac{-6+\frac{2}{3}\cdot 9}{1+\frac{2}{3}}=0;\quad D(1;0).$$

AB kesmani $\lambda=\frac{3}{2}$ nisbatda bo‘luvchi E nuqtani topamiz:

$$x_E=\frac{5+\frac{3}{2}\cdot(-5)}{1+\frac{3}{2}}=-1,\quad y_E=\frac{-6+\frac{3}{2}\cdot 9}{1+\frac{3}{2}}=3;\quad E(-1;3).$$

AB kesmani $\lambda=\frac{4}{1}=4$ nisbatda bo‘luvchi F nuqtani topamiz:

$$x_F=\frac{5+4\cdot(-5)}{1+4}=-3,\quad y_F=\frac{-6+4\cdot 9}{1+4}=6;\quad F(-3;6).$$

Bo‘linish nuqtalari: $C(3;-3), D(1;0),\ E(-1;3)$ va $F(-3;6)$.

34. Kesmaning uchlari $M(-7;-2)$ va $N(13;3)$ nuqtalardan iborat. Bu kesmani beshta teng bo‘lakka bo‘luvchi nuqtalarning koordinatalarini toping.

VI. Kesma uchining koordinatalarini uning ikkinchi uchini koordinatalari va berilgan nuqta orqali bo‘linadigan nisbat yordamida hisoblash

35. $C(3;5)$ nuqta AB kesmani $AC:CB=3:4$ kabi nisbatda bo‘ladi. Agar kesmaning oxiri $B(-1;1)$ nuqta bo‘lsa, uning uchi A nuqtani toping.

Yechilishi. Masala shartida $\lambda=\dfrac{AC}{CB}=\dfrac{3}{4},\quad x_C=3,\quad y_C=5,$ $x_B=-1, y_B=1$ berilgan. $A(x_A;y_A)$ nuqtani topish kerak. Bu qiymatlarni (1.3) formulalarga qo‘yamiz:

$$3=\frac{x_A+\frac{3}{4}\cdot(-1)}{1+\frac{3}{4}},\qquad 5=\frac{y_A+\frac{3}{4}\cdot 1}{1+\frac{3}{4}}.$$

Tenglamalarning birinchisini x_A ga, ikkinchisini y_A ga nisbatan yechib, $x_A=6,\ y_A=8$ ni topamiz. Kesmaning uchi $A(6;8)$ nuqta bo‘ladi.

36. $C(-3;1.5)$ nuqta AB kesmani $AC:CB=3:2$ kabi nisbatda bo‘ladi. Agar kesmaning oxiri $B(7;-3.5)$ nuqta bo‘lsa, uning uchi A nuqtani toping.

37. $C(-2;1)$ nuqta AB kesmani $AC:CB=2:1$ kabi nisbatda bo‘ladi. Agar kesmaning boshi $A(-10;5)$ nuqta bo‘lsa, uning oxiri B nuqtani toping.

VII. Kesmaning davomida yotuvchi nuqtaning koordinatalarini kesma uchlarining koordinatalari va berilgan kesmani uning izlanayotgan nuqtagacha davomi bo‘lgan kesmaga nisbati bo‘yicha topish

38. AB kesma $A(-9;-3)$ va $B(1;2)$ nuqtalar bilan berilgan. $AB:BC=5:3$ bo‘lishligi uchun AB kesmani qanday C nuqtagacha davom ettirish kerak?

Yechilishi. Masala shartida quyidagi berilgan: $x_A=-9, x_B=1$, $y_A=-3$, $y_A=-3$, $\lambda=\dfrac{AB}{BC}=\dfrac{5}{3}$. $C(x_C;y_C)$ nuqtani topish kerak.

AC kesmani berilgan nisbatda bo‘luvchi $B(1;2)$ nuqta uchun (1.3) tenglamalar ushbu ko‘rinishda yoziladi:

$$1=\frac{-9+\frac{5}{3}x_C}{1+\frac{5}{3}},\qquad 2=\frac{-3+\frac{5}{3}y_C}{1+\frac{5}{3}}.$$

Bu tenglamalarni x_C va y_C ga nisbatan yechib, $x_C=7, y_C=5$; $C(7;5)$ nuqtani topamiz.

39. Kesma $A(-4;7)$ va $B(-3;5)$ nuqtalar bilan berilgan. AB kesmaning davomida shunday C nuqtani topingki, $AB:BC=1:7$ bo‘lsin.

40. Kesma $A(-5;-2)$ va $B(-1;0)$ nuqtalar bilan berilgan. $AB:BC=2:5$ bo‘lishligi uchun kesmani qanday nuqtagacha davom ettirish kerak?

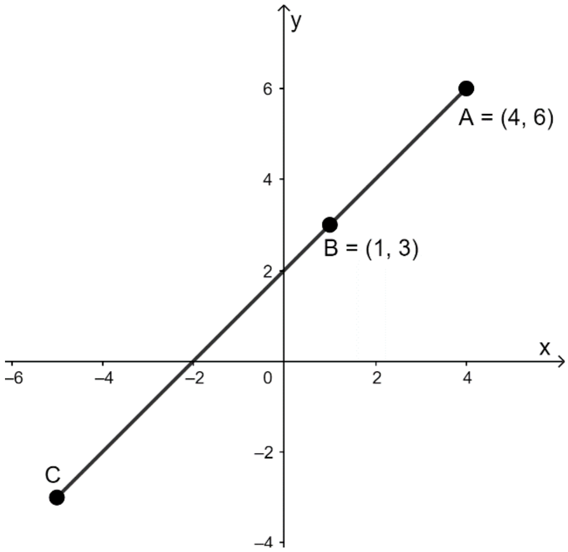
VIII. Kesmaning davomida yotuvchi nuqtaning koordinatalarini bu kesmaning uchlarini koordinatalari va berilgan kesma bilan izlanayotgan nuqtagacha davom ettirilgan kesmaning nisbati bo‘yicha topish

41. Kesma $A(4;6)$ va $B(1;3)$ nuqtalar bilan berilgan. Uzunligi AB kesmaning uzunligidan uch marta katta bo‘lgan AC kesmani hosil qilish uchun AB kesmani A dan B ga tomon qanday C nuqtagacha davom ettirish kerak?

*Yechilishi.*Masala shartida quyidagilar berilgan: $x_A = 4$, $x_B = 1$, $y_A = 6$, $y_B = 3$, $AC = 3AB$. $C(x_C; y_C)$ nuqtani topish kerak.

B nuqta AC kesmani

$$\lambda = \frac{AB}{BC} = \frac{AB}{AC - AB} = \frac{AB}{3AB - AB} = \frac{AB}{2AB} = \frac{1}{2}$$



12-rasm

nisbatda bo‘ladi (12-rasm).

(1.3) formulalar bo‘yicha $B(1;3)$ nuqta uchun quyidagini hosil qilamiz:

$$1 = \frac{4 + \frac{1}{2}x_C}{1 + \frac{1}{2}}, \qquad 3 = \frac{6 + \frac{1}{2}y_C}{1 + \frac{1}{2}}.$$

Birinchi tenglamani x_C ga, ikkinchisini y_C ga nisbatan yechamiz:

$$x_C = -5,, \quad y_C = -3; \qquad C(-5;-3).$$

42. Kesma $A(-4;7)$ va $B(0;-1)$ nuqtalar bilan berilgan. Uzunligi AB kesmaning uzunligidan bir yarim marta katta bo‘lgan AC kesmani hosil qilish uchun berilgan kesmani A dan B ga tomon qanday C nuqtagacha davom ettirish kerakligini aniqlang?

43. Kesma $M(-3;-6)$ va $N(1;-3)$ nuqtalar bilan berilgan. Uzunligi MN kesmaning uzunligidan besh marta katta bo‘lgan MP kesma hosil qilish uchun berilgan kesmani M dan N ga tomon qanday P nuqtagacha davom ettirish kerak?

IX. Kesma uchining koordinatalarini uning ikkinchi uchining koordinatalari, kesma bo‘lingan bo‘laklar soni va bo‘lish nuqtalaridan birining koordinatalari bo‘yicha hisoblash

44. AB kesma beshta teng bo‘lakka bo‘lingan. Kesmaning bir uchi $A(8;6)$ nuqtada, ikkinchi bo‘linish nuqtasi (A dan B tomon) $D(2;4)$. B nuqtani toping.

Yechilishi. Masala shartida quyidagilar berilgan: $x_A = 8$, $y_A = 6$, $x_D = 2$, $y_D = 4$. D nuqta A dan B ga qarab hisoblaganda ikkinchi bo‘linish nuqtasi. $B(x_B; y_B)$ nuqtani topish kerak, D nuqta kesmani (A dan B ga qarab)

$$\lambda = \frac{AD}{DB} = \frac{2}{5-2} = \frac{2}{3}$$

nisbatda bo‘ladi.

$D(2;4)$ nuqta uchun (1.3) formulalarni tatbiq etib topamiz:

$$2 = \frac{8 + \frac{2}{3}x_B}{1 + \frac{2}{3}}, \qquad 4 = \frac{6 + \frac{2}{3}y_B}{1 + \frac{2}{3}}.$$

Bu tenglamalarni x_B va y_B ga nisbatan yechib, topamiz: $x_B = -7$, $y_B = 1$; $B(-7;1)$.

45. MN kesma uchta teng bo‘lakka bo‘lingan. Kesmaning bir uchi $M(-3;-5)$ nuqtada, unga eng yaqin bo‘lgan bo‘lish nuqtasi $P(-2;-2)$. N nuqtani toping.

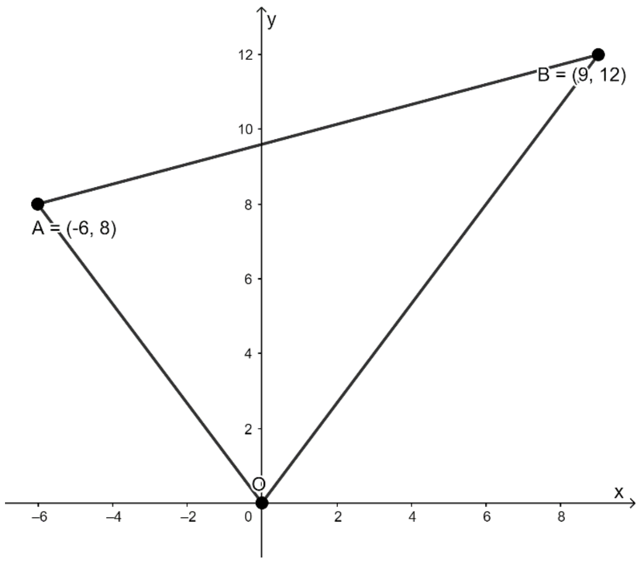
46. AB kesma yettita teng bo‘lakka bo‘lingan. Kesmaning uchlaridan biri $B(10;5)$ nuqta, A nuqtaga eng yaqin nuqta bo‘lish nuqtasi $C(-8;-1)$ dir. A nuqtani toping.

X. Berilgan ikkita nuqta orasidagi kesmani bu nuqtalardan koordinatalar boshigacha bo‘lgan masofalar qanday nisbatda bo‘lsa, shunday nisbatda bo‘luvchi nuqtaning koordinatalarini hisoblash

47. $A(-6;8)$ va $B(9;12)$ nuqtalar orasidagi kesmani bu nuqtalardan koordinatalar boshigacha bo‘lgan masofalar qanday nisbatda bo‘lsa, shunday nisbatda (A dan B ga qarab) bo‘ling.

Yechilishi. A va B nuqtalardan koordinatalar boshigacha bo‘lgan masofani topamiz (13-rasm):

$$AO=\sqrt{(-6)^2+8^2}=10,\, BO=\sqrt{9^2+12^2}=15;\ \lambda=\frac{AO}{BO}=\frac{10}{15}=\frac{2}{3}.$$



13-rasm

Izlanayotgan nuqta $M(x_M;y_M)$ bo‘lsin. (1.3) formulalar bo‘yicha topamiz:

$$x_M=0,\ y_M=9,6;\quad M(0;9,6).$$

48. $A(-16;12)$ va $B(6;-8)$ nuqtalar orasidagi kesmani bu nuqtalardan koordinatalar boshigacha bo‘lgan masofalar qanday nisbatda bo‘lsa, shunday nisbatda (A dan B ga qarab) bo‘ling.

XI. Uchburchak medianalari kesishish nuqtasining koordinatalarini uchburchak uchlarining koordinatalari bo‘yicha topish

49. Uchburchakning uchlari berilgan: $A(x_A;y_A)$, $B(x_B;y_B)$ va $C(x_C;y_C)$. Bu uchburchak medianalarining kesishish nuqtasini toping.

Yechilishi. Geometriya kursidan ma’lumki, uchburchakning medianalari bir nuqtada kesishib, har bir mediana bu nuqtada uchburchakning tegishli uchidan hisoblanganda 2:1 kabi nisbatda bo‘linadi.

BC tomonining o‘rtasi bo‘lgan D nuqtani (1.4) formulalarga ko‘ra topamiz:

$$x_D = \frac{x_B + x_C}{2}, \qquad y_D = \frac{y_B + y_C}{2}.$$

Medianalar kesishadigan M nuqtani (1.3) formulalar bo‘yicha topamiz, buning uchun AD medianani (A dan D ga qarab) $\lambda = 2:1 = 2$ kabi nisbatda bo‘lamiz:

$$x_M = \frac{x_A + \lambda x_D}{1+\lambda} = \frac{x_A + 2\cdot\frac{x_B + x_C}{2}}{1+2} = \frac{x_A + x_B + x_C}{3},$$

$$x_M = \frac{y_A + \lambda y_D}{1+\lambda} = \frac{y_A + 2\cdot\frac{y_B + y_C}{2}}{1+2} = \frac{y_A + y_B + y_C}{3}.$$

Uchburchak medianalari kesishish nuqtasining koordinatalari uchburchak uchlarining bir xil ismli koordinatalarining o‘rta arifmetigiga teng:

$$M\left(\frac{x_A + x_B + x_C}{3}, \frac{y_A + y_B + y_C}{3}\right).$$

50. Agar uchburchakning uchlari 1) $A(7;-4)$, $B(-1;8)$ va $C(-12;-1)$; 2) $A(-4;2)$, $B(2;6)$ va $C(0;-2)$ nuqtalar bo‘lsa, uchburchak medianalarining kesishish nuqtasini toring.

XII. Bir tekislikda yotgan moddiy nuqtalar sistemasi og‘irlik markazining koordinatalarini hisoblash

51. Uchlari $A(x_A;y_A), B(x_B;y_B)$ va $C(x_C;y_C)$ bo‘lgan bir jinsli uchburchak plastinkaning og‘irlik markazini toping (plastinkaning qalinligini hisobga olmang).

Yechilishi. Uchburchakning og‘irlik markazi uning medianalari kesishgan nuqtadadir. Binobarin, og‘irlik markazining koordinatalari quyidagicha bo‘ladi:

$$x_M = \frac{x_A + x_B + x_C}{3}, \qquad y_M = \frac{y_A + y_B + y_C}{3}.$$

Bir jinsli uchburchak plastinkaning og‘irlik markazi uchburchak uchlarining bir xil ismli koordinatalarining o‘rta arifmetigiga teng.

52. Uchlari $A(5;4)$, $B(-3;1)$ va $C(4;-2)$ nuqtalarda bo‘lgan uchburchak shaklidagi bir jinsli plastinkaning og‘irlik| markazini toping (plastinkaning qalinligini hisobga olmang).

53. Massalari m_A va m_B bo‘lgan $A(x_A; y_A)$ va $B(x_B; y_B)$ moddiy nuqtalardan iborat sistemaning massalar markazini toping.

Yechilishi. Massalarning izlanayotgan markazi AB kesmada, uni massalarga teskari proporsional bo‘laklarga bo‘luvchi N nuqtada yotadi, yani

$$\frac{AN}{NB} = \frac{m_B}{m_A}, \qquad \lambda = \frac{AN}{NB} = \frac{m_B}{m_A}.$$

(1.3) formulalar bo‘yicha N nuqtani topamiz:

$$x_N = \frac{x_A + \dfrac{m_B}{m_A} \cdot x_B}{1 + \dfrac{m_B}{m_A}} = \frac{m_A x_A + m_B x_B}{m_A + m_B},$$

$$y_N = \frac{y_A + \dfrac{m_B}{m_A} \cdot y_B}{1 + \dfrac{m_B}{m_A}} = \frac{m_A y_A + m_B y_B}{m_A + m_B}.$$

Agar og‘irlik kuchlari P_A va P_B hamda ularning qo‘yilish nuqtalari $A(x_A; y_A)$ va $B(x_B; y_B)$ berilgan bo‘lsa, u holda og‘irlik markazi (N nuqta) ham (1.3) formulalar bo‘yicha topiladi:

$$\lambda = \frac{AN}{NB} = \frac{P_B}{P_A}, \quad x_N = \frac{x_A + \frac{P_B}{P_A} \cdot x_B}{1 + \frac{P_B}{P_A}} = \frac{P_A x_A + P_B x_B}{P_A + P_B},$$

$$y_N = \frac{y_A + \frac{P_B}{P_A} \cdot y_B}{1 + \frac{P_B}{P_A}} = \frac{P_A y_A + P_B y_B}{P_A + P_B}.$$

54. $A(-4;0)$ nuqtada 9 kg massa, $B(4;12)$ nuqtada 3 kg massa to‘plangan. Bu sistemaning massalar markazini toping.

55. $A(x_A; y_A)$, $B(x_B; y_B)$ va $C(x_C; y_C)$ nuqtalarda m_A, m_B va m_C massalar to‘plangan. Bu sistemaning massalar markazini toping.

Yechilishi. Bu sistemaning massalar markazi $N(x_N; y_N)$ nuqta bo‘lsin. Massalari m_A va m_B bo‘lgan $A(x_A; y_A)$ va $B(x_B; y_B)$ nuqtalar uchun massalar markazi ushbu

$$M\left(\frac{m_A x_A + m_B x_B}{m_A + m_B}; \frac{m_A y_A + m_B y_B}{m_A + m_B}\right)$$

nuqta bo‘ladi (53-masalaga qarang).

Massalari $m_A + m_B$ va m_C bo‘lgan M va C nuqtalar sistemasi uchun ham massalar markazi N nuqtani xuddi shunday topamiz:

$$\lambda = \frac{MN}{NC} = \frac{m_C}{m_A + m_B}.$$

(1.3) formulalar bo‘yicha quyidagilarni hosil qilamiz:

$$x_N = \frac{x_M + \lambda x_C}{1+\lambda} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C},$$

$$y_N = \frac{y_M + \lambda x_C}{1+\lambda} = \frac{m_A y_A + m_B y_B + m_C y_C}{m_A + m_B + m_C}.$$

56. $A(-1;3)$, $B(4;3)$ va $C(6;-5)$ nuqtalarda mos ravishda 2, 3 va 5 kg massalar to‘plangan. Bu sistemaning massalar markazini toping.

3-§. Aralash masalalar

57. $A(-2;3)$ nuqtaga: 1) koordinatalar boshiga nisbatan; Ox o‘qqa nisbatan; 3) Oy o‘qqa nisbatan; 4) birinchi va uchinchi koordinata choraklarining bissektrisalariga nisbatan simmetrik bo‘lgan nuqtalarni toping.

58. Uchlari $A(-3;1)$ va $B(-1;7)$ bo‘lgan kesma berilgan. Berilgan kesmaga: 1) koordinatalar boshiga nisbatan; 2) Ox o‘qqa nisbatan; 3) Oy o‘qqa nisbatan; 4) ikkinchi va to‘rtinchi koordinata choraklarining bissektrisalariga nisbatan simmetrik bo‘lgan kesmaning uchlarini toping.

59. Uchlari $A(3;2)$, $B(7;4)$ va $C(1;6)$ bo‘lgan uchburchak berilgan. Berilgan uchburchakka: 1) koordinatalar boshiga nisbatan; 2) Ox o‘qqa nisbatan; 3) Oy o‘qqa nisbatan; 4) birinchi va uchinchi koordinata choraklarining bissektrisalariga nisbatan simmetrik bo‘lgan uchburchakning uchlarini toping.

60. Uchlari $A(-2;4)$ va $B(6;12)$ bo‘lgan kesma C nuqtada teng ikkiga bo‘linadi. C nuqtani $D(7;-4)$ nuqta bilan tutashtiruvchi kesmaning uzunligini toping.

61. Uchlari $A(-7;-1)$ va $B(9;7)$ bo‘lgan kesma C nuqtada (A dan B ga qarab) 5:3 kabi nisbatda bo‘linadi. C nuqta bilan unga koordinatalar boshiga nisbatan simmetrik bo‘lgan nuqtani tutashtiruvchi kesmaning uzunligini toping.

62. AB kesma yettita teng bo‘lakka bo‘lingan. Kesmaning bir uchi $B(13;4)$ nuqta. A dan B ga qarab hisoblaganda to‘rtinchi bo‘linish nuqtasi $F(4;1)$. A nuqtani toping.

63. Birinchi va uchinchi koordinata burchaklarining bissektrisalarida $A(7;2)$ va $B(2;-13)$ nuqtalardan teng uzoqlashgan nuqtani toping.

64. $A(3;2)$, $B(-2;1)$ va $C(1;-4)$ nuqtalar parallelogrammning uchlari, bunda A va C uning qarama-qarshi uchlari. Parallelogrammning to‘rtinchi uchi D nuqtani toping.

65. Parallelogrammning qarama-qarshi uchlari $A(-4;2)$ va $C(2;-3)$ hamda $B(0;1)$ va D nuqtalardan iborat. D nuqtani toping.

66. Parallelogrammning qo‘shni uchlari $A(-3;1)$ va $B(1;3)$ nuqtalardan iborat. Parallelogrammning diagonallari $M(1;-2)$ nuqtada kesishadi. Uning qolgan ikkita uchlarini toping.

67. Parallelogrammning qo‘shni uchlari $A(-3;0)$ va $B(0;4)$ nuqtalarda joylashgan. Parallelogrammning diagonallari $M(-1;0)$ nuqtada kesishadi. Uning qolgan ikkita uchlarini toping.

Nazorat uchun savollar

I variant

68. 1. Oy o‘qda $A(6;-1)$ va $B(-2;3)$ nuqtalardan teng uzoqlashgan M nuqtani toping.

2. Uchlari $A(-5;-1)$ va $B(5;4)$ bo‘lgan kesma (A dan B ga qarab) 2:1:2 kabi nisbatda bo‘lingan. Bo‘linish nuqtalarini toping.

3. C nuqta AB kesmani (A dan B ga qarab) 1:4 kabi nisbatda bo‘ladi. Agar kesmaning ikkinchi uchi $B(-6;-1)$ nuqta bo‘lsa, uning A uchini toping.

4. AB kesma $A(-3;-2)$ va $B(5;2)$ uchlari bilan berilgan.
$AB:BC=4:3$ bo‘lishi uchun AB kesmani qanday C nuqtagacha

davom ettirish kerak?

5. Koordinata uchlaridan va berilgan $A(4;-2)$ nuqtadan teng uzoqlashgan nuqtani toping.

II variant

69. Shunday M nuqtani topingki, undan abssissalar o‘qigacha va $A(-2;4)$ nuqtagacha bo‘lgan masofa 10 ga teng bo‘lsin.

2. Uchlari $A(7;-4)$ va $B(-8;1)$ bo‘lgan kesmani C nuqta (A dan B ga qarab) 1:4 kabi nisbatda bo‘ladi. C nuqtani toping.

3. Uchlari $A(2;1)$ va $B(11;4)$ bo‘lgan kesmani uchta teng bo‘lakka bo‘luvchi nuqtalarni toping.

4. $C(-2;1)$ nuqta AB kesmani $AC:CB=2:3$ kabi nisbatda bo‘ladi. Kesmaning bir uchi $A(-8;-1)$ nuqta bo‘lsa, uning ikkinchi uchini toping.

5. Kesma $A(-10;4)$ va $B(5;-1)$ nuqtalar bilan berilgan. $AB:BC=5:1$ bo‘lishi uchun AB ni qanday C nuqtagacha davom ettirish kerak?

2-BOB

TO’G’RI CHIZIQ

4-§. Koordinata o‘qlariga parallel bo‘lgan to‘g‘ri chiziqlarning tenglamalari. Koordinata o‘qlarining tenglamalari

Oy o‘qiga parallel bo‘lgan to‘g‘ri chiziqning tenglamasi ushbu ko‘rinishga ega:

$$x = a \qquad (2.1)$$

Bu to‘g‘ri chiziqning barcha nuqtalari Oy o‘qdan a ga teng bo‘lgan bir xil masofaga uzoqlashgan.

Ox o‘qiga parallel bo‘lgan to‘g‘ri chiziqning tenglamasi ushbu ko‘rinishga ega:

$$y = b. \qquad (2.2)$$

Bu to‘g‘ri chiziqning barcha nuqtalari Ox o‘qidan b ga teng bo‘lgan bir xil masofaga uzoqlashgan.

Agar $a > 0$ bo‘lsa, to‘g‘ri chiziq Oy o‘qdan o‘ngda joylashgan, agar $a < 0$ bo‘lsa, to‘g‘ri chiziq Oy o‘qdan chapda joylashgan bo‘ladi. Agar $a = 0$ bo‘lsa, to‘g‘ri chiziq Oy o‘q bilan ustma-ust tushadi. Bu holda Oy o‘qining tenglamasiga ega bo‘lamiz:

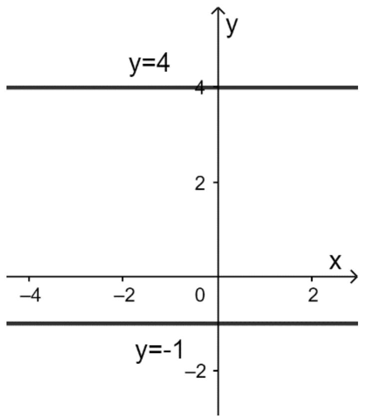
$$x = 0. \qquad (2.3)$$

Agar $b > 0$ bo‘lsa, to‘g‘ri chiziq Ox o‘qidan yuqorida, agar $b < 0$ bo‘lsa, to‘g‘ri chiziq Ox o‘qidan pastda joylashgan bo‘ladi. Agar $b = 0$ bo‘lsa, to‘g‘ri chiziq, Ox o‘q bilan ustma-ust tushadi. Bu holda Ox o‘qining tenglamasiga ega bo‘lamiz:

$$y = 0. \qquad (2.4)$$

I. Abssissalar (ordinatalar) o‘qiga parallel bo‘lgan to‘g‘ri chiziqni yasash

70. Ushbu to‘g‘ri chiziqlarni yasang: *1)* $x=3$*;* *2)* $x=-2$*;* *3)* $x=0$.



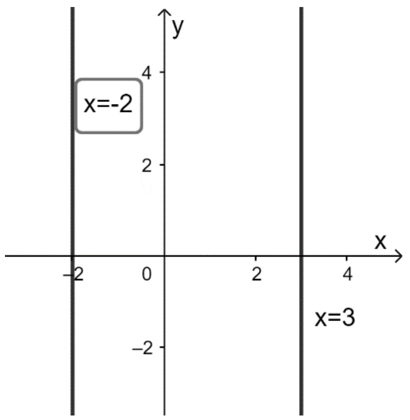
14-rasm

Yechilishi. *Ox* o‘qda *1)* $x=3$*;* *2)* $x=-2$ nuqtalarni yasaymiz. Bu nuqtalardan *Oy* o‘qda parallel to‘g‘ri chiziqlar o‘tkazamiz (14- rasm). $x=0$ to‘g‘ri chiziq *Oy* o‘qdan iborat.

71. Ushbu to‘g‘ri chiziqlarni yasang: *1)* $x=4$*;* *2)* $x=-3$.

72. Ushbu to‘g‘ri chiziqlarni yasang: *1)* $y=4$*;* *2)* $y=-1$*;* *3)* $y=0$.

Yechilishi. *Oy* o‘qda *1)* $y=4$*; 2)* $y=-1$ nuqtalarni yasaymiz. Bu nuqtalardan *Ox* o‘qda parallel to‘g‘ri chiziqlar o‘tkazamiz (15- rasm), $y=0$ to‘g‘ri chiziq *Ox* o‘qdan iborat.



15-rasm

73. Ushbu to‘g‘ri chiziqlarni yasang: *1)* $y=2$*;* *2)* $y=-4$.

II. Abssissalar (ordinatalar) o‘qiga parallel bo‘lgan va berilgan nuqtadan o‘tadigan to‘g‘ri chiziqning tenglamasini tuzish

74. *Ox* ga parallel bo‘lgan to‘g‘ri chiziq $(-2;3)$ nuqtadan o‘tadi. Bu to‘g‘ri chiziqning tenglamasini tuzing.

Yechilishi. *Ox* o‘qqa parallel bo‘lgan to‘g‘ri chiziq $y=b$ ko‘rinishga ega bo‘ladi. Izlanayotgan to‘g‘ri chiziq o‘tadigan nuqtalarning ordinatasi 3 ga teng; binobarin, to‘g‘ri chiziq tenglamasi $y=3$ yoki $y-3=0$ bo‘ladi.

75. Ox o‘qqa parallel bo‘lgan to‘g‘ri chiziq $(3;-4)$ nuqtadan o‘tadi. Bu to‘g‘ri chiziqning tenglamasini tuzing.

76. Oy o‘qqa parallel bo‘lgan to‘g‘ri chiziq $(-6;0)$ nuqtadan o‘tadi. Bu to‘g‘ri chiziqning tenglamasini tuzing.

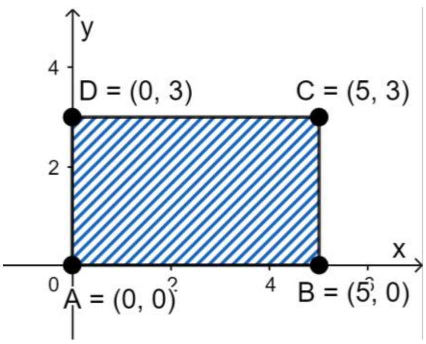
III. Koordinata o‘qlariga parallel to‘g‘ri chiziqlar bilan chegaralangan figurani yasash

77. $x=-2$, $x=0$, $y=-3$ va $y=0$ chiziqlar bilan chegaralangan figurani yasang. Bu figuraning yuzini hisoblang.

78. $x=-3$, $x=2$, $y=-1$ va $y=-5$ chiziqlar bilan chegaralan-gan figurani yasang. Bu figuraning yuzini toping.

IV. Koordinata o‘qlariga nisbatan vaziyati va o‘lchamlari bilan berilgan figurani chegaralovchi chiziqlarning tenglamalarini tuzish

79. To‘g‘ri turtburchakning tomonlari 3 va 5 ga teng. Agar to‘g‘ri turtburchak birinchi koordinata choragida joylashgan bo‘lib, uning ikkita tomoni koordinata uchlari bilan ustma-ust tushsa va shu bilan birga katta tomon Ox o‘q bilan ustma-ust tushsa, uning barcha tomonlarining tenglamalarini tuzing.



16-rasm

Yechilishi. Berilgan to‘g‘ri to‘rtburchakni yasaymiz (16-rasm). To‘g‘ri turtburchakning yasalishidan ko‘ramizki, uning uchlari ushbu koordinatalarga ega: $A(0;0)$, $B(5;0)$, $C(5;3)$ va $D(0;3)$. Endi tomonlarning tenglamalarini tuzish oson: $AB: y=0$ (Ox o‘q); $BC: x=5$ $[B(5;0)$ nuqtadan o‘tadi]; $CD: y=3$ $[D(0;3)$ nuqtadan o‘tadi]; $DA: x=0$ (Oy o‘q).

80. To‘g‘ri to‘rtburchakning tomonlari 3 va 4 ga teng. Agar to‘g‘ri to‘rtburchak uchinchi koordinata burchagida joylashgan bo‘lib, uning ikkita tomoni koordinata uchlari bilan ustma-ust tushsa va shu bilan birga tomonlarning kichigi Ox o‘q bilan ustma-ust tushsa, berilgan to‘g‘ri turtburchak tomonlarining tenglamalarini tuzing.

81. Agar kvadrat birinchi koordinata burchagida joylashgan bo‘lib, uning ikkita uchi $A(2;0)$ va $B(5;0)$ koordinatalarga ega bo‘lsa, kvadrat tomonlarining tenglamalarini tuzing.

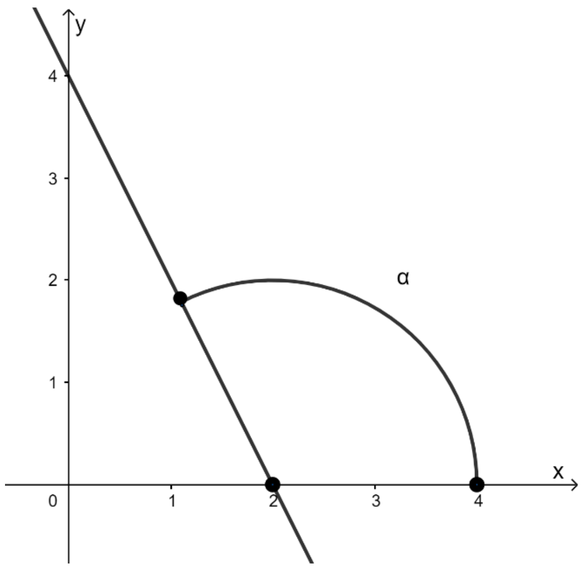
5-§. Koordinatalar boshidan o‘tuvchi to‘g‘ri chiziqning tenglamasi

Koordinatalar boshidan o‘tuvchi to‘g‘ri chiziqning tenglamasi ushbu ko‘rinishga ega:

$$y = kx, \tag{2.5}$$

bu yerda k — burchak koeffitsient; x, y — yuqoridagi tenglama bilan ifodalangan to‘g‘ri chiziqning ixtiyoriy nuqtasining koordinatalari — o‘zgaruvchi koordinatalar. k burchak koeffitsient to‘g‘ri chiziqning Ox o‘qqa og‘ish burchagining tangensiga teng:

$$k = tg\alpha. \tag{2.6}$$



17-rasm

To‘g‘ri chiziqning Ox o‘qqa og‘ish burchagi deb Ox o‘qning musbat yo’nalishini to‘g‘ri chiziqning Ox o‘q bilan kesishadigan nuqta atrofida o‘q to‘g‘ri chiziq bilan ustma-ust tushguncha soat strelkasining harakatiga qarshi yo’nalishda burish kerak bo‘ladigan burchakka aytilishini qayd qilib o‘taylik (17-rasm).

Agar to‘g‘ri chiziq Ox o‘qqa parallel bo‘lsa, u holda uning og‘ish burchagi 0°ga teng bo‘ladi. Istalgan to‘g‘ri chiziqning Ox o‘qqa og‘ish burchagi 0° va 180° $\left(0^\circ < \alpha < 180^\circ\right)$ orasidagi qiymatlarga ega bo‘ladi.

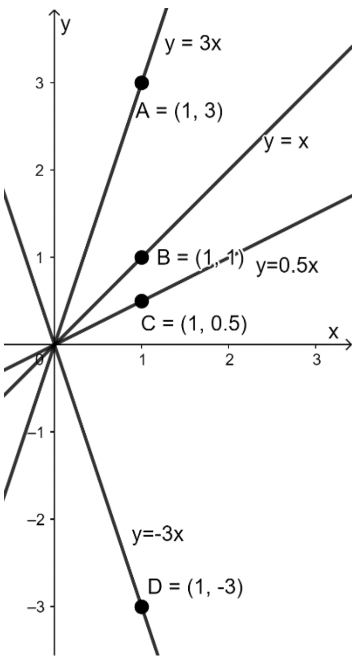
Agar $0^\circ < \alpha < 90^\circ$ (o‘tkir burchak) bo‘lsa, $k > 0$; agar $90^\circ < \alpha < 180^\circ$ (o‘tmas burchak) bo‘lsa, $k < 0$ bo‘ladi. $\alpha = 90^\circ$da k burchak koeffitsient mavjud bo‘lmaydi, chunki $tg90^\circ$ son qiymatga ega emas. Demak, Ox o‘qqa perpendikulyar bo‘lgan har qanday $x = a$ to‘g‘ri chiziq burchak koeffitsientga ega bo‘lmaydi.

Agar koordinatalar boshidan o‘tuvchi to‘g‘ri chiziqda $A(x_A; y_A)$ nuqta olsak, u holda:

$$k = tg\alpha = \frac{y_A}{x_A} \tag{2.7}$$

bo‘ladi. α burchak umumiy holda teskari funksiya orqali yoziladi:

$$\alpha = arctgk. \tag{2.8}$$



18-rasm

I va III, II va IV koordinata burchaklarining bissektrisalari mos ravishda $y = x$ va $y = -x$ tenglamalarga ega.

I. $y = kx$ to‘g‘ri chiziqni yasash

82. Ushbu to‘g‘ri chiziqlarni yasang:

1) $y = 3x$; 2) $y = x$; 3) $y = \frac{1}{2}x$; 4) $y = -3x$.

Yechilishi. Tekislikda to‘g‘ri chiziqning vaziyati ikkita nuqta bilan aniqlanadi, biroq koordinatalar boshidan o‘tuvchi to‘g‘ri chiziq uchun bitta nuqta (koordinatalar boshi) o‘zgarmas bo’ladi. Shu sababli tenglamadan bitta nuqtani topish va uni koordinatalar boshi bilan tutashtirish yetarli.

$y = 3x$ to‘g‘ri chiziqni yasaymiz. $x = 1$ deb, $y = 3 \cdot 1 = 3$ni aniqlaymiz. $A(1; 3)$ nuqtani koordinatalar boshi bilan tutashtirib, izlanayotgan to‘g‘ri chiziqni hosil qilamiz. Qolgan to‘g‘ri chiziqlarni ham shunga o‘xshash yasaymiz:

$y = x,\ B(1;1)$; $y = \frac{1}{2}x,\ C(1;\frac{1}{2})$; $y = -3x,\ D(1;-3)$ (18- rasm).

83. To‘g‘ri chiziqlarni yasang:

1) $y = 2x$; 2) $y = -x$; 3) $y = -\frac{1}{3}x$; 4) $y = -4x$.

II. Berilgan nuqtalarning $y = kx$ to‘g‘ri chiziqqa tegishliligini tekshirish

84. $A(3;6)$, $B(-1;-2)$ va $C(4;10)$ nuqtalar $y = 2x$ to‘g‘ri chiziqqa tegishli yoki tegishli emasligini tekshiring.

Yechilishi. Agar berilgan nuqtaning koordinatalari berilgan tenglamani qanoatlantirsa, ya’ni tenglamani ayniyatga aylantirsa, u holda bu nuqta berilgan to‘g‘ri chiziqqa tegishli bo‘ladi; agar berilgan nuqtaning koordinatalari tenglamani qanoatlantirmasa, nuqta to‘g‘ri chiziqqa tegishli bo‘lmaydi, ya’ni u to‘g‘ri chiziqdan tashqarida yotadi.

$y = 2x$ tenglamaga x va y o‘zgaruvchilar o‘rniga $A(3;6)$ nuqtaning koordinatalarini qo‘yib, $6 = 2\cdot 3$ ayniyatni hosil qilamiz, ya’ni $A(3;6)$ nuqtaning koordinatalari $y = 2x$ tenglamani qanoatlantiradi, binobarin, $A(3;6)$ nuqta berilgan to‘g‘ri chiziqqa tegishlidir. $B(-1;-2)$ va $C(4;10)$ nuqtalarning $y = 2x$ to‘g‘ri chiziqqa tegishli ekanligini ham xuddi shunday tekshiramiz. $B(-1;-2)$ nuqta uchun quyidagiga egamiz: $-2 = 2\cdot(-1)$, $-2 = -2$. $B(-1;-2)$ nuqta $y = 2x$ to‘g‘ri chiziqqa tegishli. $C(4;10)$ nuqta uchun: $10 \neq 2\cdot 4$. $C(4;10)$ nuqta $y = 2x$ to‘g‘ri chiziqqa tegishli emas.

85. $A(9;-3)$, $B\left(-1;\frac{1}{3}\right)$ va $C(8;4)$ nuqtalar $y = -\frac{1}{3}x$ to‘g‘ri chiziqqa tegishli yoki tegishli emasligini tekshiring.

III. $y = kx$ **ko'rinishidagi to'g'ri chiziqning tenglamasini** k **ning berilgan qiymati bo'yicha tuzish**

86. Agar koordinatalar boshidan o'tuvchi to'g'ri chiziqning burchak koeffitsienti: 1) $k = 5$; 2) $k = -3$ ga teng bo'lsa, bu to'g'ri chiziqning tenglamasini tuzing.

Yechilishi. Koordinatalar boshidan o'tuvchi to'g'ri chiziqning tenglamasini tuzish uchun bitta parametr — burchak koeffitsient $k$ni bilish kerak. Agar k berilgan bo'lsa, u holda izlanayotgan to'g'ri chiziqning tenglamasini hosil qilish uchun k ning son qiymatini $y = kx$ tenglamaga qo'yish yetarlidir. Quyidagilarga egamiz:

1) $y = 5x$ yoki $5x - y = 0$; 2) $y = 5 - 3x$ yoki $3x + y = 0$.

87. Agar koordinatalar boshidan o'tuvchi to'g'ri chiziqning burchak koeffitsienti: 1) $k = -1$; 2) $k = 4$ ga teng bo'lsa, bu to'g'ri chiziqning tenglamasini tuzing.

IV. $y = kx$ **to'g'ri chiziqning** Ox **o'qqa qiyalik burchagini berilgan** k **bo'yicha hisoblash**

88. Quyidagi to'g'ri chiziqlar Ox o'q bilan qanday burchak tashqil etishini hisoblang: 1) $y = x$; 2) $y = -x$; 3) $y = 3x$; 4) $y = -2x$.

Yechilishi. α burchakni (2.6) munosabatdan topamiz:

1) $y = x$, $k = 1$, $tg\alpha = 1$, $\alpha = 45^\circ$;

2) $y = -x$, $k = -1$, $tg\alpha = -1$, $\alpha = 135^\circ$;

3) $y = 3x$, $k = 3$, $tg\alpha = 3$, $\alpha = 71^\circ 34'$;

4) $y = -2x$, $k = -2$, $tg\alpha = -2$.

$tg\alpha_1 = 2$ munosabatdan: $\alpha_1 = 63^\circ 26'$, izlanayotgan burchak

$\alpha = 180^\circ - 63^\circ 26' = 116^\circ 34'$.

89. Quyidagi to'g'ri chiziqlar Ox o'q bilan qanday burchak tashkil etishini hisoblang:

1) $y=\frac{\sqrt{3}}{3}x$;   2) $y=-\sqrt{3}x$;   3) $y=5x$;   4) $y=-3x$.

V. Koordinatalar boshidan o‘tuvchi to‘g‘ri chiziqning tenglamasini uning Ox o‘qqa og‘ish burchagi α bo‘yicha tuzish

90. Koordinatalar boshidan o‘tuvchi va Ox o‘q bilan: 1) 0°; 2) 30°; 3) $\frac{\pi}{4}$; 4) 120°; 5) $arctg(-3)$ burchak tashkil etuvchi to‘g‘ri chiziqning tenglamasini tuzing.

Yechilishi. (2.6) munosabatdan k ni topamiz va uning qiymatini (2.5) tenglamaga keltirib qo‘yamiz:

1) $k=tg0^\circ=0,\ y=0$ — Ox o‘qning tenglamasi;

2) $k=tg30^\circ=\frac{\sqrt{3}}{3},\ y=\frac{\sqrt{3}}{3}x$ yoki $\sqrt{3}x-3y=0$;

3) $k=tg\frac{\pi}{4}=1,\ y=x$ yoki $x-y=0$;

4) $k=tg120^\circ=tg(180^\circ-60^\circ)=-tg60^\circ=-\sqrt{3}$. $y=-\sqrt{3}x$ yoki $\sqrt{3}x+y=0$;

5) $k=tg\left[arctg(-3)\right]=-3,\ y=-3x$ yoki $3x+y=0$.

91. Koordinatalar boshidan o‘tuvchi va Ox o‘q bilan: 1) 60°; 2) $\frac{\pi}{6}$; 3) 135°; 4) $arctg3$; 5) $arctg(-5)$ burchak tashqil etuvchi to‘g‘ri chiziqning tenglamasini tuzing.

VI. Koordinatalar boshidan va berilgan nuqtadan o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzish.

92. Koordinatalar boshidan va $A(-2;3)$ nuqtadan o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzing.

Yechilishi. Masala shartida quyidagilar berilgan: $x_A = -2$ va $y_A = 3$. Koordinatalar boshidan o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzish uchun k ni bilish kerak.

k ni (2.7) munosabatdan topamiz: $k = -3/2$, so‘ngra k ning topilgan qiymatini (2.5) tenglamaga qo‘yib, $y = -(3/2)x$ ni yoki $3x + 2y = 0$ ni hosil qilamiz.

k ni topish uchun (2.5) tenglamada x va y o‘zgaruvchilarning o‘rniga $A(-2;3)$ ning koordinatalarini qo‘yish ham mumkin: $3 = k(-2)$, bu yerdan $k = -\frac{3}{2}$.

93. Koordinatalar boshidan va 1) $A(3;-6)$; 2) $A(-1;5)$ nuqtadan o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzing.

VII. Koordinatalar boshidan o‘tuvchi to‘g‘ri chiziqqa tegishli nuqtaning koordinatalarini bu to‘g‘ri chiziqning burchak koeffitsienti va u nuqtadan koordinatalar boshigacha bo‘lgan masofa bo‘yicha hisoblash

94. Agar koordinatalar boshidan va A nuqtadan o‘tuvchi to‘g‘ri chiziqning burchak koeffitsienti $\frac{3}{4}$ ga teng bo‘lib, A nuqta koordinatalar boshidan 10 birlik masofada joylashgan bo‘lsa, A nuqtaning koordinatalarini toping.

Yechilishi. Masala shartida quyidagilar berilgan: $k = -\frac{3}{4}$, $d = OA = 10$. $A(x_A; y_A)$ ni topish kerak. (2.7) munosabatdan quyidagini topamiz: $\frac{y_A}{x_A} = \frac{4}{3}$. (1.2) formula bo‘yicha OA kesmaning uzunligini ifodalaymiz: $\sqrt{x_A{}^2 + y_A{}^2} = 10$. Ushbu

$$\begin{cases} \dfrac{y_A}{x_A} = \dfrac{3}{4} \\ \sqrt{x_A{}^2 + y_A{}^2} = 10 \end{cases}$$

sistemani yechib, quyidagilarni hosil qilamiz: $x_A = \pm 8$; $y_A = \pm 6$; $A_1(8;6)$; $A_2(-8;-6)$.

95. P nuqta koordinatalar boshidan 5 uzunlik birligi narida joylashgan. Koordinatalar boshi bilan P nuqtani tutashtiruvchi to‘g‘ri chiziqning burchak koeffitsienti $\dfrac{3}{4}$ ga teng. P nuqtani toping.

96. Ikkita tomoni koordinata o‘qlarining musbat yo‘nalishlari bilan ustma-ust tushadigan to‘g‘ri turtburchakning diagonali 20 uzunlik birligiga teng. Diagonalning burchak koeffitsienti $4/3$ ga teng. To‘g‘ri to‘rtburchakning uchlarini toping.

6-§. To‘g‘ri chiziqning burchak koeffitsientli va boshlang‘ich ordinatali tenglamasi

To‘g‘ri chiziqning burchak koeffitsientli va boshlang‘ich ordinatali tenglamasi quyidagi ko‘rinishga ega:

$$y = kx + b, \qquad (2.9)$$

bu yerda x va y — to‘g‘ri chiziqning istalgan nuqtasini koordinatalari — o‘zgaruvchi koordinatalar; k — burchak koeffitsient, u to‘g‘ri chiziqning Ox o‘qqa og‘ish burchagining tangensiga teng: $k = tg\alpha$, b — boshlang‘ich ordinata — to‘g‘ri chiziqning Oy o‘q bilan kesishish nuqtasining ordinatasi.

Agar $a = 0$ bo‘lsa, u holda k ham nolga teng bo‘lib, to‘g‘ri chiziq Ox o‘qqa parallel bo‘ladi $(y = a)$. Agar $\alpha = 90^\circ$ bo‘lsa, k mavjud bo‘lmay, to‘g‘ri chiziq Ox o‘qqa perpendikulyar bo‘ladi $(x = a)$.

Agar $b = 0$ bo‘lsa, u holda to‘g‘ri chiziq Oy o‘qni koordinatalar boshidan yuqoriroqda kesib o‘tadi, agar $b < 0$ bo‘lsa, u holda to‘g‘ri chiziq Oy o‘qni koordinatalar boshidan pastda kesib o‘tadi. $b=0$ bo‘lganda koordinatalar boshidan o‘tuvchi to‘g‘ri chiziq tenglamasi $y=kx$ ga ega bo‘lamiz.

I. $y=kx+b$ to‘g‘ri chiziqning koordinata uchlari bilan kesishish nuqtalarining koordinatalarini hisoblash

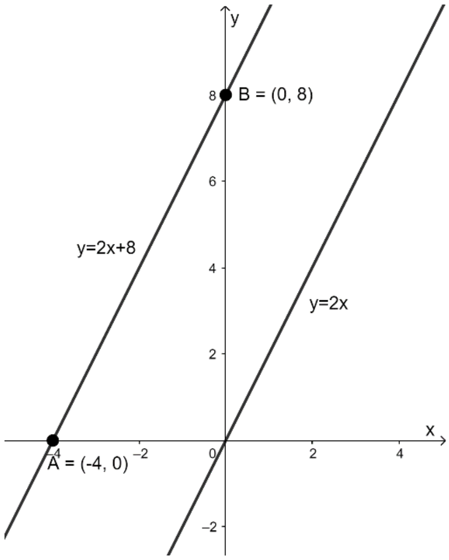
97. $y=3x-6$ to‘g‘ri chiziqning koordinata uchlari bilan kesishish nuqtasini toping.

Yechilishi. A nuqta to‘g‘ri chiziqning Ox o‘q bilan kesishish nuqtasi bo‘lsin. Berilgan tenglamada $y=0$ deb $x=2$, $A(2;0)$ ni topamiz.

B nuqta to‘g‘ri chiziqning Oy o‘q bilan kesishish nuqtasi bo‘lsin: $x=0$ bo‘lganda $y=-6$, $B(0;-6)$ bo‘ladi. Shunday to‘g‘ri chiziqning Ox va Oy o‘qlar bilan kesishish nuqtalari $A(2;0)$ va $B(0;-6)$ bo‘ladi.

98. Quyidagi to‘g‘ri chiziqlarning koordinata uchlari bilan kesishish nuqtalarini toping: 1) $y=-2x+4$; 2) $y=-x-5$.

II. $y=kx+b$ to‘g‘ri chiziqni yasash



19-rasm

99. $y=2x+8$ to‘g‘ri chiziqni yasang.

Yechilishi. 1-usul. $y=2x$ to‘g‘ri chiziqni yasaymiz. $y=2x+8$ to‘g‘ri chiziq $y=2x$ to‘g‘ri chiziqqa parallel bo‘lib, koordinatalar boshidan 8 birlik yuqoridan o‘tadi (19- rasm).

2- usul. (To‘g‘ri chiziqni uning ikkita nuqtasi bo‘yicha yasash.)

To‘g‘ri chiziqning koordinata o‘qlari bilan kesishish nuqtalarini topamiz. To‘g‘ri chiziqning Ox o‘q bilan kesishish nuqtasining ordinatasi nolga teng; berilgan tenglamada $y=0$ deb $x=-4$, $A(-4;0)$ ni topamiz. To‘g‘ri chiziqning Oy o‘q bilan kesishish nuqtasining abssissasi nolga teng; berilgan tenglamada $x=0$ deb $y=8$, $B(0;8)$ ni topamiz. A va B nuqtalarni yasaymiz va ular orqali izlanayotgan to‘g‘ri chiziqni o‘tkazamiz (19- rasm).

Ikkinchi usul amaliy jihatdan ancha qulay, shu sababli $y=kx+b$ ko‘rinishdagi to‘g‘ri chiziqlarni yasashda odatda ana shu usuldan foydalaniladi.

100. Quyidagi to‘g‘ri chiziqlarni yasang: 1) $y=2x+1$; 2) $y=3x-4$; 3) $y=-x+2$; 4) $y=-5x-10$.

III. Berilgan nuqtalarning $y=kx+b$ to‘g‘ri chiziqqa tegishliligini tekshirish

101. $M(-1;-6)$, $N(3;10)$, $P(-2;3)$ nuqtalar $y=4x-2$ to‘g‘ri chiziqqa tegishliligini tekshiring.

102. $A(0,5;-1,5)$, $B\left(0,75;-1,25\right)$ va $C(0;5)$ nuqtalar $y=x-2$ to‘g‘ri chiziqqa tegishliligini tekshiring.

IV. To‘g‘ri chiziq tenglamasini berilgan burchak koeffitsient va boshlang‘ich ordinata bo‘yicha tuzish

103. Agar to‘g‘ri chiziqning burchak koeffitsnenti $k=-3$, boshlang‘ich ordinatasi $b=2$ bo‘lsa, uning tenglamasini tuzing.

Yechilishi. To‘g‘ri chiziqning tenglamasini tuzish uchun k va b larning son qiymatlarini (2.9) tenglamaga qo‘yish kifoya: $y=-3x+2$.

104. Agar to‘g‘ri chiziqning burchak koeffitsienti $k=2/3$, boshlang‘ich ordinatasi esa $b=-1/2$ bo‘lsa, uning tenglamasini tuzing.

V. To‘g‘ri chiziqning tenglamasini uning *Ox* o‘qqa og‘ish burchagi va boshlang‘ich ordinatasi bo‘yicha tuzish

105. Boshlang‘ich ordinatasi $b=3$, *Ox* o‘qqa og‘ish burchagi *1)* $\alpha=45^\circ$*; 2)*$\alpha=120^\circ$*; 3)*$\alpha=arctg3$ bo‘lgan to‘g‘ri chiziqning tenglamasini tuzing.

Yechilishi. Masala shartida $b=3$ berilgan. k burchak koeffitsientni (2.6) formula bo‘yicha topamiz:

1) $k=tg45^\circ=1$**;** 2) $k=tg120^\circ=-tg60^\circ=-\sqrt{3}$**;**
3) $k=tg(arctg3)=3$**.**

k va b larning qiymatlarini (2.9) formulaga qo‘yib, quyidagilarni hosil qilamiz:

1) $y=x+3$*;* *2)* $y=-\sqrt{3}x+3$**;** *3)* $y=3x+3$**.**

106. Boshlang‘ich ordinatasi $b=-2$, Ox o‘qqa og‘ish burchagi esa *1)* $\alpha=30^\circ$*; 2)* $\alpha=135^\circ$*; 3)* $\alpha=arctg2$ bo‘lgan to‘g‘ri chiziqning tenglamasini tuzing.

VI. $y=kx+b$ **to‘g‘ri chiziqning** Ox **o‘qqa og‘ish burchagini hisoblash**

107. To‘g‘ri chiziqning Ox og‘ish burchagini uning tenglamasi bo‘yicha hisoblang*: 1)* $y=7x-8$*; 2)* $y=-x+1$*; 3)* $y=0,41x-2$*; 4)* $y=-2,9x+3$.

VII. Berilgan nuqtadan o‘tuvchi va berilgan boshlang‘ich ordinataga ega bo‘lgan to‘g‘ri chiziqning tenglamasini tuzish

108. $(3,4)$ nuqtadan o‘tuvchi va Oy o‘qdan $b=2$ kesma ajratuvchi to‘g‘ri chiziqning tenglamasini tuzing.

Yechilishi. (2.9) ko‘rinishdagi to‘g‘ri chiziqning tenglamasini tuzish uchun k ni topish kerak. Tenglamaga boshlang‘ich ordinataning qiymatini hamda x va y o‘zgaruvchilarning o‘rniga berilgan nuqtaning koordinatalarini qo‘yamiz*:* $4=k\cdot 3+2$, bu yerdan $k=\dfrac{2}{3}$. k va b ning qiymatlari (2. 9) tenglamaga qo‘yib, izlanayotgan tenglamani hosil qilamiz: $y=\dfrac{2}{3}x+2$ **.**

109. $(-5;-2)$ nuqtadan o‘tuvchi va $b=-12$ boshlang‘ich ordinataga ega bo‘lgan to‘g‘ri chiziqning tenglamasini tuzing.

VIII. Berilgan nuqtadan o‘tuvchi va Ox o‘q bilan berilgan og‘ish burchagini tashqil etuvchi to‘g‘ri chiziqning tenglamasini tuzish

110. $(2;6)$ nuqtadan o‘tuvchi va Ox o‘q bilan $arctg5$ burchak tashqil etuvchi to‘g‘ri chiziqning tenglamasini tuzing.

Yechilishi. To‘g‘ri chiziqning (2.9) ko‘rinishdagi tenglamasini tuzish uchun k va b ni hisoblash kerak. k burchak koeffitsientni topamiz: $k = tg(arctg5) = 5$.

b ni hisoblash uchun (2.9) tenglamaga k ning topilgan qiymatini hamda x va y o‘zgaruvchilarning o‘rniga berilgan nuqtaning koordinatalarini qo‘yamiz: $6 = 5\cdot 2 + b$, bu yerdan $b = -4$. Izlanayotgan tenglama $y = 5x - 4$.

111. $(5;-7)$ nuqtadan o‘tuvchi va Ox o‘q bilan $arctg(-2)$burchak tashqil etuvchi to‘g‘ri chiziqning tenglamasini tuzing.

112. $(-1;-4)$ nuqtadan o‘tuvchi va Oxo‘q bilan 135°li burchak tashqil etuvchi to‘g‘ri chiziqning tenglamasini tuzing.

7-§. To‘g‘ri chiziqning umumiy tenglamasi

To‘g‘ri chiziqning umumiy tenglamasi ushbu ko‘rinishga ega:

$$Ax + By + C = 0, \tag{2.10}$$

bu yerda A, B va C— o‘zgarmas koeffitsientlar; x va y — to‘g‘ri chiziqning istalgan nuqtasini koordinatalari.

To‘g‘ri chiziq umumiy tenglamasining xususiy xollari:

1) $C = 0$ bo‘lganda

$$Ax + By = 0 \tag{2.11}$$

tenglamani, ya’ni koordinatalar boshidan o‘tuvchi to‘g‘ri chiziqning tenglamasini hosil qilamiz ($y = kx$ ko‘rinishdagi tenglama, bu yerda $k = -\frac{A}{B}$);

2) $B = 0$ bo‘lganda

$$Ax + C = 0 \tag{2.12}$$

tenglamani, ya’ni Oy o‘qqa parallel bo‘lgan to‘g‘ri chiziqning tenglamasini hosil qilamiz ($x = a$ ko‘rinishdagi tenglama, bu yerda $a = -\frac{C}{A}$);

3) $A = 0$ bo‘lganda

$$By + C = 0 \tag{2.13}$$

tenglamani, yani Ox o‘qqa parallel bo‘lgan to‘g‘ri chiziqning tenglamasini hosil qilamiz ($y = b$ ko‘rinishdagi tenglama, bu yerda $b = -\frac{C}{B}$);

4) $B = 0$va $C = 0$ bo‘lganda

$$Ax = 0 \quad \text{yoki} \quad x = 0 \tag{2.14}$$

tenglamani, ya’ni Oy o‘qning tenglamasini hosil qilamiz;

5) $A = 0$ va $C = 0$ bo‘lganda

$$By = 0 \text{ yoki } \quad y = 0 \tag{2.15}$$

tenglamani, ya’ni Ox o‘qning tenglamasini hosil qilamiz.

I. $Ax + By + C = 0$ **to‘g‘ri chiziqning koordinata o‘qlari bilan kesishish nuqtalarining koordinatalarini hisoblash**

113. $4x - 3y - 12 = 0$ to‘g‘ri chiziqning koordinata o‘qlari bilan kesishish nuqtalarining koordinatalarini toping.

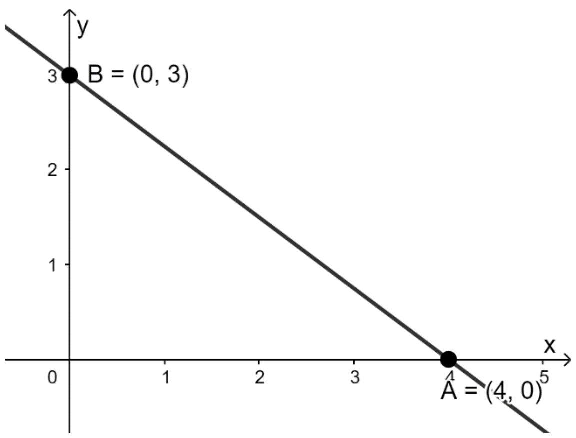
Yechilishi. A nuqta to‘g‘ri chiziqning Ox o‘q bilan kesishish nuqtasi bo‘lsin. Berilgan tenglamada $y=0$ deb $4x-12=0$ ni hosil qilamiz, bu yerdan $x=3$, $A(3;0)$.

B nuqta to‘g‘ri chiziqning Oy o‘q bilan kesishish nuqtasi bo‘lsin. $x=0$ desak, $-3y-12=0$ hosil bo‘ladi, bu yerdan $y=-4$, $B(0;-4)$. To‘g‘ri chiziqning Ox va Oy o‘qlar bilan kesishish nuqtalari: $A(3;0)$ va $B(0;-4)$.

114. Quyidagi to‘g‘ri chiziqlar-ning koordinata o‘qlari bilan kesishish nuqtalarini toping: 1) $5x+4y+20=0$; 2) $7x-2y+14=0$.

II. $Ax+By+C=0$ tenglama bilan berilgan to‘g‘ri chiziqni yasash

115. $3x+4y-12=0$ to‘g‘ri chiziqni yasang.



20-rasm

Yechilishi. To‘g‘ri chiziqni yasash uchun uning Ox va Oy o‘qlar bilan kesishish nuqtalarining koordinatalarini topamiz, $y=0$ deb quyidagini hosil qilamiz: $3x-12=0$, $x=4$, $A(4;0)$. $x=0$ bo‘lganda $4y-12=0$, $y=3$, $B(0;3)$. A va B nuqtalarni yasaymiz va ular orqali izlanayotgan to‘g‘ri chiziqni o‘tkazamiz (20- rasm).

116. Quyidagi to‘g‘ri chiziqlarni yasang: 1) $2x-5y+10=0$; 2) $4x+6y-3=0$.

III. Berilgan nuqtalarning $Ax+By+C=0$ to‘g‘ri chiziqqa tegishliligini tekshirish

117. Ushbu $A(3;14)$, $B(4;13)$, $C(-3;0)$, $D(0;7)$ nuqtalarining $7x-3y+21=0$ to‘g‘ri chiziqqa tegishliligini tekshiring.

IV. $Ax+By+C=0$ to‘g‘ri chiziqni $y=kx+b$ ko‘rinishga keltirish

118. $3x-5y+10=0$ to‘g‘ri chiziqni $y=kx+b$ ko‘rinishga keltiring.

Yechilishi. Berilgan tenglamani y ga nisbatan yechamiz:

$$5y=3x+10, \qquad y=\frac{3}{5}x+2.$$

119. Quyidagi to‘g‘ri chiziqlarni $y=kx+b$ ko‘rinishga keltiring: *1)* $3x+5y+1=0$*; 2)* $5x-2y+6=0$.

V. $Ax+By+C=0$ to‘g‘ri chiziqning Ox o‘qqa og‘ish burchagini hisoblash

120. $3x+2y+6=0$ to‘g‘ri chiziqning Ox o‘qqa og‘ish burchagini hisoblang.

Yechilishi. $3x+2y+6=0$ tenglamani y ga nisbatan yechib, $y=-\dfrac{3}{2}x-3$ ni hosil qilamiz, bu yerdan $k=-\dfrac{3}{2}=-1,5;$ biroq, $k=tg\alpha;$ demak, $tg\alpha=-1,5$. Jadvaldan $\alpha=180^\circ-56^\circ19'=123^\circ41'$ ekanligini topamiz.

121. Ushbu to‘g‘ri chiziqning Ox o‘qqa og‘ish burchagini toping: *1)* $3x+5y+20=0$*; 2)* $29x-10y+10=0$.

VI. $Ax+By+C=0$ to‘g‘ri chiziqning koordinata o‘qlari orasiga joylashgan kesmasining o‘zunligini hisoblash

122. $3x+4y-24=0$ to‘g‘ri chiziqning koordinata o‘qlari orasiga joylashgan kesmasining uzunligini hisoblang.

Yechilishi. To‘g‘ri chiziqning koordinata o‘qlari bilan kesishish nuqtalarining koordinatalarini topamiz: $y=0$, $x=8$, $A(8;0)$ va $x=0$, $y=6$, $B(0;6)$. (1.1) formula bo‘yicha AB kesmaning uzunligini hisoblaymiz:

$$AB=\sqrt{64+36}=10.$$

123. $4x+3y-36=0$ to‘g‘ri chiziqning koordinata o‘qlari orasiga joylashgan kesmasining uzunligini hisoblang.

VII. $Ax+By+C=0$ **to‘g‘ri chiziqda yotuvchi va bu to‘g‘ri chiziqda yotmaydigan ikkita nuqtadan teng uzoqlashgan nuqtaning koordinatalarini hisoblash**

124. $2x+y-6=0$ to‘g‘ri chiziqda $A(3;5)$ va $B(2;6)$ nuqtalardan teng uzoqlashgan M nuqtani toping.

Yechilishi. M nuqtaning koordinatalarini $(x_M; y_M)$ bilan belgilaymiz. (1.1) formula bo‘yicha

$$MA=\sqrt{(x_M-3)^2+(y_M-5)^2}, \quad MB=\sqrt{(x_M-2)^2+(y_M-6)^2},$$

biroq $MA=MB$, u holda:

$$\sqrt{(x_M-3)^2+(y_M-5)^2}=\sqrt{(x_M-2)^2+(y_M-6)^2}.$$

Kvadratga ko‘targandan va soddalashtirgandan so‘ng quyidagini hosil qilamiz:

$$x_M-y_M+3=0.$$

$M(x_M; y_M)$ nuqta $2x+y-6=0$ to‘g‘ri chiziqqa tegishli, demak, uning koordinatalari bu tenglamani qanoatlantiradi. Quyidagi ikkinchi tenglamaga ham egamiz:

$$2x_M+y_M-6=0$$

Ushbu

$$\begin{cases} x_M-y_M+3=0 \\ 2x_M+y_M+6=0 \end{cases}$$

tenglamalar sistemasini yechib, quyidagini hosil qilamiz:

$$x_M=1, \quad y_M=4; \qquad M(1;4).$$

125. $x-2y-4=0$ to‘g‘ri chiziqda $A(5;-1)$ va $B(2;-4)$ nuqtalardan teng uzoqlashgan M nuqtani toping.

126. $3x+4y+20=0$ to‘g‘ri chiziqda $A(-8;-3)$ va $B(-5;-6)$ nuqtalardan teng uzoqlashgan nuqtani toping.

8-§. To‘g‘ri chiziqning “kesmalardagi” tenglamasi

To‘g‘ri chiziqning o‘qlardagi kesmalar bo‘yicha tenglamasi ushbu ko‘rinishga ega:

$$\frac{x}{a}+\frac{y}{b}=1 \qquad (2.16)$$

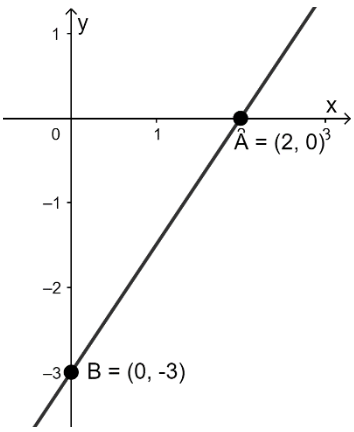
bu yerda a — to‘g‘ri chiziqning Ox o‘q bilan kesishish nuqtasining abssissasi; b — to‘g‘ri chiziqning Oy o‘q bilan kesishish nuqtasining ordinatasi; x va y — to‘g‘ri chiziqning ixtiyoriy nuqtasining koordinatalari.

I. $\frac{x}{a}+\frac{y}{b}=1$ **to‘g‘ri chiziqni yasash**

127. $\frac{x}{2}-\frac{y}{3}=1$ to‘g‘ri chiziqni yasang.

Yechilishi. Berilgan tenglamani quyidagicha qayta yozib olamiz:

$$\frac{x}{2}+\frac{y}{-3}=2.$$



21-rasm

Bundan $a=2$ va $b=-3$ ga egamiz. $A(2;0)$ va $B(0;-3)$ nuqtalarni yasaymiz. A va B nuqtalar orqali o‘tkazilgan to‘g‘ri chiziq izlanayotgan to‘g‘ri chiziq bo‘ladi (21-rasm).

128. Quyidagi to‘g‘ri chiziqlarni yasang:

$$1)\ \frac{x}{2}+\frac{y}{6}=1;\quad 2)\ \frac{x}{5}-\frac{y}{4}=1;\quad 3)\ -\frac{x}{3}+\frac{y}{2}=1;\quad 4)\ -\frac{x}{6}-\frac{y}{3}=1.$$

II. Berilgan nuqtalarning $\frac{x}{a}+\frac{y}{b}=1$ to‘g‘ri chiziqqa tegishliligini tekshirish

129. Quyidagi nuqtalarning $\frac{x}{4}-\frac{y}{2}=1$ to‘g‘ri chiziqqa tegishliligini tekshiring: 1) (4; 0); 2) (8; 2); 3) (5; 3) va 4) (6; 1).

III. $Ax+By+C=0$ to‘g‘ri chiziqni $\frac{x}{a}+\frac{y}{b}=1$ ko‘rinishga keltirish

130. $3x-4y+2=0$ to‘g‘ri chiziqni $\frac{x}{a}+\frac{y}{b}=1$ ko‘rinishga keltiring.

Yechilishi. Quyidagi almashtirishlarni bajaramiz:

$$3x-4y=-2; \quad \frac{3x}{-2}-\frac{4y}{-2}=1; \quad \frac{x}{-2/3}+\frac{y}{1/2}=1.$$

131. Quyidagi to‘g‘ri chiziqlarni $\frac{x}{a}+\frac{y}{b}=1$ ko‘rinishga keltiring:

1) $x+y-3=0$*;* *2)* $2x+3y+1=0$*;* *3)* $2x+3y-6=0$*;* *4)* $3x-4y+12=0$.

IV . $\frac{x}{a}+\frac{y}{b}=1$ to‘g‘ri chiziqni $Ax+By+C=0$ ko‘rinishga keltirish

132. $\frac{x}{4}+\frac{y}{5}=1$to‘g‘ri chiziqni $Ax+By+C=0$ ko‘rinishga keltiring.

Yechilishi. Aytilgan ko‘rinishga keltirish berilgan tenglamani butun ko‘rinishga keltirishdan iborat: $\frac{x}{4}+\frac{y}{5}=1,$ $5x+4y=20.$ $5x+4y-20=0.$

133. Quyidagi to‘g‘ri chiziqlarni $Ax+By+C=0$ ko‘rinishga keltiring:

$$1)\ \frac{x}{2}+\frac{y}{5}=1;\quad 2)\ \frac{x}{3}-\frac{y}{4}=1;\quad 3)\ \frac{x}{\frac{1}{2}}+\frac{y}{\frac{1}{3}}=1.$$

V. $y=kx+b$ **to‘g‘ri chiziqni** $\frac{x}{a}+\frac{y}{b}=1$ **ko‘rinishga keltirish**

134. $y=2x-5$ to‘g‘ri chiziqni $\frac{x}{a}+\frac{y}{b}=1$ ko‘rinishga keltiring.

Yechilishi. Almashtirishni quyidagicha bajarish mumkin:

$$2x-y=5,\quad \frac{2x}{5}-\frac{y}{5}=1,\quad \frac{x}{\frac{5}{2}}+\frac{y}{-5}=1.$$

135. Quyidagi to‘g‘ri chiziqlarni $\frac{x}{a}+\frac{y}{b}=1$ ko‘rinishga keltiring:

1) $y=x+1$*;* *2)* $y=4x-2$*;* *3)* $y=-x+3$.

VI. $\frac{x}{a}+\frac{y}{b}=1$ **to‘g‘ri chiziqni** $y=kx+b$ **ko‘rinishga keltirish**

136. $\frac{x}{3}+\frac{y}{2}=1$ to‘g‘ri chiziqni $y=kx+b$ ko‘rinishga keltiring.

Yechilishi. Masalani echish uchun $\frac{x}{3}+\frac{y}{2}=1$ tenglamani y ga nisbatan yechamiz*:* $2x+3y=6$; $3y=-2x+6$*;* demak, $y=-\frac{2}{3}x+2$.

137. Quyidagi to‘g‘ri chiziqlarni $y=kx+b$ ko‘rinishga keltiring:

$$1)\ \frac{x}{2}-\frac{y}{5}=1;\quad 2)\ \frac{x}{-4}+\frac{y}{2}=1;\quad 3)\ \frac{x}{2}+\frac{y}{3}=1.$$

VII. To‘g‘ri chiziqning $\frac{x}{a}+\frac{y}{b}=1$ ko‘rinishdagi tenglamasini uning koordinata o‘qlaridan ajratgan kesmalari bo‘yicha tuzish

138. To‘g‘ri chiziq Ox o‘qda 3 ga, Oyo‘qda 5 ga teng kesma ajratadi. Bu to‘g‘ri chiziqning tenglamasini tuzing.

Yechilishi. Masala shartida $a=3$va $b=5$ berilgan (2.16) formula bo‘yicha $\frac{x}{3}+\frac{y}{5}=1$ ga ega bo‘lamiz.

139. Agar to‘g‘ri chiziq koordinata o‘qlarini 1) $A(-2;0)$ va $B(0;3)$*; 2)* $A(3;0)$ va $B(0;-4)$ nuqtalarda kesib o‘tsa, to‘g‘ri chiziqning o‘qlardagi kesmalar bo‘yicha tenglamasini tuzing.

VIII. $\frac{x}{a}+\frac{y}{b}=1$ to‘g‘ri chiziqning bu to‘g‘ri chiziq koordinata o‘qlari bilan kesishgan nuqtalari orasidagi kesmasining uzunligini hisoblash

140. $\frac{x}{9}-\frac{y}{12}=1$ to‘g‘ri chiziqning koordinata uchlari bilan kesishish nuqtalari orasida joylashgan kesmasining uzunligini hisoblang.

Yechilishi. Masala shartidan $a=9$ va $b=-12$ ekanligi ma’lum, demak, to‘g‘ri chiziq koordinata uchlarini $A(9;0)$ va $B(0;-12)$ nuqtalarda kesib o‘tadi. Bu nuqtalar orasidagi masofani (1.1) formula bo‘yicha topamiz:

$$AB=\sqrt{(9-0)^2+(0+12)^2}=15.$$

141. Quyidagi to‘g‘ri chiziqlarning koordinata uchlari bilan kesishish nuqtalari orasida joylashgan kesmalarining o‘zunligini hisoblang.

$$1)\ \frac{x}{6}+\frac{y}{8}=1;\quad 2)\ \frac{x}{12}-\frac{y}{16}=1.$$

IX. $\frac{x}{a}+\frac{y}{b}=1$ **to‘g‘ri chiziqning** Ox **o‘qqa og‘ish burchagini hisoblash**

142. $\frac{x}{4}-\frac{y}{2}=1$ to‘g‘ri chiziqning Ox o‘q bilan hosil qilgan burchagini hisoblang.

Yechilishi. Berilgan tenglamani $y=kx+b$ ko‘rinishga keltiramiz:

$$\frac{x}{4}-\frac{y}{2}=1;\ x-2y=4;\ 2y=x-4;\ y=\frac{1}{2}x-2;\ k=\frac{1}{2}y;$$

$$\mathrm{tg}\alpha=\frac{1}{2};\ \alpha=26^{\circ}34'.$$

143. Quyidagi to‘g‘ri chiziqlarning Ox o‘q bilan tashqil etgan burchagini hisoblang:

$$1)\frac{x}{5}+\frac{y}{2}=1;\qquad 2)\frac{x}{2}-\frac{y}{8}=1.$$

9-§. To‘g‘ri chiziqlar dastasining tenglamasi. Berilgan nuqtadan berilgan yo‘nalishda o‘tuvchi to‘g‘ri chiziqning tenglamasi

To‘g‘ri chiziqlar dastasining tenglamasi quyidagi ko‘rinishnga ega:

$$y-y_A=k(x-x_A), \tag{2.17}$$

bu yerda k — burchak koeffitsient; $(y_A;x_A)$ to‘g‘ri chiziqlar o‘tadigan nuqta (dastaning markazi), x va y — o‘zgaruvchi koordinatalar. Agar k tayin son qiymatga ega bo‘lsa, u holda berilgan nuqtadan berilgan yo‘nalishda o‘tuvchi to‘g‘ri chiziqning tenglamasini hosil qilamiz.

I.Berilgan nuqtadan o‘tuvchi to‘g‘ri chiziqlar dastasining tenglamasini tuzish

144. $A(3;-1)$ nuqta orqali o‘tuvchi to‘g‘ri chiziqlar dastasining tenglamasini tuzing.

Yechilishi. (2.17) tenglamaga $A(3;-1)$ nuqtaning koordinatalarini qo‘yib, izlanayotgan tenglamani hosil qilamiz: $y+1=k(x-3)$ yoki $y-kx+3k+1=0$.

145. $A(-4;-2)$ nuqtadan o‘tuvchi to‘g‘ri chiziqlar dastasining tenglamasini tuzing.

II. To‘g‘ri chiziqlar dastasining tenglamasi bo‘yicha shu dastaning markazini topish

146. Dasta tenglamasi berilgan: $y-3=k(x+2)$. Bu to‘g‘ri chiziqlar dastasining markazini toping.

Yechilishi. Dasta tenglamasidan: $x_A=-2$; $y_A=3$, demak, to‘g‘ri chiziqlar $A(-2;3)$ nuqtadan o‘tadi.

147. Ushbu tenglamalar bilan berilgan to‘g‘ri chiziqlar dastasining markazini toping:

1) $y+4=k(x+1)$*;* *2)* $y=k(x-2)$.

III. To‘g‘ri chiziqning tenglamasini u o‘tadigan nuqtaning berilgan koordinatalari bo‘yicha va bu to‘g‘ri chiziqning burchak koeffitsienti bo‘yicha tuzish

148. $A(5;-1)$ nuqtadan o‘tuvchi va burchak koeffitsienti 4 ga teng bo‘lgan to‘g‘ri chiziqning tenglamasini tuzing.

Yechilishi. Masala shartida quyidagilar berilgan: $x_A=5$; $y_A=-1$; $k=3$. Bu qiymatlarni (2.17) tenglamaga qo‘yib topamiz: $y+1=3(x-5)$ yoki $3x-y-16=0$.

149. $(-1;-1)$ nuqtadan o‘tuvchi va burchak koeffitsienti 1 ga teng bo‘lgan to‘g‘ri chiziqning tenglamasini tuzing.

150. $(2;0)$ nuqtadan o‘tuvchi va $k=-2$ ga ega bo‘lgan to‘g‘ri chiziqning tenglamasini tuzing.

IV. To‘g‘ri chiziqning tenglamasini u o‘tadigan nuqtaning koordinatalari va bu to‘g‘ri chiziqning Ox o‘q bilan hosil qiladigan burchagi bo‘yicha tuzish

151. $(-3;-2)$ nuqtadan o‘tuvchi va Ox o‘q bilan $arctg2$ burchak tashqil etuvchi to‘g‘ri chiziqning tenglamasini tuzing.

Yechilishi. Masala shartida quyidagilar berilgan: $x_A=-3$; $y_A=-2$. $k$ni topamiz: $k=tg(arctg2)=2$. Bu qiymatlarni (2.17) tenglamaga qo‘yib:

$$y+2=2(x+3)\text{yoki } 2x-y+4=0$$

tenglamalarni hosil qilamiz.

152. $(4;-5)$ nuqtadan o‘tuvchi va Ox o‘q bilan $arctg(-3)$ burchak tashkil etuvchi to‘g‘ri chiziqning tenglamasini tuzing.

153. 1) $(2;3)$ nuqtadan o‘tuvchi va Ox o‘q bilan 45°li burchak tashqil etuvchi to‘g‘ri chiziqning tenglamasini tuzing.

2) $(0;5)$ nuqtadan o‘tuvchi va Ox o‘q bilan 135°li burchak tashqil etuvchi to‘g‘ri chiziqning tenglamasini tuzing.

10-§. Berilgan ikkita nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi

Berilgan $A(x_A;y_A)$ va $B(x_B;y_B)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning tenglamasi quyidagi ko‘rinishga ega:

$$y-y_A=\frac{y_B-y_A}{x_B-x_A}(x-x_A), \tag{2.18}$$

bu yerda x va y — o‘zgaruvchi koordinatalar. A va B nuqtalardan o‘tuvchi to‘g‘ri chiziqning burchak koeffitsienti

$$k_{AB}=\frac{y_B-y_A}{x_B-x_A} \tag{2.19}$$

munosabatdan topiladi.

I. Ikkita nuqtadan o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzish

154. $A(2;-3)$ va $B(-1;4)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzing.

Yechilishi. Masala shartida quyidagilar berilgan $x_A=2$; $x_B=-1$; $y_A=-3$ va $y_B=4$. Bu qiymatlarni (2.18) tenglamaga qo‘yib, quyidagini topamiz:

$$y+3=\frac{4+3}{-1-2}(x-2) \text{ yoki } 7x+3y-5=0.$$

155. 1) $A(-1;-1)$ va $B(-2;-2)$; 2) $A(3;0)$ va $B(0;4)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzing.

156. Uchlari: 1) $A(-3;-2)$, $B(1;5)$ va $C(8;-4)$. 2) $(-1;-3)$, $(3;5)$ va $(4;0)$ nuqtalardan iborat bo‘lgan uchburchakning tomonlari tenglamalarini tuzing.

157. 1) Uchburchak o‘zining $A(-3;4)$, $B(-4;-3)$ va $C(8;1)$ uchlari bilan berilgan. AD medananing tenglamasini tuzing.

2) Uchburchak o‘zining $A(2;5)$, $B(-6;-4)$ va $C(6;-3)$ uchlari bilan berilgan. BD medananing tenglamasini tuzing.

II. Berilgan ikkita nuqtadan o‘tuvchi to‘g‘ri chiziqning Ox o‘qqa og‘ish burchagini topish

158. $A(2;3)$ va $B(-3;1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning Ox o‘qqa og‘ish burchagini toping.

Yechilishi. Masala shartida quyidagilar berilgan; $x_A=2$; $x_B=-3$; $y_A=3$ va $y_B=1$; (2.19) formula bo‘yicha k ni topamiz:

$$k=\frac{y_B-y_A}{x_B-x_A}=\frac{1-3}{-3-2}=\frac{2}{5}=0,4.$$

bu yerdan $\alpha = arctg 0,4 = 21^{\circ}48^{'}$ ekanligi kelib chiqadi.

159. 1) $A(-3;-3)$ va $B(2;1)$; 2) $A(3;1)$ va $B(4;-2)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning Ox o‘qqa og‘ish burchagini toping.

III. Berilgan ikkita nuqtadan o‘tuvchi to‘g‘ri chiziqning koordinata o‘qlaridan ajratgan kesmalarini hisoblash

160. $A(6;2)$ va $B(-3;8)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqning koordinata o‘qlaridan ajratgan kesmalarini toping.

Yechilishi. (2.18) tenglamaga $A(6;2)$ va $B(-3;8)$ nuqtalarning koordinata-larini qo‘yib, Ox va Oy o‘qlardan izlanayotgan kesmalarni ajratuvchi to‘g‘ri chiziqning tenglamasini hosil qilamiz:

$$y-2=\frac{8-2}{-3-6}(x-6).$$

Bu tenglamani o‘qlardagi kesmalar bo‘yicha tenglama (2.16) ga keltiramiz:

$$y-2=-\frac{2}{3}(x-6), \qquad y-2=-\frac{2}{3}x+4\ ,$$

$$\frac{2}{3}x+y=6; \quad \frac{\frac{2}{3}x}{6}+\frac{y}{6}=1, \quad \frac{x}{9}+\frac{y}{6}=1.$$

O‘qlarda ajratilgan kesmalar: $a=9$ va $b=6$.

161. To‘g‘ri chiziq $A(-1;-6)$ va $B(7;2)$ nuqtalardan o‘tadi. Bu to‘g‘ri chiziqning Ox va Oy o‘qlardan ajratgan kesmalarini toping.

162. Nuqta to‘g‘ri chiziqli harakat qila borib, $A(12;-1)$ va $B(3;2)$ vaziyatlardan o‘tdi. U Oy o‘qni qaysi nuqtada kesib o‘tadi?

IV. Berilgan nuqtadan o‘tuvchi va Ox (Oy) o‘qdan berilgan kesma ajratuvchi to‘g‘ri chiziqning tenglamasini tuzish

163. $(-5;1)$ nuqtadan o‘tuvchi to‘g‘ri chiziq Oy o‘qda 6 ga teng kesma ajratadi. Bu to‘g‘ri chiziqning tenglamasini tuzing.

Yechilishi. Izlanayotgan to‘g‘ri chiziq Oy o‘qni $(0;6)$ nuqtada kesadi. Ikkita nuqtaga ega bo‘ldik: $A(-5;1)$ va $B(0;6)$. Bu nuqtalarning koordinatalarini (2.18) tenglamaga qo‘yib, izlanayotgan to‘g‘ri chiziqning tenglamasini hosil qilamiz:

$$y-1=\frac{6-1}{0+5}(x+5) \quad \text{yoki} \quad x-y+6=0.$$

164. $(-4;-1)$ nuqtadan o‘tuvchi to‘g‘ri chiziq Oy o‘qni $(0;3)$ nuqtada kesib o‘tadi. Bu to‘g‘ri chiziqning tenglamasini tuzing.

165. $(-2;4)$ nuqtadan o‘tuvchi to‘g‘ri chiziq Ox o‘qdan 2 ga teng kesma ajratadi. Bu to‘g‘ri chiziqning tenglamasini tuzing.

11-§. Ikkita to‘g‘ri chiziqning kesishishi

Agar kesishuvchi $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ to‘g‘ri chiziqlar berilgan bo‘lsa, u holda ularning kesishish nuqtasining koordinatalari berilgan tenglamalarning har birini qanoatlantirishi, ya’ni ular bu tenglamalarning umumiy ildizlari bo‘lishi kerak. Berilgan to‘g‘ri chiziqlar kesishgan nuqtaning koordinatalarini hisoblash uchun bu to‘g‘ri chiziqlar tenglamalaridan tuzilgan sistemani yechish kerak.

I.Berilgan ikkita to‘g‘ri chiziqning kesishish nuqtasining koordinatalarini hisoblash

166. $3x-4y+11=0$ va $4x-y-7=0$ to‘g‘ri chiziqlarning kesishish nuqtasini toping.

Yechilishi. Ushbu

$$\begin{cases} 3x - 4y + 11 = 0 \\ 4x - y - 7 = 0 \end{cases}$$

tenglamalar sistemasini yechib, $x = 3$ va $y = 5$ ni topamiz. Demak, bu to‘g‘ri chiziqlarning kesishish nuqtasi $(3;5)$ ekan.

167. Quyidagi to‘g‘ri chiziqlarning kesishish nuqtasini toping: 1) $y = 3x$ va $x + y + 4 = 0$; 2) $x - 2y - 8 = 0$ va $x + y - 2 = 0$.

II. Uchburchak tomonlarining tenglamalariga ko‘ra uning uchlari koordinatalarini hisoblash

168. Uchburchak tomonlarining tenglamalari berilgan: $x + 3y - 3 = 0$, $3x - 11y - 29 = 0$ va $3x - y + 11 = 0$. Bu uchburchakning uchlarini toping.

Yechilishi. Uchburchak uchlarining koordinatalarini hisoblash uchun quyidagi uchta tenglamalar sistemalarini echish kerak:

$$1)\begin{cases} 3x - 11y - 29 = 0 \\ 3x - y + 11 = 0 \end{cases}; \quad 2)\begin{cases} x + 3y - 3 = 0 \\ 3x - 11y - 29 = 0 \end{cases};$$

$$3)\begin{cases} 3x - y + 11 = 0 \\ x + 3y - 3 = 0. \end{cases}$$

Birinchi sistemaning ildizlari $x = 6$, $y = -1$, ikkinchiniki $x = -5$, $y = -4$ va uchinchiniki $x = -3$, $y = 2$. Demak, uchburchakning uchlari quyidagi nuqtalardan iborat: $(6;-1)$, $(-5;-4)$ va $(-3;2)$.

169. Agar uchburchakning tomonlari ushbu 1) $4x + 3y + 20 = 0$, $6x - 7y - 16 = 0$ va $x - 5y + 5 = 0$; 2) $7x + 3y - 25 = 0$, $2x - 7y - 15 = 0$ va $9x - 4y + 15 = 0$ tenglamalar bilan berilgan bo‘lsa, uning uchlarini toping.

12-§. Ikkita to‘g‘ri chiziq orasidagi burchak

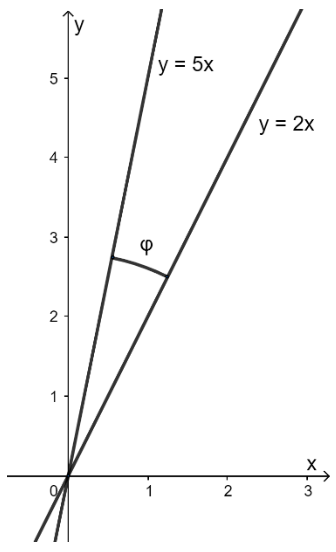
Biror *M* nuqtada kesishuvchi $y = k_1 x + b_1$ va $y = k_2 x + b_2$ to‘g‘ri chiziqlar orasidagi φ burchak quyidagi formula bo‘yicha hisoblanadi:

$$tg\varphi = \frac{k_2 - k_1}{1 + k_2 k_1}, \qquad (2.20)$$

bu yerda k_1 va k_2 — berilgan to‘g‘ri chiziqlarning burchak koeffitsientlari.

I.Ikkita to‘g‘ri chiziq orasidagi burchakni bu to‘g‘ri chiziqlarning burchak koeffitsientlari bo‘yicha topish

170. $y = 2x$ va $y = 5x$ to‘g‘ri chiziqlar orasidagi o‘tkir burchakni toping.



22-rasm

Yechilishi. Berilgan to‘g‘ri chiziqlarning burchak koeffitsientlari 2 va 5 ga teng. Ikkita to‘g‘ri chiziq orasidagi o‘tkir burchakni hisoblash uchun k_2 va k_1 larni $tg\varphi > 0$ bo‘ladigan qilib (chunki o‘tkir burchakning tangensi — musbat son) tanlash kerak. Buning uchun $k_2 = 5$ va $k_1 = 2$ deb olamiz. (2.20) formula bo‘yicha:

$$tg\varphi = \frac{5-2}{1+5\cdot 2} = \frac{3}{11} = 0,2727;$$

φ burchakni jadvaldan topamiz: $\varphi = arctg\frac{3}{11} = 15^\circ 15'$ (22-rasm).

171. Quyidagi to‘g‘ri chiziqlar orasidagi o‘tkir burchakni toping: 1) $y = -x$ va $y = 3x$; 2) $2x - 3y + 6 = 0$ va $3x - y - 3 = 0$; 3) $\frac{x}{5} + \frac{y}{2} = 1$ va $\frac{x}{3} + \frac{y}{4} = 1.$

172. Uchburchak tomonlarining tenglamalari berilgan:

1) $3x-2y-1=0$; 2) $5x+4y-31=0$ va 3) $x-8y-15=0$.

Bu uchburchakning ichki burchaklarini toping.

Yechilishi. 1)$3x-2y-1=0$ va 2) $5x+4y-31=0$ tomonlar A burchakni; 2) $5x+4y-31=0$ va 3) $x-8y-15=0$ tomonlar B burchakni; 3) $x-8y-15=0$ va 1) $3x-2y-1=0$ tomonlar C burchakni tashkil etsin. Uchburchakni yasash uchun A, B va C uchlarni topamiz, buning uchun esa ushbu tenglamalar sistemalarini yechamiz:

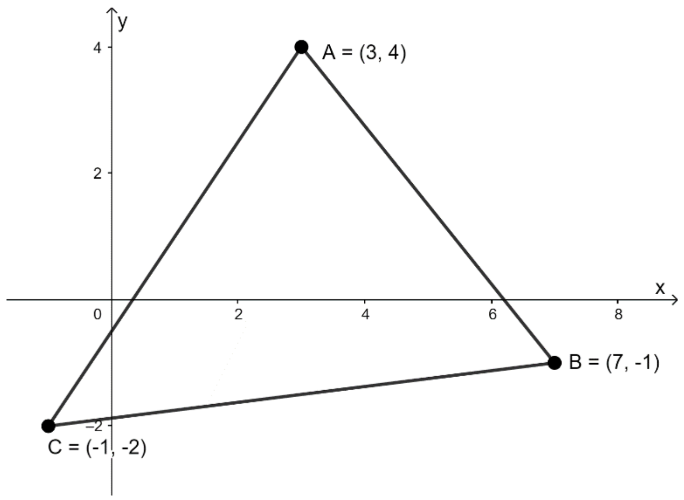
$$\begin{cases} 3x - 2y - 1 = 0 \\ 5x + 4y - 31 = 0 \end{cases},$$

$x=3,\ y=4,\ A(3;4)$;

$$\begin{cases} 5x + 4y - 31 = 0 \\ x - 8y - 15 = 0 \end{cases},$$

$x=7,\ y=-1,\ B(7;-1)$;

$$\begin{cases} x-8y-15=0 \\ 3x-2y-1=0 \end{cases},$$



23-rasm

$x=-1,\ y=-2,\ C(-1;-2)$ (23-rasm).

Tomonlarning tenglamalaridan burchak koeffitsientlarini topamiz:

AC tomon: $3x-2y-1=0,\ k_{AC}=\frac{3}{2}$;

AB tomon: $5x+4y-31=0,\ k_{AB}=-\frac{5}{4}$;

BC tomon: $x-8y-15=0,\ k_{BC}=\frac{1}{8}$.

(2.20) formula bo‘yicha A, B va C burchaklarni topamiz;

$$tgA = \frac{k_{AB} - k_{AC}}{1 + k_{AB}k_{AC}} = \frac{-\frac{5}{4} - \frac{3}{2}}{1 + (-\frac{5}{4}) \cdot \frac{3}{4}} = 3{,}143, \quad A = arctg3{,}143 = 72^\circ 21';$$

$$tgB = \frac{k_{BC} - k_{AB}}{1 + k_{BC}k_{AB}} = \frac{\frac{1}{8} - (-\frac{5}{4})}{1 + \frac{1}{8} \cdot (-\frac{3}{4})} = 1{,}63, \quad B = arctg1{,}63 = 58^\circ 28';$$

$$tgC = \frac{k_{AC} - k_{BC}}{1 + k_{AC}k_{BC}} = \frac{\frac{3}{2} - \frac{1}{8}}{1 + \frac{3}{2} \cdot \frac{1}{8}} = 1{,}1579, \quad C = arctg1{,}1579 = 49^\circ 11'$$

Tekshirish: $C = 72^\circ 21' + 58^\circ 28' + 49^\circ 11' = 180^\circ$.

173. Agar uchburchakning tomonlari ushbu 1) $7x + 4y + 9 = 0$, $x - 8y + 27 = 0$ va $2x - y - 6 = 0$; 2) $6x - y + 13 = 0$, $3x + 7y - 1 = 0$ va $3x - 8y - 31 = 0$ tenglamalar bilan berilgan bo‘lsa, uning ichki burchaklarini toping.

II. Har biri ikkita nuqta yordamida berilgan ikki to‘g‘ri chiziq orasidagi burchakni hisoblash

174. Agar to‘g‘ri chiziqlarning biri $A_1(4;2)$ va $B_1(1;-7)$ nuqtalardan, ikkinchisi esa $A_2(-1;3)$ va $B_2(8;6)$ nuqtalardan o‘tsa, bu to‘g‘ri chiziqlar orasidagi o‘tkir burchakni toping.

Yechilishi. To‘g‘ri chiziqlarning tenglamalarini tuzmasdan, (2.19) formula yordamida ularning burchak koeffitsientlarini topamiz:

$$k_{A_1B_1} = \frac{-7-2}{1-4} = 3, \qquad k_{A_2B_2} = \frac{6-3}{8-(-1)} = \frac{1}{3}, \qquad k_2 = k_{A_1B_1} = 3,$$

$$k_1 = k_{A_2B_2} = \frac{1}{3}$$

deylik. (2.20) formula bo‘yicha topamiz:

$$\mathrm{tg}\varphi=\frac{3-\frac{1}{3}}{1+3\cdot\frac{1}{3}}=1,333,\quad \varphi=arctg1,333=53^{\circ}08^{'}.$$

175. Agar to‘g‘ri chiziqlarning biri $A_1(-6;7)$ va $B_1(2;-5)$ nuqtalardan, ikkinchisi esa $A_2(-5;2)$ va $B_2(1;1)$ nuqtalardan o‘tsa, bu to‘g‘ri chiziqlar orasidagi o‘tkir burchakni toping.

176. Biri $A(3;3)$ nuqtadan, ikkinchisi esa $B(3;-2)$ nuqtadan o‘tadigan va umumiy $M(-2;-1)$ nuqtaga ega bo‘lgan to‘g‘ri chiziqlar orasidagi o‘tkir burchakni toping.

177. Uchlari $A(-6;-1)$, $B(4;6)$ va $C(2;1)$ bo‘lgan uchburchak berilgan. Bu uchburchakning ichki burchaklarini toping.

Yechilishi. (2.19) formula bo‘yicha bu uchburchak tomonlarining burchak koeffitsientlarini topamiz:

$$k_{AB}=\frac{y_B-y_A}{x_B-x_A}=\frac{6-(-1)}{4-(-6)}=\frac{7}{10};\ k_{BC}=\frac{y_C-y_B}{x_C-x_B}=\frac{1-6}{2-4}=\frac{5}{2};$$

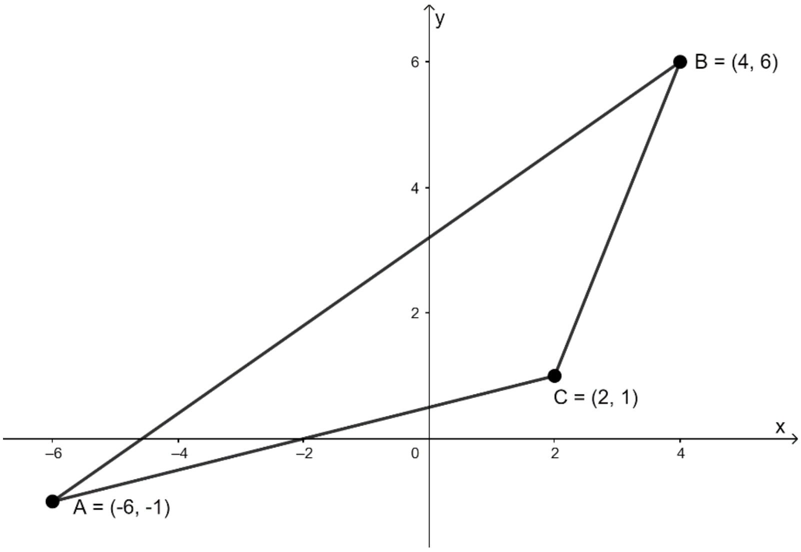
$$k_{CA}=\frac{y_A-y_C}{x_A-x_C}=\frac{-1-1}{-6-2}=\frac{1}{4}.$$

(2.20) formula bo‘yicha uchburchakning burchaklarini topamiz:

$$\mathrm{tgA}=\frac{k_{AB}-k_{CA}}{1+k_{AB}k_{CA}}=\frac{\frac{7}{10}-\frac{1}{4}}{1+\frac{7}{10}\cdot\frac{1}{4}}=0,383;\ \ A=20^{\circ}57;$$

$$\mathrm{tgB}=\frac{k_{BC}-k_{AB}}{1+k_{BC}k_{AB}}=\frac{\frac{5}{2}-\frac{7}{10}}{1+\frac{5}{2}\cdot\frac{7}{10}}=0,6545;\ \ B=33^{\circ}\ 12';$$

$$\text{tgC}=\frac{k_{BC}-k_{CA}}{1+k_{BC}k_{CA}}=\frac{\frac{5}{2}-\frac{1}{4}}{1+\frac{5}{2}\cdot\frac{1}{4}}=1,3846; \quad C=\ 54°10'.$$



24-rasm

Tekshirish: $20°57' + 33°12' + 54°10' = 108°19'$. Agar uchburchak-ning burchaklaridan biri o‘tmas bo‘lsa, bu uchburchakning burchaklari yig‘indisi 180°dan kichik bo‘lishi mumkin. Uchburchakni yasash bilan C burchakning o‘tmas ekanligiga ishonch hosil qilamiz (24-rasm).

Hisoblashda uchburchakning ichki burchagi emas, balki izlanayotgan burchak bilan qo‘shni bo‘lgan tashqi burchak topilgan edi; izlanayotgan burchak $C=180°-4°\ 10'=125\ 50'$ bo‘ladi. Bu holda uchburchakning burchaklari yig‘indisi 180°ga teng bo‘ladi: $20°57'+33°12'+125°50'=179°59'$ ($1'$ xatolikka hisoblashlarda yo‘l qo‘yilgan). Uchburchakni yasamasdan turib ham berilgan uchburchakning burchaklaridan biri o‘tmas ekanligini osongina ko‘rsatish mumkin. Uchburchakning tomonlarini hisoblaymiz:

$$AB=\sqrt{(-6-4)^2+(-1-6)^2}=\sqrt{149};$$

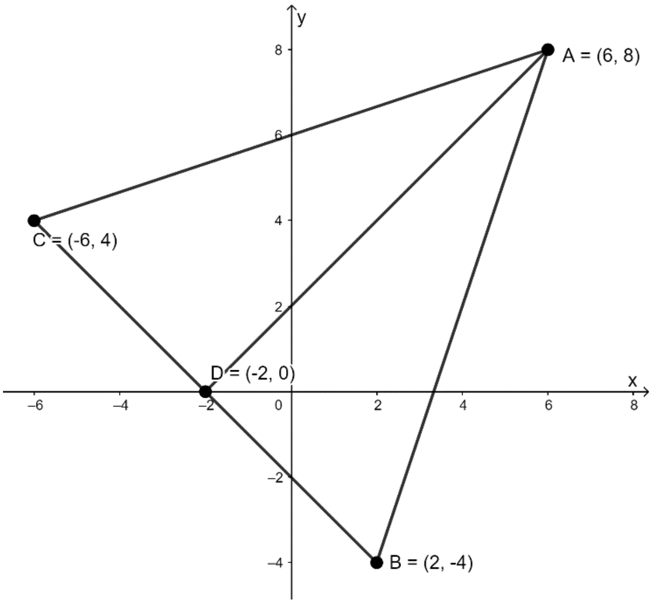
$$BC=\sqrt{(4-2)^2+(6-1)^2}=\sqrt{29};$$

$$AC=\sqrt{(-6-2)^2+(-1-1)^2}=\sqrt{68}\,.$$

Ma’lumki, o‘tmas burchakli uchburchakda katta tomonning kvadrati qolgan ikkita tomon kvadratlarining yig‘indisidan katta. Bu yerda $149\ >\ 29\ +\ 68$. AB tomon o‘tmas burchak qarshisida yotadi.

178. Agar uchburchakning uchlari 1) $A(-6;-3)$, $B(6;7)$ va $C(2;-1)$; 2) $A(0;4)$, $B(4;-2)$ va $C(-4;-2)$ nuqtalarda bo‘lsa, uning ichki burchaklarini toping.

179. Uchlari $A(6;8)$, $B(2;-4)$ va $C(-6;4)$ bo‘lgan uchburchak berilgan. AB tomon bilan A uchdan o‘tkazilgan mediana orasidagi burchakni toping.



25-rasm

Yechilishi. Med
ananing burchak koeffitsientini topish uchun BC tomonni teng ikkiga bo‘luvchi D nuqtaning koordinatalarini (1.4) formula bo‘yicha topamiz (25-rasm):

$$x_D=\frac{x_B+x_C}{2}=\frac{2-6}{2}=-2;\quad y_D=\frac{y_B+y_C}{2}=\frac{-4+4}{2}=0,\ \ D(-2;0)\,.$$

(2.19) formula bo‘yicha medananing burchak koeffitsientini topamiz:

$$k_{AD}=\frac{y_D-y_A}{x_D-x_A}=\frac{0-8}{-2-6}=1;$$

AB tomonning burchak koeffitsientini hisoblaymiz:

$$k_{AB}=\frac{y_B-y_A}{x_B-x_A}=\frac{-4-8}{2-6}=3;$$

(2.20) formula bo‘yicha $\angle BAD = \varphi$ ni topamiz:

$$\mathrm{tg}\varphi = \frac{k_{AB} - k_{AD}}{1 + k_{AB}k_{AD}} = \frac{3-1}{1+3\cdot 1} = 0,5, \qquad \varphi = arctg0,5 = 26^\circ 34'.$$

180. Uchburchakning uchlari $A(-2;2)$, $B(6;4)$ va $C(2;-6)$ nuqtalardan iborat. AC tomon bilan A uchdan o‘tkazilgan mediana orasidagi burchakni toping.

III. Berilgan ikkita nuqtadan o‘tuvchi to‘g‘ri chiziq bilan berilgan to‘g‘ri chiziq orasidagi burchakni hisoblash

181. $A(4;-3)$ va $B(2;-2)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq bilan $3x + 2y + 4 = 0$ to‘g‘ri chiziq orasidagi burchakni toping.

Yechilishi. To‘g‘ri chiziqlarning burchak koeffitsientlarini topamiz:

1) $3x + 2y + 4 = 0$, $y = -\frac{3}{2}x - 2$, $k = -\frac{3}{2}$;

2) $k_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{-2-(-3)}{2-4} = -\frac{1}{2}$.

$k_2 = -\frac{1}{2}$; $k_1 = -\frac{3}{2}$ deymiz. (2.20) formula bo‘yicha topamiz:

$$\mathrm{tg}\varphi = \frac{-\frac{1}{2} - (-\frac{3}{2})}{1 + (-\frac{1}{2})\cdot(-\frac{3}{2})} = \frac{4}{7};$$

$$\varphi = arctg\frac{4}{7} = 29^\circ 45'.$$

182. $A(1;5)$ va $B(4;-3)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq bilan $x + 2y - 4 = 0$ to‘g‘ri chiziq orasidagi o‘tkir burchakni toping.

183. $A(-1;-3)$ va $B(5;1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq bilan $2x+y+8=0$ to‘g‘ri chiziq orasidagi burchakni toping.

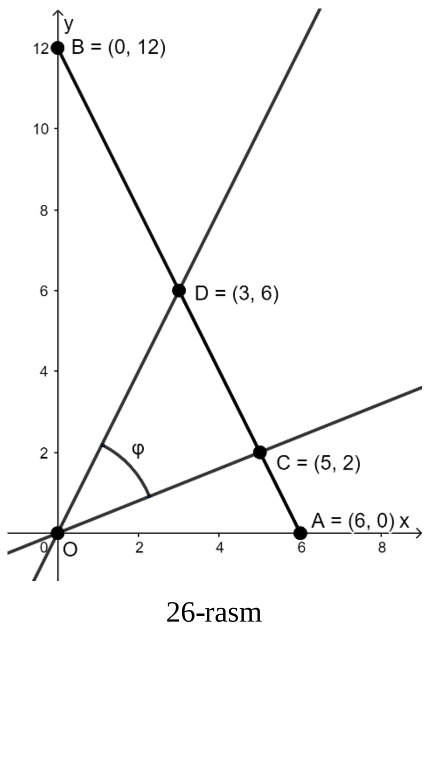
IV. Berilgan nuqtadan va berilgan to‘g‘ri chiziqning koordinata o‘qlari orasida joylashgan kesmasini berilgan nisbatda bo‘luvchi nuqtalardan o‘tuvchi ikki to‘g‘ri chiziq orasidagi o‘tkir burchakni hisoblash

184. Koordinatalar boshidan va $2x+y-12=0$ to‘g‘ri chiziqning koordinata o‘qlari orasida joylashgan kesmasini 1:2:3 nisbatda bo‘luvchi nuqtalardan o‘tuvchi to‘g‘ri chiziqlar orasidagi burchakni toping.

Yechilishi. Koordinatalar boshidan o‘tuvchi to‘g‘ri chiziqlarning burchak koeffitsientlarini hisoblash uchun $2x+y-12=0$ to‘g‘ri chiziqda bu to‘g‘ri chiziqlar o‘tadigan nuqtalarni topish kerak. $2x+y-12=0$ to‘g‘ri chiziq Ox o‘qni A nuqtada, Oy o‘qni B nuqtada kesib o‘tsin. Bu nuqtalarni topamiz:

1) $y=0$, $x=6$; $A(6;0)$; 2) $x=0$, $y=12$; $B(0;12)$.

Endi to‘g‘ri chiziqning A va B nuqtalar orasidagi kesmasini 1:2:3 kabi nisbatda bo‘luvchi C va D nuqtalarni topamiz (nuqtalarni A, C, D, B ketma-ketlikda olamiz). Masala shartida to‘g‘ri chiziq kesmasi qanday yo‘nalishda (A dan B ga qarabmi yoki B dan A ga qarabmi) bo‘linishi aytilmagan, shu sababli har ikkala yo‘nalish ham masala shartini qanoatlantiradi, ya’ni masala ikkita yechimga ega.



26-rasm

1-xil Yechilishi. Bo‘lish A dan B ga qarab, ya’ni $AC:CD:DB=1:2:3$ kabi nisbatda bajarilmoqda (26 -rasm).

(1.3) formulalar bo‘yicha C nuqtani topamiz:

$$\lambda=\frac{AC}{CB}=\frac{1}{2+3}=\frac{1}{5};$$

$$x_C=\frac{x_A+\lambda x_B}{1+\lambda}=\frac{6+\frac{1}{5}\cdot 0}{1+\frac{1}{5}}=5,$$

$$y_C=\frac{y_A+\lambda y_B}{1+\lambda}=\frac{0+\frac{1}{5}\cdot 12}{1+\frac{1}{5}}=2, \quad C(5;2).$$

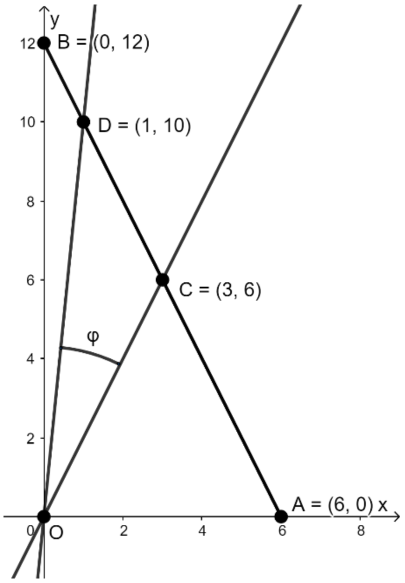
(1.4) formulalar bo‘yicha AB kesmaning o‘rtasi bo‘lgan D nuqtaning koordinatalarini hisoblaymiz:

$$x_D=\frac{x_A+x_B}{2}=\frac{6+0}{2}=3 \ ; \ y_D=\frac{y_A+y_B}{?}=\frac{0+12}{2}=6 \ , \ D(3;6).$$

OC va OD to‘g‘ri chiziqlarning burchak koeffitsientlarini (2.7) formula bo‘yicha topamiz:

$$\mathrm{tg}\varphi=\frac{k_{OD}-k_{OC}}{1+k_{OD}k_{OC}}=\frac{2-\frac{2}{5}}{1+2\cdot\frac{2}{5}}=0{,}8889,$$

$$\varphi=arctg0{,}8889=41^{\circ}38'.$$



27-rasm

2- xil Yechilishi. Kesmani bo‘lish B dan A ga qarab, ya’ni $BD:DC:CA=1:2:3$ kabi nisbatda bajarilmokda (27-rasm). D nuqtani topamiz:

$$\lambda=\frac{BD}{DA}=\frac{1}{2+3}=\frac{1}{5} \ ;$$

$$x_D=\frac{x_B+\lambda x_A}{1+\lambda}=\frac{0+\frac{1}{5}\cdot 6}{1+\frac{1}{5}}=1 \ ;$$

$$y_D=\frac{y_B+\lambda y_A}{1+\lambda}=\frac{12+\frac{1}{5}\cdot 0}{1+\frac{1}{5}}=10, \quad D(1;10).$$

BA kesmaning o‘rtasi bo‘lgan C nuqtani topamiz:

$$x_C=\frac{x_B+x_A}{2}=\frac{0+6}{2}=3 \;\;; \;\; y_C=\frac{y_B+y_A}{2}=\frac{12+0}{2}=6,\;\; C(3;6).$$

OD va OC to‘g‘ri chiziqlarnnng burchak koeffitsientlarini topamiz:

$$k_{OD}=\frac{y_D}{x_D}=\frac{10}{1}=10, \quad k_{OC}=\frac{y_C}{x_C}=\frac{6}{3}=2.$$

OD va OC to‘g‘ri chiziqlar orasidagi o‘tkir burchakni hisoblaymiz:

$$\mathrm{tg}\varphi=\frac{k_{OD}-k_{OC}}{1+k_{OD}k_{OC}}=\frac{10-2}{1+10\cdot 2}=0.381 \;\;;$$

$$\varphi=arctg0,381=26^{\circ}51'.$$

185. Koordinatalar boshidan va $x+3y-9=0$ to‘g‘ri chiziqning koordinata uchlari orasida joylashgan kesmasini (uning Ox o‘q bilan kesishish nuqtasidan Oy o‘q bilan kesishish nuqtasi tomon yunalishda) 1:3:2 kabi nisbatda bo‘luvchi nuqtalardan o‘tuvchi ikki to‘g‘ri chiziq orasidagi o‘tkir burchakni toping.

186. $A(4;3)$ nuqtadan va $x+2y-6=0$ to‘g‘ri chiziqning koordinata uchlari orasida joylashgan kesmasini uning Ox o‘q bilan kesishish nuqtasidan Oy o‘q bilan kesishish nuqtasi tomon yo‘nalishda 3:1:2 kabi nisbatda bo‘luvchi nuqtalardan o‘tuvchi ikki to‘g‘ri chiziq orasidagi o‘tkir burchakni toping.

Yechilishi. $x+2y-6=0$ to‘g‘ri chiziq Ox o‘qni A nuqtada, Oy o‘qni esa B nuqtada kesib o‘tsin. Bu nuqtalarni topamiz:

1) $y=0$, $x=6$, $A(6;0)$*;* 2*)* $x=0$, $y=3$, $B(0;3)$.

To‘g‘ri chiziqning AB kesmasini 3:1:2 kabi nisbatda bo‘luvchi C va D nuqtalarni topamiz: C nuqta uchun:

$$\lambda=\frac{AC}{CB}=\frac{3}{1+2}=1.$$

(1.4) formulalarni qo‘llanib, topamiz:

$$x_C=\frac{x_A+x_B}{2}=\frac{6+0}{2}=3\ ;\ y_C=\frac{y_A+y_B}{2}=\frac{0+3}{2}=\frac{3}{2}\ ;\ C\left(3;\frac{3}{2}\right).$$

D nuqta uchun:

$$\lambda=\frac{AD}{DB}=\frac{3+1}{2}=2.$$

(1.3) formulalar bo‘yicha:

$$x_D=\frac{x_A+\lambda x_B}{1+\lambda}=\frac{6+2\cdot 0}{1+2}=2,\ y_D=\frac{y_A+\lambda y_B}{1+\lambda}=\frac{0+2\cdot 3}{1+2}=2;\ D(2;2).$$

MC va MD to‘g‘ri chiziqlarning burchak koeffitsientlarini (2.19) formula bo‘yicha topamiz:

$$k_{MC}=\frac{y_C-y_M}{x_C-x_M}=\frac{\frac{3}{2}-3}{3-4}=\frac{3}{2}\ ;\qquad k_{DM}=\frac{2-3}{2-4}=\frac{1}{2}.$$

MC va MD to‘g‘ri chiziqlar orasidagi o‘tkir burchakni hisoblaymiz:

$$\mathrm{tg}\varphi=\frac{k_{MC}-k_{MD}}{1+k_{MC}k_{MD}}=\frac{\frac{3}{2}-\frac{1}{2}}{1+\frac{3}{2}\cdot\frac{1}{2}}=0.5714;$$

$$\varphi=arctg0,5714=29^{\circ}45'.$$

187. $C(8;7)$ nuqtadan va $3x+2y-12=0$ to‘g‘ri chiziqning koordinata o‘qlari orasida joylashgan kesmasini uchta teng bo‘lakka bo‘luvchi nuqtalardan o‘tuvchi ikki to‘g‘ri chiziq orasidagi o‘tkir burchakni toping.

188. $M(-6;-8)$nuqtadan va $2x+y+10=0$ to‘g‘ri chiziqning koordinata uchlari orasida joylashgan kesmasini (uning Oxo‘q bilan kesishish nuqtasidan Oy o‘q bilan kesishish nuqtasi tomon yunalishda) 1:2:2 kabi nisbatda bo‘luvchi nuqtalardan o‘tuvchi ikki to‘g‘ri chiziq orasidagi o‘tkir burchakni toping.

V. Berilgan nuqtadan o‘tuvchi va berilgan to‘g‘ri chiziq bilan oldindan berilgan burchak hosil qiluvchi to‘g‘ri chiziqning tenglamasini tuzish

189. $A(2;3)$ nuqtadan o‘tuvchi va $2x-y-1=0$to‘g‘ri chiziq bilan $arctg\frac{4}{3}$ burchak tashkil qiluvchi to‘g‘ri chiziqning tenglamasini tuzing.

Yechilishi. Berilgan to‘g‘ri chiziqning burchak koeffitsienti (2.20) formulada k_1 ga ham, k_2 ga ham teng bo‘lishi mumkin, shu sababli ikkita echimga egamiz: 1) $k_1=2$; 2) $k_2=-2$.

1) $k_1=2$. (2.20) formula bo‘yicha: $tg\left(arctg\frac{3}{4}\right)=\frac{k_2-2}{1+2k_1}$, yoki $\frac{3}{4}=\frac{k_2-2}{1+2k_1}$ tenglamani yechib, $k_2=-2$ ni hosil qilamiz. Izlanayotgan tenglama ushbu ko‘rinishga ega: $y-y_A=k(x-x_A)$.

Bunga A nuqtaning koordinatalarini va $k_2$ning qiymatini qo‘yamiz: $y-3=-2(x-2)$ yoki $2x+y-7=0$.

2) $k_2=2$. $tg(arctg\frac{4}{3})=\frac{2-k_1}{1+2k_1}$, $k_1=\frac{2}{11}$;

$$y-3=\frac{2}{11}(x-2) \quad \text{yoki} \quad 2x-11y+29=0.$$

190. $(-2,5)$ nuqtadan o‘tuvchi va $3x-y+4=0$ to‘g‘ri chiziq bilan $arctg\frac{1}{7}$ burchak tashqil etuvchi to‘g‘ri chiziqning tenglamasini tuzing.

191. Koordinatalar boshidan o‘tuvchi va $x-y+1=0$ to‘g‘ri chiziq bilan 45° li burchak tashqil etuvchi to‘g‘ri chiziqning tenglamasini tuzing. Bu to‘g‘ri chiziqning berilgan to‘g‘ri chiziq bilan kesishish nuqtasini toping.

VI. Koordinatalar boshidan o‘tuvchi ikkita to‘g‘ri chiziqning tenglamalarini ularning burchak koeffitsientlarini nisbati va bu to‘g‘ri chiziqlar orasidagi burchak bo‘yicha tuzish

192. Koordinatalar boshidan o‘tuvchi ikkita to‘g‘ri chiziq o‘zaro $arctg\frac{1}{3}$ burchak tashqil etadi. Bu to‘g‘ri chiziqlarning burchak koeffitsientlarini nisbati $\frac{2}{7}$ ga teng. Bu to‘g‘ri chiziqlarning tenglamalarini tuzing.

Yechilishi. Koordinatalar boshidan o‘tuvchi to‘g‘ri chiziqlarning tenglamalari $y=kx$ ko‘rinishga ega; ularning burchak koeffitsientlarini topamiz. Bu to‘g‘ri chiziqlarning burchak koeffitsientlarini k_1 va k_2 bilan belgilaymiz. Masala shartidan:

$$\frac{k_1}{k_2}=\frac{2}{7}.$$

To‘g‘ri chiziqlar orasidagi burchak $\varphi=arctg\frac{1}{3}$, bu yerdan $tg\varphi=\frac{1}{3}$ biroq $tg\varphi=\frac{k_2-k_1}{1+k_2k_1}$, ya’ni $\frac{k_2-k_1}{1+k_2k_1}=\frac{1}{3}$. Endi k_1 va k_2 ni hisoblash uchun quyidagi sistemani yechamiz:

$$\begin{cases} \dfrac{k_1}{k_2} = \dfrac{2}{7}, \\ \dfrac{k_2 - k_1}{1 + k_2 k_1} = \dfrac{1}{3}. \end{cases}$$

Birinchi tenglamadan $k_2 = \frac{7}{2} k_1$ ni topamiz.

k_2 ning qiymatini ikkinchi tenglamaga qo‘yib, $7k^2{}_1 - 15k_1 + 2 = 0$ kvadrat tenglamani hosil qilamiz, uning ildizlari:

$$(k_1)_1 = 2 \quad \text{va} \quad (k_1)_2 = \frac{1}{7}.$$

k_1 ning bu qiymatlariga k_2 ning ikkita qiymati mos keladi:

$$(k_2)_1 = \frac{7}{2} \cdot 2 = 7 \quad \text{va} \quad (k_2)_2 = \frac{7}{2} \cdot \frac{1}{7} = \frac{1}{2}.$$

Ikkita yechimga egamiz:

1) $k_1 = 2$ va $k_2 = 7$ va 2) $k_1 = 1/7$ va $k_2 = 1/2.$

Izlanayotgan to‘g‘ri chiziqlar mos ravishda quyidagilar bo‘ladi: 1) $y = 2x$ va $y = 7x$; 2) $y = \frac{1}{7}x$ va $y = \frac{1}{2}x$, ya’ni masala ikkita yechimga ega.

193. Koordinatalar boshidan o‘tuvchi ikki to‘g‘ri chiziq o‘zapo $arctg\frac{7}{9}$ burchak tashkil etadi. Bu to‘g‘ri chiziqlarning burchak koeffitsientlarining nisbati $\frac{9}{2}$ ga teng. Bu to‘g‘ri chiziqlarning tenglamalarini tuzing.

VII. Uchburchak burchagining bissektrisasi tenglamasini bu uchburchakning berilgan uchlari bo‘yicha tuzish

194. Uchburchakning uchlari berilgan: $A(2;-1)$, $B(7;-3)$ va $C(-1;-5)$. C burchak bissektrisasining tenglamasini tuzing.

Yechilishi. C burchak bissektrisasining AB tomon bilan kesishish nuqtasi M ni topamiz. Geometriya kursidan ma’lumki, uchburchak burchagining bissektrisasi burchak qarshisidagi tomonni burchakka yopishgan tomonlarga proporsional bo‘laklarga bo‘ladi. Demak,

$$\lambda=\frac{BM}{MA}=\frac{CB}{CA}.$$

(1.1) formula bo‘yicha topamiz:

$$CB=\sqrt{(-1+7)^2+(-5-3)^2}=10,$$

$$CA=\sqrt{(-1-2)^2+(-5+1)^2}=5,$$

u holda:

$$\lambda=\frac{10}{5}=2.$$

(1.3) formulalar bo‘yicha M nuqtaning koordinatalarini hisoblaymiz:

$$x_M=\frac{-7+2\cdot 2}{1+2}=-1;\quad y_M=\frac{3+2\cdot(-1)}{1+2}=\frac{1}{3},\ M\left(-1;\frac{1}{3}\right).$$

C va M nuqtalarning abssissalari teng, demak, C burchakning bissektrisasi Oy o‘qqa parallel: $x=-1$ yoki $x+1=0$.

195. Uchburchakning uchlari berilgan: $A(-6;-2)$, $B(4;8)$ va $C(2;-10)$. A burchak bissektrisasining tenglamasini tuzing.

13-§. Ikkita to‘g‘ri chiziqning parallellik sharti

Agar $y=k_1x+b_1$ va $y=k_2x+b_2$ to‘g‘ri chiziqlarning burchak koeffitsientlari teng, ya’ni

$$k_1=k_2 \qquad (2.21)$$

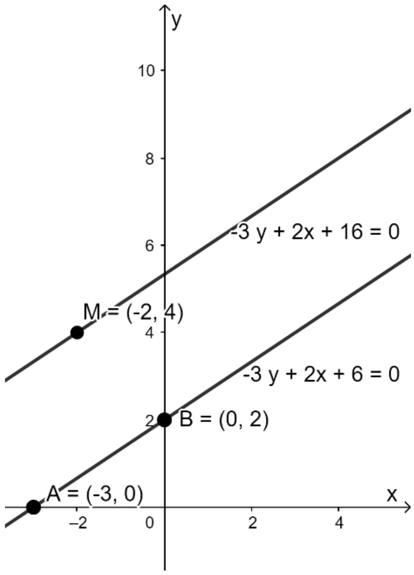
bo‘lsa, bu to‘g‘ri chiziqlar parallel bo‘ladi.

I. Berilgan nuqtadan berilgan to‘g‘ri chiziqqa parallel bo‘lib o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzish

196. $M(-2;4)$ nuqtadan $2x-3y+6=0$ to‘g‘ri chiziqqa parallel bo‘lib o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzing.

Yechilishi. $2x-3y+6=0$ to‘g‘ri chiziqni uning koordinata uchlari bilan kesishish nuqtalari bo‘yicha yasaymiz: 1) $y=0$, $x=-3$; $A(-3;0)$; 2) $x=0$, $y=2$; $B(0;2)$. $M(-2;4)$ nuqtani yasaymiz (28-rasm).

AB to‘g‘ri chiziqning burchak koeffitsientini topish uchun uning tenglamasini y ga nisbatan yechamiz: $y=\frac{2}{3}x+2$, bu yerdan burchak koeffitsient $k_{AB}=\frac{2}{3}$.



28-rasm

Izlanayotgan to‘g‘ri chiziqning burchak koeffitsienti berilgan to‘g‘ri chiziqning burchak koeffitsientiga teng bo‘ladi, chunki to‘g‘ri chiziqlar parallel, ya’ni $k_{MC}=k_{AB}=\frac{2}{3}$. Izlanayotgan to‘g‘ri chiziq $M(-2;4)$ nuqtadan o‘tadi va $k_{MC}=\frac{2}{3}$ burchak koeffitsientga ega bo‘ladi. Bu qiymatlarni berilgan nuqtadan berilgan yunalishda o‘tuvchi to‘g‘ri chiziqning tenglamasi (2.17) ga qo‘yib, $y-4=\frac{2}{3}(x+2)$ yoki $2x-3y+16=0$ tenglamalarni hosil qilamiz.

197. $A(-3;2)$ nuqtadan $5x-3y+21=0$ to‘g‘ri chiziqqa parallel bo‘lib o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzing.

198. $A(-1;-4)$ nuqtadan $\frac{x}{4}+\frac{y}{3}=1$ to‘g‘ri chiziqqa parallel bo‘lib o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzing.

II. Berilgan ikkita nuqtadan o‘tuvchi to‘g‘ri chiziqqa parallel va berilgan nuqtadan o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzish

199. $A(-2;6)$ va $B(3;-1)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqqa parallel hamda $M(-3;-1)$ nuqtadan o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzing.

200. $(-3;1)$ va $(3;2)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqqa parallel hamda $(1;-4)$ nuqtadan o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzing.

III. Berilgan to‘g‘ri chiziqlarning kesishish nuqtasidan berilgan to‘g‘ri chiziqqa parallel bo‘lib o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzish

201. $x+y-4=0$ va $x-y=0$ to‘g‘ri chiziqlarning kesishish nuqtasidan $x-4y+4=0$ to‘g‘ri chiziqqa parallel bo‘lib o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzing.

202. $\frac{x}{6}+\frac{y}{3}=1$ va $\frac{x}{3}+\frac{y}{6}=1$ to‘g‘ri chiziqlarning kesishish nuqtasidan $x-2y-6=0$ to‘g‘ri chiziqqa parallel bo‘lib o‘tgan to‘g‘ri chiziqning tenglamasini tuzing.

14-§. Ikkita to‘g‘ri chiziqning perpendikulyarlik sharti

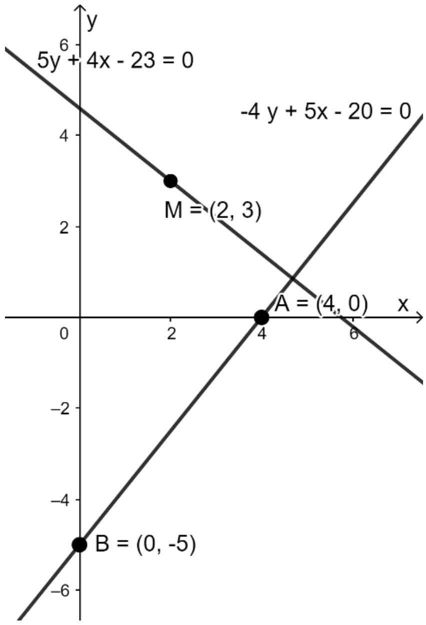
Agar $y=k_1x+b_1$ va $y=k_2x+b_2$ to‘g‘ri chiziqlarning burchak koeffitsientlari kattaliklari bo‘yicha teskari, ishoralari bo‘yicha qarama-qarshi, ya’ni:

$$k_2=-\frac{1}{k_1}\text{, ya'ni } k_1k_2=-1 \qquad (2.22)$$

bo‘lsa, bu to‘g‘ri chiziqlar perpendikulyar bo‘ladi.

I. Berilgan nuqtadan berilgan to‘g‘ri chiziqqa perpendikulyar bo‘lib o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzish

203. $5x-4y-20=0$ to‘g‘ri chiziqqa perpendikulyar bo‘lib, $M(2;3)$ nuqtadan o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzing.



29-rasm

Yechilishi. $5x-4y-20=0$ to‘g‘ri chiziqni uning koordinata o‘qlari bilan kesishish nuqtalari: 1) $y=0$, $x=4$; $A(4;0)$ va 2) $x=0$, $y=-5$; $B(0;-5)$ bo‘yicha yasaymiz va $M(2;3)$ nuqtani yasaymiz (29-rasm). AB to‘g‘ri chiziqning burchak koeffitsienti: $k_{AB}=\dfrac{5}{4}$ Izlanayotgan to‘g‘ri chiziqning burchak koeffitsientini (2.22) formula bo‘yicha topamiz: $k_{MC}=-\dfrac{1}{k_{AB}}=-\dfrac{5}{4}$. $k_{MC}=-\dfrac{5}{4}$ ni va M nuqtaning koordinatalarini (2.18) tenglamaga qo‘yib, $y-3=-\dfrac{5}{4}(x-2)$ yoki $4x+5y-23=0$ ni hosil qilamiz.

204. $M(4;-3)$ nuqtadan $5x-2y+10=0$ to‘g‘ri chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziqning tenglamasini tuzing.

205. $M(-4;1)$ nuqtadan $\dfrac{x}{5}-\dfrac{y}{6}=1$ to‘g‘ri chiziqqa perpendikulyar bo‘lib o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzing.

206. Koordinatalar boshidan $2x+3y-12=0$ to‘g‘ri chiziqqa perpendikulyar bo‘lib o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzing.

II.Berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziqqa perpendikulyar bo‘lgan va berilgan nuqtadan o‘tadigan to‘g‘ri chiziqning tenglamasini tuzish

207. $(-2;6)$ va $(3;-3)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqqa perpendikulyar bo‘lgan va $(2;4)$ nuqtadan o‘tadigan to‘g‘ri chiziqning tenglamasini tuzing.

208. To‘g‘ri chiziq $(-4;1)$ va $(2;-5)$ nuqtalardan o‘tadi. Bu to‘g‘ri chiziqning Oy o‘q bilan kesishish nuqtasi orqali unga perpendikulyar bo‘lgan ikkinchi bir to‘g‘ri chiziq o‘tadi. Bu to‘g‘ri chiziqlarning tenglamalarini tuzing.

209. Koordinatalar boshidan o‘tuvchi va Ox o‘qni $(2;0)$ nuqtada, Oy o‘qni $(0;-6)$ nuqtada kesib o‘tuvchi to‘g‘ri chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziqning tenglamasini tuzing.

III. Berilgan to‘g‘ri chiziqlarning kesishish nuqtasidan berilgan to‘g‘ri chiziqqa perpendikulyar bo‘lib o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzish

210. $x+2y+4=0$ va $3x-y-9=0$ to‘g‘ri chiziqlarning kesishish nuqtasidan $x+y-7=0$ to‘g‘ri chiziqqa perpendikulyar bo‘lib o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzing.

211. To‘g‘ri chiziq Ox o‘qni $(-2;0)$ nuqtada, Oy o‘qni $(0;-3)$ nuqtada kesib o‘tuvchi to‘g‘ri chiziqqa perpendikulyar bo‘lib, $x+y-5=0$ va $x-y+3=0$ to‘g‘ri chiziqlarning kesishish nuqtasidan o‘tadi. Bu to‘g‘ri chiziqning tenglamasini tuzing.

IV. Berilgan nuqtalarni tutashtiruvchi kesmaning o‘rtasidan unga perpendikulyar bo‘lib o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzish

212. To‘g‘ri chiziq $A(-2;1)$ va $B(4;4)$ nuqtalarni tutashtiruvchi kesmaning o‘rtasidan bu kesmaga perpendikulyar bo‘lib o‘tadi. Bu to‘g‘ri chiziqning tenglamasini tuzing.

Yechilishi. Izlanayotgan to‘g‘ri chiziq o‘tadigan nuqtani topamiz. Bu nuqta AB kesmaning o‘rtasidir (uni C bilan belgilaymiz):

$$x_C=\frac{x_A+x_B}{2}=\frac{-2+4}{2}=1, \quad y_C=\frac{y_A+y_B}{2}=\frac{1+4}{2}=\frac{5}{2}; \quad C(1;\frac{5}{2}).$$

Izlanayotgan to‘g‘ri chiziqning burchak koeffitsientini hisoblash uchun AB to‘g‘ri chiziqning burchak koeffitsientini (2.19) formula bo‘yicha topamiz:

$$k_{AB}=\frac{y_B-y_A}{x_B-x_A}=\frac{4-1}{4-(-2)}=\frac{1}{2}.$$

bu yerdan izlanayotgan to‘g‘ri chiziqning burchak koeffitsienti $k=-2$ ekanligi kelib chiqadi.

(2.17) teiglamaga $k=-2$ qiymatni va $C\left(1;\frac{5}{2}\right)$ nuqtaning koordinatalarini qo‘yib

$$y-\frac{5}{2}=-2(x-1), \text{ yoki } \ 4x+2y-9=0$$

tenglamalarni hosil qilamiz.

213. To‘g‘ri chiziq $(-1;4)$ va $(3;-2)$ nuqtalarni tutashtiruvchi kesmaning o‘rtasidan bu kesmaga perpendikulyar bo‘lib o‘tadi. Bu to‘g‘ri chiziqning tenglamasini tuzing.

214. To‘g‘ri chiziq $3x-7y+21=0$ to‘g‘ri chiziqning koordinata o‘qlari orasiga joylashgan kesmasining o‘rtasidan bu kesmaga perpendikulyar bo‘lib o‘tadi. Bu to‘g‘ri chiziqning tenglamasini tuzing.

V. Uchburchak balandliklarining tenglamalarini ular uchlarining koordinatalari bo‘yicha tuzish

215. Uchlari $A(4;2)$, $B(6;-5)$ va $C(-5;4)$ nuqtalar bo‘lgan uchburchak berilgan. Uchburchak balandliklarining tenglamalarini tuzing.

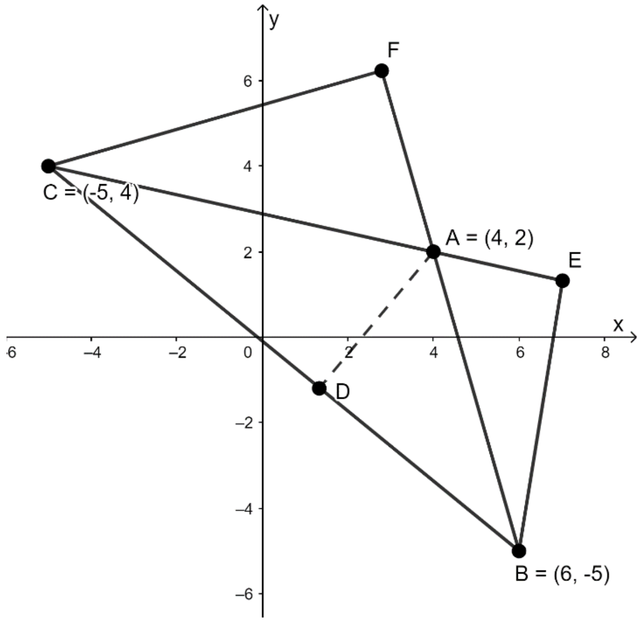
Yechilishi. Uchburchakning balandliklarini mos ravishda AD, BE va CF bilan belgilaymiz (30-rasm).

Uchburchak tomonlari burchak koeffitsientlarini (2.19) formula bo‘yicha hisoblaymiz:

1) $k_{BC}=\dfrac{y_C-y_B}{x_C-x_B}=\dfrac{4-(-5)}{-5-6}=-\dfrac{9}{11}$;

2) $k_{AC}=\dfrac{y_C-y_A}{x_C-x_A}=\dfrac{4-2}{-5-4}=-\dfrac{2}{9}$;

3) $k_{AB}=\dfrac{y_B-y_A}{x_B-x_A}=\dfrac{-5-2}{6-4}=-\dfrac{7}{2}$.



30-rasm

Uchburchak balandliklarining burchak koeffitsientlarini (2.22) formula bo‘yicha hisoblaymiz:

1) $k_{AD}=-\dfrac{1}{k_{AD}}=\dfrac{11}{9}$; 2) $k_{BE}=-\dfrac{1}{k_{AC}}=\dfrac{9}{2}$; 3) $k_{CF}=-\dfrac{1}{k_{AB}}=\dfrac{2}{7}$.

Uchburchak balandliklarining tenglamalarini uning berilgan uchlarining koordinatalari va tegishli balandliklarning burchak koeffitsientlari bo‘yicha (2.17) formulaga ko‘ra hisoblaymiz:

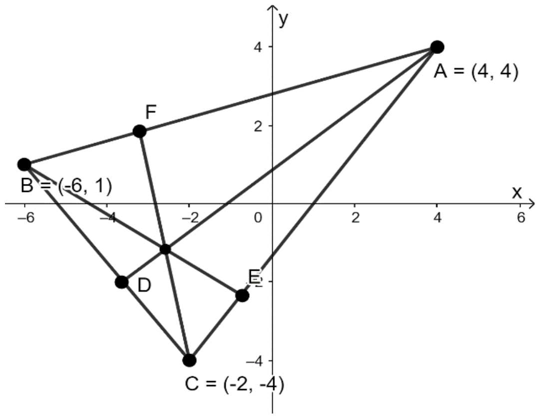
$AD: y-y_A=k_{AD}(x-x_A)$, $y-2=\dfrac{11}{9}(x-4)$ yoki $11x-9y-26=0$;

$BE: y-y_B=k_{BE}(x-x_B)$, $y+4=\dfrac{9}{2}(x-6)$ yoki $9x-2y-64=0$;

$CF: y-y_C=k_{CF}(x-x_C)$, $y-4=\dfrac{2}{7}(x+5)$ yoki $2x-7y+38=0$.

216. Uchlari 1) $A(-4;2)$, $B(6;5)$ va $C(1;-4)$; 2) $(2;-3)$, $(7;2)$ va $(-8;-2)$ nuqtalarda bo‘lgan uchburchak balandliklarining tenglamalarini tuzing.

VI. Uchburchakning tomonlari tenglamalari bo‘yicha uning balandliklari tenglamalarini tuzish



31-rasm

217. Uchburchak tomonlarining tenglamalari berilgan:

$3x - 10y + 28 = 0,\quad 5x + 4y + 26 = 0$ va $4x - 3y - 4 = 0.$ Uchburchak balandliklarining tenglamalarini tuzing.

Yechilishi. Aytaylik, birinchi tenglama AB tomonning, ikkinchi tenglama BC tomonning va uchinchi tenglama CA tomonning tenglamasi bo‘lsin. A, B va C uchlardan tegishli tomonlarga tushirilgan balandliklarni AD, BE va CF bilan belgilaylik. Ushbu tenglamalar sistemasini yechib, uchburchak uchlarining koordinatalarini topamiz (31-rasm):

$$\begin{cases} 3x - 10y + 28 = 0, \\ 5x + 4y + 26 = 0; \end{cases} B(-6;1); \qquad \begin{cases} 3x - 10y + 28 = 0, \\ 4x - 3y - 4 = 0; \end{cases} A(4;4);$$

$$\begin{cases} 5x + 4y + 26 = 0 \\ 4x - 3y - 4 = 0; \end{cases} C(-2;-4)$$

Tomonlar tenglamalaridan ularning burchak koeffitsientlarini topamiz:

$$k_{AB} = \frac{3}{10},\quad k_{BC} = -\frac{5}{4}\quad \text{va}\quad k_{CA} = \frac{4}{3}.$$

Uchburchak balandliklarining burchak koeffitsientlari (2.22) formula bo‘yicha quyidagicha bo‘ladi:

$$k_{AD} = -\frac{1}{k_{BC}} = \frac{4}{5},\qquad k_{BE} = -\frac{1}{k_{CA}} = -\frac{3}{4}\qquad \text{va}\qquad k_{CF} = -\frac{1}{k_{AB}} = -\frac{10}{3}.$$

(2.17) formula bo‘yicha uchburchak balandliklarining tenglamalarini to‘zamiz:

$AD: y - y_A = k_{AD}(x - x_A)$, $y - 4 = \frac{4}{5}(x-4)$ yoki $4x - 5y + 4 = 0$;

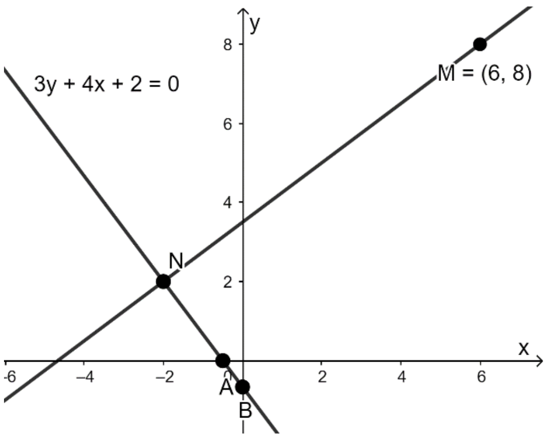
$BE: y - y_B = k_{BE}(x - x_B)$, $y - 1 = -\frac{3}{4}(x+6)$ yoki $3x + 4y + 14 = 0$;

$CF: y - y_C = k_{CF}(x - x_C)$, $y + 4 = -\frac{10}{3}(x+2)$ yoki $10x + 3y + 32 = 0$.

218. Uchburchak tomonlarining tenglamalari berilgan: 1) $11x + 2y - 21 = 0$, $8x - 3y + 7 = 0$ va $3x + 5y + 21 = 0$; 2) $2x - y + 5 = 0$; $x + y - 5 = 0$ va $x - 2y - 5 = 0$. Uchburchak balandliklarining tenglamalarini tuzing.

VII. Berilgan nuqtadan berilgan to‘g‘ri chiziqqacha bo‘lgan masofani hisoblash

219. $M(6;8)$ nuqtadan $4x + 3y + 2 = 0$ to‘g‘ri chiziqqacha bo‘lgan masofani hisoblang.



32-rasm

Yechilishi. M nuqtadan berilgan to‘g‘ri chiziqqacha bo‘lgan masofa M nuqtadan shu to‘g‘ri chiziqqa tushirilgan perpendikulyar kesmasining o‘zunligiga teng (32-rasm). Bu perpendikulyarning tenglamasini tuzamiz va uning berilgan to‘g‘ri chiziq bilan kesishish nuqtasini topamiz. Berilgan to‘g‘ri chiziqning (bu to‘g‘ri chiziqni AB deylik) burchak koeffitsienti: $k_{AB} = -\frac{4}{3}$ ga teng. MN perpendikulyarning burchak koeffitsientini esa (2.22) formula bo‘yicha topamiz: $k_{MN} = -\frac{1}{k_{AB}} = \frac{3}{4}$.

Perpendikulyarning tenglamasini (2.17) formula bo‘yicha tuzamiz:

$$y-8=\frac{3}{4}(x-6) \quad \text{yoki} \quad 3x-4y+14=0.$$

N nuqtaning koordinatalarini hisoblash uchun ushbu sistemani yechamiz:

$$\begin{cases} 4x+3y+2=0\ ; \\ 3x - 4y+14=0; \end{cases} \quad N(-2;2)$$

MN masofani (1.1) formula bo‘yicha topamiz:

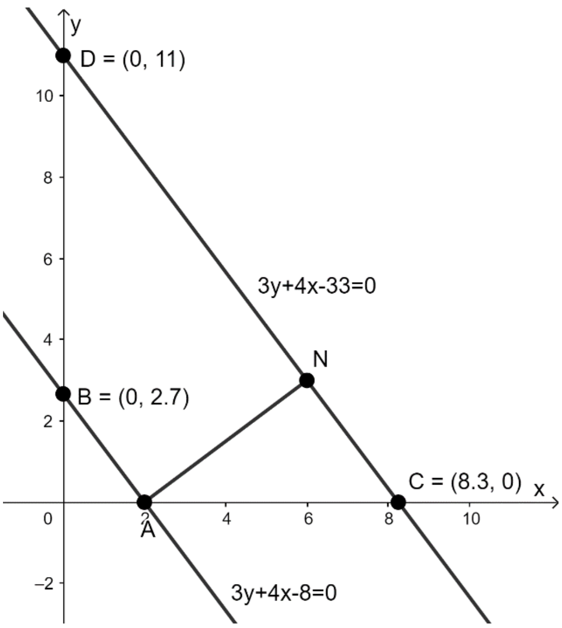
$$MN=\sqrt{(6+2)^2+(8-2)^2}=10.$$

220. $M(-2;4)$ nuqtadan $4x-3y-5=0$ to‘g‘ri chiziqqacha bo‘lgan masofani toping.

221. $(4;6)$ nuqtadan $3x+4y+14=0$ to‘g‘ri chiziqqacha bo‘lgan masofani toping.

VIII. Ikkita parallel to‘g‘ri chiziq orasidagi masofani hisoblash

222. $4x+3y-8=0$ va $4x+3y-33=0$ parallel to‘g‘ri chiziqlar orasidagi masofani toping.



33-rasm

Yechilishi. To‘g‘ri chiziqlarning istalgan biridagi ixtiyoriy nuqta orqali unga ikkinchi to‘g‘ri chiziq bilan kesishguncha perpendikulyar o‘tkazamiz. Bu perpendikulyarning berilgan to‘g‘ri chiziqlar bilan kesishish nuqtalari koordinatalarini hisoblab, bu nuqtalar orasidagi masofani topamiz. Endi to‘g‘ri chiziqlarni yasaymiz mumkin. $4x+3y-8=0$ to‘g‘ri chiziq koordinata o‘qlari bilan $A(2;0)$ va

$B\left(0;\dfrac{8}{3}\right)$ nuqtalarda, $4x+3y-33=0$ to‘g‘ri chiziq esa $C\left(\dfrac{33}{4};0\right)$ va $D(0;1)$ nuqtalarda kesishadi (33-rasm). Hisoblashlarni echishning yuqoridagi rejasi bo‘yicha bajaramiz.

1. $4x+3y-8=0$ to‘g‘ri chiziqda uning Ox o‘q bilan kesishish nuqtasi $A(2;0)$ ni olamiz.

2. AB to‘g‘ri chiziqning burchak koeffitsientini topamiz: $k_{AB}=-\dfrac{4}{3}$.

3. A nuqtadan o‘tib, AB to‘g‘ri chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziqning burchak koeffitsientini (2.22) formula bo‘yicha topamiz (bu perpendikulyarni AN opqali belgilaymiz): $k_{AN}=-\dfrac{1}{k_{AB}}=\dfrac{3}{4}$.

4. Bu perpendikulyarning tenglamasini (2.17) formula bo‘yicha tuzamiz:

$y-0=\dfrac{3}{4}(x-2)$ yoki $3x-4y-6=0$.

5. CD va AN to‘g‘ri chiziqlarning tenglamalaridan iborat sistemani yechib, AN perpendikulyarning CD to‘g‘ri chiziq bilan kesishish nuqtasini topamiz:

$$\begin{cases} 4x+3y-33=0, \\ 3x-4y-6=0; \end{cases} \qquad N(6;3)$$

6 . (1.1) formula bo‘yicha A va N nuqtalar orasidagi masofani hisoblaymiz:

$$AN=\sqrt{(2-6)^2+(0-3)^2}=5.$$

Parallel to‘g‘ri chiziqlar orasidagi masofa 5 ga teng.

223. 1) $4x+3y+33=0$ va $4x+3y-17=0$; 2) $2x+5y-101=0$ va $2x+5y+68=0$ parallel to‘g‘ri chiziqlar orasidagi masofani toping.

15- §. Aralash masalalar

224. k koeffitsientining qanday qiymatida $y=kx+9$ to‘g‘ri chiziq $x-y+5=0$ va $x+2y+2=0$ to‘g‘ri chiziqlarning kesishish nuqtasidan o‘tadi?

225. To‘g‘ri chiziq $M(2;5)$ nuqtadan Ox o‘qqa $arctg3$ burchak ostida o‘tadi. Bu to‘g‘ri chiziqda abssissasi -2 bo‘lgan nuqtani toping.

226. $x+2y-4=0$ to‘g‘ri chiziqning koordinata o‘qlari orasida joylashgan kesmasini koordinatalar boshidan o‘tuvchi ikkita to‘g‘ri chiziq 1:2:1 kabi nisbatda bo‘ladi. Bu to‘g‘ri chiziqlarning tenglamalarini tuzing.

227. $\frac{x}{9}+\frac{y}{3}=1$ to‘g‘ri chiziqning koordinata o‘qlari orasida joylashgan kesmasini koordinatalar boshidan o‘tuvchi ikkita to‘g‘ri chiziq uchta teng bo‘lakka bo‘ladi. Bu to‘g‘ri chiziqlarning tenglamalarini tuzing.

228. Uchburchak tomonlarining tenglamalari berilgan: $6x-5y+8=0$, $4x+y-38=0$ va $x-3y-3=0$. Uning medianalari tenglamalarini toping.

229. Uchburchak tomonlarining tenglamalari berilgan $4x-5y+22=0$, $5x-2y+2=0$ va $x+3y+14=0$. Bu uchburchak medianalarining kesishish nuqtasidan va $(1;-3)$ nuqtadan o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzing.

230. $2x+3y-18=0$ to‘g‘ri chiziqda shunday nuqtani topingki, u Ox o‘qdan Ox o‘qqa qaraganda uch baravar narida bo‘lsin.

231. Koordinatalar boshidan o‘tuvchi va Ox o‘q bilan $y=0,2x$ to‘g‘ri chiziq Ox bilan hosil qilgan burchakka qaraganda ikki marta katta burchak hosil qiluvchi to‘g‘ri chiziqning tenglamasini tuzing.

232. $(8;5)$ nuqtadan o‘tuvchi va Ox o‘q bilan $x-4y+4=0$ to‘g‘ri chiziq Ox o‘q bilan hosil qilgan burchakka qaraganda ikki marta katta burchak hosil qiladigan to‘g‘ri chiziqning tenglamasini tuzing.

233. $(-7;8)$ nuqtadan o‘tuvchi va $3x-5y+15=0$ to‘g‘ri chiziq bilan 45^0 li burchak hosil qiluvchi to‘g‘ri chiziqlarning tenglamalarini toping.

234. $5x-4y-20=0$ to‘g‘ri chiziqning koordinata o‘qlari bilan kesishgan nuqtalaridan unga o‘tkazilgan ikkita perpendikulyarning tenglamalarini toping.

235. Uchburchak uchlari bilan berilgan: $A(-5;-2)$, $B(7;6)$ va $C(5;-4)$. Quyidagilarni toping: 1) AB tomonning tenglamasi; 2) A uchdan BC tomonga o‘tkazilgan mediananing tenglamasi; 3) C uchdan AB tomonga o‘tkazilgan balandlikning tenglamasi; 4) B va C burchaklar; 5) bu uchburchakning og‘irlik markazi.

236. $x-y-7=0$ va $x-y+3=0$ parallel to‘g‘ri chiziqlar berilgan. Bu to‘g‘ri chiziqlarga parallel bo‘lib, ular orasidagi masofani boshlang‘ich ordinatasi kichik bo‘lgan to‘g‘ri chiziqdan boshlang‘ich ordinatasi katta bo‘lgan to‘g‘ri chiziq tomon yunalishda 3:2 nisbatda bo‘luvchi to‘g‘ri chiziqning tenglamasini tuzing.

237. $A(-4;2)$ va $B(8;4)$ nuqtalardan o‘tuvchi to‘g‘ri chiziqqa AB masofani (A dan B ga qarab) 3:4 kabi nisbatda bo‘luvchi nuqtada perpendikulyar o‘tkazilgan. Perpendikulyarning tenglamasini tuzing.

238. Rombning ikkita tomonining tenglamasi $3x-10y+37=0$ va $9x+2y-17=0$ hamda diagonallaridan birining tenglamasi $3x-2y-19=0$ berilgan. Rombning qolgan ikkita tomonining va ikkinchi diagonalining tenglamalarini toping.

239. Parallelogrammning ikkita tomoni tenglamasi $3x-2y+12=0$ va $x-3y+11=0$ hamda uning diagonallarini kesishish nuqtasi $(2;2)$ berilgan. Parollelogrammning qolgan ikkita tomonining va diagonallarining tenglamalarini tuzing.

240. Parallelogrammning bir uchidan chiqqan ikkita tomonining tenglamasi $5x-3y+28=0$ va $x-3y-4=0$ hamda bu uchga qarama-qarshi uchning koordinatalari $(10;6)$ berilgan. Parallelogrammning qolgan ikkita tomonining va diagonallarining tenglamalarini tuzing.

241. Kvadratning ikkita qarama-qarshi uchi $A(-1;1)$ va $C(5;3)$ nuqtalarda joylashgan. Bu kvadrat tomonlarining va diagonallarining tenglamalarini tuzing.

242. Agar to‘g‘ri burchakli teng yonli uchburchak gipotenuzasining tenglamasi $x-2y-3=0$ bo‘lib, to‘g‘ri burchakning uchi $C(1;6)$ nuqtada bo‘lsa, bu uchburchak katetlarining tenglamalarini tuzing.

243. Yorug‘lik nuri $A(3;10)$ nuqtadan chiqib, $2x+y-6=0$ to‘g‘ri chiziqdan qaytadi va qaytgandan so‘ng $B(7;2)$ nuqtadan o‘tadi. Tushuvchi va qaytuvchi nurlarning tenglamalarini tuzing.

Nazorat uchun misollar

I variant

244. Uchburchak uchlari bilan berilgan: $A(-7;3)$, $B(2;-1)$ va $C(-1;-5)$. Quyidagilarni toping: 1) BC tomonga parallel bo‘lgan AN tomonning tenglamasi; 2) AD medainaning tenglamasi; 3) BF balandlikning tenglamasi; 4) B burchak; 5) CP bissektrisaning tenglamasi.

II variant

245. Uchburchak uchlari bilan berilgan: $A(-8;-2)$, $B(2;10)$ va $C(4;4)$. Quyidagilarni toping: 1) AC tomonga parallel bo‘lgan BN tomonning tenglamasi; 2) CD medainaning tenglamasi; 3) AE balandlikning tenglamasi; 4) B burchak; 5) bu uchburchakning og‘irlik markazi.

3- BOB

TEKISLIKDA NUQTALARNING GEOMETRIK O‘RNI. IKKINCHI TARTIBLI EGRI CHIZIQLAR

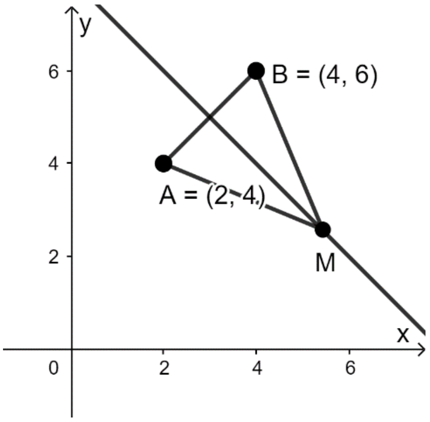
16- §. Tekislikda nuqtalarning geometrik o‘rni

Tekislikda bir xil xossalarga ega bo‘lgan va bu xossalari ularni tekislikning boshqa nuqtalaridan ajratib turuvchi nuqtalar to‘plami tekislikda *nuqtalarning geometrik o‘rni* deyiladi.

x va y o‘zgaruvchili tenglamaga tekislikda nuqtalarning geometrik o‘rni sifatida biror chiziq mos kelib, uning koordinatalari bu tenglamani qanoatlantiradi. Aksincha, tekislikda nuqtalarning geometrik o‘rnidan iborat chiziqda x va y o‘zgaruvchili biror tenglama mos keladi.

Masala shartiga ko‘ra tekislikdagi nuqtalar geometrik o‘rnining tenglamasini tuzish uchun x va y o‘zgaruvchi kattaliklar (geometrik o‘rinning ixtiyoriy nuqtasining koordinatalari) bilan masalada berilgan o‘zgarmas kattaliklar (parametrlar) orasida munosabat o‘rnatish va bu munosabatni tenglama qilib yozish kerak.

246. Tekislikda $A(2;4)$ va $B(4;6)$ nuqtalardan teng uzoqlashgan nuqtalarning geometrik o‘rnini toping.



34-rasm

Yechilishi. Geometriya kursidan ma’lumki, berilgan kesmaga uning o‘rtasidan o‘tkazilgan perpendikulyar shu kesma uchlaridan teng uzoqlashgan nuqtalarning geometrik o‘rnidir. Bu xossadan tenglama tuzish uchun foydalanamiz.

$M(x;y)$ nuqta shu nuqtalarning geometrik o‘rniga tegishli bo‘lsin (34- rasm), u holda $MA = MB$.

(1.1) formula bo‘yicha:

$$MA = \sqrt{(x-2)^2 + (y-4)^2} \quad \text{va} \quad MB = \sqrt{(x-4)^2 + (y-6)^2}$$

yoki

$$\sqrt{(x-2)^2 + (y-4)^2} = \sqrt{(x-4)^2 + (y-6)^2}.$$

Buning chap va o‘ng qismlarini kvadratga ko‘targandan so‘ng

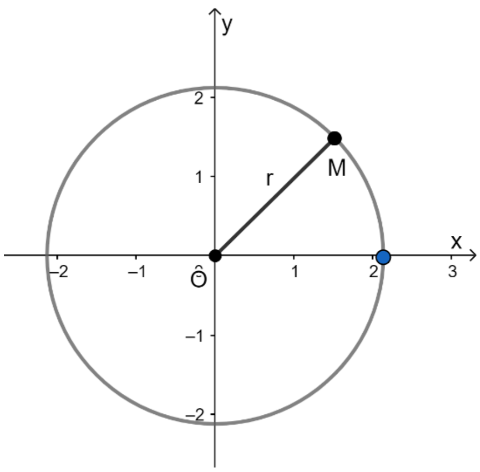
$$(x-2)^2 + (y-4)^2 = (y-4)^2 + (y-6)^2$$

tenglikni hosil qilamiz.

Bu tenglamani soddalashtirib,

$$x + y - 8 = 0$$

tenglamani hosil qilamiz.



35-rasm

Masala shartida ko‘rsatilgan xossaga ega bo‘lgan nuqtalarning geometrik o‘rni to‘g‘ri chiziqdan iboratdir.

247. Tekislikda $A(-4;2)$ va $B(6;-8)$ nuqtalardan teng uzoqlashgan nuqtalarning geometrik o‘rni tenglamasini tuzing.

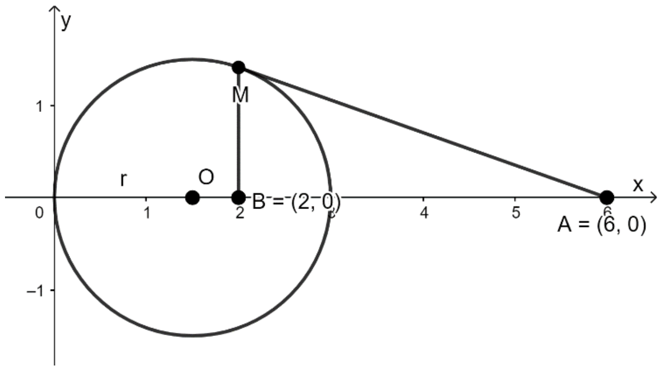
248. Tekislikda koordinatalar boshidan r masofaga uzoqlashgan nuqtalarning geometrik o‘rnini toping.

Yechilishi. Masala shartidan, izlanayotgan nuqtalarning geometrik o‘rniga tegishli bo‘lgan ixtiyoriy $M(x;y)$ nuqta uchun $OM = r$ tenglikning o‘rinli bo‘lishi kelib chiqadi, biroq

$$OM = \sqrt{x^2 + y^2}, \quad \sqrt{x^2 + y^2} = r, \quad x^2 + y^2 = r^2.$$

Nuqtalarning bu geometrik o‘rni markazi koordinatalar boshida bo‘lgan aylanadir (35-rasm).

249. Tekislikdagi shunday nuqtalarning geometrik o‘rni tenglamasini tuzingki, ularning har biridan $A(6;0)$ nuqtagacha bo‘lgan masofa ulardan $B(2;0)$ nuqtagacha bo‘lgan masofadan uch marta katta bo‘lsin.



36-rasm

Yechilishi. Masala shartidan, izlanayotgan geometrik o‘ringa tegishli istalgan $M(x;y)$ nuqta uchun $MA = 3\cdot MB$ tenglik o‘rinli ekanligi kelib chiqadi (36-rasm).

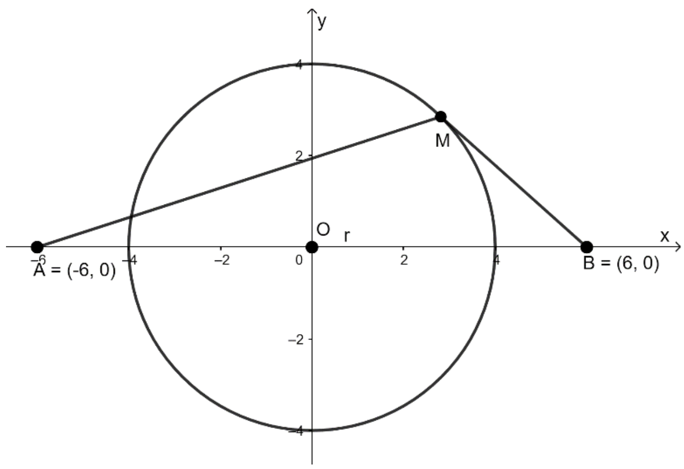
(1.1) formula bo‘yicha topamiz:

$$MA = \sqrt{(x-6)^2 + y^2} \text{ va } MB = \sqrt{(x-2)^2 + y^2}$$

yoki

$$\sqrt{(x-6)^2 + y^2} = 3\sqrt{(x-2)^2 + y^2};$$

$$\sqrt{(x-6)^2 + y^2} = 9(x-2)^2 + 9y^2.$$



37-rasm

Soddalashtirishlardan so‘ng:

$$x^2 + y^2 - 3x = 0.$$

250. Tekislikda shunday nuqtalarning geometrik o‘rni tenglamasini tuzingki, ularning har biridan $A(12;0)$ nuqtagacha bo‘lgan masofa ulardan $B(3;0)$ nuqtagacha bo‘lgan masofadan ikki marta katta bo‘lsin.

251. Tekislikda berilgan $A(-6;0)$ va $B(6;0)$ nuqtalargacha bo‘lgan masofalari kvadratlarining yig‘indisi o‘zgarmas kattalik 104 ga teng bo‘lgan nuqtalarning geometrik o‘rni tenglamasini tuzing.

Yechilishi. Masala shartidan, izlanayotgan nuqtalarning geometrik o‘rniga tegishli bo‘lgan ixtiyoriy $M(x;y)$ nuqta uchun $MA^2+MB^2=104$ munosabatning o‘rinli ekanligi kelib chiqadi (37-rasm). (1.1) formula bo‘yicha

$$MA=\sqrt{(x+6)^2+y^2}, MB=\sqrt{(x-6)^2+y^2}$$

yoki:

$$(\sqrt{(x+6)^2+y^2})^2+(\sqrt{(x-6)^2+y^2})^2=104\,;$$

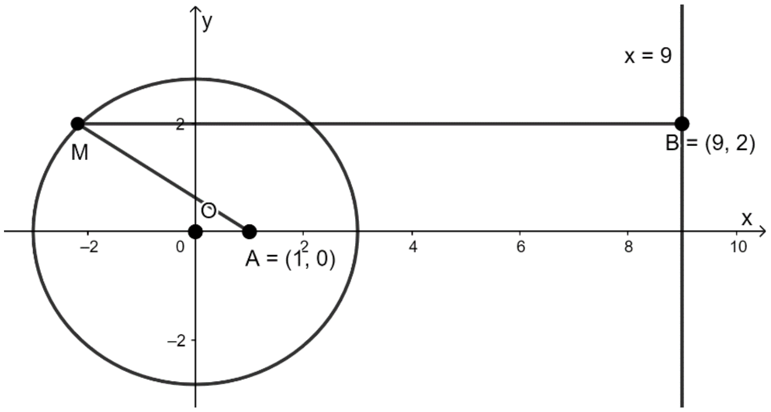
$$(x+6)^2+y^2+(x-6)^2+y^2=104$$

tenglamaga ega bo‘lamiz. Soddalashtirishlardan so‘ng:

$$x^2+y^2=16$$

tenglamani hosil qilamiz.

252. Tekislikda berilgan $A(0;-2)$ va $B(0;2)$ nuqtalargacha bo‘lgan masofalari kvadratlarining yig‘indisi o‘zgarmas kattalik 33 ga teng bo‘lgan nuqtalarning geometrik o‘rni tenglamasini tuzing.



38-rasm

253. Tekislikda $A(1;0)$ nuqtagacha va $x=9$ to‘g‘ri chiziqqacha bo‘lgan masofalariniig nisbati $\lambda=\dfrac{1}{3}$ ga teng bo‘lgan nuqtalarning geometrik o‘rni tenglamasini tuzing.

Yechilishi. Masala shartidan, izlanayotgan nuqtalarning geometrik o‘rniga tegishli bo‘lgan ixtiyoriy $M(x;y)$ nuqta uchun $\frac{MA}{MB}=\frac{1}{3}$ munosabatning o‘rinli ekanligi kelib chiqadi (38-rasm).

(1.1) formula bo‘yicha:

$$MA=(x-1)^2+y^2;$$

$$MB=\sqrt{(x-9)^2+(y-y)^2}=\sqrt{(x-9)^2}=|x-9|.$$

MB ning qiymatini absolyut kattaligi bo‘yicha olamiz, chunki kesmaning uzunligi musbat sondir:

$$\frac{\sqrt{(x-1)^2+y^2}}{|x-9|}=\frac{1}{3}; \quad 3\sqrt{(x-1)^2+y^2}=|x-9|.$$

Chap va o‘ng tomonlarni kvadratga ko‘taramiz:

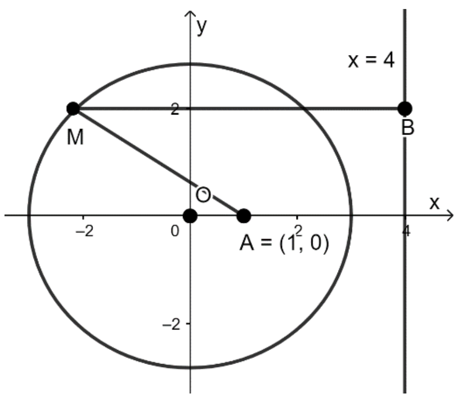
$$9(x-1)^2+9y^2=|x-9|^2.$$

Soddalashtirishlardan so‘ng:

$$8x^2+9y^2=72 \text{ yoki } \frac{x^2}{9}+\frac{y^2}{8}=1$$

tenglamani hosil qilamiz.

254. Tekislikda $A(3;0)$ nuqtagacha va $x=12$ to‘g‘ri chiziqqacha bo‘lgan masofalarining nisbati $\lambda=\frac{1}{2}$ ga teng bo‘lgan nuqtalarning geometrik o‘rni tenglamasini toping.



39-rasm

255. O‘zining tekislikdagi harakati davomida $A(1;0)$ nuqtaga $x=4$ to‘g‘ri chiziqqa nisbatan ikki marta yaqinda qoladigan M nuqtaning harakat traektoriyasi tenglamasini tuzing.

Yechilishi. Masala shartidan, izlanayotgan nuqtalarning geometrik o‘rniga tegishli bo‘lgan ixtiyoriy $M(x;y)$ nuqta uchun $2\cdot MA = MB$ tenglik o‘rinli bo‘lishi kelib chiqadi (39-rasm).

(1.1) formula bo‘yicha:

$$MA = \sqrt{(x-1)^2 + y^2} \quad \text{va}$$

$$MB = \sqrt{(x-4)^2 + (y-y)^2} = \sqrt{(x-4)^2} = |x-4|$$

yoki $\quad 2\sqrt{(x-1)^2 + y^2} == |x-4|$.

Chap va o‘ng tomonlarni kvadratga ko‘taramiz:

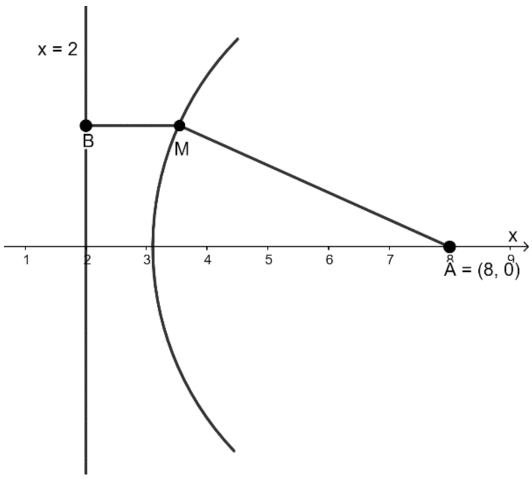
$$4(x-1)^2 + 4y^2 = (x-4)^2 .$$

Soddalashtirishlardan so‘ng:

$$3x^2 + 4y^2 - 12 \quad \text{yoki} \quad \frac{x^2}{4} + \frac{y^2}{3} = 1$$

tenglamani hosil qilamiz.

256. O‘zining tekislik bo‘ylab harakati davomida $y=9$ to‘g‘ri chiziqdan $A(0;1)$ nuqtaga qaraganda uch marta uzoqda qoladigan M nuqtaning traektoriya tenglamasini tuzing.



40-rasm

257. Tekislikda shunday nuqtalarning geometrik o‘rni tenglamasini tuzingki, ularning har biridan $x=2$ to‘g‘ri chiziqqacha bo‘lgan masofa ulardan $A(8;0)$ nuqtagacha bo‘lgan masofaga qaraganda ikki marta yaqin bo‘lsin.

Yechilishi. Masala shartidan, nuqtalarning geometrik o‘rniga tegishli bo‘lgan ixtiyoriy $M(x;y)$ nuqta uchun $2\cdot MB = MA$ tenglik o‘rinli bo‘lishi kelib chiqadi (40- rasm).

(1.1) formula bo‘yicha:

$$MB = \sqrt{(x-2)^2 + (y-y)^2} = \sqrt{(x-2)^2} = |x-2| \text{ va}$$

$$MA = \sqrt{(x-8)^2 + y^2}$$

yoki

$$2\cdot|x-2| = \sqrt{(x-8)^2 + y^2}\ .$$

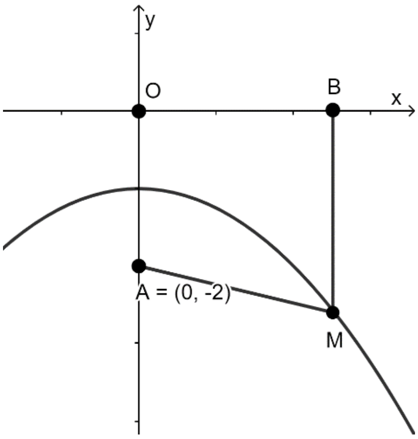
Kvadratga ko‘targandan va soddalashtirgandan so‘ng:

$$3x^2 - y^2 = 48 \quad \text{yoki} \quad \frac{x^2}{16} - \frac{y^2}{48} = 1$$

tenglamani hosil qilamiz.

258. O‘zining tekislik bo‘yicha harakati davomida $x=1$ chiziqqa $A(9;0)$ nuqtaga qaraganda uch marta yaqin qoladigan $M(x;y)$ nuqtaning traektoriya tenglamasini tuzing.

259. Tekislikda Ox o‘qdan va $A(0;-2)$ nuqtadan teng uzoqlashgan nuqtalarning geometrik o‘rni tenglamasini tuzing.



41-rasm

Yechilishi. Masala shartidan, nuqtalarning geometrik o‘rniga tegishli bo‘lgan ixtiyoriy $M(x;y)$ nuqta uchun $MA = MB$ tenglik o‘rinli bo‘lishi kelib chiqadi(41-rasm). (1.1) formula bo‘yicha:

$$MA = \sqrt{x^2 + (y+2)^2}; \quad MB = |y| \quad \text{yoki}$$

$$\sqrt{x^2 + (y+2)^2} = |y|\ .$$

Chap va o‘ng tomonlarni kvadratga ko‘tarib, so‘ngra soddalashtirib:

$$x^2+4y+4=0$$

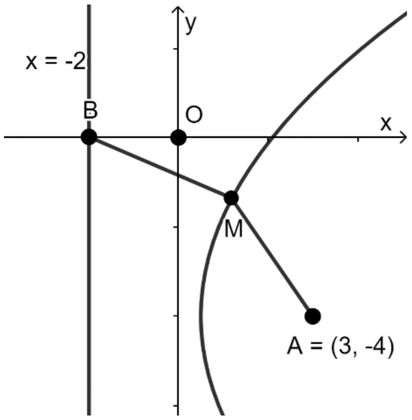
tenglamani hosil qilamiz.

260. Tekislikda Oy o‘qdan va $A(3;0)$ nuqtadan teng uzoqlashgan nuqtalarning geometrik o‘rni tenglamasini tuzing.

261. Tekislikda $y=1$ to‘g‘ri chiziq va $A(0;-3)$ nuqtadan teng uzoqlashgan nuqtalarning geometrik o‘rni tenglamasini yozing.

262. Tekislikda har biridan $x=-2$ to‘g‘ri chiziqqacha bo‘lgan masofa ulardan $A(3;-4)$ nuqtagacha bo‘lgan masofaga teng bo‘lgan nuqtalarning geometrik o‘rni tenglamasini tuzing.

Yechilishi. Masala shartidan, nuqtalarning geometrik o‘rniga tegishli bo‘lgan ixtiyoriy $M(x;y)$ nuqta uchun $MA=MB$ tenglik o‘rinli bo‘lishi kelib chiqadi (42- rasm). (1.1) formula bo‘yicha:



42-rasm

$$MA=\sqrt{(x^2-3)+(y+4)^2} \quad \text{va}$$

$$MB=\sqrt{(x^2+2)+(y-y)^2}=|x+2|$$

yoki $\sqrt{(x^2-3)+(y+4)^2}=|x+2|$.

Chap va o‘ng tomoilarni kvadratga ko‘tarib va soddalashtirib,

$$x^2+8y-10x+21=0$$

tenglamani hosil qilamiz.

263. Tekislikda har biri $A(-2;3)$ nuqtadan va $x=4$ to‘g‘ri chiziqdan teng uzoqlashgan nuqtalarning geometrik o‘rni tenglamasini tuzing.

264. Tekislikda har biri $y=-2$ to‘g‘ri chiziqdan va $A(-3;4)$ nuqtadan teng uzoqlashgan nuqtalarning geometrik o‘rni tenglamasini toping.

265. Agar tekislikda harakatlanayotgan nuqtaning harakati davomida undan $A(2;-1)$ nuqtagacha bo‘lgan masofaning kvadrati har doim undan Ox o‘qqacha bo‘lgan masofaning kvadratiga teng bo‘lsa, nuqtaning traektoriya tenglamasini tuzing.

266. Agar tekislikda berilgan nuqtaning harakati davomida undan $A(-3;4)$ nuqtagacha bo‘lgan masofaning kvadrati har doim undan Ox o‘qqacha bo‘lgan masofa kvadratining ikkilanganiga teng bo‘lsa, bu nuqtaning harakat traektoriyasi tenglamasini toping.

Yechilishi. Masala shartidan, nuqta traektoriyasining ixtiyoriy $M(x;y)$ nuqtasi uchun $(MA)^2=2(MB)^2$ tenglik o‘rinli bo‘lishi kelib chiqadi.

(1.1) formula bo‘yicha:

$$MB=\sqrt{(x+3)^2+(y-4)^2} \quad \text{va} \quad MB=|y|$$

yoki

$$(\sqrt{(x+3)^2+(y-4)^2})^2=2y^2, \quad (x+3)^2+(y-4)^2=2y^2$$

Soddalashtirgandan so‘ng ushbuni hosil qilamiz:

$$x^2-y^2+6x-8y+25=0.$$

267. Agar tekislikdagi nuqtalar geometrik o‘rni har bir nuqtasining abssissasi bu nuqtaning ordinatasi va bu nuqtani $A(1;0)$ nuqta bilan tutashtiruvchi kesmaning uzunligi orasida o‘rta proporsional bo‘lsa, bu nuqtalarning geometrik o‘rnini toping.

Yechilishi. a va b sonlarning o‘rta proporsionali deb $m=\sqrt{ab}$ songa aytilishini eslatib o‘tamiz. Masala shartidan, geometrik o‘ringa tegishli bo‘lgan ixtiyoriy $N(x;y)$ nuqta uchun ushbu tenglik o‘rinli bo‘lishi kelib chiqadi:

$$OB=\sqrt{BN\bullet AN},\quad OB=|x|,\quad BN=|y|,\quad AN=\sqrt{(x-1)^2+y^2}$$

Bu qiymatlarni birinchi tenglikka qo‘yib topamiz:

$$|x|=\sqrt{|y|\sqrt{(x-1)^2+y^2}}$$

Ifodani radikallardan qutqarib, soddalashtirgandan so‘ng, ushbuga ega bo‘lamiz:

$$x^2=|y|\sqrt{(x-1)^2+y^2};\, x^4=y^2(x-1)^2+y^2$$

yoki:

$$x^4-y^4=y^2(x-1)^2.$$

268. Agar geometrik o‘rinning har bir nuqtasini koordinatalar boshi bilan tutashtiruvchi kesma bu nuqtaning abssissasi va ordinatasi orasida o‘rta proporsional bo‘lsa, tekislikdagi bu nuqtalarning geometrik o‘rnini toping.

269. Tekislikda shunday nuqtalarning geometrik o‘rnini topingki, bu geometrik o‘rinning nuqtasini $A(-1;-2)$ nuqta bilan tutashtiruvchi to‘g‘ri chiziqning burchak koeffitsienti xuddi shu nuqtaning o‘zini $B(-4;2)$ nuqta bilan tutashtiruvchi to‘g‘ri chiziqning burchak koeffitsientidan uch marta katta bo‘lsin.

Yechilishi. Masala shartidan, izlanayotgan nuqtalarning geometrik o‘rniga tegishli bo‘lgan ixtiyoriy $M(x;y)$ nuqta uchun $k_{MA}=3k_{MB}$ tenglik o‘rinli bo‘lishi kelib chiqadi.

(2.19) formula bo‘yicha yozamiz:

$$k_{MA}=\frac{y+2}{x+1};\quad k_{MB}=\frac{y-2}{x+4}$$

k_{MA} va k_{MB} ning qiymatini yuqoridagi tenglikka qo‘yib,

$$\frac{y+2}{x+1}=3\frac{y-2}{x+4}$$

tenglikni hosil qilamiz

Soddalashtirishlardan sung,

$$2xy - 8x - y - 14 = 0$$

tenglamaga ega bo‘lamiz.

270. Tekislikda shunday nuqtalarning geometrik o‘rnini topingki, bu geometrik o‘rinning istalgan nuqtasini $A(2;3)$ nuqta bilan tutashtiruvchi to‘g‘ri chiziqning burchak koeffitsnenti geometrik o‘rinning xuddi shu nuqtasini $B(5;1)$ nuqta bilan tutashtiruvchi to‘g‘ri chiziqning burchak koeffitsientidan ikki marta kichik bo‘lsin.

17- §. Aylana

Aylana deb tekislikda berilgan nuqta (markaz)dan bir xil masofa (radius)ga uzoqlashgan nuqtalarning geometrik o‘rniga aytiladi.

Markazi koordinatalar boshida va radiusi r bo‘lgan aylananing tenglamasi

$$x^2 + y^2 = r^2 \tag{3.1}$$

bo‘ladi.

Markazi $O_1(a,b)$ nuqtada va radiusi r bo‘lgan aylananing tenglamasi

$$(x-a)^2 + (y-b)^2 = r^2 \tag{3.2}$$

bo‘ladi.

Aylananing umumiy ko‘rinishdagi tenglamasi:

$$Ax^2 + Ay^2 + Bx + Cy + D = 0 \tag{3.3}$$

Umumiy ko‘rinishdagi aylananing xususiy holi:

$$x^2+y^2+Mx+Ny+P=0 \tag{3.4}$$

(3. 1) — (3. 4) tenglamalarda x va y o‘zgaruvchi koordinatalar — aylananing istalgan nuqtasini koordinatalari. (3.3) va (3.4) tenglamalarda A, B, C, D, M, N va P — o‘zgarmas koeffitsientlar.

Shuni nazarda tutish kerakki, xususiy hollarda (3.3), shuningdek, (3.4) tenglamalarda aylananing koordinata o‘qlariga nisbatan vaziyatiga qarab, B, C yoki D va mos ravishda M, N va P koeffitsientlarning har qaysisi alohida yoki ikkitasi bir paytda nolga teng bo‘lib qolishi mumkin. Aylananing (3.4) tenglamasida M va N koeffitsientlar bilan aylana markazi $O_1(a,b)$ ning koordinatalari orasida ushbu sodda

$$a=-\frac{M}{2};\quad b=-\frac{M}{2} \tag{3.5}$$

munosabat, shuningdek, M, N va P bilan aylana radiusi r o‘rtasida

$$r=\frac{\sqrt{M^2+N^2-4P}}{2} \tag{3.6}$$

munosabat mavjud (305- masalaning Yechilishiga qarang).

I.Berilgan nuqtalarning aylanaga tegishliligini tekshirish

271. $(2;4),(7;1)$ va $(0;2)$ nuqtalarning $x^2+y^2-2x-4y=0$ aylanaga tegishliligini tekshirib ko‘ring.

272. $(-4;3)$ va $(5;0)$ nuqtalarning $x^2+y^2-4x-2y-35=0$ aylanaga tegishliligini tekshirib ko‘ring.

II. Markazi berilgan nuqtada bo‘lgan va radiusi berilgan aylananing tenglamasini tuzish

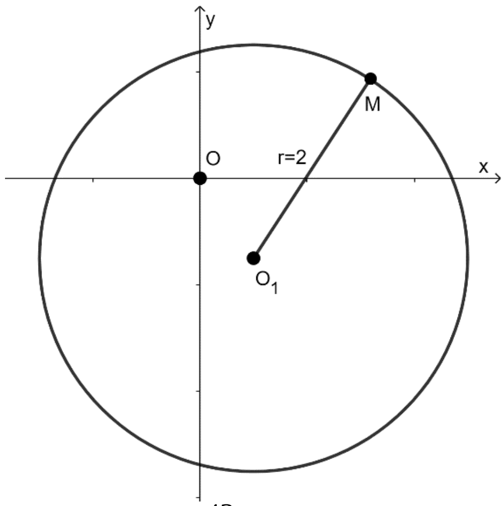
273. Markazi $\left(\frac{1}{2};-\frac{3}{4}\right)$ nuqtada va radiusi 2 ga teng bo‘lgan aylananing tenglamasini tuzing. Bu aylanani yasang.

Yechilishi. Masala shartidan:

$$a = \frac{1}{2}; \quad b = -\frac{3}{4} \text{ va } r = 2.$$

Bu qiymatlarni (3.2) tenglamaga qo‘yib, ushbuni hosil qilamiz:

$$\left(x - \frac{1}{2}\right)^2 + \left[y - \left(-\frac{3}{4}\right)\right]^2 = 2^2$$



43-rasm

Kvadratga ko‘targandan va ozod hadni chap tomonga o‘tkazgandan so‘ng aylananing (3.3) ko‘rinishdagi tenglamasini hosil qilamiz:

$$16x^2 + 16y^2 - 16x + 24y - 51 = 0.$$

Aylanani yasash: 1) aylananing markazini, ya’ni $O_1\left(\frac{1}{2}; -\frac{3}{4}\right)$ nuqtani yasaymiz: 2) O_1 markazdan 2 ga teng radius bilan aylana chizamiz (43-rasm).

274. Markazi koordinatalar boshida va radiusi 3 ga teng bo‘lgan aylananing tenglamasini tuzing.

275. Markazi $(-2; -5)$ nuqtada va radiusi 3 ga teng bo‘lgan aylananing tenglamasini tuzing. Bu aylanani yasang.

III. Markazi berilgan nuqtada bo‘lgan va berilgan nuqtadan o‘tadigan aylananing tenglamasini tuzish

276. Markazi $(5; -7)$ nuqtada bo‘lgan va $(2; -3)$ nuqtadan o‘tadigan aylananing tenglamasini tuzing.

Yechilishi. Masala shartidan izlanayotgan aylananing radiusi noma’lum ekanligini ko‘ramiz. Uni ikki usul bilan topish mumkin.

1-usul. Aylananing (3.2) tenglamasiga markazning $(5;-7)$ koordinatalarini va berilgan nuqtaning $(2;-3)$ koordinatalarini *x* va *u* o‘zgaruvchilarning o‘rniga qo‘yamiz:

$$(2-5)^2+\left[-3-(-7)\right]^2=r^2; 9+16=r^2$$

bu yerdan $r=5$.

2-usul. Radiusni aylana markazidan uning birorta berilgan nuqtasigacha bo‘lgan masofa sifatida topamiz. (1.1) formula bo‘yicha:

$$r=\sqrt{(2-5)^2+\left[-3-(-7)\right]^2}=5$$

Endi (3.2) tenglamaga markazning koordinatalarini va radiusning qiymatini qo‘yamiz:

$$(x-5)^2+\left[y-(-7)\right]^2=5^2$$

Soddalashtirib,

$$x^2+y^2-10x+14y+49=0$$

tenglamani hosil qilamiz.

277. Markazi $(-1;4)$ nuqtada bo‘lgan va $(3;5)$ nuqtadan o‘tadigan aylananing tenglamasini tuzing.

278. Markazi $(-3;0)$ nuqtada bo‘lgan va $(2;4)$ nuqtadan o‘tadigan aylananing tenglamasini tuzing.

IV. Aylananing tenglamasini uning diametri uchlarining koordinatalari bo‘yicha tuzish

279. Diametrining uchlari 1) $(0;3)$ va $(6;-7)$; 2) $(-2;3)$ va $(2;5)$ koordinatalarga ega bo‘lgan aylananing tenglamasini tuzing.

V. Aylana tenglamasini koordinata o‘qlari orasidagi kesmasi shu aylananing diametri bo‘lib xizmat qiladigan to‘g‘ri chiziqning tenglamasi bo‘yicha tuzish

280. Diametri $4x+3y-24=0$ to‘g‘ri chiziqning koordinata o‘qlari orasida joylashgan kesmasidan iborat bo‘lgan aylananing tenglamasini tuzing.

281. $5x-4y+40=0$ to‘g‘ri chiziqning koordinata o‘qlari orasida joylashgan kesmasi aylana uchun diametr bo‘lib xizmat qiladi. Aylananing tenglamasini tuzing.

VI. Markazi berilgan nuqtada bo‘lgan va koordinatalar boshidan o‘tadigan aylananing tenglamasini tuzish

282. Koordinatalar boshidan o‘tadigan va markazi 1) $(-2;3)$; 2) $(3;-5)$ nuqtada bo‘lgan aylananing tenglamasini tuzing.

VII. Berilgan aylananing koordinata o‘qlari bilan kesishish nuqtalarining koordinatalarini hisoblash

283. $3x^2+3y^2-18x+10y-48=0$ aylananing koordinata uchlari bilan kesishish nuqtalarini toping.

Yechilishi. Aylana abssissalar o‘qi bilan ordinatalari nolga teng bo‘lgan nuqtalarda kesishadi. Berilgan aylananing tenglamasida *u* ni nolga tenglab,

$$3x^2-18x-48=0 \text{ yoki } x^2-6x-16=0$$

tengliklarni hosil qilamiz.

Kvadrat tenglamani yechib,

$$x_1=-2; \quad x_2=8$$

yechimlarni topamiz.

Demak, aylana abssissalar o‘qi bilan $(-2;0)$ va $(8;0)$ nuqtalarda kesishar ekan. Aylana ordinatalar o‘qi bilan abssissalari nolga teng bo‘lgan nuqtalarda kesishadi.

Aylana tenglamasida $x=0$ deb, ushbu tenglamani hosil qilamiz:

$$3y^2-10y-48=0$$

Bu kvadrat tenglamani yechib,

$$y_1=-\frac{8}{3};\quad y_2=6$$

yechimlarni topamiz.

Demak, aylana ordinatalar uqi bilan $\left(0;-\frac{8}{3}\right)$ va $(0;6)$ nuqtalarda kesishar ekan.

284. $x^2+y^2+4x+y-12=0$ aylananing koordinata o‘qlari bilan kesishish nuqtalarining koordinatalarini toping.

VIII. Berilgan aylananing berilgan to‘g‘ri chiziq bilan kesishish nuqtalarining koordinatalarini hisoblash

285. $x^2+y^2-4x+2y-29=0$ aylana bilan $x-y-1=0$ to‘g‘ri chiziqning kesishish nuqtalarining koordinatalarini toping.

Yechilishi. Aylana va to‘g‘ri chiziqning kesishish nuqtalarining koordinatalarini topish uchun ushbu tenglamalar sistemasini echish kerak:

$$\begin{cases} x^2+y^2-4x+2y-29=0 \\ x-y-1=0 \end{cases}$$

Bu sistemaning ildizlari:

$$x_1=-3,\quad y_1=-4 \quad \text{va} \quad x_2=5,\quad y_2=4.$$

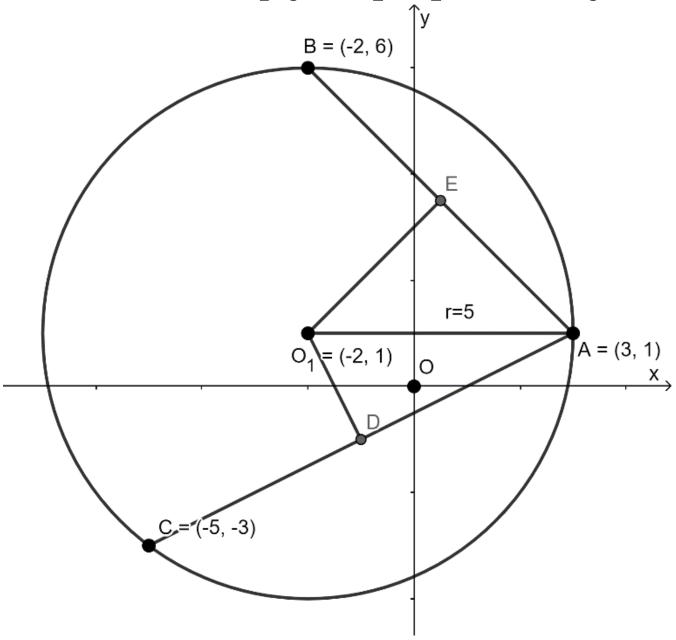
Aylana va to‘g‘ri chiziq $(-3;-4)$ va $(5;4)$ nuqtalarda kesishadi.

286. $x^2+y^2-8x-2y-8=0$ aylana bilan $4x+3y-19=0$ to‘g‘ri chiziqning kesishish nuqtalarining koordinatalarini toping.

IX. Berilgan uchta nuqtadan o‘tadigan aylananing tenglamasini tuzish

287. $A(3;1)$, $B(-2;6)$ va $C(-5;-3)$ nuqtalardan o‘tadigan aylananing tenglamasini tuzing.

Yechilishi. Geometriya kursidan ma’lumki, izlanayotgan aylananing markazi berilgan nuqtalarni tutashtiruvchi istalgan ikkita kesmaning o‘rtasidan o‘tkazilgan perpendikulyarlarning kesishish nuqtasida yotadi. Demak, aylana markazining koordinatalarini topish uchun har qaysi perpendikulyarniig tenglamasini tuzish va bu tenglamalardan iborat sistemani echish kerak. Perpendikulyarning tenglamasini tuzish uchun esa perpendikulyar o‘tadigan nuqtaning koordinatalarini va perpendikulyarning burchak koeffitsientini bilish kerak.



44-rasm

1-usul. 1. (1.4) formulalar bo‘yicha AC va AB kesmalarning o‘rtalari bo‘lgan D va E nuqtalarning koordinatalarini topamiz (44-rasm):

$$x_D=\frac{3+(-5)}{2}=-1,\quad y_D=\frac{1+(-3)}{2}=-1,\quad D(-1;-1)$$

$$x_E=\frac{3+(-2)}{2}=\frac{1}{2},\ y_E=\frac{1+6}{2}=\frac{7}{2},\ E(\frac{1}{2};\frac{7}{2}).$$

2. (2.19) va (2.22) formulalardan foydalanib, AC, DO_1, AB va EO_1 to‘g‘ri chiziqlarning burchak koeffitsientlarini topamiz:

$$k_{AC}=\frac{y_C-y_A}{x_C-x_A}=\frac{1}{2};\quad k_{DO_1}=-\frac{1}{k_{AC}}=-2\quad ;$$

$$k_{AB}=\frac{y_B-y_A}{x_B-x_A}=-1;\quad k_{BO_1}=-\frac{1}{k_{AB}}=1.$$

3. (2.17) formulani tatbiq etib, DO_1 va EO_1 perpendikulyarlarning tenglamalarini to‘zamiz:

$$y-(-1)=-2[x-(-1)],\qquad 2x+y+3=0;$$

$$y-\frac{7}{2}=1\cdot\left(x-\frac{1}{2}\right),\qquad x-y+3=0.$$

4. DO_1 va EO_1 perpendikulyarlarning tenglamalaridan tuzilgan sistemani yechib, aylana markazi O_1 ning koordinatalarini topamiz:

$$\begin{cases}2x+y+3=0\\ x-y+3=0\end{cases};\quad x=-2,\quad y=1;\quad O(-2;1)$$

5. (1.1) formuladan foydalanib, aylananing AO_1 radiusini va (3.2) formuladan foydalanib, uning tenglamasini tuzamiz:

$$r=AO_1=\sqrt{(-2-3)^2+(1-1)^2}=5$$

$$(x+2)^2+(y-1)^2=5^2$$

yoki

$$x^2+y^2+4x-2y-20=0.$$

2-usul. Aylananing markazi $O_1(a;b)$ nuqta bo‘lsin, bitta aylananing radiuslari bo‘lgani uchun $O_1A=O_1B=O_1C$.

(1.1) formula bo‘yicha:

$$O_1A=\sqrt{(a-3)^2+(b-1)^2};\quad O_1B=\sqrt{(a+2)^2+(b-6)^2};$$

$$O_1C=\sqrt{(a+5)^2+(b+3)^2}.$$

Ushbu tenglamalar sistemasini to‘zamiz hamda uni a va b noma’lumlarga nisbatan yechamiz:

$$\begin{cases} \sqrt{(a-3)^2+(b-1)^2} = \sqrt{(a+2)^2+(b-6)^2} \\ \sqrt{(a-3)^2+(b-1)^2} = \sqrt{(a+5)^2+(b+3)^2} \end{cases}$$

Soddalashtirishlardan so‘ng quyidagi sistemani hosil qilamiz:

$$\begin{cases} a-b+3=0 \\ 2a+b+3=0 \end{cases}.$$

Sistemani yechib,

$$a=-2; \quad b=1; \quad O_1(-2;1)$$

natijalarga ega bo’lamiz.

Aylananing radiusi 1-usuldagidek topiladi, aylana tenglamasi avvalgi ko‘rinishda bo‘ladi.

288. Quyidagi nuqtalardan o‘tadigan aylananing tenglamasini tuzing: 1) $A(2;8)$, $B(4;-6)$ va $C(-12;-6)$; 2) $A(-2;-6)$, $B(-3;1)$ va $C(4;2)$.

X. Tomonlari berilgan to‘g‘ri chiziqlardan iborat bo‘lgan uchburchakka tashqi chizilgan aylananing tenglamasini tuzish

289. Tomonlari ushbu to‘g‘ri chiziqlardan iborat bo‘lgan uchburchakka tashqi chizilgan aylananing tenglamasini tuzing:

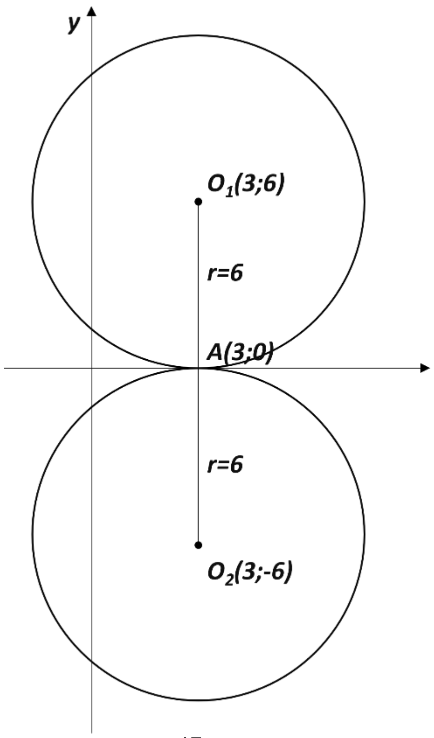
1) $x-y+4=0$, $3x+y-16=0$ va $x+y-2=0$;

2) $2x-y+2=0$, $x-3y-14=0$ va $x+y-2=0$;

3) $4x-3y-17=0$, $7x+y-61=0$ va $x-7y-73=0$.

XI. Abssissalar (ordinatalar) o‘qiga berilgan nuqtada urinadigan va berilgan radiusga ega bo‘lgan aylananing tenglamasini tuzish

290. Abssissalar o‘qiga $A(3;0)$ nuqtada urinadigan va radiusi 6 ga teng bo‘lgan aylananing tenglamasini tuzing.



45-rasm

Yechilishi. Aylananing markazi $O_1(a;b)$ nuqtada bo‘lsin (45-rasm). Urinish nuqtasining va aylana markazining abssissasi bir xil: $a = 3$.

(3.2) tenglamaga berilgan qiymatlarni qo‘yib, aylana markazining ordinatasi b ni topamiz:

$$(3-3)^2 + (0-b)^2 = 6^2; b = \pm 6$$

ya’ni ikkita markazga egamiz:

$$O_1(3;6) \quad \text{va} \quad O_2(3;-6) \quad .$$

Bu yerdan berilgan shartlarni qanoatlantiruvchi ikkita aylana tenglamasini hosil qilamiz:

$$(x-3)^2 + (y-6)^2 = 6^2$$

$$(x-3)^2 + (y+6)^2 = 6^2$$

yoki

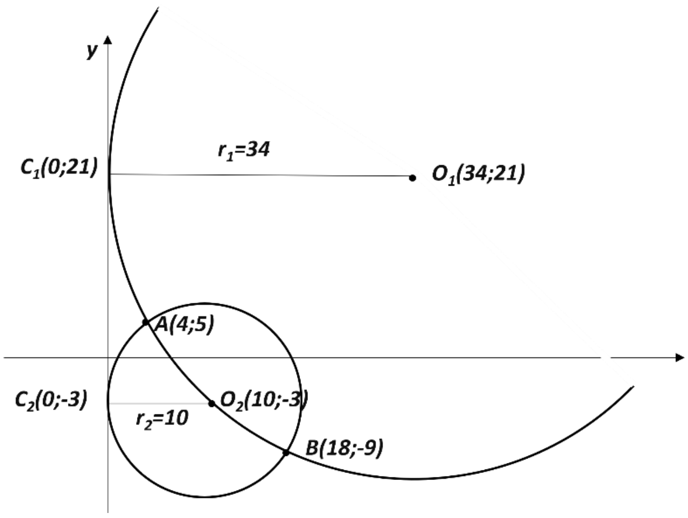
$$x^2 + y^2 - 6x - 12y + 9 = 0$$

$$x^2 + y^2 - 6x + 12y + 9 = 0$$

291. Ordinatalar o‘qiga $A(0;4)$ nuqtada urinadigan va radiusi 5 ga teng bo‘lgan aylananing tenglamasini tuzing.

XII. Abssissalar (ordinatalar) o‘qiga urinadigan va berilgan ikkita nuqtadan o‘tadigan aylana tenglamasini tuzish

292. Ordinatalar o‘qiga urinadigan hamda $A(4;5)$ va $B(18;-9)$ nuqtalardan o‘tadigan aylananing tenglamasini tuzing.



46-rasm

Yechilishi. Izlanayotgan aylananing markazi $O_1(a;b)$ nuqtada bo‘lsin (46- rasm). Urinish nuqtasi C ga (uning koordinatalari $C(0;b)$ bo‘ladi) radius o‘tkazamiz. Aylananing radiusi:

$$r = |a|.$$

Yuqoridagi (1.1) formukani qo’llanib, ushbu sistemani hosil qilamiz:

$$\begin{cases} \sqrt{(a-4)^2+(b-5)^2} = |a|, \\ \sqrt{(a-18)^2+(b+9)^2} = |a|. \end{cases}$$

Kvadratga ko’targandan va soddalashtirganadan so’ng, quyidagiga ega bo’lamiz:

$$\begin{cases} b^2-10b-8a+41=0, \\ b^2+18b-36a+405=0. \end{cases}$$

Sistemani yechib, $b_1 = 21, b_2 = -3$ va $a_1 = 34, a_2 = 10$ qiymatlarni topamiz, demak, ikkita markaz $Q_1(34;21)$ va $Q_2(10;-3)$ ga hamda ikkita radius $r_1 = 34$ va $r_2 = 10$ ga egamiz. Shunday qilib, masala shartini ushbu ikkita aylana qanoatlantiradi:

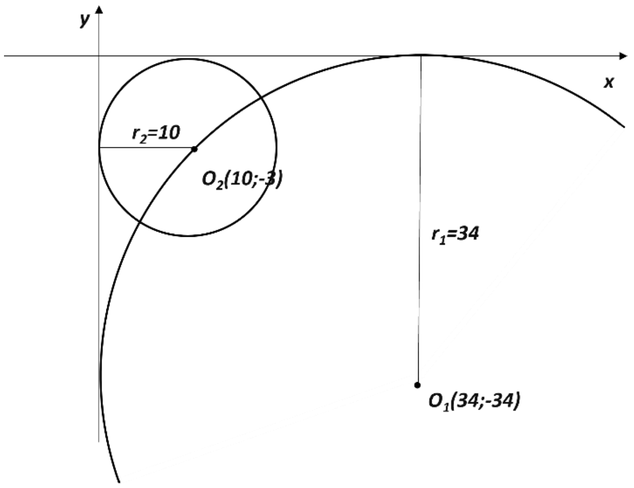
$$(x-34)^2+(y-2)^2 = 34^2 \text{ va } (x-10)^2+(y+3)^2 = 10^2 \text{ yoki}$$

$$x^2+y^2-68x-42y+441=0 \text{ va } x^2+y^2-20x+6y+9=0.$$

293. Abssissalar o'qiga urinadigan hamda $A(7;8)$ va $B(6;9)$ nuqtalardan o'tadigan aylananing tenglamasini tuzing.

XIII. Koordinata o'qlariga urinadigan va berilgan nuqtadan o'tadigan aylananing tenglamasini tuzish

294. Koordinata o'qlariga urinadigan va $A(18;-4)$ nuqtadan o'tadigan aylananing tenglamasini tuzing.



47-rasm

Yechilishi. Koordinata o'qlariga urinadigan va to'rtinchi koordinata burchagining nuqtasidan o'tadigan aylananing markazi $Q_1(a;-a)$ koordinalarga ega bo'ladi, bu yerda $a>0$. Aylananing radiusi $r=a$ (47-rasm).

Endi (1.1) formula bo'yicha $\sqrt{(a-18)^2+(-a+4)^2}=a$ tenglik o'rinli. Kvadratga ko'targandan so'ng va soddalashtirishlardan so'ng $a^2-44a+340=0$ kvadrat tenglamani hosil qilamiz, bu yerdan $a_1=34, a_2=10$. Ikkita markaz $Q_1(34;-34)$ va $Q_2(10;-10)$ ga hamda ikkita radius $r_1=34$ va $r_2=10$ ga egamiz.

Shunday qilib, masala shartini ushbu ikkita aylana qanoatlantiradi: $(x-34)^2+(y+34)^2=34^2$ va $(x-10)^2+(y+10)^2=10^2$ yoki $x^2+y^2-68x+68y+1156=0$ va $x^2+y^2-20x+20y+100=0.$

295. Koordinata o'qlariga urinadigan va $A(8;9)$ nuqtadan o'tadigan aylananing tenglamasini tuzing.

Ko'rsatma. Aylana birinchi koordinata burchagining nuqtasidan o'tganligi uchun uning markazining koordinatalari $Q_1(a;a),\ a \neq 0$ bo'ladi.

XIV. Berilgan ikkita nuqtadan o'tadigan va markazi abssissalar (ordinatalar) o'qida bo'lgan aylananing tenglamasini tuzish

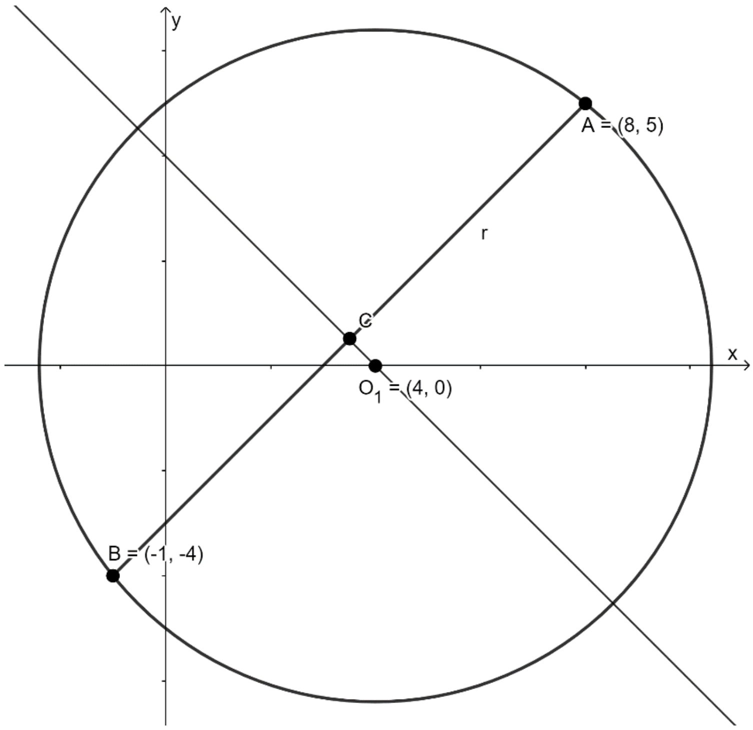
296. $A(8;5)$ va $B(-1;-4)$ nuqtalardan o'tadigan va markazi abssissalar o'qida bo'lgan aylananing tenglamasini tuzing.

Yechilishi. 1-usul. Aylananing markazi $Q_1(a;0)$ nuqta bo'lsin, u holda $Q_1A = Q_1B.$ (1.1) formulani tatbiq etsak:

$$\sqrt{(a-8)^2+(0-5)^2} = \sqrt{(a+1)^2+(0+4)^2}.$$

Bu ifodani soddalashtirib, topamiz: $18a = 72,\quad a = 4,$ ya'ni $Q_1(4;0).$

Aylananing radiusi $r = OA = \sqrt{(4-8)^2+(0-5)^2} = \sqrt{41}$ bo'ladi.



48-rasm

Aylana tenglamasi quyidagi ko'rinishga keladi:

$$\sqrt{(x-4)^2+(y-0)^2}=\left(\sqrt{41}\right)^2$$

yoki

$$x^2+y^2-8x-25=0.$$

2- usul. Yechish rejasi: 1) AB kesmaning o‘rtasi bo‘lgan C nuqtaning koordinatalari topiladi (48-rasm); 2) AB va CO_1 to‘g‘ri chiziqlarning burchak koeffitsientlari hisoblanadi; 3) CO_1 to‘g‘ri chiziqning tenglamasi tuziladi; 4) CO_1 to‘g‘ri chiziq bilan abssissalar o‘qining kesishish nuqtasini koordinatalari topiladi (tenglamalar sistemasi echiladi); 5) aylananing radiusi topiladi va uning tenglamasi tuziladi:

$$1)\ x_C=\frac{x_A+x_B}{2}=\frac{8+(-1)}{2}=\frac{7}{2},$$

$$y_C=\frac{y_A+y_B}{2}=\frac{5+(-4)}{2}=\frac{1}{2};\quad C\left(\frac{7}{2};\frac{1}{2}\right)$$

$$2)\ k_{AB}=\frac{y_B-y_A}{x_B-x_A}=\frac{-4-5}{-1-8}=1,\quad k_{OC}=-\frac{1}{k_{AB}}=-1$$

$$3)\ y-y_C=k_{OC}(x-x_C),\qquad y-\frac{1}{2}=-1\left(x-\frac{7}{2}\right)$$

yoki $\quad x+y-4=0$;

$$4)\ \begin{cases} x+y-4=0 \\ y=0 \end{cases}\quad \text{(abssissalar o‘qining tenglamasi)};$$

$$x=4,\quad y=0,\quad O_1(4;0)$$

$$5)\ r=O_1A=\sqrt{41};\ \sqrt{(x-4)^2+(y-0)^2}=\left(\sqrt{41}\right)^2 \quad \text{yoki}$$

$$x^2+y^2-8x-25=0.$$

297. $A(3;7)$ va $B(5;-1)$ nuqtalardan o‘tadigan va markazi ordinatalar o‘qida bo‘lgan aylananing tenglamasini tuzing.

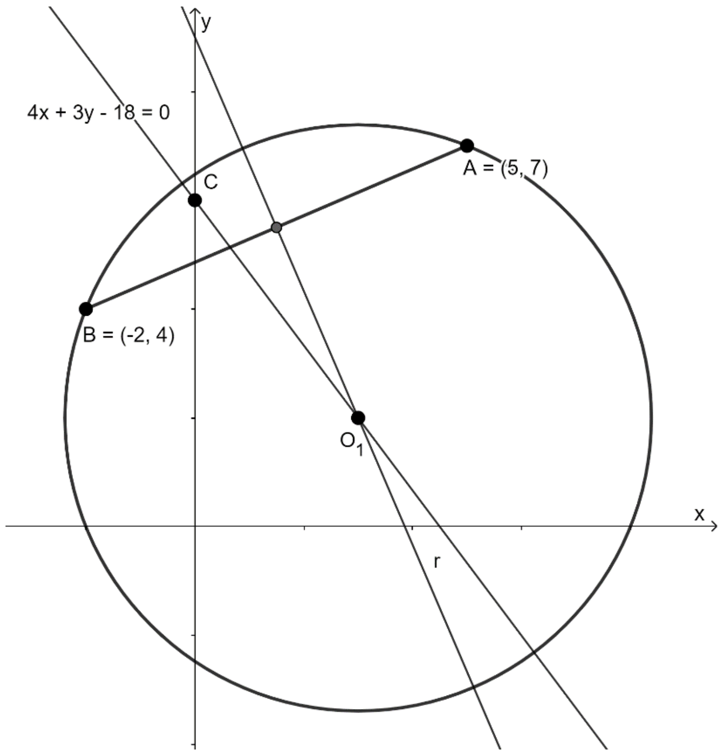
XV. Markazi berilgan to‘g‘ri chiziqda bo‘lgan va berilgan ikkita nuqtadan o‘tadigan aylananing tenglamasini tuzish

298. $A(5;7)$ va $B(-2;4)$ nuqtalardan o‘tuvchi aylananing markazi

$$4x+3y-18=0$$

to‘g‘ri chiziqda yotsa, uning tenglamasini tuzing.

Yechilishi. Geometriya kursidan ma’lumki, izlanayotgan



49-rasm

aylananing markazi AB vatarning o‘rtasidan o‘tkazilgan CO_1 perpendikulyarda yotadi (49-rasm). Masala shartidan bu aylananing markazi berilgan to‘g‘ri chiziqda yotishi kelib chiqadi. Demak, aylananing markazini topish uchun CO_1 perpendikulyar va berilgan to‘g‘ri chiziq tenglamalaridan iborat sistemani echish kifoya.

1- usul. Yechish rejasi: 1) AB kesmaning o‘rtasi bo‘lgan C nuqtaning koordinatalari topiladi 2) AB va CO_1 to‘g‘ri chiziqlarning burchak koeffitsientlari topiladi; 3) CO_1 ning tenglamasi tuziladi; 4) CO_1 to‘g‘ri chiziq va berilgan to‘g‘ri chiziq tenglamalaridan iborat sistema echiladi; 5) Izlanayotgan aylananing radiusi $r = O_1A$ ni topiladi va aylananing tenglamasi tuziladi.

$$1)\ x_C = \frac{x_A + x_B}{2} = \frac{5+(-2)}{2} = \frac{3}{2};\quad y_C = \frac{y_A + y_B}{2} = \frac{7+4}{2} = \frac{11}{2};\quad C\left(\frac{3}{2};\frac{11}{2}\right)$$

$$2)\ k_{AB} = \frac{y_A + y_B}{x_A + x_B} = \frac{7-2}{5-(-2)} = \frac{3}{7},\quad k_{CO} = -\frac{1}{k_{AB}} = -\frac{3}{7}$$

$$3)\ y - y_C = k_{OC}(x - x_C),\quad y - \frac{11}{2} = -\frac{7}{3}\left(x - \frac{2}{3}\right)\ \text{yoki}$$

$$7x + 3y - 27 = 0$$

$$4)\ \begin{cases} 7x + 3y - 27 = 0 \\ 4x + 3y - 18 = 0, \end{cases}\quad x = 3,\quad y = 2;\quad O_1(3;2)$$

$$5)\ r = O_1A = \sqrt{(5-3)^2 + (7-2)^2} = \sqrt{29}\ ;$$

$$(x-3)^2 + (y-2)^2 = (\sqrt{29})^2$$

$$\text{yoki}\qquad x^2 + y^2 - 6x - 4y - 16 = 0\,.$$

2- usul. Izlanayotgan aylananing markazi $O_1(a;b)$ nuqta bo‘lsin, O_1A va O_1B bu aylananing radiuslari; demak, $O_1A = O_1B$:

$$\sqrt{(a-5)^2 + (b-7)^2} = \sqrt{(a+2)^2 + (b-4)^2}\,.$$

Soddalashtirishlardan so‘ng:

$$7a + 3b - 27 = 0\,.$$

Izlanayotgan aylananing markazi $4x + 3y - 18 = 0$ to‘g‘ri chiziqda yotadi, demak, aylana markazining koordinatalari bu tenglamani qanoatlantirishi kerak:

$$4a + 3b - 18 = 0\,.$$

Quyidagi:

$$\begin{cases} 7a+3b-27=0 \\ 4a+3b-18=0 \end{cases}$$

tenglamalar sistemasini yechib, $a=3$, $b=2$; $O_1(3;2)$ ni hosil qilamiz.

Radiusni hisoblash va aylana tenglamasini tuzish birinchi usuldagidek bajariladi.

299. $A(-8;3)$ va $B(2;7)$ nuqtalardan o‘tuvchi aylananing markazi $x+4y+16=0$ to‘g‘ri chiziqda yotsa, uning tenglamasini tuzing.

300. $M(3;2)$ va $N(-1;-6)$ nuqtalardan o‘tuvchi aylananing markazi koordinatalar o‘qini $A(2;0)$ va $B(0;-4)$ nuqtalarda kesib o‘tuvchi to‘g‘ri chiziqda yotsa, aylananing tenglamasini tuzing.

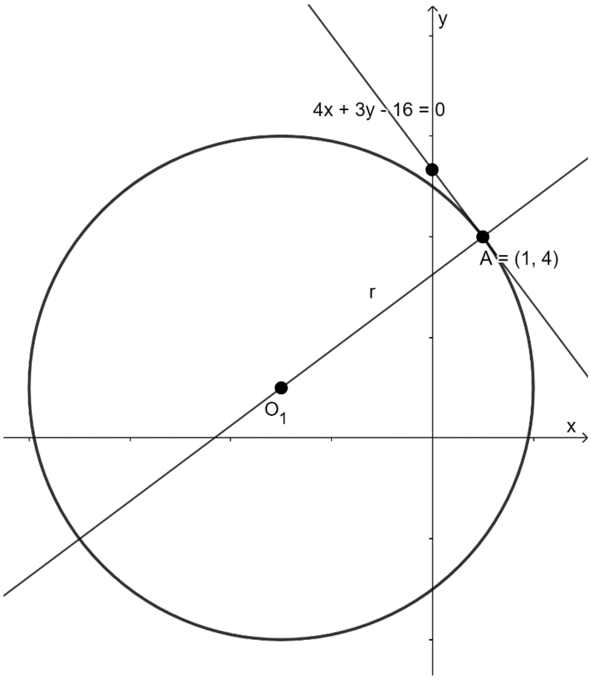
XVI. Markazi berilgan nuqtada bo‘lgan va berilgan to‘g‘ri chiziqqa urinadigan aylananing tenglamasini tuzish

301. Aylananing markazi $O_1(-3;1)$ nuqtada joylashgan. Agar bu aylana $4x+3y-16=0$ to‘g‘ri chiziqqa urinsa, uning tenglamasini tuzing.

Yechilishi. Aylananing tenglamasini tuzish uchun uning radiusini topish kerak. Urinish nuqtasiga o‘tkazilgan radius urinmaga perpendikulyardir. Demak, urinma tenglamasiga ko‘ra O_1A radiusning tenglamasini topishimiz mumkin, chunki aylananing markazi O_1 berilgan. Urinma va O_1A radius tenglamalaridan iborat sistemani yechib, urinish nuqtasi A ni va so‘ngra radiusni topamiz.

Yechish rejasi: 1) urinma va O_1A radiusning burchak koeffitsientlarini hisoblash (50-rasm), 2) O_1A radiusning tenglamasini tuzish. 3) urinma va O_1A radiusning tenglamalaridan iborat sistemani echish (urinish nuqtasi *A* ning koordinatalarini topish) ; 4) radiusni topish va aylananing tenglamasini tuzish.

1) $4x+3y-16=0, \quad k=-\dfrac{3}{4}; \quad k_{OA}=-\dfrac{1}{k}=\dfrac{3}{4}$.



50-rasm

2) $y-y_C=k_{OA}(x-x_O), \quad y-1=\dfrac{3}{4}(x+3)$ yoki $3x-4y+13=0$

3) $\begin{cases} 4x+3y-16=0 \\ 3x+4y+13=0, \quad x=1, \quad y=4; \quad A(1;4) \end{cases}$;

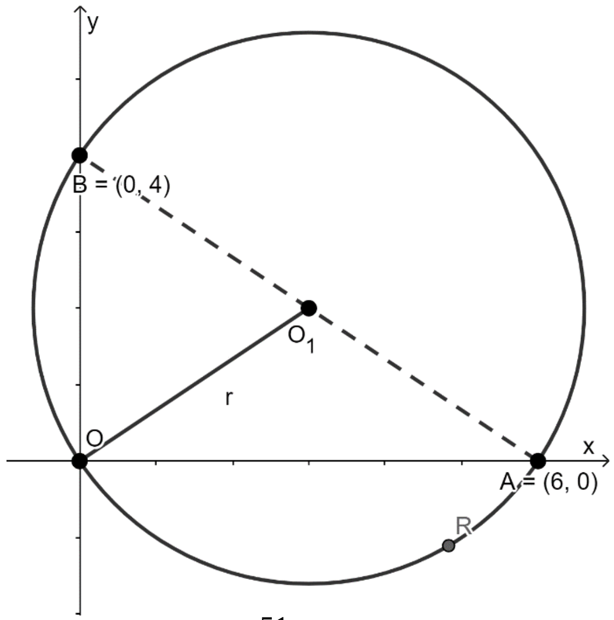
4) $r=O_1A=\sqrt{(-3-1)^2+(1-4)^2}=5; (x+3)^2+(y-4)^2=5^2$

yoki $x^2+y^2+6x-2y-15=0$.

302. Aylananing markazi $(-1;-4)$ nuqtada joylashgan. Agar bu aylana koordinata o‘qlarini $A(2,25;0)$ va $B(0;3)$ nuqtalarda kesib o‘tuvchi to‘g‘ri chiziqqa urinsa, uning tenglamasini tuzing.

XVII. Koordinatalar boshidan o‘tuvchi va koordinata o‘qlarini berilgan nuqtalarda kesib o‘tuvchi aylananing tenglamasini tuzish

303. Koordinatalar boshidan o‘tuvchi va koordinata o‘qlarini $A(6;0)$ va $B(0;4)$ nuqtalarda kesib o‘tuvchi aylananing tenglamasini tuzing.



51-rasm

Yechilishi. 1- usul. Aylananing tenglamasini tuzish uchun uning markazini va radiusini topish kerak. Aylana markazi $O_1(a;b)$ nuqtada joylashgan bo‘lsin, u holda $O_1A = O_1B$ (51-rasm):

$$\sqrt{(a-6)^2+(b-0)^2}=\sqrt{(a-0)^2+(b-4)^2};$$

soddalashtirishlardan so‘ng,

$$3a-2b-5=0$$

tenglikni hosil qilamiz.

Ikkinchi tenglamani tuzamiz:

$$O_1O=O_1B;\quad \sqrt{a^2+b^2}=\sqrt{(a-0)^2+(b-4)^2}.$$

Soddalashtirishlardan so‘ng,

$$b-2=0;\quad b=2$$

ni topamiz.

Ushbu sistemani yechamiz:

$$\begin{cases}3a-2b-5=0\\ b=2\end{cases};\qquad a=3,\quad b=2;\quad O_1(3;2)$$

Aylananing radiusini hisoblaymiz:

$$r=O_1A=\sqrt{a^2+b^2}=\sqrt{3^2+2^2}=\sqrt{13}.$$

Aylana tenglamasini tuzamiz:

$$\sqrt{(x-3)^2+(y-2)^2}=(\sqrt{13})^2$$

yoki

$$x^2+y^2-6x-4y=0.$$

2-usul. Aylananing markazi OA va OB vatarlarning o‘rtalaridan o‘tkazilgan perpendikulyarlarning kesishish nuqtasida yotadi. Bu perpendikulyarlarning tenglamalari mos ravishda $x=3$ va $y=2$ bo‘ladi (Oy va Ox o‘qlarga parallel bo‘lgan to‘g‘ri chiziqlarning tenglamalari). Perpendikulyarlarning kesishish nuqtasi $O_1(3;2)$ — izlanayotgan aylananing markazi bo‘ladi.

Aylananing radiusi va tenglamasi 1-usuldagidek topiladi.

304. Koordinatalar boshidan o‘tadigan va Ox, Oy o‘qlardan mos ravishda 4 va 3 kesmalar ajratadigan aylananing tenglamasini tuzing.

XVIII. Aylananing berilgan tenglamasi bo‘yicha uning radiusini va markazining koordinatalarini hisoblash

305. $x^2+y^2+Mx+Ny+P=0$ aylananing radiusini va markazining koordinatalarini toping.

Yechilishi. Berilgan tenglamani quyidagicha qayta yozib olamiz:

$$x^2+Mx+y^2+Ny=-P.$$

x^2+Mx va y^2+Ny ikkihadlarni to‘lik kvadratga to‘ldirib yozamiz:

$$x^2+2\cdot\frac{M}{2}x+\left(\frac{M}{2}\right)^2+y^2+2\cdot\frac{N}{2}y+\left(\frac{N}{2}\right)^2=\left(\frac{M}{2}\right)^2+\left(\frac{N}{2}\right)^2-P$$

yoki

$$\left(x+\frac{M}{2}\right)^2+\left(y+\frac{N}{2}\right)^2=\frac{M^2+N^2-4P}{4}.$$

(3.2) ko‘rinishdagi tenglamani hosil kildik, bu yerdan (3.5) va (3.6) munosabatlarni hosil qilamiz:

$$\begin{cases} a=-\dfrac{M}{2}, \\ b=-\dfrac{N}{2}, \end{cases} \quad r=\frac{\sqrt{M^2+N^2-4P}}{2}.$$

Bu munosabatlardan kelgusida foydalanib, (3.4) tenglamadan aylana markazining koordinatalari $(a;b)$ ni va r radiusni osongina topishimiz mumkin.

306. Ushbu

$$x^2+y^2-8x-10y-8=0$$

aylananing markazi koordinatalarini va radiusini toping. Bu aylanani yasang.

Yechilishi. 1- usul. Berilgan tenglamani quyidagicha qayta yozib olamiz:

$$x^2-8x+y^2-10y=8.$$

x^2-8x va y^2-10y ikkixadlarni tulik kvadratga tuldirib yozsak:

$$x^2-2\cdot 4x+4^2+y^2+2\cdot 5y+5^2=8+4^2+5^2 \text{ yoki}$$
$$\sqrt{(x-4)^2+(y-5)^2}=49.$$

bu yerdan $a=4$, $b=5$; $r=7$, ya’ni markazi (4; 5) nuqtada va radiusi 7 ga teng bo‘lgan aylanaga egamiz.

2-usul. 305-masalada hosil qilingan (3.5) va (3.6) munosabatlardan foydalanamiz.

Berilgan aylana uchun quyidagilarga egamiz:

$$M = -8,\ N = -10,\ P = -8,$$

bu yerdan $a = -\frac{-8}{2} = 4$, $\qquad b = -\frac{-10}{2} = 5$,

$$r = \frac{\sqrt{(-8)^2 + (-10)^2 - 4(-8)}}{2} = \frac{\sqrt{196}}{2} = 7.$$

Aylanani yasash: aylananing markazi $O_1(4;5)$ nuqtani yasab, 7 ga teng radius bilan aylana chizamiz.

307. Ushbu

1) $x^2 + y^2 + 6x - 10y + 13 = 0$; 2) $x^2 + y^2 + 12y + 13 = 0$

aylananing radiusini va markazining koordinatalarini toping.

308. $4x^2 + 4y^2 - 4x + 20y - 23 = 0$ aylananing markazi koordinatalarini va radiusini toping.

Ko‘rsatma. Berilgan tenglamani (3.4) ko‘rinishga keltiring.

309. Quyidagi aylanalarning markazi koordinatalarini va radiusini toping:

1) $9x^2 + 9y^2 + 42x - 54y - 95 = 0$;

2) $x^2 + y^2 - 4x - 10y + 29 = 0$;

3) $x^2 + y^2 + 6x + 14y + 81 = 0$.

XIX. Berilgan ikkita aylananing markazlari orasidagi masofani hisoblash

310. Quyidagi aylanalarning markazlari orasidagi masofani toping:

1) $x^2 + y^2 - 10x + 16y + 80 = 0$ va $x^2 + y^2 + 6x + 4y - 12 = 0$;

2) $x^2 + y^2 + 4x - 12y + 36 = 0$, $\quad x^2 + y^2 - 8x + 10y + 5 = 0$.

XX. Berilgan ikkita aylananing markazlaridan o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzish

311. Quyidagi aylanalarning markazlaridan o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzing:

$$1)\ x^2+y^2-8x-4y+11=0 \text{ va}$$

$$x^2+y^2+4x+12y+4=0$$

$$2)\ x^2+y^2+4x-6y-23=0 \text{ va}$$

$$x^2+y^2-10x-14y+58=0 .$$

XXI. Berilgan aylananing Ox o‘q bilan berilgan burchak hosil qiluvchi diametrining tenglamasini tuzish

312. $x^2+y^2+6x-4y-12=0$ aylanada diametr Ox o‘q bilan 60° li burchak tashkil etadi. Diametrning tenglamasini tuzing.

Yechilishi. Diametrning tenglamasini tuzish uchun u o‘tadigan nuqtani va bu diametrning burchak koeffitsientini bilish kerak. Bu nuqta aylananing markazi $O_1(a;b)$ bo‘ladi. (3.5) munosabatlardan quyidagiga egamiz:

$$a=-\frac{M}{2}=-\frac{6}{2}=-3,\ b=-\frac{N}{2}=-\frac{-4}{2}=2,\ O_1(-3;2).$$

Burchak koeffitsient $k=tga$ munosabatdan topiladi:

$$k=tg60^\circ=\sqrt{3}.$$

Diametrning tenglamasi:

$$y-2=\sqrt{3}(x+3) \quad \text{yoki} \quad \sqrt{3}x-y+3\sqrt{3}+2=0 .$$

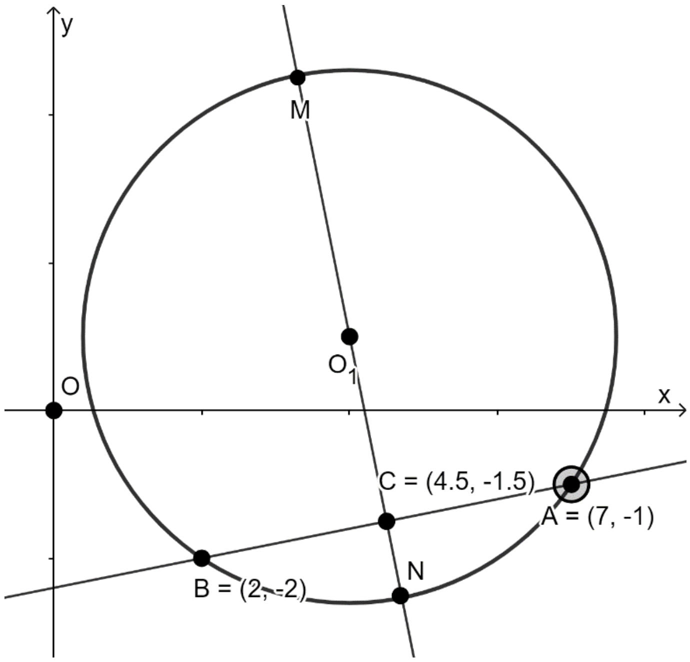
313. $x^2+y^2-2x+4y-4=0$ aylananing diametri Ox o‘qning musbat yo‘nalishi bilan 135° li burchak tashkil etadi. Bu diametrning tenglamasini tuzing.

XXII. Berilgan aylananing berilgan vatarga perpendikulyar bo‘lgan diametrining tenglamasini tuzish

314. $x^2+y^2-8x-2y+4=0$ aylana berilgan. $x-5y-12=0$ vatarga perpendikulyar bo‘lgan diametrning tenglamasini tuzing.

Yechilishi. 1- usul. Aylananing vatar bilan kesishish nuqtalarining koordinatalarini topamiz. Buning uchun quyidagi tenglamalar sistemasini yechamiz:

$$\begin{cases} x^2+y^2-8x-2y+4=0 \\ x-5y-12=0 \end{cases}$$



52-rasm

Bu sistemaning ildizlari:

$x_1=7;\quad y_1=-1$ va

$x_2=2;\quad y_2=-2$.

Vatar aylana bilan kesishgan ikkita nuqtaga egamiz: $A(7;-1)$ va $B(2;-2)$ (52-rasm).

Vatarga perpendikulyar bo‘lgan diametr vatarning o‘rtasi bo‘lgan C nuqtadan o‘tadi. C nuqtaning koordinatalarini topamiz:

$$x_C=\frac{x_A+x_B}{2}=\frac{7+2}{2}=\frac{9}{2},$$

$$y_C=\frac{y_A+y_B}{2}=\frac{-1+(-2)}{2}=-\frac{3}{2}, \qquad C(\frac{9}{2};-\frac{3}{2}).$$

AB vatar va MN diametrning burchak koeffitsientlarini aniqlaymiz:

$$k_{AB}=\frac{y_B-y_A}{x_B+x_A}=\frac{-2-(-1)}{2-7}=\frac{1}{5};$$

$$k_{MN}=-\frac{1}{k_{AB}}=-5 .$$

Diametrning tenglamasini tuzamiz:

$$y-y_C=k_{MN}(x-x_C),\quad y-(-\frac{3}{2})=-5(x-\frac{9}{2})$$

yoki

$$5x+y-21=0 .$$

2- usul. (3.5) munosabatlardan aylana markazini topamiz:

$$a=-\frac{M}{2}=-\frac{-8}{2}=4,\ \ b=-\frac{N}{2}=-\frac{-2}{2}=1;\ \ O_1(4;1) .$$

AB vatar va bu vatarga perpendikulyar bo‘lgan MN diametrning burchak koeffitsientlarini hisoblaymiz*:*

$$x-5y-12=0,\ \ y=\frac{1}{5}x-\frac{12}{5},\ \ k_{AB}=\frac{1}{5};$$

$$k_{MN}=-\frac{1}{k_{AB}}=-5$$

Diametrning tenglamasini tuzamiz:

$$y-y_{O_1}=k_{MN}(x-x_{O_1}),\ \ y-1=-5(x-4)$$

yoki

$$5x+y-21=0 .$$

315. $x^2+y^2+4x-6y=0$ aylana berilgan. $2x-3y+13=0$ vatarga perpendikulyar bo‘lgan diametrning tenglamasini tuzing.

XXIII. Berilgan aylananing berilgan nuqtasiga o‘tkazilgan radiusining tenglamasini tuzish

316. $x^2+y^2-6x+2y-19=0$ aylananing $A(5;-6)$ nuqtasiga o‘tkazilgan radiusining tenglamasini tuzing.

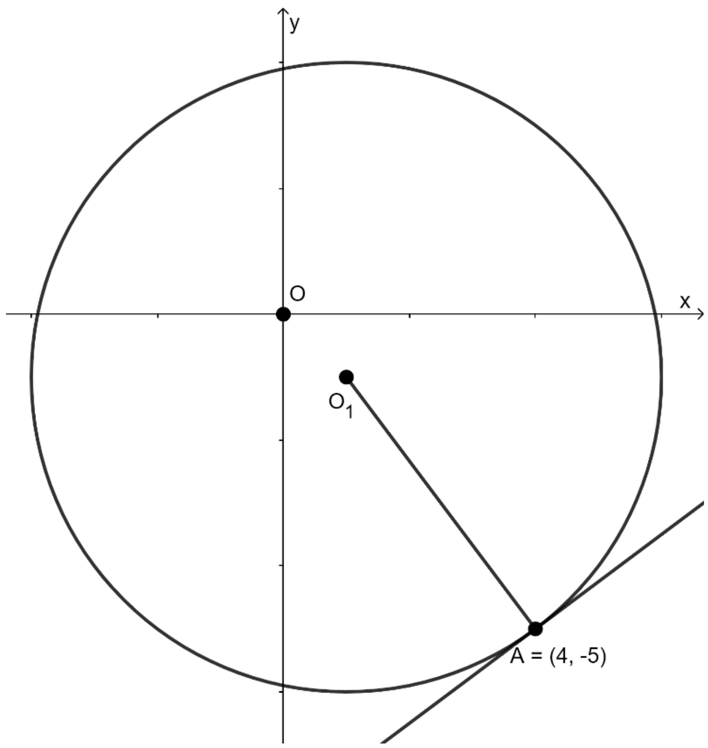
317. $x^2+y^2-6x-9=0$ aylananing $A(6;3)$ nuqtasiga o‘tkazilgan radiusining tenglamasini tuzing.

XXIV. Berilgan aylananing berilgan nuqtasiga o‘tkazilgan urinmaning tenglamasini tuzish

318. $x^2+y^2-2x+2y-23=0$ aylananing $A(4;-5)$ nuqtasiga o‘tkazilgan urinmaning tenglamasini tuzing.

Yechilishi. Urinma urinish nuqtasiga o‘tkazilgan radiusga perpendikulyardir. Urinmaning tenglamasini tuzish uchun uning burchak koeffitsientini va urinish nuqtasini bilish kerak. Urinmaning burchak koeffitsientini radiusning burchak koeffitsientidan aniqlaymiz.

(3.5) munosabatlardan aylananing markazi $O_1(a;b)$ nuqtani aniqlaymiz:



53-rasm

$$a=-\frac{M}{2}=-\frac{-2}{2}=1,$$
$$b=-\frac{N}{2}=-\frac{2}{2}=-1; \qquad O_1(1;-1), \text{ (53-rasm).}$$

O_1A radius va AB urinmaning burchak koeffitsientlarini topamiz:

$$k_{O_1A}=\frac{y_A-y_{O_1}}{x_A-x_{O_1}}=\frac{-5-(-1)}{4-1}=-\frac{4}{3}$$
$$k_{AB}=-\frac{1}{k_{O_1A}}=\frac{3}{4}$$

Urinmaning tenglamasini to‘zamiz:

$$y-y_A=k_{AB}(x-x_A), \quad y-(-5)=\frac{3}{4}(x-4)$$

yoki

$$3x-4y-32=0.$$

319. $x^2+y^2-10y+20=0$ aylananing $A(-2;0)$ nuqtasiga o‘tkazilgan urinmaning tenglamasini tuzing.

XXV. Berilgan ikkita aylananing umumiy vatari tenglamasini tuzish

320. O‘zaro kesishuvchi ushbu ikkita

$$x^2+y^2+2x+2y-23=0$$
$$x^2+y^2-26x-2y+45=0$$

aylananing umumiy vatari tenglamasini tuzing.

Yechilishi. Quyidagi

$$\begin{cases} x^2+y^2+2x+2y-23=0 \\ x^2+y^2-26x-2y+45=0 \end{cases}$$

tenglamalar sistemasini yechib, aylanalarning kesishish nuqtalari A va B ning koordinatalarini topamiz. Bu sistemaning ildizlari:
$x_1=3,\ \ y_1=-4$; $x_2=2,\ \ y_2=3.$

Aylanalarning kesishish nuqtalari: $A(3;-4)$ va $B(2;3)$.

AB vatarning burchak koeffitsientini topamiz:

$$k_{AB}=\frac{y_B-y_A}{x_B-x_A}=\frac{3-(-4)}{2-3}=-7.$$

AB vatarning tenglamasini to‘zamiz:

$$y-y_B=k_{AB}(x-x_B),\quad y-3=-7(x-2)$$

yoki

$$7x+y-17=0.$$

321. O‘zaro kesishuvchi ushbu

$$x^2+y^2-6y=0,\quad x^2+y^2-12x=0$$

aylanalarning umumiy vatari tenglamasini tuzing.

XXVI. Berilgan nuqtadan o‘tuvchi va berilgan aylanaga konsentrik bo‘lgan aylananing tenglamasini tuzish

322. $A(4;-7)$ nuqtadan o‘tuvchi va $x^2+y^2+4x-2y-11=0$ aylanaga konsentrik bo‘lgan aylananing tenglamasini tuzing.

Yechilishi. Izlanayotgan aylananing markazi berilgan aylananing markazi $O_1(a;b)$ bilan umumiy, radiusi esa O_1A ga teng.

(3.5) munosabatdan aylana markazining koordinatalarini topamiz:

$$a = -\frac{M}{2} = -\frac{4}{2} = -2, \quad b = -\frac{N}{2} = -\frac{-2}{2} = 1;$$

$$O_1(-2;1)$$

Aylananing radiusini topamiz:

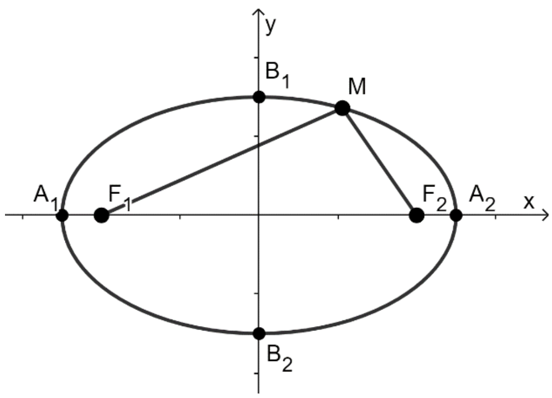
$$r = O_1A = \sqrt{(-2-4)^2 + \left[1-(-7)\right]^2} = 10.$$

Aylana tenglamasini to‘zamiz:

$$\left[x-(-2)\right]^2 + (y-1)^2 = 10^2 \quad \text{yoki} \quad x^2 + y^2 + 4x - 2y - 95 = 0.$$

323. $A(5;6)$ nuqtadan o‘tuvchi va $x^2 + y^2 - 2x + 6y + 1 = 0$ aylanaga konsentrik bo‘lgan aylananing tenglamasini tuzing.

18-§. Ellips



54-rasm

Har biridan fokuslar deb ataluvchi berilgan ikkita nuqtagacha bo‘lgan masofalari yig‘indisi o‘zgarmas miqdor bo‘lgan (va fokuslar orasidagi masofadan katta bo‘lgan) nuqtalarning geometrik o‘rni ***ellips*** deb ataladi.

Ellipsning tenglamasi:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \qquad (a > b), \tag{3.7}$$

bu yerda a - katta yarim o‘qning uzunligi; b - kichik yarim o‘qning uzunligi (54-rasm).

a, b va c parametrlar orasidagi munosabat:

$$a^2 - b^2 = c^2 \tag{3.8}$$

bu yerda c - fokuslar orasidagi masofaning yarmi.

Ellipsning ekssentrisiteti:

$$\varepsilon = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a} < 1 \qquad (3.9)$$

Agar ellipsning fokuslari Oy o‘qda $(0;\pm c)$ nuqtalarda yotsa (55- rasm), u holda uning tenglamasi ushbu ko‘rinishda bo‘ladi:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \qquad (a > b) \qquad (3.10)$$

(3.7) va (3.10) formulalarda x va y — o‘zgaruvchi koordinatalar — ellipsning istalgan nuqtasining koordinatalari.

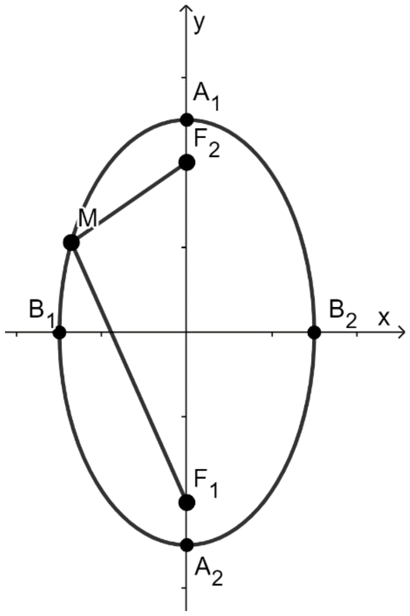
(3.7) ellipsda uning qarama-qarshi uchlarining koordinatalari $(\pm a;0)$ va $(0;\pm b)$ bo‘yicha bevosita katta o‘q uzunligi $2a$ va kichik o’q o‘zunligi $2b$ topiladi va, aksincha, ellipsning katta o‘qi uzunligi $2a$ va kichik o‘qi o‘zunligi $2b$ bo‘yicha ellipsning tegishli uchlari $(\pm a;0)$ va $(0;\pm b)$ topiladi.

Ellips fokuslarining koordinatalari $(\pm c;0)$ bo‘yicha fokuslar orasidagi masofa $2c$ va fokuslar orasidagi masofa $2c$ bo‘yicha fokuslarning koordinatalari $(\pm c;0)$ topiladi.

Ellipsni uning $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ tenglamasiga ko‘ra yasash qo‘yidagi sxema bo‘yicha bajariladi:

1. Ellipsning a va b yarim uchlari topiladi hamda $A_1(-a;0),\ A_2(a;0),\ B_1(0;-b)$ *ва* $B_2(0;b)$ koordinatali nuqtalar yasaladi.

2. Bu nuqtalar orqali Ox va Oy o‘qlarga parallel to‘g‘ri chiziqlar o‘tkaziladi, markazi koordinatalar boshida va tomonlari $2a$ va $2b$ bo‘lgan to‘g‘ri to‘rtburchak hosil qilinadi.



55-rasm

3. Ellipsni to‘g‘ri to‘rtburchakning ichida A_1, B_2, A_2 va B_1 nuqtalardan o‘tadigan egri chiziq kabi (qo‘lda) chiziladi (56-rasm).

Ellipsga doir barcha masalalarda ellipsning simmetriya o‘qlari koordinata o‘qlari bilan ustma-ust tushadi deb faraz qilinadi.

I.Berilgan nuqtalarning berilgan ellipsga tegishliligini tekshirish

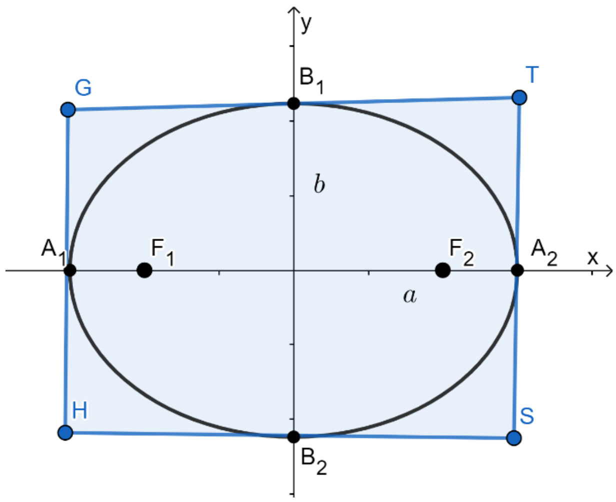
324. 1) $A(9;4)$, $B(12;3)$ va $C(1;6)$ nuqtalarning $\frac{x^2}{125}+\frac{y^2}{25}=1$ ellipsga tegishliligini tekshiring.

2) $A\left(\sqrt{10};2\right)$va $B(4;1)$ nutsqtalarning $\frac{x^2}{18}+\frac{y^2}{9}=1$ ellipsga tegishliligini tekshiring.

II. Ellipsning tenglamasini uning o‘qlarini o‘zunliklari bo‘yicha tuzish

325. Agar fokuslari Ox o‘qda bo‘lgan ellipsning o‘qlari $2a=12$ va $2b=8$ berilgan bo‘lsa, uning tenglamasini tuzing.

Yechilishi. Ellipsning tenglamasini tuzish uchun uning parametrlari a va b ni bilish kerak. Masala shartidan: $a=6$ va $b=4$, a va b ning bu qiymatlarini ellipsning (3.7) tenglamasiga qo‘yib



56-rasm

$$\frac{x^2}{6^2}+\frac{y^2}{4^2}=1$$ yoki
$$\frac{x^2}{36}+\frac{y^2}{16}=1$$

tenglamani hosil qilamiz.

326. Agar fokuslari Ox o‘qda bo‘lgan ellipsning o‘qlari $2a=8$ va $2b=6$ berilgan bo‘lsa, uning tenglamasini tuzing.

327. Agar fokuslari Oy o‘qda bo‘lgan ellipsning o‘qlari $2a=10$ va $2b=4$ berilgan bo‘lsa, uning tenglamasini tuzing.

III . Ellipsning tenglamasini uning ikkita qarama-qarshi uchlarining koordinatalari (katta yoki kichik o‘qlarining uzunliklari) va fokuslarining koordinatalari (fokuslari orasidagi masofa) bo‘yicha tuzish

328. Agar ellipsning ikkita uchi $A_1(-6;0)$ va $A_2(+6;0)$ nuqtalarda, fokuslari esa $(\pm 4;0)$ koordinatalar bilan berilgan bo‘lsa, uning tenglamasini tuzing.

Yechilishi. Masala shartidan $a=6$ va $c=4$ ekanligi kelib chiqadi. (3. 8) formula bo‘yicha topamiz: $b^2 = 6^2 - 4^2 = 20$. a va b^2 ning qiymatlarini (3.7) tenglamaga qo‘yib, izlanayotgan ellips tenglamasini hosil qilamiz:

$$\frac{x^2}{36}+\frac{y^2}{20}=1 .$$

329. Agar ellipsning ikkita uchi $A_1(-5;0)$ va $A_2(+5;0)$ nuqtalarda, fokuslari esa (± 3 ; 0) koordinatalar bilan berilgan bo‘lsa, uning tenglamasini tuzing.

330. Agar ellipsning ikkita uchi $B_1(0;-8)$ va $B_2(0;+8)$) nuqtalarda, fokuslari esa (±5; 0) koordinatalar bilan berilgan bo‘lsa, uning tenglamasini tuzing.

331. Agar ellipsning ikkita uchi (-8 ; 0) va (8 ; 0) nuqtalarda, fokuslari esa (0 ; + 6) koordinatalar bilan berilgan bo‘lsa, uning tenglamasini tuzing.

Yechilishi. Masala shartidan fokuslar Oy o‘qda yotishi kelib chiqadi, u holda $b=8$, $c=6$.

(3.8) formula bo‘yicha: $a^2 = 8^2 + 6^2 = 100$. a^2 va b ning qiymatlarini (3.10) teiglamaga qo‘ysak:

$$\frac{x^2}{64} + \frac{y^2}{100} = 1.$$

332. Agar ellipsning ikkita uchi $(0;\pm4)$ nuqtalarda joylashgan, fokuslari esa $(0;\pm2)$ koordinatalar bilan berilgan bo‘lsa, uning tenglamasini tuzing.

333. Agar ellipsning fokuslari orasidagi masofa 6 ga (fokuslar Ox o‘qda yotadi), katta o‘qi 10 ga teng bo‘lsa, bu ellipsning tenglamasini tuzing.

Yechilishi. Masala shartidan: $a = 5$ va $c = 3$. (3.8) formula bo‘yicha: $b^2 = 5^2 - 3^2 = 16$. a va b^2 ning qiymatlarini (3.7) tenglamaga qo‘yib, izlanayotgan ellipsning tenglamasini hosil qilamiz:

$$\frac{x^2}{25} + \frac{y^2}{16} = 1.$$

334. Agar ellipsning fokuslari orasidagi masofa 10 ga (fokuslar Ox o‘qda yotadi), katta o‘qi 12 ga teng bo‘lsa, bu ellipsning tenglamasini tuzing.

335. Agar ellipsning fokuslarini koordinatalari (± 2; 0) bo‘lib, kichik o‘qi 8 ga teng bo‘lsa, bu ellipsning tenglamasini tuzing.

336. Fokuslari $\left(0;\pm\sqrt{5}\right)$ nuqtalarda yotgan va katta o‘qi 6 ga teng bo‘lgan ellipsning tenglamasini tuzing.

Yechilishi. Fokuslar Oy o‘qda yotadi, demak, $a = 3$. (3.8) formula bo‘yicha:

$$b^2 = 3^2 - (\sqrt{5})^2 = 4.$$

a va b^2 ning qiymatlarini (3.10) tenglamaga qo‘yib,

$$\frac{x^2}{4}+\frac{y^2}{9}=1$$

ni hosil qilamiz.

337. Fokuslari $\left(0;\pm\sqrt{3}\right)$ nuqtalarda yotgan va katta o‘qi $4\sqrt{7}$ ga teng bo‘lgan ellipsning tenglamasini tuzing.

IV. Ellipsning tenglamasini uning fokuslari koordinatalari (fokuslari orasidagi masofa) va ekssentrisiteti bo‘yicha tuzish

338. Fokuslari $(\pm 4;0)$ nuqtalarda yotgan va ekssentrisiteti $\varepsilon=0,8$ bo‘lgan ellipsning tenglamasini tuzing.

Yechilishi. Masala shartidan $c=4$, $\varepsilon=\frac{c}{a}=0,8$. c ning qiymatini ikkinchi tenglikka qo‘yib, $a=5$ ni hosil qilamiz. (3.8) formula bo‘yicha: $b^2=5^2-4^2=9$. (3.7) tenglama bo‘yicha yozamiz:

$$\frac{x^2}{25}+\frac{y^2}{9}=1.$$

339. Fokuslari $\left(\pm\sqrt{3};0\right)$ nuqtalarda yotgan va ekssentrisiteti $\varepsilon=\frac{1}{3}$ bo‘lgan ellipsning tenglamasini tuzing.

340. Agar ellipsning fokuslari Ox o‘qda bo‘lib, ular orasidagi masofa 12 , ekssentrisiteti esa $\varepsilon=0,6$ bo‘lsa, bu ellipsning tenglamasini tuzing.

V. Ellipsning tenglamasini uning ikkita qarama-qarshi uchlarining koordinatalari (katta yoki kichik o‘qining o‘zunligi) va ekssentrisiteti bo‘yicha tuzish

341. Fokuslari Ox o‘qda bo‘lgan va katta o‘qi 14 ga, ekssentrisiteti esa $\varepsilon=2/3$ ga teng bo‘lgan ellipsning tenglamasini tuzing.

Yechilishi. Masala shartidan: 1) $a=7$; $\varepsilon=\frac{c}{a}=\frac{2}{3}$. a ning qiymatini ikkinchi munosabatga qo‘yib, $c=\frac{14}{3}$ ni topamiz. (3.8) formula bo‘yicha b^2 ni topamiz: $b^2=7^2-\left(\frac{14}{3}\right)^2=\frac{245}{9}$. (3.7) tenglama bo‘yicha:

$$\frac{x^2}{7^2}+\frac{y^2}{\frac{245}{9}}=1 \quad \text{yoki} \quad \frac{x^2}{49}+\frac{9y^2}{245}=1.$$

342. Agar ellipsning fokuslari Ox o‘qda bo‘lib, katta o‘qi 10 ga, ε ekssentrisiteti esa 0,6 ga teng bo‘lsa, bu ellipsning tenglamasini tuzing.

343. Agar ellipsning fokuslari Ox o‘qda bo‘lib, kichik o‘qi 16 ga, ε ekssentrisiteti esa 0,6 ga teng bo‘lsa, bu ellipsning tenglamasini tuzing.

VI. Ellipsning tenglamasini uning yarim o‘qlarining yig‘indisi (ayirmasi) va fokuslari orasidagi masofa bo‘yicha tuzish

344. Agar fokuslari Ox o‘qida bo‘lgan ellipsning yarim o‘qlarining yig‘indisi 8 ga, fokuslari orasidagi masofa esa 8 ga teng bo‘lsa, uning tenglamasini tuzing.

Yechilishi. Masala shartaga ko‘ra quyidagiga egamiz: 1) $a+b=8$; 2) $c=4$. (3.8) formulaga c ning qiymatini qo‘yamnz: $b^2=a^2-16$ tenglamalardan iborat sistemani yechib, $a=5$, $b=3$ ni topamiz. a va b ning topilgan qiymatlarini (3.7) tenglamaga qo‘yib, izlanayotgan ellipsning tenglamasini hosil qilamiz:

$$\frac{x^2}{25}+\frac{y^2}{9}=1.$$

345. Agar ellipsning yarim o‘qlari yig‘indisi 25 ga teng bo‘lsa va fokuslari (± 5; 0) koordinatalarga ega bo‘lsa, uning tenglamasini tuzing.

346. Agar ellipsning yarim uchlari yig‘indisi 9 ga teng bo‘lsa va fokuslari (± 6 ; 0) koordinatalarga ega bo‘lsa uning tenglamasini tuzing.

VII. Ellipsning tenglamasini uning katta (kichik) o‘qi o‘zunligi va bu ellips o‘tadigan nuqtaning koordinatalari bo‘yicha tuzish

347. Agar ellipsning fokuslari Ox o‘qda bo‘lib, ellips $M(-6;4)$ nuqtadan o‘tsa va uning kichik o‘qi 10 ga teng bo‘lsa, bu ellipsning tenglamasini tuzing.

Yechilishi. Masala shartiga ko‘ra $2b=10$, bu yerdan $b=5$. a ni hisoblash uchun ellipsning (3.7) tenglamasida x va y o‘zgaruvchilarning o‘rniga $M(-6;4)$ nuqtaning koordinatalarini va $b=5$ qiymatni qo‘yamiz, $a^2=100$ hosil bo‘ladi.

a^2 va b ning qiymatlarini (3.7) tenglamaga qo‘yib, ellipsning izlanayotgan tenglamasini hosil qilamiz:

$$\frac{x^2}{100}+\frac{y^2}{25}=1.$$

348. Agar ellipsning fokuslari Ox o‘qda bo‘lib, u $(12;-12)$ nuqtadan o‘tsa va katta o‘qi 40 ga teng bo‘lsa, bu elllipsning tenglamasini tuzing.

349. Fokuslari Ox o‘qda bo‘lgan ellips $\left(2;2\sqrt{2}\right)$ nuqtadan o‘tadi. Uning kichik o‘qi 6 ga teng. Bu ellipsning tenglamasini tuzing.

VIII. Berilgan ikkita nuqtadan o‘tuvchi ellipsning tenglamasini tuzish

350. $A\left(\sqrt{3};\sqrt{6}\right)$ va $B\left(3;\sqrt{2}\right)$ nuqtalardan o‘tuvchi ellipsning tenglamasini tuzing. Ellipsning fokuslari Ox o‘qda yotadi.

Yechilishi. Ellipsning tenglamasini tuzish uchun a va b parametrlarni topish kerak. Ellipsning (3.7) tenglamasiga berilgan nuqtalarning koordinatalarini qo‘yib, almashtirishlardan so‘ng ushbu sistemani hosil qilamiz:

$$\begin{cases} 3b^2 + 6a^2 = a^2b^2 \\ 9b^2 + 2a^2 = a^2b^2 \end{cases}.$$

Bu sistemadan $a^2 = 12$ va $b^2 = 8$ ni topamiz.

Ellipsning (3.7) tenglamasidan foydalanib, izlanayotgan tenglamani hosil qilamiz:

$$\frac{x^2}{12} + \frac{y^2}{8} = 1.$$

351. $A(6;4)$ va $B(8;3)$ nuqtalardan o‘tuvchi ellipsning tenglamasini tuzing. Ellipsning fokuslari *Ox* o‘qda yotadi.

352. $\left(\sqrt{2};2\right)$va $\left(2;\sqrt{3}\right)$ nuqtalardan o‘tuvchi ellipsning tenglamasini tuzing. Ellipsning fokuslari Ox o‘qda yotadi.

IX. Ellips uchlarining koordinatalarini (o‘qlarining o‘zunliklarini) ellipsning tenglamisi bo‘yicha hisoblash

353. $\frac{x^2}{49} + \frac{y^2}{16} = 1$ ellips berilgan. Ellips uchlarining koordinatalarini va o‘qlarining uzunliklarini toping.

Yechilishi. Ellips tenglamasidan:

$$a^2 = 49, \quad a = \pm 7; \quad b^2 = 16, \quad b = \pm 4.$$

Bu yerdan uchlarning koordinatalari: $A(\pm 7;0)$ va $B(0;\pm 4)$. O‘qlarning o‘zunliklari mos ravishda $2a = 2 \cdot 7 = 14$ va $2b = 2 \cdot 4 = 8$ ga teng.

354. Quyidagi ellipslarning uchlari koordinatalarini va o‘qlarining uzunliklarini toping:

$$1)\ \frac{x^2}{25}+\frac{y^2}{9}=1; \quad 2)\ \frac{x^2}{16}+\frac{y^2}{81}=1 .$$

X. Ellipsning fokuslari koordinatalarini (fokuslari orasidagi masofani) uning tenglamasi bo‘yicha hisoblash

355. $\frac{x^2}{100}+\frac{y^2}{36}=1$ ellips berilgan. Ellips fokuslarining koordinatalarini va fokuslari orasidagi masofani toping.

Yechilishi. Ellips tenglamasidan $a^2=100$ va $b^2=36$ ga egamiz. (3.8) formula bo‘yicha:

$$c=\pm\sqrt{100-36}=\pm 8,$$

bu yerdan fokuslarning koordinatalari $F(\pm 8;0)$ va fokuslar orasidagi masofa $2c=2\cdot 8=16$.

356. Quyidagi ellipslarning fokuslari koordinatalarini va fokuslari orasidagi masofani toping:

$$1)\ \frac{x^2}{12}+\frac{y^2}{3}=1; \quad 2)\ \frac{x^2}{10}+\frac{y^2}{26}=1 .$$

XI. Ellipsning ekssentrisitetini uning tenglamasi bo‘yicha topish

357. $\frac{x^2}{100}+\frac{y^2}{51}=1$ ellips berilgan. Bu ellipsning ekssentrisitetini hisoblang.

Yechilishi. Ellips tenglamasidan: $a^2=100$ va $b^2=51$. (3.8) formuladan foydalanib, c ni topamiz: $c=\sqrt{100-51}=7$.

Ekssentrisitetni (3.9) formula bo‘yicha topamiz: $\varepsilon=\frac{7}{10}=0,7$.

358. Quyidagi ellipslarning ekssentrisitetini hisoblang:

$$1)\ \frac{x^2}{25}+\frac{y^2}{9}=1; \qquad 2)\ \frac{x^2}{7}+\frac{y^2}{16}=1.$$

XII. Berilgan ellipsning berilgan to‘g‘ri chiziq bilan kesishish nuqtalarining koordinatalarini hisoblash

359. $\frac{x^2}{100}+\frac{y^2}{25}=1$ ellipsning $x+2y-14=0$ to‘g‘ri chiziq bilan kesishish nuqtalarining koordinatalarini toping.

Yechilishi. Ellips va to‘g‘ri chiziqning kesishish nuqtalarining koordinatalarini hisoblash uchun quyidagi tenglamalar sistemasini echish kerak:

$$\begin{cases} \frac{x^2}{100}+\frac{y^2}{25}=1 \\ x+2y-4=0 \end{cases}.$$

Bu sistemaning ildizlari $x_1=8,\ y_1=3$ va $x_2=6,\ y_2=4$. Ellips va to‘g‘ri chiziq (8;3) va (6;4) nuqtalarda kesishadi.

360. 1) $\frac{x^2}{225}+\frac{y^2}{25}=1$ ellips va $x+3y-21=0$ to‘g‘ri chiziqning;

2) $\frac{x^2}{25}+\frac{y^2}{9}=1$ ellips va $3x+5y-21=0$ to‘g‘ri chiziqning kesishish nuqtalarining koordinatalarini toping.

XIII. Berilgan to‘g‘ri chiziqning berilgan ellips ichida joylashgan kesmasining o‘zunligini hisoblash

361. $x+4y-28=0$ to‘g‘ri chiziqning $\frac{x^2}{400}+\frac{y^2}{25}=1$ ellips ichida joylashgan kesmasining o‘zunligini toping.

Yechilishi. To‘g‘ri chiziqning ellips ichida joylashgan kesmasining o‘zunligini hisoblash uchun to‘g‘ri chiziq va ellipsning kesishish nuqtalari koordinatalarini topamiz, buning uchun ushbu tenglamalar sistemasini yechamiz:

$$\begin{cases} x+4y-28=0 \\ \dfrac{x^2}{100}+\dfrac{y^2}{25}=1 \end{cases}.$$

Bu sistemaning ildizlari: $x_1=12,\ y_1=4$ va $x_2=16,\ y_2=3$. To‘g‘ri chiziqning ellips bilan kesishish nuqtalari: $A(12;4)$ va $B(16;3)$. (1.1) formula bo‘yicha AB kesmaning o‘zunligini topamiz:

$$AB=\sqrt{(16-12)^2+(3-4)^2}=\sqrt{17}.$$

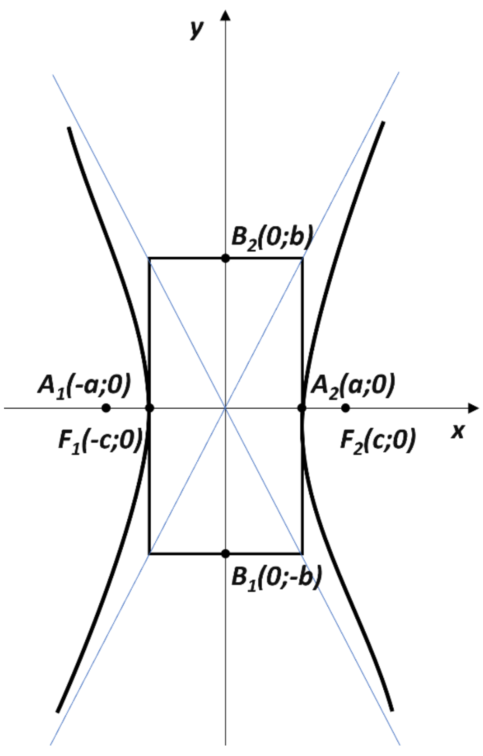
362. $x-2y-2=0$ to‘g‘ri chiziqning $\dfrac{x^2}{100}+\dfrac{y^2}{25}=1$ ellips ichida joylashgan kesmasining o‘zunligini toping.

19- §. Giperbola

Har biridan fokuslar deb ataluvchi ikkita nuqtagacha bo‘lgan masofalari ayirmasining absolyut qiymati o‘zgarmas, hamda fokuslar orasidagi masofadan kichik va nolga teng bo‘lmagan kattalik bo‘lgan nuqtalarning geometrik o‘rni **giperbola** deb ataladi.

Giperbola tenglamasi:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1, \qquad (3.11)$$



57-rasm

bu yerda a - haqiqiy yarim o‘qning uzunligi; b – mavhum yarim o‘qning uzunligi (57- rasm). a, b va c parametrlar orasidagi munosabat:

$$b^2=c^2-a^2, \qquad (3.12)$$

bu yerda c - fokuslar orasidagi masofaning yarmi.

Giperbolaning ekssentrisiteti:

$$\varepsilon=\frac{c}{a}=\frac{\sqrt{a^2+b^2}}{a}>1. \qquad (3.13)$$

Giperbola asimptotalarining tenglamalari:

$$y=\pm\frac{b}{a}x. \qquad (3.14)$$

Teng tomonli giperbolaning tenglamasi:

$$x^2-y^2=a^2. \qquad (3.15)$$

Teng tomonli giperbola asimptotalarining tenglamalari:

$$y=\pm x. \qquad (3.16)$$

(3.11) va (3.15) formulalarda x va y - o‘zgaruvchi koordinatalar - giperbolaning istalgan nuqtasining koordinatalari. (3.14) va (3.16) formulalarda x va y - asimptotaning istalgan nuqtasining koordinatalari.

Giperbolaning haqiqiy uchlarini koordinatalarini $A_1(-a;0)$ va $A_2(a;0)$ bo‘yicha bevosita haqiqiy o‘qning o‘zunligi $2a$ topiladi va mavhum uchlarning koordinatalari $B_1(0;-b)$ va $B_2(0;b)$ bo‘yicha mavhum o‘qning o‘zunligi $2b$ topiladi. Aksincha, haqiqiy va mavhum o‘qlarning o‘zunliklari bo‘yicha giperbolaning haqiqiy uchlari $(\pm a;0)$ va mavhum uchlari $(0;\pm b)$ topiladi.

Giperbola fokuslarining koordinatalari $(\pm c;0)$ bo‘yicha fokuslar orasidagi masofa $2c$ va fokuslar orasidagi masofa $2c$ bo‘yicha fokuslarning koordinatalari $(\pm c;0)$ topiladi.

Fokuslari Oy o‘qda joylashgan giperbola

$$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1 \quad \text{yoki} \quad \frac{x^2}{b^2}-\frac{y^2}{a^2}=-1 \qquad (3.17)$$

ko‘rinishga ega bo‘ladi, bu yerda a - haqiqiy yarim o‘q, b mavhum yarim o‘q (58- rasm).

Fokuslari Oy o‘qda bo‘lgan giperbola asimptotalarining tenglamalari ushbu

$$x = \pm \frac{b}{a} y \qquad (3.18)$$

ko‘rinishni oladi.

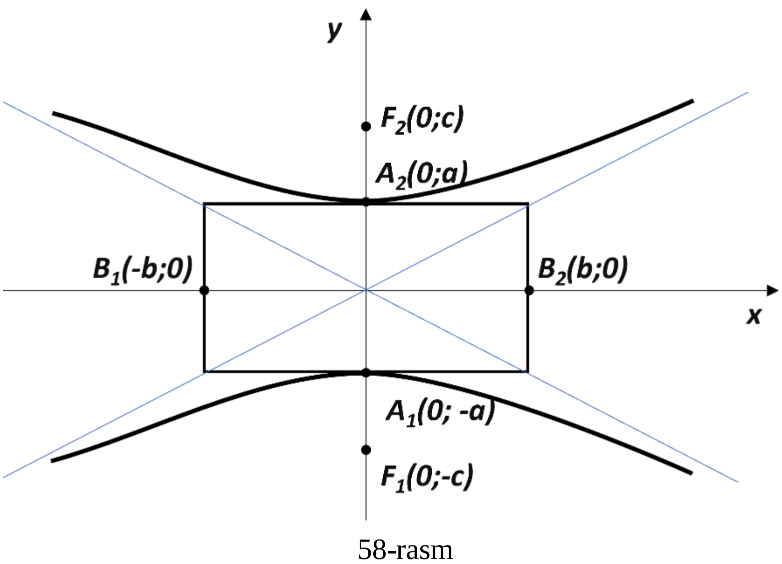
Fokuslari Oy o‘qda bo‘lgan giperbola uchun (3.12) va (3.13) formulalar o‘zgarishsiz qoladi.

(3.11) va (3.17) tenglamalar bilan ifodalangan giperbolalar ***qo‘shma*** deyiladi. Fokuslari Oy o‘qda bo‘lgan teng tomonli giperbolaning tenglamasi:

$$y^2 - x^2 = a^2 \qquad (3.19)$$

Giperbolani uning $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ tenglamasi bo‘yicha yasash quyidagi sxema bo‘yicha bajariladi:

1) giperbolaning yarim o‘qlari a va b topiladi hamda $A_1(-a;0)$, $A_2(a;0)$, $B_1(0;-b)$ va $B_2(0;b)$ koordinatali nuqtalar yasaladi;



58-rasm

2) bu nuqtalardan Ox va Oy o‘qlarga parallel to‘gri chiziqlar o‘tkazilib, markazi koordinatalar boshida bo‘lgan, $2a$ va $2b$ tomonli to‘g‘ri to‘rtburchak hosil qilinadi;

3) bu to‘g‘ri to‘rtburchakning

diagonallari chiziladi va ularning har birini ikkala tomonga davom ettirilib (3.14) giperbolaning asimptotalarini hosil qilinadi;

4) giperbolaning tarmog‘ini $A_1(-a;0)$ va $A_2(a;0)$ uchlardan o‘tuvchi va koordinatalar boshidan o‘zoqlashgan sari asimptotalarga yaqinlashib keluvchi egri chiziq kabi (qo‘lda) chiziladi (58-rasm).

Fokuslari Oy o‘qda bo‘lgan (3.17) giperbola xam shunga o‘xshash yasaladi.

Giperbolaga doir barcha masalalarda giperbolaning simmetriya o‘qi koordinata o‘qlari bilan ustma-ust tushadi deb faraz qilinadi.

I.Berilgan nuqtalarning giperbolaga tegishliligini tekshirish

363. 1) $A(5;6)$, $B(2\sqrt{5};4)$ va $C(3;5)$ nuqtalar $\frac{x^2}{16}-\frac{y^2}{64}=1$ giperbolaga; 2) $A(\sqrt{10};-2)$, $B(\sqrt{3};-5)$ va $C(-3\sqrt{2};6)$ nuqtalar $\frac{x^2}{9}-\frac{y^2}{36}=1$ giperbolaga tegishliligini tekshiring.

II. Giperbolaning tenglamasini uning o‘qlari o‘zunliklari bo‘yicha tuzish

364. Fokuslari Ox o‘qda bo‘lib, haqiqiy o‘qi 16 ga, mavhum o‘qi zsa 8 ga teng bo‘lgan giperbolaning tenglamasini tuzing.

Yechilishi. Giperbola tenglamasini tuzish uchun a va b parametrlarni bilish kerak. Masala shartidan: $2a=16,\ a=8$ va $2b=8,\ b=4$. a va b ning bu qiymatlarini giperbolaning (3.11) tenglamasiga qo‘yib,

$$\frac{x^2}{64}-\frac{y^2}{16}=1$$

ni hosil qilamiz.

365. Fokuslari Ox o‘qda bo‘lib, haqiqiy o‘qi 24 ga, mavhum o‘qi esa 40 ga teng bo‘lgan giperbolaning tenglamasini tuzing.

III. Giperbola tenglamasini uning haqiqiy o‘qi o‘zunligi (uning ikkita uchi koordinatalari) va fokuslari orasidagi masofa (fokuslarining koordinatalari) bo‘yicha tuzish

366. Agar giperbolaning uchlari $A_1(-3;0)$ va $A_2(3;0)$ nuqtalarda, fokuslari $(\pm 5;0)$ nuqtalarda bo‘lsa, uning tenglamasini tuzing.

Yechilishi. Masala shartidan $a=3$ va $c=5$ ekanligi kelib chiqadi. (3.12) formula bo‘yicha: $b^2=5^2-3^2=16$. a va b^2 ning qiymatlarini (3.11) tenglamaga qo‘yib,

$$\frac{x^2}{9}-\frac{y^2}{16}=1$$

ni hosil qilamiz.

367. Agar giperbolaning uchlari $A_1(-3;0)$ va $A_2(3;0)$ nuqtalarda, fokuslari $(\pm 3\sqrt{5};0)$ nuqtalarda bo‘lsa, uning tenglamasini tuzing.

368. Agar giperbolaning fokuslari Ox o‘qda bo‘lib, haqiqiy o‘qi 12 ga, fokuslari orasidagi masofa esa 20 ga teng bo‘lsa, bu giperbolaning tenglamasini tuzing.

IV. Giperbolaning tenglamasini uning fokuslari koordinatalari (fokuslari orasidagi masofa) va ekssentrisiteti bo‘yicha tuzish

369. Giperbola tenglamasini uning fokuslarini koordinatalari $F(\pm 20;0)$ va ekssentrisiteta $\varepsilon=\frac{5}{3}$ bo‘yicha tuzing.

Yechilishi. Masala shartidan : $c=20$, $\varepsilon=\frac{c}{a}=\frac{5}{3}$. c ning qiymatini ikkinchi tenglikka qo‘yib , $\frac{20}{a}=\frac{5}{3}$ ni topamiz, bu yerdan $a=12$.

(3.12) formuladan : $b^2 = 20^2 - 12^2 = 256$. a va b^2 ning qiymatlarini (3.11) tenglamaga qo‘yib,

$$\frac{x^2}{144} - \frac{y^2}{256} = 1$$

ni hosil qilamiz.

370. Giperbolaning fokuslarini koordinatalari va ekssentrisiteti berilgan:

1) $F(\pm 2\sqrt{2}; 0),\quad \varepsilon = 2$;   2) $F(\pm 3\sqrt{3}; 0),\quad \varepsilon = \frac{1}{2}\sqrt{6}$

Uning tenglamasini tuzing.

V. Giperbolaning tenglamasini uning haqiqiy o‘qi (mavhum o‘qi) o‘zunligi va ekssentrisiteti bo‘yicha tuzish

371. Fokuslari Ox o‘qda bo‘lib, haqiqiy o‘qi 12 ga, ekssentrisiteti $\frac{4}{3}$ ga teng bo‘lgan giperbolaning tenglamasini tuzing.

Yechilishi. Masala shartidan , $2a = 12$, bu yerdan $a = 6$ va $\varepsilon = \frac{4}{3}$.

(3.13) formula bo‘yicha c ni hisoblaymiz $\frac{4}{3} = \frac{c}{6}$, $c = 8$. (3.12) formuladan foydalanib , $b^2 = 8^2 - 6^2 = 28$ ni topamiz. a va b^2 ning qiymatlarini (3.11) formulaga qo‘yib , izlanayotgan tenglamani hosil qilamiz:

$$\frac{x^2}{36} - \frac{y^2}{28} = 1.$$

372. Fokuslari Ox o‘qda bo‘lib , haqiqiy o‘qi 14 ga, ekssentrisiteti $\frac{9}{7}$ ga teng bo‘lgan giperbolaning tenlamasini tuzing.

373. Fokuslari Ox o‘qda bo‘lib, mavhum o‘qi 8 ga, ekssentrisiteti $\frac{5}{3}$ ga teng bo‘lgan giperbolaning tenglamasini tuzing.

Yechilishi. Masala shartidan: $2b=8$, bu yerdan $b=4$ va $\varepsilon=\frac{5}{3}$. (3.13) formula bo‘yicha a^2 ni topamiz:

$$\frac{5}{3}=\frac{\sqrt{a^2+4^2}}{a}; \quad 25a^2=9a^2+144,$$

bu yerdan $a^2=9$. a^2 va b ning qiymatlarini (3.11) tenglamaga qo‘yib ,

$$\frac{x^2}{9}-\frac{y^2}{16}=1$$

ni hosil qilamiz.

374. Fokuslari Ox o‘qda bo‘lib, mavhum o‘qining o‘zunligi 8 ga, ekssentrisigeti esa $\frac{3\sqrt{5}}{5}$ ga teng bo‘lgan giperbolaning tenglamasini tuzing.

VI. Giperbolaning tenglamasini uning yarim o‘qlari yig‘indisi (ayirmasi) va fokuslari orasidagi masofa bo‘yicha tuzish

375. Fokuslari Ox o‘qda bo‘lgan giperbolaning yarim o‘qlari (haqiqiy va mavhum o‘qlarini) yig‘indisi 14 ga va fokuslari orasidagi masofa 20 ga teng bo‘lsa , uning tenglamasini tuzing.

Yechilishi. Masala shartida quyidagilar berilgan: $a+b=14$ va $2c=20$, bu yerdan $c=10$. (3.12) formula bo‘yicha: $10^2=a^2+b^2$. Quyidagi tenglamalar sistemasini yozamiz:

$$\begin{cases} a+b=14 \\ a^2+b^2=100 \end{cases}.$$

Bu sistemaning ildizlari: $a_1 = 6,\ \ b_1 = 8$ va $a_2 = 8,\ \ b_2 = 6$. Demak, masala shartini quyidagi ikkita giperbola qanoatlantirar ekan:

$$1)\ \frac{x^2}{36} - \frac{y^2}{64} = 1 \quad \text{va} \quad 2)\ \frac{x^2}{64} - \frac{y^2}{36} = 1 .$$

376. Agar fokuslari Ox o‘qda bo‘lgan giperbolaning: 1) yarim o‘qlarining (haqiqiy va mavhum o‘qlarining) yig‘indisi 7 ga va fokuslari orasidagi masofa 10 ga teng bo‘lsa; 2) yarim o‘qlarining (haqiqiy va mavhum o‘qlarining) ayirmasi 4 ga va fokuslari orasidagi masofa 40 ga teng bo‘lsa, bu giperbolaning tenglamasini tuzing.

VII. Giperbolaning tenglamasini uning haqiqiy (mavhum) o‘qi o‘zunligi va bu giperbola o‘tadigan nuqtaning koordinatalari bo‘yicha tuzish

377. Fokuslari Ox o‘qda bo‘lgan giperbolaning haqiqiy o‘qini uzunligi 8 ga teng bo‘lsa va u $(8;6)$ nuqtadan o‘tsa, bu giperbolaning tenglamasini tuzing.

Yechilishi. Masala shartida $2a = 8$ berilgan, bu yerdan $a = 4$. (3.11) tenglamaga $a = 4$ qiymatni va $(8;6)$ nuqtaning koordinatalarini x va y o‘zgaruvchilarning o‘rniga qo‘yamiz va hosil bo‘lgan tenglamadan b^2 ni topamiz:

$$\frac{8^2}{4^2} - \frac{6^2}{b^2} = 1 \quad \text{yoki} \quad 4b^2 - 36 = b^2 ,$$

bu yerdan $b^2 = 12$. (3.11) tenglamaga a va b^2 ning topilgan qiymatlarini qo‘yib, izlanayotgan tenglamani hosil qilamiz:

$$\frac{x^2}{16} - \frac{y^2}{12} = 1 .$$

378. Fokuslari Ox o‘qda bo‘lgan giperbola haqiqiy o‘qining uzunligi 16 ga teng bo‘lsa va u $(-10;-3)$ nuqtadan o‘tsa, bu giperbolaning tenglamasini tuzing.

379. Fokuslari Ox o‘qda bo‘lgan giperbola mavhum o‘qining uzunligi 12 ga teng bo‘lsa va u $(20;8)$ nuqtadan o‘tsa, bu giperbolaning tenglamasini tuzing.

VIII. Berilgan ikkita nuqtadan o‘tuvchi giperbolaning tenglamasini tuzish

380. Fokuslari Ox o‘qda bo‘lgan hamda $(6;3)$ va $\left(5\sqrt{2};-4\right)$ nuqtalardan o‘tadigan giperbolaning tenglamasini tuzing.

Yechilishi. Giperbola tenglamasini tuzish uchun a^2 va b^2 ni topish kerak. Berilgan nuqtalarning koordinatalarini (3.11) tenglamaga qo‘yib, ushbu tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} \dfrac{6^2}{a^2} - \dfrac{3^2}{b^2} = 1 \\ \dfrac{\left(5\sqrt{2}\right)^2}{a^2} - \dfrac{(-4)^2}{b^2} = 1, \end{cases}$$

so’ngra sistemani yechib, $a^2 = 18$ va $b^2 = 9$ ni topamiz.

a^2 va b^2 ning qiymatlarini (3.11) tenglamaga qo‘yib, giperbolaning izlanayotgan tenglamasini hosil qilamiz:

$$\frac{x^2}{18} - \frac{y^2}{9} = 1 .$$

381. Fokuslari Ox o‘qda bo‘lgan va 1) $\left(-6;-\sqrt{7}\right)$ va $\left(6\sqrt{2};4\right)$; 2) $\left(-8;2\sqrt{2}\right)$ va $(6;-1)$ nuqtalardan o‘tadigan giperbolaning tenglamasini tuzing.

IX. Giperbolaning tenglamasini uning asimptotalari va fokuslarining koordinatalari bo‘yicha yasash

382. Agar giperbola fokuslarining koordinatalari $(\pm 8;0)$ bo‘lib, asimptotalari $y = \pm\sqrt{3}x$ tenglama bilan berilgan bo‘lsa, bu giperbolaning tenglamasini tuzing.

Yechilishi. Masala shartida $c=8$ va $\frac{b}{a}=\pm\sqrt{3}$ berilgan [(3.14) formula)]. Bu ma'lumotlar bo'yicha a va b ni topamiz. (3.12) formula bo'yicha $64=a^2+b^2$ ni yozamiz. Ushbu tenglamalar sistemasiga egamiz:

$$\begin{cases} a^2+b^2=64 \\ \frac{b}{a}=\pm\sqrt{3} \end{cases}.$$

Bu sistemadan topilgan $a^2=16$ va $b^2=48$ ni (3.11) tenglamaga qo'yib,

$$\frac{x^2}{16}-\frac{y^2}{48}=1$$

ni hosil qilamiz.

383. Giperbola fokuslarining koordinatalari va asimptotalarining tenglamalari bo'yicha giperbolaning tenglamasini tuzing:

$$1)\ F(\pm5;0),\quad y=\pm\frac{4}{3}x\ ;\quad 2)\ F(\pm3;0),\quad y=\pm\sqrt{2}x\ .$$

X. Giperbolaning tenglamasini uning asimptotalari va giperbola o'tadigan nuqtaning koordinatalari bo'yicha tuzish

384. Agar giperbolaning asimptotalari $y=\pm\frac{\sqrt{6}}{3}x$ tenglamalar bilan berilgan bo'lib, giperbolaning o'zi $(6;-4)$ nuqtadan o'tadigan bo'lsa, uning tenglamasini tuzing.

Yechilishi. Masala shartida $\frac{b}{a}=\pm\frac{\sqrt{6}}{3}$ berilgan [(3.14) formula]. (3.11) tenglamada x va y ning o‘rniga $(6;-4)$ koordinatalarni qo‘yamiz va quyidagi tenglamalar sistemasini tuzamiz:

$$\begin{cases} \frac{6^2}{a^2}-\frac{(-4)^2}{b^2}=1 \\ \frac{b}{a}=\pm\frac{\sqrt{6}}{3} \end{cases},$$

bu sistemadan $a^2=12$ va $b^2=8$ ni topamiz.

a^2 va b^2 ning qiymatlarini (3.11) tenglamaga qo‘yib,

$$\frac{x^2}{12}-\frac{y^2}{8}=1$$

ni hosil qilamiz.

385. Giperbola asimptotalarining tenglamalari va giperbola o‘tadigan nuqtaning koordinatalari berilgan:

1) $y=\pm\frac{\sqrt{3}}{3}x,\ \left(9;3\sqrt{2}\right)$; 2) $y=\pm\frac{\sqrt{2}}{2}x,\ \left(-4;-2\right)$;

3) $y=\pm\frac{\sqrt{3}}{2}x,\ \left(4\sqrt{3};3\sqrt{3}\right)$.

Giperbolaning tenglamasini tuzing.

XI. Giperbolaning tenglamasi bo‘yicha uning asimptotalari tenglamalarini tuzish

386. $\frac{x^2}{144}-\frac{y^2}{256}=1$ giperbola berilgan. Uning asimptotalari tenglamalarini tuzing.

Yechilishi. Giperbola tenglamasidan a va b ni topamiz: $a=12$, $b=16$. a va b ning qiymatlarini asimptotalarning tenglamasiga qo‘yib, $y=\pm\frac{16}{12}x$ yoki $y=\pm\frac{4}{3}x$ ni hosil qilamiz.

387. Quyidagi giperbolalarning asimptotalari tenglamalarini tuzing:

$$1)\ \frac{x^2}{64}-\frac{y^2}{36}=1, \quad 2)\ \frac{x^2}{9}-\frac{y^2}{8}=1.$$

XII. Berilgan nuqtadan o‘tuvchi teng tomonli giperbolaning tenglamasini tuzish

388. $(-5;4)$ nuqtadan o‘tuvchi va fokuslari Ox o‘qda bo‘lgan teng tomonli giperbolaning tenglamasini tuzing.

Yechilishi. Berilgan nuqtaning koordinatalarini teng tomonli giperbolaning (3.15) tenglamasiga qo‘yib, $(-5)^2-4^2=a^2$, $a^2=9$ ni hosil qilamiz. Giperbola tenglamasi: $x^2-y^2=9$.

389. $(8;2)$ nuqtadan o‘tuvchi va fokuslari Ox o‘qda bo‘lgan teng tomonli giperbolaning tenglamasini tuzing.

390. $\left(-\sqrt{3};-\sqrt{5}\right)$ nuqtadan o‘tuvchi va fokuslari Oy o‘qda bo‘lgan teng tomonli giperbolaning tenglamasini tuzing.

XIII. Giperbola uchlarining koordinatalarini (uning uchlari o‘zunliklarini) uning tenglamasi bo‘yicha hisoblash

391. Giperbola tenglamasi berilgan: $\frac{x^2}{81}-\frac{y^2}{144}=1$. Uning uchlari koordinatalarini toping.

Yechilishi. Giperbola tenglamasidan a ni topamiz: $a^2=81$, $a=\pm 9$. Giperbolaning uchlari $(-9;0)$ va $(9;0)$ nuqtalarda joylashgan.

392. Giperbola tenglamasi berilgan: $\frac{x^2}{36}-\frac{y^2}{14}=1$. Bu giperbola uchlarining koordinatalarini toping.

393. Quyidagi giperbolalar uchlarining uzunliklarini toping:

$$1)\ \frac{x^2}{25}-\frac{y^2}{49}=1;\quad 2)\ \frac{x^2}{12}-\frac{y^2}{81}=1.$$

XIV. Giperbola fokuslarining koordinatalarini (fokuslari orasidagi masofani) giperbolaning tenglamasi bo‘yicha hisoblash

394. Giperbola tenglamasi berilgan $\frac{x^2}{15}-\frac{y^2}{49}=1\cdot$ Bu giperbola fokuslarining koordinatalarini toping.

Yechilishi. Giperbola tenglamasidan: $a^2=15,\ \ b^2=49$. (3.12) formula bo‘yicha:

$$c^2=15+49=64\ ,\quad c=\pm 8\ .$$

Fokuslar $(-8;0)$ va $(8;0)$ nuqtalarda joylashgan.

395. Giperbola tenglamasi berilgan: $\frac{x^2}{14}-\frac{y^2}{22}=1\cdot$ Bu giperbola fokuslarining koordinatalarini toping.

396. Quyidagi giperbolalarning fokuslari orasidagi masofani toping:

$$1)\quad \frac{x^2}{28}-\frac{y^2}{36}=1;\quad 2)\ \frac{x^2}{12}-\frac{y^2}{24}=1.$$

XV. Giperbolaning ekssentrisitetini uning tenglamasi bo‘yicha hisoblash

397. Giperbola tenglamasi berilgan: $\frac{x^2}{25}-\frac{y^2}{11}=1$. Uning ekssentrisitetini toping.

Yechilishi. Giperbola tenglamasidan: $a^2=25,\ \ b^2=11$.
Ekssentrisitet (3,13) formula bo‘yicha hisoblanadi:

$$\varepsilon=\frac{\sqrt{25+11}}{5}=\frac{6}{5}\ .$$

398. Quyidagi giperbolalarning ekssentrisitetini toping:

$$1)\ \frac{x^2}{9}-\frac{y^2}{7}=1;\quad 2)\ \frac{x^2}{25}-\frac{y^2}{24}=1.$$

XVI. Fokuslari *Ou* o‘qda bo‘lgan giperbola

399. $\frac{x^2}{36}-\frac{y^2}{64}=-1$ giperbolaning uchlarini, fokuslarini, ekssentrisitetini va asimptotalarini toping.

Yechilishi. Berilgan giperbola (3.17) ko‘rinishga ega:

$$\frac{x^2}{b^2}-\frac{y^2}{a^2}=-1,$$

ya’ni uning fokuslari Oy o‘qda yotadi. Tenglamadan:
$a^2=64,\ \ a=\pm 8$ va $b^2=36,\ \ b=\pm 6$. Giperbolaning uchlari $A_1(0;-8)$ va $A_2(0;8)$ nuqtalarda bo‘ladi.

(3.12) formula bo‘yicha: $c^2=64+36=100,\ \ c=\pm 10$; demak, fokuslar $F_1(0;-10)$ va $F_2(0;10)$ nuqtalarda joylashgan.

Ekssentrisitetni (3.13) formula bo‘yicha hisoblaymiz:

$$\varepsilon=\frac{5}{4}.$$

Giperbolaning asimptotalarini (3.18) formula bo‘yicha topamiz:

$$x=\pm\frac{3}{4}y.$$

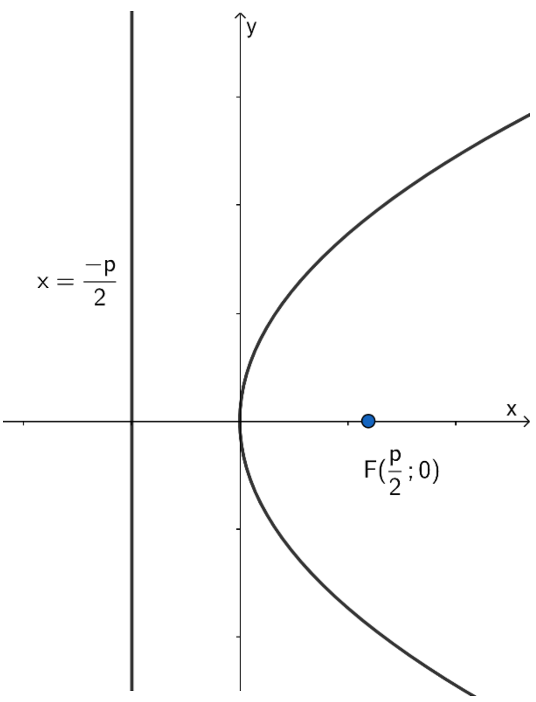
400. $\frac{x^2}{9}-\frac{y^2}{16}=-1$ giperbolaning uchlarini, fokuslarini, ekssentrisitetini va asimptotalarini toping.

20-§. Uchi koordinatalar boshida bo‘lgan parabola

Fokus deb ataluvchi berilgan nuqtadan va direktrisa deb ataluvchi berilgan to‘g‘ri chiziqdan (fokus direktrisada yotmaydi) teng uzoqlashgan nuqtalarning geometrik o‘rni parabola deb ataladi. Uchi koordinatalar boshida, simmetriya o‘qi Ox o‘q bo‘lgan va tarmoqlari o‘ngga qarab yo‘nalgan parabolaning tenglamasi quyidagi ko‘rinishda bo‘ladi:

$$y^2 = 2px \ (p > 0), \qquad (3.20)$$

bu yerda p - parabolaning parametri - fokusdan direktrisagacha bo‘lgan masofa; x va y - o‘zgaruvchi koordinatalar - parabolaning istalgan nuqtasini koordinatalari.



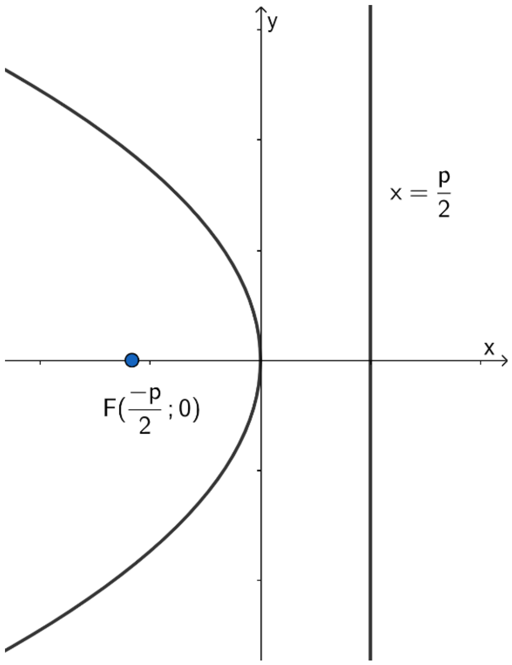
59-rasm

Parabolaning direktrisasi tenglamasi: $x = -\dfrac{p}{2}$ (59-rasm). Uchi koordinatalar boshida, simmetriya o‘qi Ox o‘q bo‘lgan va tarmoqlari chapga qarab yo‘nalgan parabolaning tenglamasi quyidagi ko‘rinishda bo‘ladi:

$$y^2 = -2px \ (p > 0). \qquad (3.21)$$

Uning direktrisasining tenglamasi:

$$x = \frac{p}{2} \qquad \text{(60-rasm).}$$



60-rasm

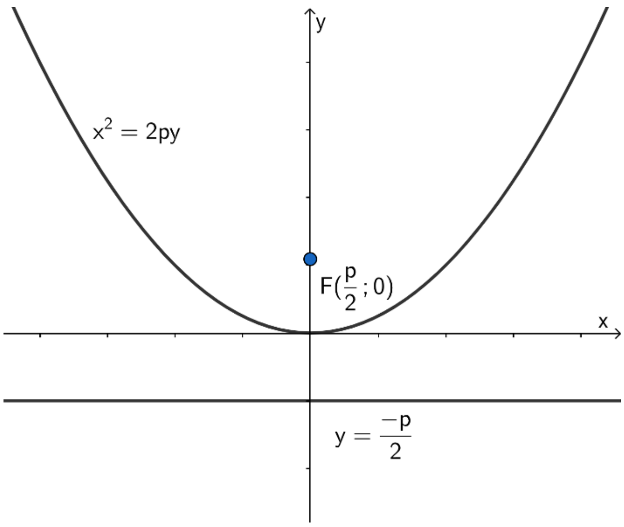
Uchi koordinatalar boshida, simmetriya o‘qi bo‘lib Oy o‘q xizmat qiladigan va tarmoqlari yuqoriga qarab yo‘nalgan parabolaning tenglamasi quyidagi ko‘rinishga ega:

$$x^2 = 2py \ (p > 0). \tag{3.22}$$

Uning direktrisasining tenglamasi:

$$y = -\frac{p}{2}$$

Uchi koordinatalar boshida, simmetriya o‘qi bo‘lib Oy o‘q xizmat qiladigan va tarmoqlari pastga qarab yo‘nalgan parabolaning tenglamasi quyidagi ko‘rinishga ega:

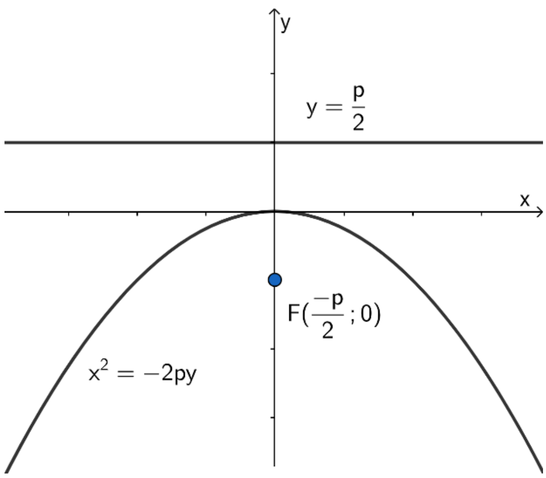


61-rasm

$$x^2 = -2py \ (p > 0). \tag{3.23}$$

Uning direktrisasining tenglamasi:

$$y = \frac{p}{2}.$$



62-rasm

Bu paragrafning barcha masalalarida koordinata o‘qlaridan biri parabolaning simmetriya o‘qi deb faraz qilinadi.

I. Berilgan nuqtalarniig berilgan parabolaga tegishliligini tekshirish

401. $A(1;-2)$, $B(4;4)$ va $C(1;3)$ nuqtalarning $y^2=4x$ parabolaga tegishliligini tekshiring.

402. $A(-3;-3)$, $B(3\sqrt{3};-9)$ va $C(5;-8)$ nuqtalarning $x^2=-3y$ parabolaga tegishliligini tekshiring.

II. Uchi koordinatalar boshida bo'lgan parabolaning tenglamasini uning fokuslari koordinatalari bo'yicha tuzish

403. Agar uchi koordinatalar boshida bo'lgan parabolaning fokusi $F(3;0)$ nuqtada yotsa, bu parabolaning tenglamasini tuzing.

Yechilishi. Parabolaning fokusi Ox ning musbat yarim o'qida yotibdi, demak, parabolaning umumiy ko'rinishdagi tenglamasi (3.20) tenglama bilan beriladi; uning fokusining koordinatalari $F_1\left(\frac{p}{2};0\right)$. p ni topamiz: $\frac{p}{2}=3,\quad p=6$. p ning qiymatini (3.20) tenglamaga qo'yib, $y^2=2\cdot 6x=12x,\quad y^2=12x$ ni hosil qilamiz.

404. Agar uchi koordinatalar boshida bo'lgan parabolaning fokusi: 1) $F(5;0)$; 2) $F(-4;0)$; 3) $F(0;2)$) va 4) $F(0;-3)$ nuqtada yotsa, bu parabolaning tenglamasini tuzing.

III. Uchi koordinatalar boshida bo'lgan parabolaning tenglamasini uning direktrisasi tenglamasi bo'yicha tuzish

405. Agar uchi koordinatalar boshida bo'lgan parabolaning direktrisasi $x=-4$ to'gri chiziqdan iborat bo'lsa, uning tenglamasini tuzing.

Yechilishi. Direktrisadan koordinatalar boshigacha bo'lgan masofa $\frac{p}{2}$ ga teng, demak, $\frac{p}{2}=4$, $p=8$.

Bu parabolaning tenglamasi umumiy ko'rinishda (3.20) tenglama bilan beriladi, chunki direktrisaning abssissasi manfiy.

(3.20) tenglamaga *r* parametrning qiymatini qo‘yib, parabolaning tenglamasini hosil qilamiz:

$$y^2 = 16x\ .$$

406. Agar uchi koordinatalar boshida bo‘lgan parabolaning direktrisasi: 1) $x = -2$; 2) $x = 3$; 3) $y = -4$; 4) $y = 1$ to‘g‘ri chiziqlardan iborat bo‘lsa, uning tenglamasini tuzing.

IV. Uchi koordinatalar boshida, Ox (Oy) o‘qqa nisbatan simmetrik va berilgan nuqtadan o‘tuvchi parabolaning tenglamasini tuzish

407. Uchi koordinatalar boshida, Ox o‘qqa nisbatan simmetrik va $A(1;-2)$ nuqtadan o‘tuvchi parabolaning tenglamasini tuzing.

Yechilishi. Izlanayotgan parabolaning simmetriya o‘qi bo‘lib Oy o‘q xizmat qiladi va parabola $A(1;-2)$ nuqtadan, ya’ni to‘rtinchi chorakda yotuvchi nuqtadan o‘tadi, demak, parabola umumiy ko‘rinishda (3.20) tenglama bilan beriladi. Bu tenglamaga $A(1;-2)$ nuqtaning koordinatalarini qo‘yib, $(-2)^2 = 2p \cdot 1,\ \ p = 2$ ni topamiz, (3.20) tenglamaga p ning qiymatini qo‘ygach, parabolaning tenglamasini hosil qilamiz:

$$y^2 = 4x\ .$$

408. Uchi koordinatalar boshida, Ox o‘qqa nisbatan simmetrik va 1) $(5;-3)$; 2) $(-4;2)$; 3) $(-2;2)$ nuqtadan o‘tuvchi parabolaning tenglamasini tuzing.

409. Uchi koordinatalar boshida, Oy o‘qqa nisbatan simmetrik va $A(4;2)$ nuqtadan o‘tuvchi parabolaning tenglamasini tuzing.

Yechilishi. Parabolaning simmetriya o‘qi Oy o‘q bo‘ladi, parabola esa $A(4;2)$ nuqtadan o‘tadi, demak, izlanayotgan parabola umumiy ko‘rinishda (3.22) tenglama bilan beriladi. Bu tenglamaga $A(4;2)$ nuqtaning koordinatalarini qo‘yib, p ni topamiz: $p = 4$. (3.22) tenglamaga p ning qiymatini qo‘ygandan so‘ng, parabolaning tenglamasini hosil qilamiz:

$$x^2 = 8y \ .$$

410. Uchi koordinatalar boshida, Oy o‘qqa nisbatan simmetrik va 1) $(2;-3)$; 2) $(-3;1)$ nuqtadan o‘tuvchi parabolaning tenglamasini tuzing.

V. Koordinatalar boshidan o‘tuvchi parabolaning tenglamasi bo‘yicha uning direktrisasi tenglamasini tuzish

411. Parabola tenglamasi berilgan: $y^2 = 6x$. Uning direktrisasi tenglamasini tuzing.

Yechilishi. Parabolaning $y^2 = 6x$ tenglamasidan: $2p = 6$, $p = 3$. $y^2 = 2px$ parabolaning direktrisasi tenglamasi $x = -\frac{p}{2}$ ko‘rinishga ega. Demak, direktrisaning tenglamasi $x = -\frac{3}{2}$ yoki $2x + 3 = 0$ bo‘ladi.

412. 1) $y^2 = 8x$; 2) $y^2 = -9x$; 3) $x^2 = 4y$; 4) $x^2 = -10y$ parabolaning direktrisasi tenglamasini tuzing.

VI. Koordinatalar boshidan o‘tuvchi parabolaning tenglamasi bo‘yicha uning fokusi koordinatalarini hisoblash.

413. Parabola tenglamasi berilgan: $y^2 = 12x$. Uning fokuslari koordinatalarini toping.

Yechilishi. Koordinatalar boshidan fokusgacha bo‘lgan masofa $\frac{p}{2}$ ga teng. $y^2 = 12x$ parabola tenglamasidan: $2p = 12$, $p = 6$ va $\frac{p}{2} = 3$. Demak, $F(3;0)$.

414. Parabolaning quyida berilgan tenglamasi bo‘yicha uning fokusi koordinatalarini hisoblang: 1) $y^2 = 6x$; 2) $y^2 = -4x$; 3) $x^2 = 14y$; 4) $x^2 = -5y$.

VII. Uchi koordinatalar boshida bo‘lgan parabolaning direktrisasi tenglamasi bo‘yicha uning fokusi koordinatalarini hisoblash

415. Agar uchi koordinatalar boshida bo‘lgan parabola direktrisasining tenglamasi $x = -3$ bo‘lsa, uning fokusi koordinatalarini toping.

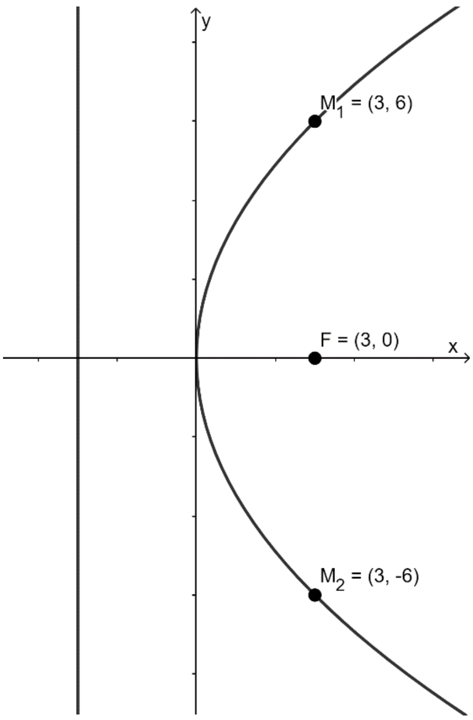
Yechilishi. Koordinatalar boshidan direktrisagacha bo‘lgan masofa koordinatalar boshidan fokusgacha bo‘lgan masofaga teng va u ham bo‘lsa $\frac{p}{2}$ ga teng. Direktrisaning $x = -3$ tenglamasydan $\frac{p}{2} = 3$ ekanligi kelib chiqadi. Tenglamasi $x = -\frac{p}{2}$ bo‘lgan direktrisaga fokusi musbat abssissa $F(3;0)$ ga ega bo‘lgan $y^2 = 2px$ parabola mos keladi.

416. Uchi koordinatalar boshida bo‘lgan parabolaning direktrisasi 1) $x = 2$; 2) $x = -5$; 3) $y = 4$; 4) $y = -6$ tenglama bilan berilgan bo‘lsa, parabola fokusining koordinatalarini toping.

VIII. Uchi koordinatalar boshida bo‘lgan parabolaning fokusidan uning o‘qiga perpendikulyar bo‘lib o‘tuvchi vatarning o‘zunligini hisoblash

417. $y^2 = 12x$ parabola berilgan. Parabolaning fokusidan uning o‘qiga perpendikulyar bo‘lib o‘tuvchi vatarning uzunligini toping.

Yechilishi. Vatar parabolaning fokusi orqali o‘tib, parabola o‘qiga perpendikulyardir, shu sababli vatarning parabola bilan kesishish nuqtalarining abssissasi fokusning abssissasi bilan umumiy bo‘ladi (63-rasm). Parabola tenglamasidan fokusning koordinatalarini topamiz:



63-rasm

$$y^2 = 12x\ ;\quad 2p = 12\ ;\quad \frac{p}{2} = 3\ ,\quad F(3;0)\ .$$

Vatarning parabola bilan kesishish nuqtalarining ordinatalarini topish uchun parabola tenglamasiga $x = 3$qiymatni (vatarning parabola bilan kesishish nuqtalarining abssissasi) qo‘yamiz: $y^2 = 12\cdot 3 = 36$, bu yerdan $y_{1,2} = \pm 6$. Demak, vatarning parabola bilan kesishish nuqtalari $M_1(3;6)$ va $M_2(3;-6)$ ekan. M_1M vatarning uzunligi $2FM_1 = 2\cdot 6 = 12$ ga teng.

418. $y^2 = 20x$ parabola berilgan. Parabolaning fokusidan o‘tib, uning o‘qiga perpendikulyar bo‘lib o‘tgan vatarning uzunligini toping.

IX. Uchi koordinatalar boshida bo‘lgan parabolaning berilgan to‘g‘ri chiziq bilan kesishish nuqtalarining koordinatalarini topish

419. $y^2 = 16x$ parabolaning $4x - 3y + 8 = 0$ to‘g‘ri chiziq bilan kesishish nuqtalarini toping.

Yechilishi. Parabola va to‘g‘ri chiziqning kesishish nuqtalarini topish uchun ushbu

$$\begin{cases} y^2 = 16x \\ 4x - 3y + 8 = 0 \end{cases}$$

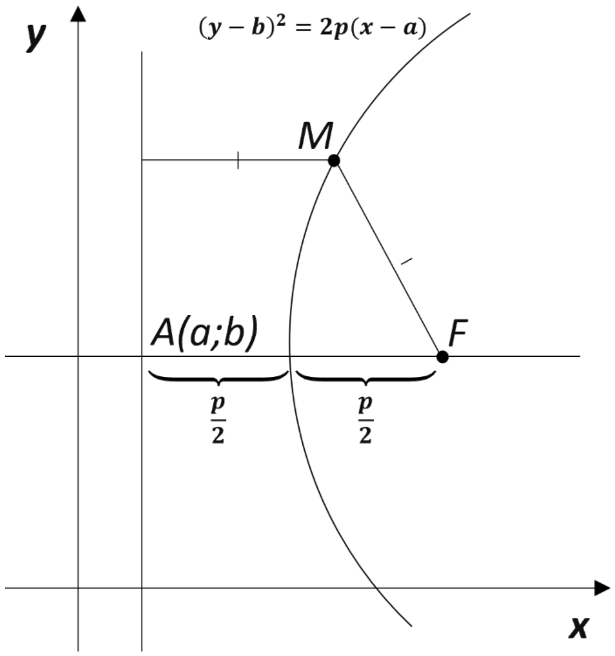
sistemani yechamiz. Bu sistemaning ildizlari $x_1 = 1,\ \ y_1 = 4$ va $x_2 = 4,\ \ y_2 = 8$. Demak, parabola va to‘g‘ri chiziq $(1;4)$ va $(4;8)$ nuqtalarda kesishadi.

420. 1) $y^2 = 16x$ parabolaning $2x - y + 2 = 0$ to‘gri chiziq bilan; 2) $y^2 = 4x$ parabolaning $2x - 3y + 4 = 0$ to‘g‘ri chiziq bilan kesishish nuqtalarini toping.

X. Uchlari koordinatalar boshida bo‘lgan ikkita parabolaning kesishish nuqtalarining koordinatalarini hisoblash

421. $y^2 = 9x$ va $x^2 = 9y$ parabolalarning kesiishsh nuqtalarini toping.

Yechilishi. Berilgan parabolalarning kesishish nuqtalarini topish uchun ushbu



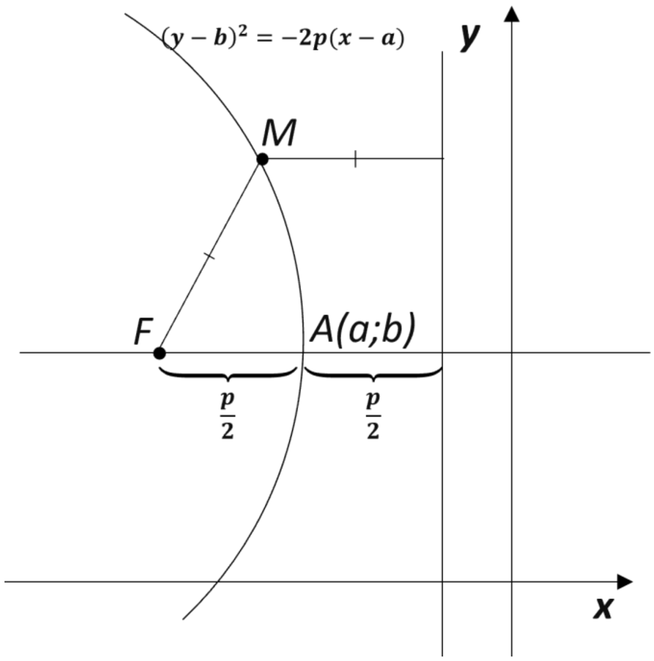
64-rasm

$$\begin{cases} y^2 = 9x \\ x^2 = 9y \end{cases}$$

tenglamalar sistemasini yechamiz. Sistemani yechib $x_1 = 0, \quad y_1 = 0$ va $x_2 = 9, \quad y_2 = 9$ ni hosil qilamiz. Demak, parabolallr (0; 0) va (9; 9) nuqtalarda kesishadi.

422. $y = x^2$ va $x = y^2$ parabolalarning kesishish nuqtalarini toping.

21-§. Uchi ixtiyoriy nuqtada bo‘lgan parabola



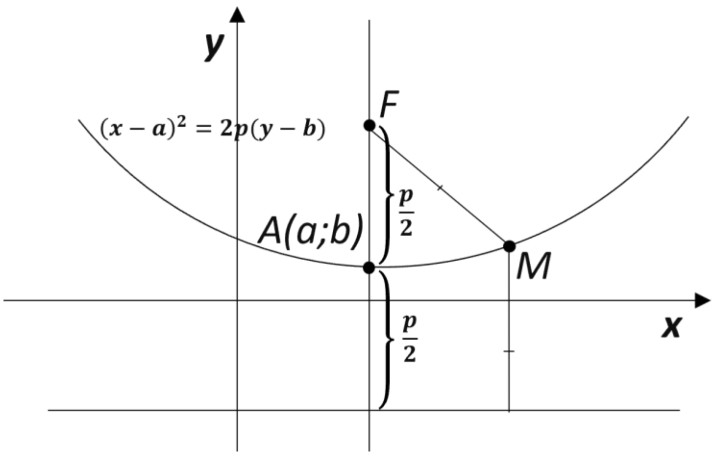
65-rasm

Uchi $(a;b)$ nuqtada, simmetriya o‘qi Ox o‘qqa parallel bo‘lgan va tarmoqlari o‘ngga yo‘nalgan parabolaning tenglamasi quyidagi ko‘rinishga ega (64-rasm):

$$(y-b)^2 = 2p(x-a). \tag{3.24}$$

Uchi $(a;b)$ nuqtada, simmetriya o‘qi Ox o‘qqa parallel bo‘lgan va tarmoqlari chap tomonga yo‘nalgan parabolaning tenglamasi (65- rasm):

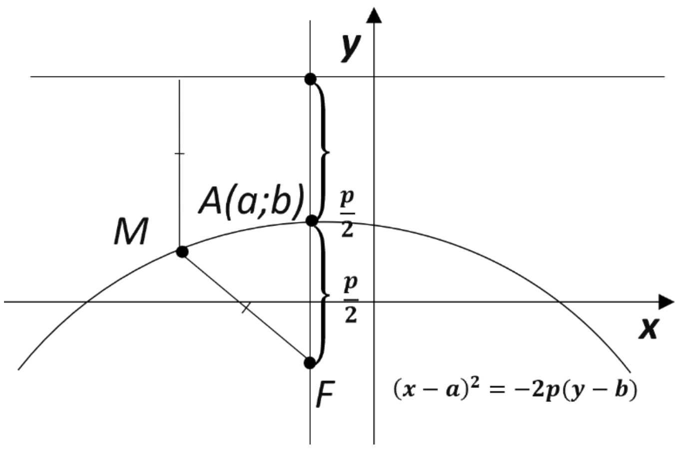
$$(y-b)^2=-2p(x-a). \tag{3.25}$$



66-rasm

Uchi $(a;b)$ nuqtada, simmetriya o‘qi Oy o‘qqa parallel bo‘lgan va tarmoqlari yuqoriga yo‘nalgan parabolaning tenglamasi (66-rasm):

$$(x-a)^2=2p(y-b). \tag{3.26}$$



67-rasm

Uchi $(a;b)$ nuqtada, simmetriya o‘qi Oy o‘qqa parallel bo‘lgan va tarmoqlari pastga yo‘nalgan parabolaning tenglamasi (67-rasm):

$$(x-a)^2=-2p(y-b). \tag{3.27}$$

Yuqoridagi tenglamalarning har qaysisida parabolaning parametri $p>0$ — parabolaning fokusidan uning direktrisasigacha bo‘lgan masofa.

I. Uchi $(a;b)$ nuqtada, simmetriya o‘qi Ox (Oy) o‘qqa parallel bo‘lgan va berilgan nuqtadan o‘tadigan parabolaning tenglamasini tuzish

423. Uchi $A(1;2)$ nuqtada, simmetriya o‘qi Ox o‘qqa parallel bo‘lgan va $M(4;8)$ nuqtadan o‘tadigan parabolaning tenglamasini tuzing.

Yechilishi. Masala shartida (3.24) ko‘rinishdagi parabola berilgan, chunki $M(4;8)$ nuqta parabola uchidan o‘ngda joylashgan, shuning uchun parabolaning tarmoqlari ham o‘ng tomonga yo‘nalgan. p parametrni hisoblash uchun (3.24) tenglamaga $A(1;2)$ uchning va $M(4;8)$ nuqtaning koordinatalarini qo‘yamiz: $(8-2)^2=2p(4-1)$, bu yerdan $p=6$. (3.24) tenglamaga p ning topilgan qiymatini va $A(1;2)$ uchning koordinatalarini qo‘yib, izlanayotgan tenglamani hosil qilamiz:

$$y^2-4y-12x+16=0.$$

424. Uchi $A(-4;-2)$ nuqtada, simmetriya o‘qi Ox o‘qqa parallel bo‘lgan va $M(1;3)$ nuqtadan o‘tadigan parabolaning tenglamasini tuzing.

425. Uchi $A(2;4)$ nuqtada, simmetriya o‘qi Oy o‘qqa parallel bo‘lgan va $M(-6;8)$ nuqtadan o‘tadigan parabolaning tenglamasini tuzing.

426. Uchi $A(-2;-4)$ nuqtada, simmetriya o‘qi Ox o‘qqa parallel bo‘lgan va koordinatalar boshidan o‘tadigan parabolaning tenglamasini tuzing.

427. Uchi $A(5;-5)$ nuqtada, simmetriya o‘qi Oy o‘qqa parallel bo‘lgan va koordinatalar boshidan o‘tadigan parabolaning tenglamasini tuzing.

428. Uchi $A(-2;1)$ nuqtada, simmetriya o‘qi Oy o‘qqa parallel bo‘lgan va $M(5;-6)$ nuqtadan o‘tadigan parabolaning tenglamasini tuzing.

Yechilishi. Masala shartida (3.27) ko‘rinishdagi parabola berilgan. $M(5;-6)$ nuqta parabola uchidan pastda joylashganligi uchun parabolaning tarmoqlari ham pastga yo‘nalgan. p parametrni hisoblash uchun (3.27) tenglamaga $A(-2;1)$ uchining va $M(5;-6)$ nuqtaning koordinatalarini qo‘yamiz: $(5+2)^2=-2p(-6-1)$, bu yerdan $p=\frac{7}{2}$. p ning topilgan qiymatini va $A(-2;1)$ uchning koordinatalarini (3.27) tenglamaga qo‘yib, izlanayotgan tenglamani hosil qilamiz:

$$x^2+4x+7y-3=0.$$

429. Uchi $A(3;-1)$ nuqtada, simmetriya o‘qi Ox o‘qqa parallel bo‘lgan va $M(-3;-3)$ nuqtadan o‘tadigan parabolaning tenglamasini tuzing.

430. Uchi $(3;5)$ nuqtada, simmetriya uqi Oy o‘qqa parallel bo‘lgan va koordinatalar boshidan o‘tadigan parabolaning tenglamasini tuzing.

II. Parabolaning tenglamasini uning uchi va fokusining koordinatalari bo‘yicha tuzish

431. Agar parabolaning uchi $A(2;3)$ nuqtada va fokusi $F(6;3)$ nuqtada bo‘lsa, bu parabolaning tenglamasini tuzing.

Yechilishi. Masalada (3.24) ko‘rinishdagi parabola berilgan, chunki parabolaning o‘qi Ox o‘qqa parallel (uchning va fokusning ordinatalari teng, demak, ular Ox ga parallel bo‘lgan to‘g‘ri chiziqda yotadilar) va parabola tarmoqlari o‘ng tomonga yo‘nalgan (uning fokusi parabola uchidan o‘ngda joylashgan).

Parabola tenglamasini tuzish uchun p parametrni topamiz. Parabolaning uchidan fokusigacha bo‘lgan masofaga $\frac{p}{2}$ teng.

Masala shartidan $\frac{p}{2} = |6-2| = 4$ (fokus va uchning abssissalari ayirmasi absolyut kattaligi bo‘yicha olinadi, chunki $p > 0$), bu yerdan $p = 8$.

(3.24) tenglamaga $A(2;3)$ nuqtaning koordinatalarini va p ning topilgan qiymatini qo‘yib, parabolaning izlanayotgan tenglamasini hosil qilamiz: $y^2 - 6y - 16x + 41 = 0$.

432. Uchi A nuqtada va fokusi F nuqtada bo‘lgan parabolaning tenglamasini tuzing:

1) $A(4;6)$, $F(-2;6)$; 2) $A(3;-2)$, $F(3;0)$; 3) $A(-1;1)$ $F(-1;-4)$.

III. Parabolaning tenglamasini uning uchi koordinatalari va direktrisasi tenglamasi bo‘yicha tuzish

433. Uchi $A(4;6)$nuqtada, direktrisasi $x=-2$ bo‘lgan parabolaning tenglamasini tuzing.

Yechilishi. Masala shartida (3.24) ko‘rinishdagi parabola berilgan, chunki uning direktrisasi ($x=-2$)Ox o‘qda perpendikulyar, va demak, parabolaning o‘qi Ox o‘qqa parallel va uning tarmoqlari o‘ng tomonga yo‘nalgan (direktrisa uchdan chaproqda joylashgan). Parabola tenglamasini tuzish uchun p parametrni topamiz. Parabolaning uchidan direktrisagacha bo‘lgan masofa $\frac{p}{2}$ ga teng. Masala shartidan, $\frac{p}{2}$ direktrisa va parabola uchi absiissalarining absolyut qiymatlari yig‘indisiga tengligi, ya’ni $\frac{p}{2}=|-2|+4=6$ ekanligi kelib chiqadi, bu yerdan $p=12$. (3.24) tenglamaga $A(4;6)$ nuqtaning koordinatalari va p ning topilgan qiymatini qo‘yib, parabolaning izlanayotgan tenglamasini hosil qilamiz:

$$y^2-12y-24x+132=0\ .$$

434. Uchi $A(1;-3)$nuqtada, direktrisasi $x=5$ bo‘lgan parabolaning tenglamasini tuzing.

Ko‘rsatma. $\frac{p}{2}$ kattalik direktrisa va uning abssissalari ayirmasiga teng.

435. Uchi $A(-2;4)$ nuqtada va direktrisasi $y=-2$bo‘lgan parabolaning tenglamasini tuzing.

436. Uchi $A(-3;5)$nuqtada va direktrisasi $y=7$ bo‘lgan parabolaning tenglamasini tuzing.

437. Ox o‘qqa nisbatan simmetrik bo‘lgan va uchi $A(-2;0)$ nuqtada, direktrisasi esa Oy o‘qdan iborat bo‘lgan parabolaning tenglamasini tuzing.

Yechilishi. Masala shartida (3.25) ko‘rinishdagi parabola berilgan, chunki uchning abssissasi Oy o‘q bilan ustma ust tushuvchi direktrisadan chapda yotadi va parabolaning tarmoqlari chap tomonga yo‘nalgan. $\frac{p}{2}=|-2|=2$ (parabola uchidan koordinatalar boshigacha bo‘lgan masofa), bu yerdan $p=4$. (3.25) tenglyamaga $A(-2;0)$ uchning koordinatalarini va p parametrning topilgan qiymatini qo‘yib

$$y^2+8x+16=0$$

ni xosil qilamiz.

438. Ox o‘qqa nisbatan simmetrik bo‘lgan va uchi $(3;0)$ nuqtada, direktrisasi esa Oy o‘qdan iborat bo‘lgan parabolaning tenglamasini tuzing.

439. Ox o‘qqa nisbatan simmetrik bo‘lgan va uchi $A(-4;0)$ nuqtada, direktrisasi esa $x=2$ to‘gri chiziqdan iborat bo‘lgan parabolaning tenglamasini tuzing.

440. Oy o‘qqa nisbatan simmetrik bo‘lgan va uchi $A(0;2)$ nuqtada, direktrisasi esa Ox o‘qdan iborat bo‘lgan parabolaning tenglamasini tuzing.

441. Oy o‘qqa nisbatan simmetrik bo‘lgan va uchi $A(0;-2)$ nuqtada, direktrisasi esa $y=-5$ to‘g‘ri chiziqdan iborat bo‘lgan parabolaning tenglamasini tuzing.

442. Oy o‘qqa nisbatan simmetrik bo‘lgan va uchi $A(0;-3)$ nuqtada, direktrisasi esa Ox o‘qdan iborat bo‘lgan parabolaning tenglamasini tuzing.

IV. Parabolaning tenglamasini uning fokusi koordinatalari va direktrisasining tenglamasi bo‘yicha tuzish

443. Fokusi $F(4;3)$ nuqtada bo‘lgan, direktrisasi esa $x=-2$ tenglama bilan berilgan parabolaning tenglamasini tuzing.

Yechilishi. Masala shartida (3.24) ko‘rinishdagi parabola berilgan, chunki direktrisa Ox o‘qqa perpendikulyar va, demak, parabolaning o‘qi Ox o‘qqa parallel, tarmoqlari esa o‘ng tomonga yo‘nalgan (direktrisa fokusdan chapda joylashgan) p parametrni topamiz. Fokusdan direktrisagacha bo‘lgan masofa $p=|-2|+4=6$ ga teng. Parabola uchining koordinatalarini topamiz. Fokusdan uchgacha (shuningdek, direktrisadan uchgacha ham) bo‘lgan masofa $\frac{p}{2}=\frac{6}{2}=3$ ga teng, biroq fokusning abssissasi 4 ga teng, binobarin, parabola uchining abssissasi $a=4-3=1$ bo‘lib, ordinatasi esa fokusning ordinatasi bilan bir xil (parabolaning o‘qi Ox o‘qqa parallel): $b=3$, $A(1;3)$. p ning topilgan qiymatini va $A(1;3)$ ni (3.24) tenglamaga qo‘yib, parabolaning tenglamasini hosil qilamiz:

$$y^2-6y-12x+21=0\ .$$

444. Fokusi $F(6;-1)$ nuqtada bo‘lgan, direktrisasi esa $x=2$ tenglama bilan berilgan parabolaning tenglamasini tuzing.

445. Fokusi koordinatalar boshida bo‘lgan, direktrisasi esa $x=-4$ tenglama bilan berilgan parabolaning tenglamasini tuzing.

446. Fokusi $(2;2)$ nuqtada bo‘lgan, direktrisasi esa $y=-4$ tenglama bilan berilgan parabolaning tenglamasini tuzing.

447. Fokusi koordinatalar boshida bo‘lgan, direktrisasi esa $y=4$ tenglama bilan berilgan parabolaning tenglamasini tuzing.

V. Parabola uchining koordinatalarini uning tenglamasi bo‘yicha hisoblash

448. $y=Ax^2+Bx+C$ tenglama bilan berilgan parabola uchining koordinatalarini toping.

Yechilishi. Berilgan tenglamani $(x-a)^2=2p(y-b)$ ko‘rinishdagi tenglamaga keltiramiz. $y=Ax^2+Bx+C$ tengalamaning ikkala tomonini A ga bo‘lamiz:

$$\frac{y}{A}=x^2+\frac{B}{A}x+\frac{C}{A}\ .$$

O‘ng tomonni to‘liq kvadratga to‘ldiramiz, buning uchun tenglamaning chap va ung tomoniga $\left(\frac{B}{2A}\right)^2$ ni qo‘shamiz va $\frac{C}{A}$ ozod hadni chap tomonga o‘tkazamiz:

$$\frac{y}{A}-\frac{C}{A}+\frac{B^2}{4A^2}=x^2+2x\frac{B}{2A}+\frac{B^2}{4A^2}\ .$$

Quyidagicha almashtiramiz:

$$\frac{1}{A}\left(y-\frac{4AC-B^2}{4A}\right)=\left(x+\frac{B}{2A}\right)^2\ ,$$

So’ngra

$$\frac{4AC-B^2}{4A}=b\ \ ,-\frac{B}{2A}=a\quad \text{va}\quad \frac{1}{A}=2p$$

deb olib, (3.26) ko‘rinishdagi parabolani hosil qilamiz.

$$y=Ax^2+Bx+C$$

ko‘rinishdagi parabolaning uchi $\left(\frac{B}{2A};-\frac{B^2-4AC}{4A}\right)$ nuqtada bo‘ladi.

Bu munosabatlardan parabola uchining koordinatalarini hisoblashda foydalanish mumkin: masalan $y=3x^2-2x+4$ parabola uchining koordinatalari (bu yerda $A=3,\ B=-2$ va $C=4$) quyidagicha bo‘ladi:

$$a=-\frac{B}{2A}=\frac{1}{3},\ \ b=-\frac{B^2-4AC}{4A}=3\frac{2}{3};\ \ A\left(\frac{1}{3};3\frac{2}{3}\right).$$

449. Parabola tenglamasi berilgan: $x^2-8x-4y+28=0$. Parabola uchining koordinatalarini toping.

Yechilishi. Bu tenglamani (3.26) ko‘rinishga keltiramiz, buning uchun quyidagi almashtirishlarni bajaramiz:

$$x^2-8x=4y-28;$$

$$x^2-2\cdot 4x+4^2=4y-28+4^2;\ \ (x-4)^2=4(y-3),$$

bu yerdan

$$a=4,\ \ b=3,\ \ 2p=4;\ \ A(4;3)\ .$$

450. 1) $x^2-6x-6y-21=0$; 2) $x^2+8x+5y+21=0$ parabola uchining koordinatalarini toping.

451. Parabola tenglamasi berilgan: $y^2-6y-12x+33=0$. Parabola uchining koordinatalarini toping.

Yechilishi. Bu tenglamani (3. 24) ko‘rinishga keltiramiz, buning uchun 449-masalada bajarilgan almashtirishlarga o‘xshash almashtirishlarni bajaramiz:

$$y^2-6y-12x+33=0\ ;\ \ y^2-2\cdot 3y+3^2=12x-33+3^2;$$

$$(y-3)^2=12(x-2)\ ,$$

bu yerdan

$$a=2,\ \ b=3,\ \ 2p=12;\ \ A(2;3)\ .$$

452. Parabola tenglamasi berilgan: $y^2+6y+3x+15=0$. Parabola uchining koordinatalarini toping.

VI. Fokusning koordinatalarini parabola tenglamasi bo‘yicha hisoblash

453. $y^2+4y-24x+76=0$ parabola fokusining koordinatalarini hisoblang.

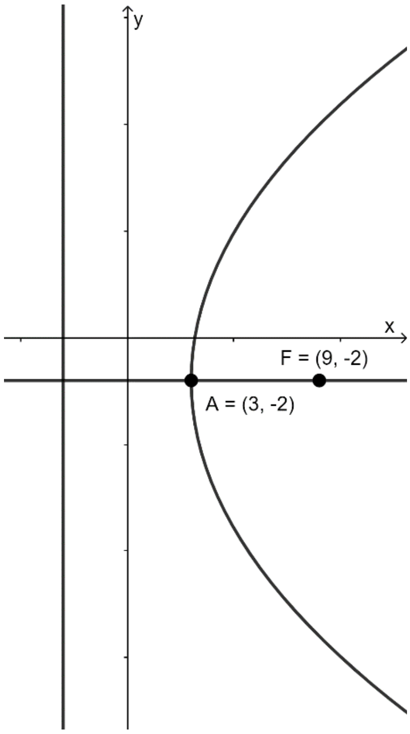
Yechilishi. Parabola tenglamasini (3.24) ko‘rinishga keltiramiz:

$$y^2+4y=24x-76\ ;\ \ y^2+2\cdot 2y+2^2=24x-76+2^2\ ,$$

$$(y+2)^2=24(x-3)\ .$$

bu yerda

$$A(3;-2)\ ,\ \ 2p=24\ ,\ \ p=12\ .$$



68-rasm

Parabolaning uchidan fokusgacha bo‘lgan masofa $\frac{p}{2}=\frac{12}{2}=6$ ga teng. Fokusning abssissasi $3+\frac{p}{2}=3+6=9$ ga teng.

Fokus parabolaning uchidan o‘ngda joylashgan, chunki parabolaning tarmoqlari o‘ngga qaragan; fokusning ordinatasi esa uchning ordinatasiga teng, chunki parabolaning o‘qi Ox o‘qqa paralleldir (68-rasm), u holda $F(9;-2)$.

454. 1) $y^2-8y-8x-8=0$; 2) $y^2-12x-36=0$. Parabola fokusining koordinatalarini hisoblang.

455. $x^2-4x-16y+68=0$ parabola fokusining koordinatalarini hisoblang.

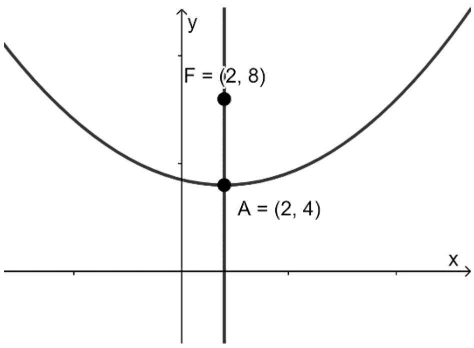
Yechilishi. Parabola tenglamasini (3.26) ko‘rinishga keltiramiz:

$$x^2-4x=16y-68\,;\quad x^2-2\cdot 2x+2^2=16y-68+2^2\,;$$

$$(x-2)^2=16(y-4)\,,$$

bu yerdan

$$A(2;4)\,,\quad 2p=16\,,\quad p=8\,.$$



69-rasm

Parabolaning uchidan fokusgacha bo‘lgan masofa $\frac{p}{2}=\frac{8}{2}=4$ ga, fokusning ordinatasi $4+\frac{p}{2}=4+4=8$ ga teng.

Fokus parabolaning uchidan yuqorida yotadi, chunki parabolaning tarmoqlari yuqoriga qaragan, fokusning abssissasi esa uchning abssissasiga teng, chunki parabolaning o‘qi Oy o‘qqa parallel (69-rasm): $F(2;8)$.

456. 1) $x^2+10x+8y+41=0$; 2) $x^2-6y-9=0$ parabola fokusining koordinatalarini hisoblang.

VII. Parabolaning simmetriya o‘qi tenglamasini parabola tenglamasi bo‘yicha tuzish

457. $y^2-6y-8x-7=0$ parabola berilgan. Parabola o‘qining tenglamasini tuzing.

Yechilishi. $y^2-6y-8x-7=0$ parabolaning o‘qi uning uchi orqali Ox o‘qqa parallel holda o‘tadi. Parabola tenglamasini (3.24) ko‘rinishga keltirib, parabola uchining koordinatalarini hisoblaymiz:

$$y^2-6y=8x+7\ ;\quad y^2-2\cdot 3y+3^2=8x+7+3^2\ ;$$
$$(y-3)^2=8(x+2),$$

bu yerdan

$$a=-2\ ,\quad b=3\ ;\quad A(-2;3)\ .$$

Parabolaning o‘qi $A(-2;3)$ nuqtadan o‘tadi, demak, uning tenglamasi: $y=3$.

458. 1) $y^2-10y-10x+5=0$; 2) $x^2+16x-18y+100=0$ parabola o‘qining tenglamasini tuzing.

VIII. Parabolaning direktrisasi tenglamasini uning tenglamasi bo‘yicha tuzish

459. $y^2-4y-20x+24=0$ parabola berilgan. Uning direktrisasi tenglamasini tuzing.

Yechilishi. Parabolaning direktrisasi uning uchidan $\frac{p}{2}$ masofada parabola o‘qiga perpendikulyar holda o‘tadi. Parabola tenglamasidan p ni topamiz:

$$y^2-4y=20x-24\ ;\quad y^2-2\cdot 2y+4=20x-24+4\ ;$$
$$\left(y-2\right)^2=20\left(x-1\right),$$

bu yerdan

$$a=1\ ,\quad b=2\ ;\quad A(1;2)$$
$$2p=20\ ,\quad \frac{p}{2}=5\ .$$

Parabolaning simmetriya o‘qi Oy o‘qqa parallel va uning tarmoqlari o‘ng tomonga yo‘nalgan, demak, parabolaning direktrisasi uning uchidan chaproqda o‘tadi. Bundan tashqari, u koordinatalar boshidan ham chaproqda o‘tadi, chunki parabolaning uchidan Oy o‘qqacha bo‘lgan masofa 1 ga, direktrisagacha bo‘lgan masofa esa 5 ga teng. Direktrisaning abssissasi $\frac{p}{2}-1=5-1=4$ ayirmaning manfiy ishora bilan olinganiga teng, shu sababli direktrisaning tenglamasi $x=-4$ bo‘ladi.

460. 1) $y^2-2y-10x+11=0$; 2) $y^2+8y+8x+32=0$; 3) $x^2-6x+2y+7=0$ parabolaning direktrisasi tenglamasini tuzing.

IX. Parabolani uning tenglamasi bo‘yicha yasash

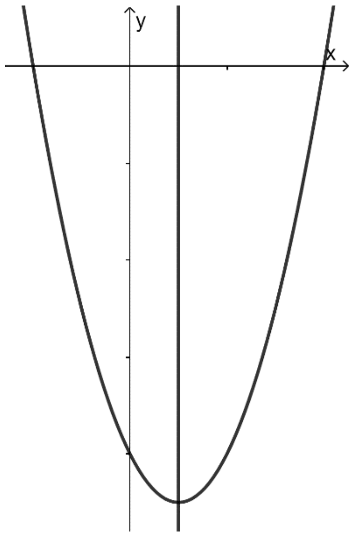
461. $x^2-2x-y-8=0$ parabolani yasang.

Yechilishi. 1- usul. Tenglamani $y=Ax^2+Bx+C$ ko‘rinishga keltirib, $y=x^2-2x-8$ ni hosil qilamiz.

448-masalada $y=Ax^2+Bx+C$ parabolani tekshirganda uning uchini koordinatalari uchun $\left(-\frac{B}{2A};\frac{B^2-4AC}{4A}\right)$ ifoda hosil qilingan edi. Ulardan berilgan parabola uchining koordinatalarini hisoblash uchun foydalanamiz. $A=1,\ B=-2$ va $C=-8$ ga egamiz. Bo‘larni yuqoridagi ifodaga qo‘yamiz:

$$a=-\frac{-2}{2\cdot 1}=1;$$

$$b=-\frac{(-2)^2-4\cdot 1\cdot(-8)}{4\cdot 1}=-9.$$



70-rasm

Parabolaning uchi $(1;-9)$ nuqtada yotadi. Parabolaning Ox va Oy o‘qlap bilan kesishish nuqtalarini topamiz: $(-2;0)$, $(4;0)$ va $(0;-8)$. Bir qator xarakterli nuqtalarni hosil qildik: $(-2;0)$, $(0;-8)$, $(1;-9)$ va $(4;0)$, bu nuqtalar bo‘yicha $x=1$ o‘qqa nisbatan simmetrik bo‘lgan parabolani yasaymiz (70-rasm).

2- usul. $y=x^2-2x-8$ tenglamani (3.26) ko‘rinishdagi tenglamaga keltirib, parabolaning uchini topamiz:

$$x^2-2x=y+8;\quad x^2-2x+1=y+8+1;$$

$$(x-1)^2=y+9,$$

bu yerdan

$$a=1,\quad b=-9;\quad A(1;-9).$$

Qo‘shimcha nuqtalar birinchi usulda ko‘rsatilgandek topiladi.

3- usul (Bu usuldan parabola Ox o‘qni kesib o‘tganda foydalaniladi). $y=0$ deb $x^2-2x-8=0$ tenglamani (70-rasm) hosil qilamiz, uning ildizlari $x_1=-2$ va $x_1=4$. Parabola uchining abssissasi parabolaning Ox o‘q bilan kesishish nuqtalari abssissalarining yarim yig‘indisiga teng:

$$x_{уч}=\frac{x_1+x_2}{2}=\frac{-2+4}{2}=1.$$

Uchning ordinatasini topish uchun uchning abssissasini berilgan teiglamaga qo‘yamiz: $y=1^2-2\cdot 1-8=-9$; $A(1;-9)$.

Qushimcha nuqtalar birinchi usuldagidek topiladi.

462. 1) $x^2+2x-y-8=0$; 2) $x^2+8x+4y=0$; 3) $y^2-4x+2y=0$; 4) $x^2-6x-6y-3=0$ parabolani yasang (so‘nggi parabolaning Ox o‘q bilan kesishish nuqtalarining abssissalarini 0,1 gacha aniqlikda taqribiy hisoblang).

22-§. Aralash masalalar

463. $x^2+y^2-12x-6y+29=0$ va $x^2+y^2+4x+6y+4=0$ aylanalarning markazlari orqali Ox o‘q bilan kesishguncha to‘g‘ri chiziq o‘tkazilgan. Bu to‘g‘ri chiziqning Ox o‘q bilan hosil qilgan burchagini hisoblang.

464. $x^2+y^2-4x-16y+32=0$ aylananing markazidan va $\frac{x^2}{36}+\frac{y^2}{20}=1$ ellipsning fokuslaridan o‘tuvchi to‘g‘ri chiziqlar orasidagi burchakni toping.

465. $x^2+y^2-6x-12y+36=0$ aylananing markazidan $\frac{x^2}{36}+\frac{y^2}{16}=1$ ellipsning katta va kichik o‘qlari qanday burchak ostida ko‘rinishini aniqlang.

466. $x^2+y^2+2x-6y-40=0$ aylanani $3x-y+16=0$ to‘g‘ri chiziq kesib o‘tadi va bu to‘g‘ri chiziqning aylana ichidagi kesmasi aylanaga ichki chizilgan to‘g‘ri to‘rtburchakning tomoni bo‘lib xizmat qiladi. Bu to‘g‘ri to‘rtburchak tomonlarining tenglamalarini tuzing.

467. $\frac{x^2}{8}+\frac{y^2}{2}=1$ ellips bilan $x^2+y^2=5$ aylananing kesishish nuqtalarini toping.

468. $x^2+y^2=4$ aylanaga muntazam uchburchak ichki chizilgan bo‘lib, uning uchlaridan biri $(0;2)$ koordinatalarga ega. Uchburchakning qolgan ikkita uchining koordinatalarini toping.

469. $\frac{x^2}{25}+\frac{y^2}{16}=1$ va $\frac{x^2}{24}+\frac{y^2}{49}=1$ ellipslarning fokuslarini tutashtiruvchi to‘g‘ri chiziqlarning tenglamalarini tuzing.

470. $\frac{x^2}{36}+\frac{y^2}{9}=1$ ellipsga ichki chizilgan kvadratning yuzini hisoblang.

471. $\frac{x^2}{16}+\frac{y^2}{12}=1$ ellipsga ichki chizilgan va ikkita qarama-qarshi tomoni bu ellipsning fokuslaridan o‘tadigan to‘g‘ri to‘rtburchakning yuzini toping.

472. Uchlari $\frac{x^2}{20}+\frac{y^2}{8}=1$ ellipsning fokuslarida, fokuslari esa bu ellipsning uchlarida bo‘lgan giperbolaning tenglamasini tuzing.

473. $\frac{x^2}{25}-\frac{4y^2}{25}=1$ giperbolaning uchidan uning asimptotasigacha bo‘lgan masofani toping.

474. $\left(5\sqrt{2};4\sqrt{2}\right)$ nuqtadan o‘tuvchi va fokuslari teng tomonli $x^2-y^2=18$ giperbolaning fokuslarida yotuvchi ellipsning tenglamasini tuzing.

475. Koordinatalar boshida umumiy uchga ega bo‘lgan hamda fokuslari $F_1\left(3;0\right)$ va $F_2\left(0;\frac{3}{8}\right)$ nuqtalarda bo‘lgan ikkita parabolaning kesishish nuqtalarini toping.

476. $x^2+y^2=20$ aylana $x^2=8y$ parabolani kesib o‘tadi. Ularning umumiy vatari tenglamasini tuzing.

477. O nuqtadan jism gorizontga o‘tkir burchak ostida otilgan. U parabola yoyi chizib, O nuqtadan 40 *m* nariga tushdi. Agar jism erishgan maksimal balandlik 25 *m* ga teng bo‘lsa, parabolik traektoriyaning parametrini toping (havoning qarshiligini hisobga olmang).

Nazorat uchun masalalar

I v a r i a n t

478. 1. $x^2+y^2-4x+2y-24=0$ aylananing $A(-3;1)$ nuqtasiga o‘tkazilgan radiusning tenglamasini tuzing.

2. Agar fokuslari Ox o‘qda bo‘lgan ellipsning fokuslari orasidagi masofa 20 ga, ekssentrisigeti esa $\frac{5}{6}$ ga teng bo‘lsa, bu ellipsning tenglamasini tuzing.

3. $\frac{x^2}{81}-\frac{y^2}{63}=1$ giperbola berilgan. Uning ekssentrisitetini toping.

4. $y^2-2y+16x+65=0$ parabola berilgan. Parabola o‘qining tenglamasini tuzing.

II v a r i a n t

479. 1. $x^2+y^2-2x+4y-13=0$ aylanaga $A(-2;1)$ nuqtada o‘tkazilgan urinmaning tenglamasini tuzing.

2. $\frac{x^2}{625}+\frac{y^2}{400}=1$ ellips berilgan. Uning ekssentrisitetini toping.

3. Fokuslari Ox o‘qda bo‘lgan giperbolaning tenglamasini fokuslar orasidagi masofa $2c=90$ va asimptotalarning tenglamalari $y=\pm\frac{4}{3}x$ bo‘yicha tuzing.

4. $x^2+6x+20y-51=0$ parabola berilgan. Parabola o‘qining tenglamasini tuzing.

DIFFERENSIAL HISOB ELEMENTLARI

4-BOB

LIMITLAR

23-§. Limitlarni hisoblash

1. Agar masala shartida berilgan miqdor har xil sonli qiymatlarni qabo‘l qilsa, bu miqdor *o‘zgaruvchi miqdor* deyiladi.

2. Agar x o‘zgaruvchi absolyut qiymati bo‘yicha hech qachon biror musbat son A dan katta bo‘lmasa, uni *chegaralangan* deyiladi: $|x| < A$.

3. Agar α o‘zgaruvchi miqdor o‘zining o‘zgarishi jarayonida avvaldan berilgan har qanday kichik musbat son ε dan absolyut qiymat bo‘yicha kichik bo‘lsa va bundan keyingi o‘zgarishida ham o‘sha sondan kichikligicha qolsa, α *cheksiz kichik miqdor* deyiladi: $|\alpha| < \varepsilon$.

4. Agar x o‘zgaruvchi miqdor o‘zining o‘zgarishi jarayonida avvldan berilgan har qanday musbat son N dan , bu son qanchalik katta bo‘lmasin, katta bo‘lsa, va bundan keyingi o‘zgarishida ham o‘sha sondan kattaligini qolsa, x *cheksiz katta miqdor* deyladi: $|x| > N$.

5. Agar $x - a$ ayirmaning absolyut qiymati x ning o‘zgarish jarayonida avvaldan berilgan har qanday musbat kichik son ε dan kichik bo‘lsa va x ning bundan keyingi o‘zgarishida ham bu sondan kichikligicha qolsa, a o‘zgarmas x ning *limiti* deyiladi.

Agar a o‘zgarmas x ning limiti bo‘lsa, x miqdor a ga intiladi va bu $\lim x = a$ yoki $x \to a$ ko‘rinishda yoziladi.

6. $x \to a$ da $f(x) \to b$ bo‘lsa (shu bilan birga x miqdor a ga teng qiymatni qabul qilmaydi), b son $f(x)$ funksiyaning $x = a$ dagi limiti deyiladi.

Cheksiz kichik miqdor, cheksiz katta miqdor va o‘zgaruvchining limiti quyidagi (7-10) munosabatlar bilan bog‘langan, ular (3,4,5) ta’riflardan kelib chiqadi.

7. Cheksiz kichik miqdorning limiti nolga teng (agar α cheksiz kichik miqdor bo‘lsa, u holda $\lim\alpha = 0$).

8 . O‘zgaruvchi bilan uning limiti orasidagi ayirma cheksiz kichikdir (agar $\lim x = a$ bo‘lsa, $x - a = z$) .

9. Cheksiz kichik miqdorga teskari bo‘lgan miqdor cheksiz katta miqdor bo‘ladi (agar a cheksiz kichik miqdor bo‘lsa, $\frac{1}{a}$ cheksiz katta miqdor bo‘ladi, ya’ni agar $\alpha \to 0$ bo‘lsa, $\frac{1}{a} \to \infty$) .

10. Cheksiz katta miqdorga teskari bo‘lgan miqdor cheksiz kichik miqdordir (agar x cheksiz katta miqdor bo‘lsa, $\frac{1}{x}$ cheksiz kichik miqdordir, ya’ni agar $x \to \infty$ bo‘lsa, $\frac{1}{x} \to 0$) .

11. Cheksiz kichik miqdorlarning asosiy xossalari:

1.Istalgan chekli sondagi cheksiz kichik miqdorlarning algebraik yig‘indisi cheksiz kichik miqdordir.

II. Chegaralangan miqdorning cheksiz kichik miqdorga ko‘paytmasi cheksiz kichik miqdordir.

12. Limitlar haqidagi teoremalar:

I . O‘zgarmasning limiti uning o‘ziga teng.

II. Limitga ega bo‘lgan chekli sondagi o‘zgaruvchi miqdorlar algebraik yig‘indisining limita bu o‘zgaruvchilar limitlarining yig‘indisiga teng:

$$\lim(u + v - w) = \lim u + \lim v - \lim w .$$

III. Limitga ega bo‘lgan chekli sondagi o‘zgaruvchi miqdorlar ko‘paytmasining limiti bu o‘zgaruvchilar limitlarining ko‘paytmasiga teng:

$$\lim(uv) = \lim u \lim v.$$

2. Limitga ega bo‘lgan o‘zgaruvchi miqdorning butun musbat darajasining limiti bu o‘zgaruvchi limitining shu darajasiga teng:

$$\lim u^n = (\lim u)^n.$$

3. Limitga ega bo‘lgan o‘zgaruvchi miqdorning butun musbat darajali ildizining limiti bu o‘zgaruvchi limitining shu darajali ildiziga teng:

$$\lim \sqrt[n]{u} = \sqrt[n]{\lim u}.$$

IV. Limitga ega bo‘lgan ikkita o‘zgaruvchi miqdor bo‘linmasining limiti agar bo‘luvchining limiti nolga teng bo‘lmasa, bo‘linuvchi limitining bo‘luvchi limitiga bo‘linganiga teng:

$$\lim\left(\frac{u}{v}\right) = \frac{\lim u}{\lim v} \quad \left(\lim v \neq 0\right).$$

I. Funksiyaning limitini argumentning limit qiymatini funksiya ifodasiga bevosita qo‘yib topish

Agar limiti argumentning biror limit qiymatga intilganda topiladigan funksiya uchun limitlar haqidagi teoremalarni, qo‘llash mumkin bo‘lsa, u holda limitni hisoblash bu limit qiymatni funksiyaga qo‘yishga keltiriladi.

480. $\lim\limits_{x\to 2}\left(5x^3 - 6x^2 + x - 5\right)$.

Yechilishi . II va I teoremalarni hamda III teoremaning 1 va 2 - natijalarini ketma-ket tatbiq qilib, topamiz:

$$\lim_{x\to 2}\left(5x^3 - 6x^2 + x - 5\right) = \lim_{x\to 2}\left(5x^3\right) - \lim_{x\to 2}\left(6x^2\right) + \lim_{x\to 2}\left(x\right) - \lim_{x\to 2}\left(5\right)$$
$$= 5\cdot 2^3 - 6\cdot 2^2 + 2 - 5 = 5\cdot 8 - 6\cdot 4 + 2 - 5 = 40 - 24 + 2 - 5 = 13.$$

Teoremalar va natijalarni tatbiq etish odatda hayolda bajariladi, shu sababli yechishning mufassal yozuvi tushirib qoldiriladi. Argumentning limit qiymati $x=2$ ni funksiya ifodasiga keltirib qo‘yish yana o‘sha natijaga olib keladi va yozuv juda qisqa bo‘ladi:

$$\lim_{x\to2}\left(5x^3-6x^2+x-5\right)=5\cdot2^3-6\cdot2^2+2-5=13.$$

481. Limitlarni hisoblang:

1.) $\lim\limits_{x\to3}\left(x^3-x-5\right)$;

2) $\lim\limits_{x\to-1}(x^3-x^2+1)$; 3) $\lim\limits_{x\to-1}(2x^3-5x^2+x-4)$;

4) $\lim\limits_{x\to0}(3x^3+x^2-8x+10)$

482. $\lim\limits_{x\to1}\left[7x-2)(4x-3)(5x+1)\right]$

Yechilishi. II va I teoremalarni hamda III teoremaning 1 - natijacini ketma-ket tatbiq qilib, topamiz:

$$\lim_{x\to1}\left[(7x+2)(4x-3)(5x+1)\right]=$$

$$\left[(7x+2)(\lim4x-3)x(\lim5x+1)\right]_{x\to1}=\left[(7\cdot1+2)(4\lim x-3)x\right.$$

$$\left.(5\lim x+1)\right]_{x\to1}=(7\cdot1+2)(4\cdot1-3)(5\cdot1+1)=54$$

Bu misolda ham yechishni hayolda bajarish mumkin va x argumentning o‘rniga uning limit qiymatini qo‘yib, hisoblash mumkin:

$$\lim_{x\to1}\left[(7x+2)(4x-3)(5x+1)\right]=(7\cdot1+2)(4\cdot1-3)(5\cdot1+1)=9\cdot1\cdot6=54.$$

483. Limitlarni hisoblang:

$$\lim_{x\to2}\left[\left(x^2-1\right)(x-3)(x+5)\right];$$

$$\lim_{x\to0}\left[(2x-4)x(x-1)(x+2)\right]$$

484.Limitni hisoblang: $\lim\limits_{x\to 2}\dfrac{x^2-x+1}{x-3}$.

Yechilishi. IV teoremani tatbiq kilish mumkinligini tekshirish uchun argumentning limit qiymatida bo‘luvchi nolga teng bo‘lmasligiga ishonch hosil qilish kerak. $x=2$ da bo‘luvchi $x-3=2-3=-1$. Demak, bo‘linmaning limiti haqidagi teoremani tatbiq qilish mumkin.

IV, II va 1 teoremalarni hamda III teoremaning 2- natijasini ketma-ket tatbiq qilib, topamiz:

$$\lim_{x\to 2}\frac{x^2-x+1}{x-3}=\left[\frac{\lim(x^2-x+1)}{\lim(x-3)}\right]_{x\to 2}$$

$$=\left[\frac{\lim x^2-\lim x+1}{\lim x-3}\right]=\left[\frac{(\lim x)^2-\lim x+1}{\lim x-3}\right]_{x\to 2}=$$

$$=\frac{2^3-2+1}{2-3}=-3$$

Argumentning limit qiymati $x=2$ ni bevosita funksiya ifodasiga keltirib ko‘yish xam shu natijaga olib keladi:

$$\lim_{x\to 2}\frac{x^2-x+1}{x-3}=\frac{2^2-2+1}{2-3}=-3$$

.

485. Limitlarni hisoblang:

1) $\lim\limits_{x\to -1}\dfrac{(x+3(x-2)}{x+2}$, 2) $\lim\limits_{x\to 1}\dfrac{\sqrt{x+1}}{\sqrt{x}-1}$.

II. Bo‘luvchining limiti nolga teng bo‘lganda funksiya limitini hisoblash

Funksiyaning limitini argumentning urniga uning limit qiymatini bevosita qo‘yish yo‘li bilan topish xar doim ham mumkin bo‘lavermaydi, birok bundan funksiyaning limitini hisoblab bo‘lmaydi, degan xulosa kelib chiqmaydi. Bunday hollarda funksnyani unga limitlar haqidagi teoremalarni tatbiq etib bo‘ladigan qilib o‘zgartirish talab etiladi.

a) bo‘luvchining limiti nolga teng, bo‘linuvchining limiti esa nolga teng bo‘lmagan hol.

486. $\lim\limits_{x\to 2}\dfrac{5}{4x-8}$.

Yechilishi. Bo‘luvchining limiti nolga teng:

$$\lim_{x\to 2}(4x-8)=4\cdot 2-8=0.$$

IV teoremani tatbiq qilib bo‘lmaydi, chunki nolga bo‘lish mumkin emas.

Agar $\lim\limits_{x\to 2}(4x-8)=0$ bo‘lsa, u holda $4x-8$ ifoda cheksiz kichik miqdor, unga teskari bo‘lgan $\dfrac{1}{4x-8}$ miqdor esa cheksiz kattadir. Demak, $x\to 2$ da $\dfrac{1}{4x-8}\cdot 5$ ko‘paytma cheksiz katta miqdor bo‘ladi, ya’ni $\lim\limits_{x\to 2}\dfrac{5}{4x-8}=\infty$.

487. 1) $\lim\limits_{x\to 3}\dfrac{3}{2x-6}\cdot 5$ 2) $\lim\limits_{x\to 0}\dfrac{1}{x^2+2x}$

b) bo‘luvchi va bo‘linuvchining limitlari nolga teng bo‘lgan hol.

488. $\lim\limits_{x\to 0}\dfrac{3x^2-2x}{2x^2-5x}$

Yechilishi. Suratning limiti $\lim\limits_{x\to 0}(3x^2-2x)=0$ va maxrajning limiti $\lim\limits_{x\to 0}(2x^2-5x)=0$.

Funksiyaning limitini argumentning o’rniga uning limit qiymatini bevosita qo‘yib hisoblash mumkin emas, chunki $x\to 0$ da ikkita cheksiz kichik miqdorning nisbatiga ega bo‘lamiz $\left(\frac{0}{0}\right.$ nisbat ma’noga ega emas).

Kasrni nolga intiluvchi umumiy ko‘paytuvchiga qisqartirish mumkin bo‘lishi, binobarin IV teoremani tatbiq qilish mumkin bo‘lishi uchun surat va maxrajni ko‘paytuvchilarga ajratamiz. Shuni nazarda tutish kerakki, bu yerda nolga qisqartirish ko‘zda tutilmaydi, chunki nolga bo‘lish mumkin emas. Funksiya limitining ta’rifiga ko‘ra (6 - punkt) x argument o‘zining limit qiymatiga bu qiymatni hech qachon qabo‘l qilmay intiladi, shu sababli limitga o‘tishdan oldin nolga intiluvchi ko‘paytuvchiga qisqartirish mumkin.

Qo‘yidagiga egamiz:

$$\lim_{x\to 0}\frac{3x^2-2x}{2x^2-5x}=\lim_{x\to 0}\frac{x(3x-2}{x(2x-5)}=\lim_{x\to 0}\frac{3x-2}{2x-5}=$$

$$=\left[\frac{3\lim x-2}{2\lim x-5}\right]_{x\to 0}=\frac{3\cdot 0-2}{2\cdot 0-5}=\frac{2}{5}$$

489.1) $\lim\limits_{x\to 0}\dfrac{2x^3-2x^2}{5x^3-4x^2}$ 2) $\lim\limits_{x\to 0}\dfrac{3x^3+x}{x}$

490. $\lim\limits_{x\to 3}\dfrac{x^2-5x+6}{3x-9}$

Yechilishi. Mahraj va suratning limitlari nolga teng:

$\lim\limits_{x\to 3}(x^2-5x+6)=3^2--5\cdot 3+6=0$

$\lim\limits_{x\to 3}(3x-9)=3\cdot 3-9=0$ Surat – kvadrat uchhaddir, uni

$$ax^2+bx+c=a(x-x_1)\cdot(x-x_2)$$

(bu yerda x_1 va x_2 uchhadning ildizlari) formula bo‘yicha ko‘paytuvchilarga ajratamiz. Maxrajni ham ko‘paytuvchilarga ajratib, kasrni $x-3$ ga qisqartiramiz. IV, II va I teoremalarni tatbiq qilib, topamiz:

$$\lim_{x\to 3}\frac{x^2-5x+6}{3x-9}=\lim_{x\to 3}\frac{(x-3)(x-2)}{3(x-3)}=\left[\frac{\lim x-2}{3}\right]_{x\to 3}=$$

$$=\frac{3-2}{3}=\frac{1}{3}$$

491.1) $\lim\limits_{x\to 3}\frac{x-3}{x^2-9}$ 2) $\lim\limits_{x\to \frac{3}{2}}\frac{4x^2-9}{2x+3}$

3) $\lim\limits_{x\to 5}\frac{x^2-8x+15}{x^2-25}$ 4) $\lim\limits_{x\to 1}\frac{x^3-1}{x-1}$

492. $\lim\limits_{x\to 2}\frac{3x^2-8x+4}{5x^2-14x+8}$

Yechilishi. Suratning limiti

$$\lim_{x\to 2}(3x^2-8x+4)=3\cdot 2^2-8\cdot 2+4=0$$

va maxrajning limiti

$$\lim_{x\to 2}(5x^2-14x+8)=5\cdot 2^2-14\cdot 2+8=0$$

Surat va maxrajni chiziqli ko‘paytuvchilarga ajratamiz. Umumiy ko‘paytuvchiga qisqartirgandan so‘ng IV, II va I teoremalarni hamda III teoremaning 1-natijasini tatbiq qilib topamiz:

$$\lim_{x\to 2}\frac{3x^2-8x+4}{5x^2-14x+8}=\lim_{x\to 2}\frac{3(x-2)\left(x-\frac{2}{3}\right)}{5(x-2)\left(x-\frac{4}{5}\right)}=$$

$$=\lim_{x\to 2}\frac{3x-2}{5x-4}=\frac{3\cdot 2-2}{5\cdot 2-4}=\frac{2}{3}.$$

493. **1).** $\lim\limits_{x\to5}\dfrac{x^2-7x+10}{x^2-9x+20}$ 2) $\lim\limits_{x\to-3}\dfrac{2x^2+x-15}{3x^2+7x-6}$

494. $\lim\limits_{x\to2}\dfrac{x^3-7x+6}{x^3-5x^2+2x+8}$

Yechilishi. Suratning limiti

$$\lim_{x\to2}(x^3-7x+6)=2^3-7\cdot2+6=0$$

va maxrajning limiti

$$\lim_{x\to2}(x^2-5x^2+2x+8)=2^3-5\cdot2^2+2\cdot2+8=0.$$

Surat va maxraji argumentning limit qiymati $x=a$ da nolga aylanadigan ko‘phadlardan iborat bo‘lgan kasrning limitini hisoblaganda Bezu teoremasidan foydalanish mumkin, bu teoremaga ko‘ra har ikkala ko‘pxad $x-a$ ga qolldiqsiz bo‘linadi. Surat va maxrajni $x-a$ ikkihadga bo‘lib (limit qiymat $x=2$), IV, II va I teoremalarni hamda III teoremaning 1 va 2- natijalarini tatbiq qilib, topamiz:

$$\lim_{x\to2}\frac{x^3-7x+6}{x^3-5x^2+2x+8}=\lim_{x\to2}\frac{(x-2)(x^2+2x-3)}{(x-2)(x^2-3x-4)}=\lim_{x\to2}\frac{x^2+2x-3}{x^2-3x-4}=$$

$$\left[\frac{(\lim x)^2+2\lim x-3}{(\lim x)^2-3\lim x-4}\right]_{x\to2}=\frac{2^2+2\cdot2-3}{2^2-3\cdot2-4}=-\frac{5}{6}$$

495. 1) $\lim\limits_{x\to1}\dfrac{x^3+2x^2-x-2}{x^2-13x+12}$ 2) $\lim\limits_{x\to-3}\dfrac{x^3-3x^2-13x+15}{x^3-3x^2-10x+24}$

496. $\lim\limits_{x\to0}\dfrac{x}{\sqrt{5-x}-\sqrt{5+x}}$

Yechilishi. Suratning limiti $\lim\limits_{x\to0}x=0$ va maxrajning limiti

$$\lim_{x\to0}(\sqrt{5-x}-\sqrt{5+x})=\sqrt{5-0}-\sqrt{5+0}=0$$

Surat va maxrajni maxrajning qo‘shmasi bo‘lgan $\sqrt{5-x}+\sqrt{5+x}$ ko‘paytuvchiga ko‘paytiramiz, so‘ngra kasrni x ga qisqartiramiz. IV, II teoremalarni va III teoremaning 3-natijasini tatbiq qilib, topamiz:

$$\lim_{x\to 0}\frac{x}{\sqrt{5-x}-\sqrt{5+x}}=\lim_{x\to 0}\frac{x\left(\sqrt{5-x}+\sqrt{5+x}\right)}{\left(\sqrt{5-x}-\sqrt{5+x}\right)\left(\sqrt{5-x}+\sqrt{5+x}\right)}=$$

$$=\lim_{x\to 0}\frac{x\left(\sqrt{5-x}+\sqrt{5+x}\right)}{-2x}=\lim_{x\to 0}\frac{\sqrt{5-x}+\sqrt{5+x}}{-2}=$$

$$=\left[\frac{\sqrt{5-\lim x}+\sqrt{5+\lim x}}{-2}\right]_{x\to 0}=\frac{\sqrt{5}+\sqrt{5}}{-2}=-\sqrt{5}.$$

497. 1) $\lim\limits_{x\to 6}\dfrac{x-6}{\sqrt{x+3}-3}$; 2) $\lim\limits_{x\to 0}\dfrac{\sqrt{x+1}-1}{x}$.

498. 1) $\lim\limits_{x\to 9}\dfrac{3-\sqrt{x}}{4-\sqrt{2x-2}}$; 2) $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt{x}-1}$.

Ko‘rsatma. Surat va maxrajni: 1) $\left(3+\sqrt{x}\right)\left(4+\sqrt{2x-2}\right)$ ko‘paytmaga ko‘paytiring, so‘ngra kasrni $9-x$ ga qisqartiring;

2) $\left[\left(\sqrt[3]{x}\right)^2+\sqrt[3]{x}+1\right]\left(\sqrt{x}+1\right)$ ga ko‘paytiring, so‘ngra kasrni $x-1$ ga qisqartiring.

499. $\lim\limits_{x\to -2}\left(\dfrac{1}{x+2}-\dfrac{12}{x^2+8}\right)$.

Yechilishi. Masala shartidan $x\to -2$ da funksiya ikkita cheksiz katta miqdorning ayirmasidan iborat ekanligi kelib chiqadi. Kasrlarni ayirib, surat va maxraji $x\to -2$ da nolga intiluvchi kasrni hosil qilamiz. Kasrni $x+2$ ga qisqartirib, IV, II va I teoremalarni hamda III teoremaning 1 va *2*-natijalarini tatbiq etib topamiz:

$$\lim_{x\to-2}\left(\frac{1}{x+2}-\frac{12}{x^3+8}\right)=\lim_{x\to-2}\frac{x^2-2x-8}{x^3+8}=$$

$$=\lim_{x\to-2}\frac{(x+2)(x-4)}{(x+2)\left(x^2-2x+4\right)}=\lim_{x\to-2}\frac{x-4}{x^2-2x+4}=$$

$$=\frac{-2-4}{(-2)^2-2\cdot(-2)+4}=-\frac{1}{2}.$$

500. $\lim\limits_{x\to3}\left(\frac{6}{x^2-9}-\frac{1}{x-3}\right)$.

501. $\lim\limits_{x\to\frac{-\pi}{2}}\frac{\cos^2 x}{1+\sin^3 x}$.

Yechilishi. Suratning limiti

$$\lim_{x\to\frac{\pi}{2}}\cos^2 x=\cos^2\left(\frac{\pi}{2}\right)=\cos^2\frac{\pi}{2}=0$$

va maxrajning limiti

$$\lim_{x\to\frac{\pi}{2}}(1+\sin^3 x)=1+\sin^3\left(\frac{\pi}{2}\right)=1-\sin^3\frac{\pi}{2}=1-1=0.$$

Surat va maxrajni ko‘paytuvchilarga ajratib va kasrni $1+\sin x$ ga qisqartirib topamiz:

$$\lim_{x\to-\pi/2}\frac{\cos^2 x}{1+\sin^3 x}=\lim_{x\to-\pi/2}\frac{1-\sin^2 x}{(1+\sin x)\left(1-\sin x+\sin^2 x\right)}=$$

$$=\lim_{x\to-\pi/2}\frac{1-\sin x}{1-\sin x+\sin^2 x}=\frac{1-\sin\left(-\pi/2\right)}{1-\sin\left(-\pi/2\right)+\sin^2\left(-\pi/2\right)}=$$

$$=\frac{1+\sin\left(\pi/2\right)}{1+\sin\left(\pi/2\right)+\sin^2\left(\pi/2\right)}=\frac{1+1}{1+1+1}=\frac{2}{3}.$$

502. 1) $\lim\limits_{x\to\pi}\frac{\sin^2 x}{1+\cos x}$; 2) $\lim\limits_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{tgx-1}$.

503. $\lim\limits_{x\to\frac{\pi}{4}}\frac{1-\sqrt{1+ctgx}}{ctgx}$.

Ko'rsatma. Surat va maxrajni $1+\sqrt{1+ctgx}$ ga ko'paytiring kasrni $ctgx$ ga qisqartiring.

504. $\lim\limits_{x\to 0}\dfrac{tgx}{\sqrt{1-tgx}-1}$.

505. $\lim\limits_{x\to\infty}(x^3-6x^2+5x-1)$.

Yechilishi . Birinchi uchta qo'shiluvchi $x\to\infty$ da limit $\dfrac{6}{x},\dfrac{5}{x^2}\,ва\,\dfrac{1}{x^3}$ hadlar $x\to\infty$ da cheksiz kichik miqdorlardir [(10) munosabat], ularning limitlari nolga teng.

506. 1) $\lim\limits_{x\to\infty}(x^2-5x+6)$; 2) $\lim\limits_{x\to-\infty}(x^3+3x^2)$

a) $\dfrac{1}{\infty}$ b) $\dfrac{\infty}{\infty}$, c) $\infty-\infty$ ko'rinishdagi aniqmasliklarni ochish.

$x\to\infty$ da bo'luvchi cheksiz katta miqdor, bo'linuvchi esa o'zgarmas miqdor bo'lgan hol.

507. $\lim\limits_{x\to\infty}\dfrac{5}{4x+1}$.

Yechilishi. $4x+1$ bo'luvchi $x\to\infty$ da cheksiz o'sadi, ya'ni cheksiz katta miqdordir, unga teskari bo'lgan miqdor $\dfrac{1}{4x+1}$ esa cheksiz kichikdir.

Cheksiz kichik miqdorning chegaralangan miqdorga (o'zgarmas cheklangan miqdorning xususiy holi) ko'paytmasi $\dfrac{1}{4x+1}\cdot 5$ cheksiz kichik miqdordir va uning $x\to\infty$ dagi limiti nolga teng. Demak $\lim\limits_{x\to\infty}\dfrac{5}{4x+1}=0$.

508. 1) $\lim\limits_{x\to\infty}\dfrac{2}{x^2+3x}\cdot 5$ 2) $\lim\limits_{x\to\infty}\left(5+\dfrac{2}{x}-\dfrac{3}{x^2}\right)$

b) $x\to\infty$ da bo'linuvchi va bo'luvchi cheksiz katta miqdorlar bo'lgan hol.

509. $\lim\limits_{x\to\infty}\dfrac{2x+3}{5x+1}$.

Yechilishi. $x\to\infty$ da bo‘linuvchi va bo‘luvchi cheksiz katta miqdorlar. Shu sababli IV teoremani bevosita tatbiq etganda $\dfrac{\infty}{\infty}$ ko‘rinishdagi ifodani hosil qilamiz, biroq $\dfrac{\infty}{\infty}$ nisbat hech qanday sonni ifodalamaydi va aniqmaslik deb ataladi. Bu funksiyaning limitini hisoblash uchun bo’linuvchi va bo’luvchini x ga qisqartirish kerak:

$$\frac{2x+3}{5x+1}=\frac{2+\frac{3}{x}}{5+\frac{1}{x}}.$$

$\dfrac{3}{x}$ va $\dfrac{1}{x}$ qo’shiluvchilar $x\to\infty$ da cheksiz kichik miqdorlar, demak, ularning limitlari nolga teng.

IV, II va I teoremalarni tatbiq qilib, topamiz:

$$\lim_{x\to\infty}\frac{2x+3}{5x+1}=\lim_{x\to\infty}\frac{2+\frac{3}{x}}{5+\frac{1}{x}}=\frac{2+0}{5+0}=\frac{2}{5}.$$

510. 1) $\lim\limits_{x\to\infty}\dfrac{3x}{x-2}$; 2) $\lim\limits_{x\to\infty}\dfrac{x-8}{2x-2}$.

511. $\lim\limits_{x\to\infty}\dfrac{2x^3-3x^2+1}{x^3+4x^2+2x}$.

Yechilishi. Kasrning surat va mahrajini x^2 ga bo’lamiz:

$$\frac{2x^3-3x^2+1}{x^3+4x^2+2x}=\frac{2-\frac{3}{x}+\frac{1}{x^3}}{1+\frac{4}{x}+\frac{2}{x^2}}.$$

$x\to\infty$ da $\dfrac{3}{x}$, $\dfrac{1}{x^3}$, $\dfrac{4}{x}$ va $\dfrac{2}{x^2}$ qo’shiluvchilar cheksiz kichik miqdorlar va ularning limitlari nolga teng bo’ladi. Shunday qilib:

$$\lim_{x\to\infty}\frac{2x^3-3x^2+1}{x^3+4x^2+2x}=\lim_{x\to\infty}\frac{2-\frac{3}{x}+\frac{1}{x^3}}{1+\frac{4}{x}+\frac{2}{x^2}}=\frac{2-0+0}{1+0+0}=2.$$

512. 1) $\lim\limits_{x\to\infty}\frac{3x^2-5x+4}{x^2+2x+3}$; 2) $\lim\limits_{x\to\infty}\frac{4x^3-x^2}{x^3+2x^2-2}$.

513. $\lim\limits_{x\to\infty}\frac{x^4-6x^2+5}{3x^3+7x^2-2}$.

Yechilishi. Surat va mahrajni argumentning mahrajdagi eng yuqori darajasiga, ya'ni x^3 ga bo'lamiz:

$$\lim_{x\to\infty}\frac{x^4-6x^2+5}{3x^3+7x^2-2}=\lim_{x\to\infty}\frac{x-\frac{6}{x}+\frac{5}{x^3}}{3+\frac{7}{x}-\frac{2}{x^3}}.$$

$x\to\infty$ da: $\lim\limits_{x\to\infty}\left(x-\frac{6}{x}+\frac{5}{x^3}\right)=\infty$ va $\lim\limits_{x\to\infty}\left(3+\frac{7}{x}-\frac{2}{x^3}\right)=3.$

Mahraj cheklangan miqdor, shuning uchun

$$\lim_{x\to\infty}\frac{x^4-6x^2+5}{3x^3+7x^2-2}=\infty.$$

514. 1) $\lim\limits_{x\to\infty}\frac{x^6+x^7}{x^4+x^5}$; 2) $\lim\limits_{x\to\infty}\frac{x^7-x^6+4}{x^3+2x^2+x}$; 3) $\lim\limits_{x\to\infty}\frac{5\cdot 3^x+9}{3\cdot 3^x-7}$.

Ko'rsatma. Surat va mahrajni 3^x ga bo'ling.

c) $x\to\infty$ da kamayuvchi va ayriluvchi cheksiz katta miqdorlar bo'lgan hol.

515. $\lim\limits_{x\to\infty}\left(x-\sqrt{x^2-5x}\right)$.

Yechilishi.

$$\lim_{x\to\infty}\left(x-\sqrt{x^2-5x}\right)=\lim_{x\to\infty}\frac{\left(x-\sqrt{x^2-5x}\right)\left(x+\sqrt{x^2-5x}\right)}{x+\sqrt{x^2-5x}}=$$

$$=\lim_{x\to\infty}\frac{x^2-x^2+5x}{x+\sqrt{x^2-5x}}=\lim_{x\to\infty}\frac{5x}{x+\sqrt{x^2-5x}}=$$

$$=\lim_{x\to\infty}\frac{5}{1+\sqrt{1-\frac{5}{x}}}=\frac{5}{2}=2,5.$$

516. 1) $\lim\limits_{x\to\infty}\left(\sqrt{x^2-5x}-x\right)$; 2) $\lim\limits_{x\to\infty}\left(\sqrt{x^2+5x}-x\right)$.

24-§. Cheksiz kichik miqdorlarni taqqoslash. Ekvivalent cheksiz kichik miqdorlar. $x\to 0$ da $\frac{\sin x}{x}$ nisbatning limiti

1.Ikkita cheksiz kichik miqdor α va β ni bir-biri bilan taqqoslash uchun ular nisbatining limiti topiladi. Agar bunda:

1) $\lim\frac{\alpha}{\beta}=0$ bo'lsa, u holda α cheksiz kichik miqdor β ga nisbatan yuqori tartibli cheksiz kichik miqdor deyiladi;

2) $\lim\frac{\alpha}{\beta}=\infty$ bo'lsa, u holda α cheksiz kichik miqdor β ga nisbatan quyi tartibli cheksiz kichik miqdor deyiladi;

3) $\lim\frac{\alpha}{\beta}=a$ bo'lsa (a -o'zgarmas son, $a\neq 0$), u holda α va β cheksiz miqdorlar bir xil tartibli cheksiz miqdorlar deyiladi;

4) $\lim\frac{\alpha}{\beta}=1$ bo'lsa, u holda α va β cheksiz miqdorlar ekvivalent cheksiz miqdorlar deyiladi.

α va β cheksiz kichik miqdorlarning ekvivalengligi ushbu taqribiy ifoda orqali yoziladi:

$$\alpha\approx\beta.$$

2. Bu paragrafdagi mashqlarni bajarishda limitlar haqidagi teoremalar va cheksiz kichik miqdorlarning asosiy xossalaridan tashqari yana quyidagilardan ham foydalaniladi:

a) ekvivalent cheksiz kichik miqdorlarning xossasi. Agar $\alpha \approx \alpha_1$ va $\beta \approx \beta_1$ bo‘lsa, u holda

$$\lim \frac{\alpha}{\beta} = \lim \frac{\alpha_1}{\beta_1} = \lim \frac{\alpha_1}{\beta} = \lim \frac{\alpha}{\beta_1} \qquad (4.1)$$

ya’ni ikkita cheksiz kichik miqdor nisbatining limitini hisoblashda ularning har birini (yoki birortasini) unga ekvivalent bo‘lgan boshqa cheksiz kichik miqdor bilan almashtirish mumkin;

b) $$\lim_{x \to 0} \frac{\sin x}{x} = 1, \quad \lim_{x \to 0} \frac{x}{\sin x} = 1 \qquad (4.2)$$

517. Qo‘yidagi cheksiz kichik miqdorlarni $x(x \to 0)$ cheksiz kichik miqdor bilan taqqoslang: 1) x^2; 2) $\sqrt[3]{2x}$; 3) $8x$; 4) $\sin 3x$; 5) $tg 2x$; 6) $\sin x \cos x$.

Yechilishi. Berilgan har bir cheksiz miqdorni x cheksiz kichik miqdor bilan taqqoslash uchun ularning cheksiz kichik miqdor x ga nisbatining limitini topish kerak:

1) $$\lim_{x \to 0} \frac{x^2}{x} = \lim_{x \to 0} x = 0.$$

bu yerda x^2 miqdor x cheksiz miqdorga nisbatan yuqori tartibli cheksiz kichik miqdordir (1 - hol);

2) $$\lim_{x \to 0} \frac{\sqrt[3]{2x}}{x} = \lim_{x \to 0} \sqrt[3]{\frac{2x}{x^3}} = \lim_{x \to 0} \sqrt[3]{\frac{2}{x^2}} = \infty$$

bu yerda $8x$ miqdor x bilan bir xil tartibga ega bo‘lgan cheksiz kichik miqdordir (3- hol);

4) $$\lim_{x \to 0} \frac{\sin 3x}{x}$$

(4.2) formulani tatbiq qilish uchun surat va maxrajni 3 ga ko‘paytiramiz:

$$\lim_{x \to 0} \frac{\sin 3x}{x} = \lim_{x \to 0} \frac{3 \sin 3x}{3x} = 3 \lim_{x \to 0} \frac{\sin 3x}{3x} 3 \cdot 1 = 3.$$

bu yerda $\sin 3x$ miqdor x bilan bir xil tartibga ega bo‘lgan cheksiz kichik miqdordir (3- hol);

5) $\lim\limits_{x\to 0}\dfrac{tg2x}{x}=\lim\limits_{x\to 0}\dfrac{\dfrac{\sin 2x}{\cos 2x}}{x}=\lim\limits_{x\to 0}\dfrac{\sin 2x}{x\cos 2x}=$

$$=\lim_{x\to 0}\frac{\sin 2x}{x}\cdot\lim_{x\to 0}\frac{1}{\cos 2x}=\lim_{x\to 0}\frac{2\sin 2x}{2x}\cdot\lim_{x\to 0}\frac{1}{\cos 2x}=$$

$$=2\lim_{x\to 0}\frac{\sin 2x}{2x}\cdot\lim_{x\to 0}\frac{1}{\cos 2x}=2\cdot 1\cdot 1=2,$$

bu yerda $tg2x$ miqdor x bilan bir xil tartibga ega bo‘lgan cheksiz kichik miqdordir (3- hol);

6) $\lim\limits_{x\to 0}\dfrac{\sin x\cos x}{x}=\lim\limits_{x\to 0}\dfrac{\sin x}{x}\lim\limits_{x\to 0}cox=1\cdot 1=1$.

$\sin x\cos x$ va x ekvivalent cheksiz kichik miqdorlar, chunki ular nisbatining limiti 1 ga teng (4- hol).

518. Quyidagi cheksiz kichik miqdorlarni cheksiz kichik miqdor $x(x\to 0)$ bilan taqqoslang:

1) x^3; 2) $\sqrt{6x}$; 3) $5x$; 4) $\sin\dfrac{x}{3}$; 5) tgx .

519. $x\to 0$ da quyidagi cheksiz kichik miqdorlarning ekvivalentligini isbot qiling: 1) $\sin ax\approx ax$, 2) $tg\approx ax$, 3) $arc\sin ax\approx ax$, 4) $\sqrt{6x+1}-1\approx 3x$; 5) $\sin x\approx tgx$; 6) $\sin^2 x\approx x^2$.

Yechilishi. Agar ikkita cheksiz kichik miqdor nisbatining limiti 1 ga teng bo‘lsa, ular ekvivalent bo‘ladi.

3) $arc\sin ax=\alpha$ bo‘lsin, u holda $\sin\alpha=ax$; agar $x\to 0$ bo‘lsa, u holda $\alpha\to 0$ da

$$\lim_{x\to 0}\frac{\arcsin ax}{ax}=\lim_{x\to 0}\frac{\alpha}{\sin\alpha}=1;$$

4) $\lim\limits_{x\to 0}\dfrac{\sqrt{6x+1}-1}{3x}=\lim\limits_{x\to 0}\dfrac{\left(\sqrt{6x+1}-1\right)\left(\sqrt{6x+1}+1\right)}{3x\left(\sqrt{6x+1}+1\right)}=$

$$=\lim_{x\to 0}\frac{6x+1-1}{3x\left(\sqrt{6x+1}+1\right)}=\lim_{x\to 0}\frac{6x}{3x\left(\sqrt{6x+1}+1\right)}=$$

$$=\lim_{x\to 0}\frac{2}{\sqrt{6x+1}+1}=\lim_{x\to 0}\frac{2}{\sqrt{6\cdot 0+1}+1}=\frac{2}{2}=1;$$

5) $\lim\limits_{x\to 0}\frac{\sin x}{tgx}=\lim\limits_{x\to 0}\frac{\sin x}{\sin x}=\lim\limits_{x\to 0}\cos x=1;$

6) $\lim\limits_{x\to 0}\frac{\sin^2 x}{x^2}=\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)^2=\left(\lim\limits_{x\to 0}\frac{\sin x}{x}\right)^2=1^2=1.$

520. $x\to 0$ da quyidagi cheksiz kichik miqdorlarning ekvivalentligini isbot qiling:

1) $\sin\frac{2}{x}\approx\frac{x}{2}$; 2) $tg\sqrt{x}\approx\sqrt{x}$ 3) $arctgax\approx ax$

4) $\sqrt{4x+1}-1\approx 2x$; 5) $tg^3x\approx x^3$.

Limitlarni hisoblang.

521. $\lim\limits_{x\to 0}\frac{\sin 4x}{3x}$.

Yechilishi. 1-usul.

$$\lim_{x\to 0}\frac{\sin 4x}{3x}=\lim_{x\to 0}\frac{4\sin 4x}{4\cdot 3x}=\lim_{x\to 0}\frac{4\sin 4x}{4\cdot 3x}=\frac{4}{3}\lim_{x\to 0}\frac{\sin 4x}{4x}=\frac{4}{3}1=\frac{4}{3}.$$

2-usul. 519-masalada $\sin\alpha x\approx ax$ ekanligi ko‘rsatilgan edi, u holda $\alpha=4$ da $\sin 4x\approx 4x$. (4.1) xossaga ko‘ra

$$\lim_{x\to 0}\frac{\sin 4x}{3x}=\lim_{x\to 0}\frac{4x}{3x}=\frac{4}{3}.$$

522. 1) $\lim\limits_{x\to 0}\frac{3x}{\sin 2x}$ 2) $\lim\limits_{x\to 0}\frac{5a}{\cos a}$

523. $\lim\limits_{x\to 0}\dfrac{\sin^3 2z}{z^3}$

Yechilishi. (4.1) xossaga ko‘ra,

$$\lim_{x\to 0}\frac{\sin^3 2z}{z^3}=\lim_{z\to 0}\left(\frac{\sin 2z}{z}\right)^3=\left(\lim_{x\to 0}\frac{\sin 2z}{z}\right)^3=\left(\lim_{z\to 0}\frac{2z}{z}\right)=2^3=8.$$

524. $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{x^2}=\lim\dfrac{\sin^3 3z}{z^3}$.

525. $\lim\limits_{x\to 0}\dfrac{\sin^3 x}{x^2}$.

Yechilishi. $\lim\limits_{x\to 0}\dfrac{\sin^3 x}{x^2}=\lim\limits_{x\to 0}\left(\dfrac{\sin^2 x}{x^2}\cdot\sin x\right)=$

$$=\lim_{x\to 0}\left[\left(\frac{\sin x}{x}\right)^2\cdot\sin x\right]=\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^2\cdot\lim_{x\to 0}\sin x=1^2\cdot 0=0.$$

526. 1) $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{x}$ 2) $\lim\limits_{x\to 0}\dfrac{\sin^3 x}{x^3}$.

527. $\lim\limits_{x\to 0}\dfrac{\cos x-\cos 3x}{x}$.

Yechilishi. Kosinuslar ayirmasini ko‘paytmaga almashtirib, topamiz:

$$\lim_{x\to 0}\frac{cos-\cos 3x}{x}=2\lim_{x\to 0}\frac{\sin 2x\sin x}{x}=2\lim_{x\to 0}\frac{\sin 2x}{x}\lim_{x\to 0}\sin x=2\cdot 2\cdot 0=0.$$

528. 1) $\lim\limits_{x\to 0}\dfrac{\sin 2x\cos x}{x}$ 2) $\lim\limits_{x\to 0}\dfrac{\sin 2x\cdot\sin x}{x^2}$

3) $\lim\limits_{x\to 0}\dfrac{\sin 3x+\sin x}{x}$ 4) $\lim\limits_{x\to 0}\dfrac{\cos 2x-\cos x}{x}$.

529. $\lim\limits_{x\to 0}\dfrac{\cos x-1}{x^2}$.

Yechilishi.

$$\lim_{x\to 0}\frac{\cos x-1}{x^2}=-\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x^2}=-2\left(\lim_{x\to 0}\frac{\sin\frac{x}{2}}{x}\right)^2=$$

$$=-2\left(\lim_{x\to 0}\frac{\frac{x}{2}}{x}\right)^2=-2\cdot\left(\frac{1}{2}\right)^2=-\frac{1}{2}.$$

530. $\lim\limits_{x\to 0}\dfrac{\sin x-tgx}{x}$.

531. $\lim\limits_{x\to 0}\dfrac{\sin 2x}{\sin 3x}$.

Yechilishi. (4.1) xossani qo'llaymiz:

$$\lim_{x\to 0}\frac{\sin 2x}{\sin 3x}=\lim_{x\to 0}\frac{2x}{3x}=\frac{2}{3}.$$

532. 1) $\lim\limits_{x\to 0}\dfrac{\sin 4x}{\sin x}$; 2) $\lim\limits_{x\to 0}\dfrac{\sin 4x}{tg2x}$.

533. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{ctgx}{x-\frac{\pi}{2}}$.

Yechilishi. $x-\dfrac{\pi}{2}=y$ almashtirish bajaramiz, u holda $x=\dfrac{\pi}{2}+y$.

Agar $x\to\dfrac{\pi}{2}$ bo'lsa, u holda $y\to 0$. Topamiz:

$$\lim_{x\to\frac{\pi}{2}}\frac{ctgx}{x-\frac{\pi}{2}}=\lim_{y\to0}\frac{ctg\left(\frac{\pi}{2}+y\right)}{y}=-\lim_{y\to0}\frac{tg\,y}{y}$$

$$=-\lim_{y\to0}\frac{\sin y}{y}\lim_{y\to0}\frac{1}{\cos y}=-1\cdot1=-1.$$

534. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cos x}{\frac{\pi}{2}-x}.$

535. $\lim\limits_{x\to0}\dfrac{\arcsin 3x}{2x}.$

Yechilishi. $\arcsin 3x=\alpha$ desak, $\sin\alpha=3x$. Quyidagicha almashtiramiz:

$$\frac{\arcsin 3x}{2x}=\frac{3\arcsin 3x}{2\cdot3x}=\frac{3}{2}\frac{\alpha}{2\sin\alpha},$$

u holda

$$\lim_{x\to0}\frac{\arcsin 3x}{2x}=\lim_{\alpha\to0}\left(\frac{3}{2}\frac{\alpha}{\sin\alpha}\right)=\frac{3}{2}\frac{1}{\lim\limits_{\alpha\to0}\frac{\alpha}{\sin\alpha}}=\frac{3}{2}\cdot\frac{1}{1}=\frac{3}{2}.$$

536. 1) $\lim\limits_{z\to0}\dfrac{\arcsin 2z}{2z}$; 2) $\lim\limits_{x\to0}\dfrac{\arcsin 5x}{5x}.$

25-§. e soni. Natural logarifmlar.

1.Quyidagi limit munosabat o'rinli

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=\lim_{a\to0}(1+\alpha)^{\frac{1}{\alpha}}=e. \qquad (4.3)$$

bu yerda e irratsional sondir ($e = 2,718...$). e ning aniqroq qiymati ($e = 2,7182818$). Asosi e bo'lgan logarifmlar *natural logarifmlar* deb ataladi, ular uchun $\ln$ belgi kirirtilgan.

2. O'nli va natural logarifmlar quyidagi munosabatlar bilan bog'langan:

$$\lg N = M \ln N; \tag{4.4}$$

$$\ln N = \frac{1}{M} \lg N, \tag{4.5}$$

bu yerda M — natural logarifmlardan o'nli logarifmlarga o'tish moduli:

$$M = \lg e = \lg 2,718 \approx 0,4343;$$

$$\frac{1}{M} = \ln 10 \approx 2,303.$$

(4.4) va (4.5) formulalarni hisoblashlarga tatbiq qilish uchun ularga boshqacha ko'rinish berish mumkin:

$$\lg N = 0,4343 \ln N; \tag{4.6}$$

$$\ln N = 2,303 \lg N. \tag{4.7}$$

537. 1)7; 2)0,12 sonlarining natural logarifmlarini toping.

Yechilishi. (4.7) formula bo'yicha topamiz:

$$1) \ln 7 = 2,303 \times \lg 7 = 2,303 \cdot 0,8451 = 1,946;$$

$$2) \ln 0,12 = 2,303 \cdot \lg 0,12 = 2,303 \cdot \bar{1},0792$$

$$= 2,303 \cdot (-0,9208) = -2,121.$$

538. Quyidagi sonlarning natural logarifmlarini toping: 1)3; 2)4; 3)5; 4)10; 5)0,3; 6)0,8; 7)5,8; 8)0,24; 9)15,6 .

539. Sonlarining natural logarifmlari: 1)0,2624; 2)2,1401 ga ko'ra ularning o'nli logarifmlarini toping.

Yechilishi. (4.6) formula bo'yicha topamiz:

$$1)\lg N = 0,4343 \times 0,2624 = 0,1140;$$

$$2)\lg N = 0,4343 \times 2,1401 = 0,9294.$$

540. Sonlarining natural logarifmlari: 1)2,0794; 2)3,6889; 3)1,959 ga ko'ra ularning o'nli logarifmlarini toping.

541. O'nli logarifmlar jadvalidan foydalanib hisoblang:

$1)e^3;\ \ 2)\sqrt{e};\ \ \ 3)e^{-3}.$

Yechilishi.

$$1)\lg e^3 = 3\lg e = 3 \cdot 0,4343 = 1,3029; \quad e^3 = 20,08;$$

$$2)\lg\sqrt{e} = \frac{1}{2}\lg e = \frac{1}{2} \cdot 0,4343 = 0,2171; \quad \sqrt{e} = 1,648;$$

$$3)\lg e^{-3} = -3\lg e = -3 \cdot 0,4343 = -1,3029 = 2,6971;$$

$$e^{-3} = 0,04978.$$

542. O'nli logarifmlar jadvalidan foydalanib hisoblang:

$1)e^5;\ \ 2)\sqrt[3]{e};\ \ \ 3)e^{-2}$

543. Jadvallardan foydalanib hisoblang:

$1)\ln 100;\ \ 2)\ln 0,001;\ \ \ 3)\ln\sqrt{10}\,.$

Yechilishi.

$$1)\ln 100 = \ln 10^2 = 2\ln 10 = 2 \cdot 2,303 = 4,606;$$

$$2)\ln 0,001 = \ln 10^{-3} = -3\ln 10 = -3 \cdot 2,303 = -6,909;$$

$$1)\ln\sqrt{10} = \frac{1}{2}\ln 10 = \frac{1}{2} \cdot 2,303 = 1,151.$$

544. Jadvallardan foydalanib hisoblang:

$1)\ln 1000;\quad 2)\ln 0,01;\quad 3)\ln \sqrt[3]{100}$

Limitlarni hisoblang.

545. $\lim\limits_{x\to\infty}\left(1+\dfrac{3}{x}\right)^{x}.$

Yechilishi. Quyidagi almashtirishlarni bajaramiz va (4.3) formuladan foydalanib, limitni topamiz:

$$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{x}=\lim_{x\to\infty}\left[\left(1+\frac{1}{\frac{x}{3}}\right)^{\frac{x}{3}}\right]^{3}=\left[\lim_{x\to\infty}\left(1+\frac{1}{\frac{x}{3}}\right)^{\frac{x}{3}}\right]^{3}=e^{3}.$$

546. $\lim\limits_{x\to\infty}\left(1+\dfrac{2}{3x}\right)^{x}.$

547. $\lim\limits_{x\to 0}\left(1+2x\right)^{\frac{5}{x}}.$

Yechilishi. Quyidagi almashtirishlarni bajaramiz va (4.3) formulani tatbiq qilib, topamiz:

$$\lim_{x\to 0}\left(1+2x\right)^{\frac{5}{x}}=\lim_{x\to 0}\left(1+\frac{1}{\frac{1}{2x}}\right)^{5\cdot\frac{1}{x}\cdot\frac{2}{x}\cdot 2}=\lim_{\frac{1}{2x}\to\infty}\left[\left(1+\frac{1}{\frac{1}{2x}}\right)^{\frac{2}{x}}\right]^{10}=e^{10}.$$

548. $1)\lim\limits_{z\to 0}\left(1+4z\right)^{\frac{3}{5z}};\qquad 2)\lim\limits_{z\to\infty}\left(1+\dfrac{1}{z}\right)^{-z}.$

549. $\lim\limits_{x\to\infty}\left(\dfrac{x}{1+x}\right)^{x}.$

Yechilishi. Quyidagi almashtirishlarni bajaramiz va (4.3) formulani tatbiq qilib, limitni topamiz:

$$\lim_{x\to\infty}\left(\frac{x}{1+x}\right)^{x}=\lim_{x\to\infty}\left(\frac{x+1}{x}\right)^{-x}=\lim_{x\to\infty}\left[\left(1+\frac{1}{x}\right)^{x}\right]^{-1}.$$

$$=\left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\right]^{-1}=e^{-1}=\frac{1}{e}.$$

550. 1) $\lim\limits_{x\to\infty}\left(\frac{2x}{2x+1}\right)^{x}$; 2) $\lim\limits_{x\to\infty}\left(\frac{2x+3}{2x+1}\right)^{x+1}$.

26-§. Aralash masalalar.

Limitlarni hisoblang.

551. $\lim\limits_{x\to 2}\frac{4x^2-7x-2}{5x^2-11x+2}$. **552.** $\lim\limits_{x\to -3}\frac{x^3+6x^2+15x+18}{x^3+5x^2+5x-3}$.

553. $\lim\limits_{z\to 0}\frac{2z}{\sqrt{4+z}-\sqrt{4-z}}$. **554.** $\lim\limits_{x\to 4}\frac{2-\sqrt{x}}{3-\sqrt{2x+1}}$.

555. $\lim\limits_{x\to 8}\frac{\sqrt[3]{x}-2}{\sqrt{x}-2\sqrt{2}}$. **556.** $\lim\limits_{z\to\frac{\pi}{2}}\frac{1-\sin^3 z}{\cos^2 z}$.

557. $\lim\limits_{x\to 0}\frac{\sqrt{1+tgx}-1}{2tgx}$. **558.** 1) $\lim\limits_{n\to\infty}\frac{n^2+a^2}{n^3+a^3}$;

2) $\lim\limits_{n\to\infty}\frac{4n+1}{\sqrt{4n^2+1}}$; 3) $\lim\limits_{n\to\infty}\frac{1+2+3+...+n}{1+3+5+...+(2n-1)}$.

559. 1) $\lim\limits_{x\to\infty}\frac{x^4-x^2+2}{x^3-x+1}$; 2) $\lim\limits_{x\to\infty}\left(\sqrt{x^2+x}-x\right)$.

560. $\lim\limits_{x\to\infty}\dfrac{5-4\cdot 3^x}{3+2\cdot 3^x}$. **561.** $\lim\limits_{t\to 0}\dfrac{\sin^3 2t}{3t^2}$.

562. $\lim\limits_{x\to 0}\dfrac{\cos 3x-\cos 5x}{8x^2}$. **563.** $\lim\limits_{z\to 0}\dfrac{\sin z+\sin 3z}{2z}$.

564. $\lim\limits_{x\to 0}\dfrac{\arcsin 3x}{5x}$. **565.** $\lim\limits_{x\to 0}\dfrac{arctg 2x}{4x}$.

566. $\lim\limits_{x\to 0}\left(1+\dfrac{2x}{3}\right)^{\frac{2}{3x}}$. **567.** $\lim\limits_{x\to 0}\left(1+\dfrac{5}{3x}\right)^{2x}$.

568. 1) $\lim\limits_{x\to\infty}\left(1+\dfrac{k}{x}\right)^{x}$; 2) $\lim\limits_{x\to\infty}\left(1-\dfrac{3}{x}\right)^{x}$.

Nazorat uchun misollar

I variant

569. Limitlarni hisoblang:

1) $\lim\limits_{x\to 5}\dfrac{3x^2-17x+10}{3x^2-16x+5}$; 2) $\lim\limits_{x\to 5}\dfrac{5-x}{3-\sqrt{2x-1}}$;

3) $\lim\limits_{x\to 0}\dfrac{1-\sqrt{1-\sin x}}{\sin x}$; 4) $\lim\limits_{x\to\infty}\dfrac{2x^3+x+1}{3x^3+x^2+1}$; 5) $\lim\limits_{x\to\infty}\left(1-\dfrac{2}{x}\right)^{x}$.

II variant

570. Limitlarni hisoblang:

1) $\lim\limits_{x\to 1}\dfrac{4x^2-7x+3}{3x^2-2x-1}$; 2) $\lim\limits_{x\to 0}\dfrac{x}{\sqrt{3+x}-\sqrt{3-x}}$;

3) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+tgx}-1}{3tgx}$; 4) $\lim\limits_{x\to\infty}\dfrac{5x^4-x^3+2x}{x^4-8x^3+1}$; 5) $\lim\limits_{x\to\infty}\left(1+\dfrac{3}{x}\right)^{-x}$.

5-BOB

FUNKSIYA TUSHUNCHASI

27-§. Funksional bog'lanish simvolikasi.

1. Agar x o'zgaruvchining qabul qilishi mumkin bo'lgan har bir qiymatiga y o'zgaruvchining aniq qiymati mos kelsa, y o'zgaruvchi x ning funksiyasi deyiladi.

2. O'zgaruvchi y(funksiya) va o'zgaruvchi x(argument) orasidagi funksional bog'lanish simvolik ravishda

$$y = f(x)$$

tenglik orqali yoziladi, bu yerda f belgi y ni hosil qilish uchun x ustida bajariladigan amallar to'plamini bildiradi.

Funksional bog'lanish yana jadval yoki grafik usul bilan ham berilishi mumkin.

3. Asosiy elementar funksiyalar deb quyidagi funksiyalarga aytiladi: 1) $y = x^a$ darajali funksiya; 2) $y = a^x$, $a > 0$ ko'rsatkichli funksiya, 3) $y = \log_a x$, $a > 0$ logarifmik funksiya; 4) $y = \sin x, y = \cos x;\ y = tgx; y = ctgx$ triginimetrik funksiyalar; 5) $y = \arcsin x$; $y = \arccos x; y = arcctgx$ teskari triginometrik funksiyalar.

Asosiy elementar funksiyalardan chekli sondagi arifmetrik amallar va operatsiyalar yordamida tuzilgan va bitta formula bilan berilgan funksiyalar elementar funksiyalar deyiladi. Masalan:

$$y = 2\sqrt{x}\cos 3x, \qquad y = \lg\frac{x+\sin x}{tg\sqrt{x}-1}$$

Argumenti ustida chekli sondagi arifmetik amallar (qo'shish, ayrish, ko'paytirish, bo'lish va ratsional darajaga ko'tarish) bajariladigan funksiyalar algebraic funksiyalar deyiladi. Masalan:

$$y = x^2 - 2\sqrt[3]{x} + 1; \quad y = \frac{5x^3 - 1}{x+2}$$

Algebraik bo'lmagan funksiyalar funksiyalar transcendent funksiyalar deyiladi. Masalan:

$$y = 3^x; \; y = \log_3\sqrt{x}; \;\; y = tg^2 x; \; y = \arcsin\sqrt{x-1}$$

4. Funksiyaning argumentning berilgan son qiymatiga mos keluvchi qiymati bu funksiyaning hususiy qiymati deyiladi. Masalan, $y = f(x)$ funksiya $x = a$ da $y = f(a)$ qiymatga ega.

Funksiyaning hususiy qiymatlarini hisoblash.

571. $f(x) = x^3 - 2x^2 + x - 1$ funksiya berilgan. $f(0)$, $f(1)$, $f(-1)$, $f(2)$ ni toping.

Yechilishi. $f(0)$ ni hisoblash uchun berilgan funksiyada x argumentning o'rniga uning $x = 0$ qiymatini qo'yish kerak. Demak,

$$f(0) = 0^3 - 2\cdot 0^2 + 0 - 1 = -1.$$

Huddi shunga o'xshash: $f(1) = -1$; $f(-1) = -5$ va $f(2) = 1$.

572. 1. $F(x) = x^4 - x^3 + 2x^2 + 4$ funksiya berilgan. $F(0)$, $F(-1)$ va $F(2)$ ni toping.

2. $s(t) = t^2 - 6t + 8$ funksiya berilgan. $s(0)$, $s(2)$ va $s(-1)$ ni toping.

573. 1. $f(x) = x^4 - x^2 + 1$ funksiya berilgan. $f(1) = f(-1)$ ekanligini ko'rsating.

2. $f(x) = x^4 + x^2 + 5$ funksiya berilgan. $f(2) = -f(-2)$ ekanligini ko'rsating.

574. 1. $f(x)=x^3+x$ funksiya berilgan. $f(1)=-f(-1)$ ekanligini ko'rsating.

2. $f(x)=x^5+x^3$ funksiya berilgan. $f(2)=-f(-2)$ ekanligini ko'rsating.

575. 1. $f(x)=1-\sin^3 x$ funksiya berilgan. 1) $f\left(\frac{\pi}{6}\right)$, 2) $f\left(-\frac{\pi}{2}\right)$, 3) $f\left(\frac{3\pi}{2}\right)$, 4) $f(0)$ ni toping.

2. $f(x)=\cos^2 x$ funksiya berilgan. 1) $f(0)$, 2) $f\left(\frac{\pi}{4}\right)$, 3) $f\left(\frac{\pi}{2}\right)$, 4) $f(\pi)$ ni toping.

28-§. Funksiyaning aniqlanish va o'zgarish sohalari.

1. x argumentning $y=f(x)$ funksiyasi haqiqiy qiymatlarga ega bo'ladigan barcha haqiqiy qiymatlari to'plami (son o'qining barcha nuqtalari) $y=f(x)$ funksiyaning aniqlanish (mavjudlik) sohasi deyiladi.

2. Funksiyaning o'zgarish sohasi deb y qabul qilishi mumkin bo'lgan barcha haqiqiy sonlar to'plamiga aytiladi.

3.Funksiyaning eng ko'p uchrab turadigan aniqlanish sohalari – interval va kesma(yopiq interval)dir.

$a<x<b$ tengsizliklarni qanoatlantiradigan barcha haqiqiy sonlar to'plami interval deb ataladi, u qisqacha (a,b) simvol bilan belgilanadi, a va b nuqtalar intervalga kirmaydi.

$a \le x \le b$ tengsizliklarni qanoatlantiradigan barcha haqiqiy sonlar to'plami kesma(yopiq interval) deb ataladi, u qisqacha $[a,b]$ simvol bilan belgilanadi, a va b nuqtalar kesmaga kiradi.

$a \le x < b$ va $a < x \le b$ tengsizliklarni qanoatlantiruvchi barcha haqiqiy sonlar to'plamlari yarim ochiq intervallar deyiladi va mos ravishda $[a,b)$ va $(a,b]$simvollar bilan belgilanadi.

$-\infty < x < a$, $-\infty < x \le a$, $a < x < +\infty$ $a \le x < +\infty$ va $-\infty < x < +\infty$ tengsizliklarni qanoatlantiradigan barcha haqiqiy sonlar to'plamlari cheksiz intervallar deb ataladi va mos ravishda $(-\infty, a)$, $(-\infty, a]$, $(a, +\infty)$ $[a, +\infty)$ va $(-\infty, +\infty)$ simvollar bilan belgilanadi.

4. Asosiy elementar funksiyalarning aniqlanish va o'zgarish sohalari 1-jadvalda keltirilgan.

Asosiy elementar funksiyalarning aniqlanish va o'zgarish sohalari

1-jadval

№	Funksiya	Funksiyaning aniqlanish sohasi	Funksiyaning o'zgarish sohasi
1	$y = x^n$, $n -$ natural son	$(-\infty, +\infty)$	n juft bo'lganda $[0, +\infty)$, n toq bo'lganda $(-\infty, +\infty)$
2	$y = \sqrt[2n]{x}$	$[0, +\infty)$	$[0, +\infty)$
3	$y = \sqrt[2n+1]{x}$	$(-\infty, +\infty)$	$(-\infty, +\infty)$
4	$y = a^x$	$(-\infty, +\infty)$	$(0, +\infty)$

5	$y=\lg x$	$(0,\ +\infty)$	$(-\infty,\ +\infty)$
6	$y=\sin x$	$(-\infty,\ +\infty)$	$[-1,+1]$
7	$y=\cos x$	$(-\infty,\ +\infty)$	$[-1,+1]$
8	$y=tgx$	$\left((2n-1)\frac{\pi}{2},(2n+1)\frac{\pi}{2}\right),$ $n=0,\pm1,\pm2,...$	$(-\infty,\ +\infty)$
9	$y=ctgx$	$(n\pi,(n+1)\pi),$ $n=0,\pm1,\pm2,...$	$(-\infty,\ +\infty)$
0	$y=\arcsin x$	$[-1,+1]$	$\left[-\frac{\pi}{2},+\frac{\pi}{2}\right]$
1	$y=\arccos x$	$[-1,+1]$	$[0,\pi]$
2	$y=arctgx$	$(-\infty,\ +\infty)$	$\left(-\frac{\pi}{2},+\frac{\pi}{2}\right)$
3	$y=arcctgx$	$(-\infty,\ +\infty)$	$(0,\pi)$

Funksiyalarning aniqlanish sohalarini toping.

576. $y=x^2$

Yechilishi. x ga hech qanday cheklanishlar qo'yilmaydi, shu sababli $y=x^2$ funksiya $(-\infty,+\infty)$ intervalda aniqlangan.

577. 1) $y=x^3$; 2) $y=x^2-1$; 3) $y=x^3+1$. $y=ax^2+bx+c$ kvadrat funksiyani har xil tekshirishlarda $ax^2+bx+c>0$ va $ax^2+bx+c<0$ ko'rinishdagi tengsizliklarni yechish talab etiladi.

Bunday tengsizliklarning yechimlari 2-jadvalda keltirilgan.

578. $y=\dfrac{1}{x}$

Yechilishi. $x=0$ da y son qiymatga ega bo'lmaydi (nolga bo'lish mumkin emas). Barcha qiymatlar uchun (0 dan tashqari) y haqiqiy qiymatlarga ega, shuning uchun aniqlanish sohasi $(-\infty,0)$ va $(0,+\infty)$ intervallar bo'ladi.

579. $y=\dfrac{1}{2x-6}$

Yechilishi. Funksiya x ning kasr maxraji nolga aylanadigan qiymatlaridan boshqa barcha qiymatlari uchun aniqlangan. $2x-6=0$ tenglamani yechib, $x=3$ da maxraj nolga aylanishini ko'ramiz. Demak, funksiyaning aniqlanish sohasi 3 dan tashqari barcha haqiqiy sonlar ekan, ya'ni $(-\infty,3)$ va $(3,+\infty)$lardan iborat.

580. 1) $y=\dfrac{1}{4x-2}$; 2) $y=\dfrac{x+2}{2x-8}$; 3) $y=\dfrac{x^2-4}{x+2}$.

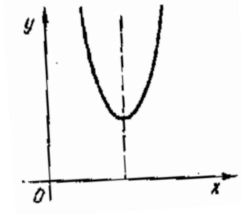
Asosiy elementar funksiyalarning aniqlanish va o'zgarish sohalari.

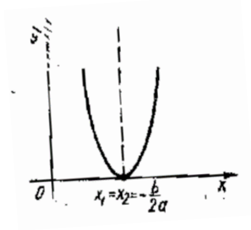
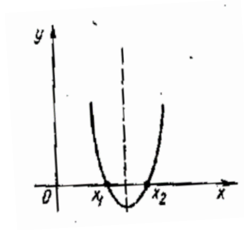
1 - jadval

№	Funksiyalar	Funksiyaning aniqlanish sohasi	Funksiyaning o'zgarish sohasi
1	$y=x^n$, n-natural son	$(-\infty,+\infty)$	n-juft bo'lganda $\left[0,+\infty\right)$, n- toq bo'lganda $(-\infty,+\infty)$
2	$y=\sqrt[2n]{x}$	$\left[0,+\infty\right)$	$\left[0,+\infty\right)$
3	$y=\sqrt[2n+1]{x}$	$(-\infty,+\infty)$	$(-\infty,+\infty)$
4	$y=a^x$	$(-\infty,+\infty)$	$\left(0,+\infty\right)$

5	$y=\lg x$	$(0,+\infty)$	$(-\infty,+\infty)$
6	$y=\sin x$	$(-\infty,+\infty)$	$[-1,+1]$
7	$y=\cos x$	$(-\infty,+\infty)$	$[-1,+1]$
8	$y=tgx$	$\left((2n-1)\frac{\pi}{2},(2n+1)\frac{\pi}{2}\right)$ $n=0,\pm1,\pm2,...$	$(-\infty,+\infty)$
9	$y=ctgx$	$(n\pi,(n+1)\pi)$, $n=0,\pm1,\pm2,...$	$(-\infty,+\infty)$
10	$y=\arcsin x$	$[-1,+1]$	$\left[-\frac{\pi}{2},+\frac{\pi}{2}\right]$
11	$y=\arccos x$	$[-1,+1]$	$[0,\pi]$
12	$y=arctgx$	$(-\infty,+\infty)$	$\left(-\frac{\pi}{2},+\frac{\pi}{2}\right)$
13	$y=arcctgx$	$(-\infty,+\infty)$	$(0,\pi)$

2 – jadval

№	$D=b^2-4ac$ discriminantning qiymati	$y=ax^2+bx+c$ funksiyaning grafigi, bu yerda a,b va c-haqiqiy sonlar, $a>0$	a argumentning	
I.	$b^2-4ac<0$			

I I.	$b^2-4ac=0$			
I II.	$b^2-4ac>0$			

581. $y=\dfrac{1}{x^2-5x+6}$

Yechilishi. Funksiya argumentning kasrning maxrajini nolga aylanadigan qiymatlaridan boshqa barcha qiymatlari uchun aniqlangan. x^2-5x+6 tenglamani yechib, $x_1=2$ $x_2=3$ ni topamiz. Aniqlanish sohasi $(2,3)$ va $(3,+\infty)$intervallar.

582. 1) $y=\dfrac{1}{1-x^2}$; 2) $y=\dfrac{1}{x^2-x-12}$;

3) $y=\dfrac{4x-1}{3x^2-5x-2}$; 4) $y=\dfrac{x-1}{x^2-9x+20}$.

583. $y=\sqrt{x}$

Yechilishi. Kavdrat ildiz manfiy bo'lmagan sonlar uchun aniqlangan. Shuning uchun $y=\sqrt{x}$ funksiya x ning $0\le x$ tengsizlikni qanoatlantiradigan barcha qiymatlari uchun aniqlangan, ya'ni uning aniqlanish sohasi $[0,+\infty)$ yarim ochiq interval.

584. $y=\sqrt{2x-4}$

Yechilishi. $2x-4\geq 0$ tengsizlikni yechib, $x\geq 2$ ni hosil qilamiz. Aniqlanish sohasi $\left[2,+\infty\right)$.

585. 1) $y=\sqrt{1-x}$; 2) $y=\sqrt{18-6x}$; 3) $y=\sqrt{3x-12}$

586. $y=\sqrt{x}+\sqrt{x-1}$.

Yechilishi. Har bir qo'shiluvchining aniqlanish sohasi alohida-alohida topamiz. Bu sohalarning umumiy qismi funksiyaning aniqlanish sohasi bo'ladi.

Aniqlanish sohasi $\sqrt{x}$ uchun $x\geq 0$ va $\sqrt{x-1}$ uchun $x\geq 1$. U holda $\sqrt{x}+\sqrt{x-1}$ yig'indi uchun aniqlanish sohasi $x\geq 1$ yoki $\left[1,+\infty\right)$ bo'ladi.

587. 1) $y=\sqrt{x}+\sqrt{4-x}$; 2) $y=\sqrt{x-2}+\sqrt{x-5}$.

588. $y=3\sqrt{5-x}-\dfrac{4}{\sqrt{x-3}}$.

Yechilishi. Aniqlanish sohasi $\sqrt{5-x}$ uchun $x\leq 5$ va $\sqrt{x-3}$ uchun $x\geq 0$. $x-3\neq 0$, chunki nolga bo'lish mumkin emas. Aniqlanish sohasi $\left(3,5\right]$ yaqim ochiq interval.

589. $y=\sqrt{7-x}+\dfrac{1}{\sqrt{x-1}}$.

590. $y=\sqrt{x^2-2x-8}$

Yechilishi. x^2-2x-8 uchhad $x_1=-2$ va $x_2=4$ ildizlarga ega. $x^2-2x-8\geq 0$. Diskriminant $D>0$(2-jadval, III hol).

Tengsizlikning yechimlaridan funksiyaning aniqlanish sohasi $\left(-\infty,-2\right]$ va $\left[4,+\infty\right)$ yaqim ochiq intervallar ekanligi kelib chiqadi.

591. 1) $y=\sqrt{x^2+8x+15}$; 2) $y=\sqrt{\left(2-x\right)\cdot\left(5+x\right)}$.

592. $y=\sqrt{\dfrac{3x-2}{2x+6}}$.

Yechilishi. Funksiya x ning $\sqrt{\dfrac{3x-2}{2x+6}}\geq 0$ tengsizlikni qanoatlantiruvchi barcha qiymatlari uchun aniqlangan, bu tengsizlik esa quyidagi hollarda bajariladi:

1) $\begin{cases} 3x-2\geq 0 \\ 2x+6>0 \end{cases}$, va 2) $\begin{cases} 3x-2\leq 0 \\ 2x+6<0 \end{cases}$

(1) sistemadan:

$\begin{cases} x\geq \dfrac{2}{3}, \\ x>3, \end{cases}$ bu yerdan $x\geq \dfrac{2}{3}$.

(2) sistemadan:

$\begin{cases} x\leq \dfrac{2}{3}, \\ x<-3, \end{cases}$ bu yerdan $x<-3$.

Demak, funksiyaning aniqlanish sohasi $(-\infty,-3)$ interval va $\left[\dfrac{2}{3},+\infty\right)$ yaqim ochiq intervaldan iborat.

593. 1) $y=\sqrt{\dfrac{x-8}{12-x}}$; 2) $y=\sqrt{\dfrac{4x-8}{3-6x}}$.

594. $y=\ln\left(\frac{5x}{x-1}-2\right)$.

Yechilishi. Natural (shuningdek o'nli) logarifmlar faqat musbat sonlar uchun aniqlangan. Quyidagi tengsizliklarga egamiz:

$$\frac{5x}{x-1}-2>0 \text{ yoki } \frac{5x-2x+2}{x-1}>0,\ \frac{3x+2}{x-1}>0.$$

Bu tengsizliklar quyidagi hollarda bajariladi:

1) $\begin{cases}3x+2>0\\ x-1>0\end{cases}$, va 2) $\begin{cases}3x+2<0\\ x-1<0\end{cases}$

(1) sistemadan:

$\begin{cases}x>-\dfrac{2}{3},\\ x>1,\end{cases}$ bu yerdan $x>1$.

(2) sistemadan:

$\begin{cases}x<-\dfrac{2}{3},\\ x<1,\end{cases}$ bu yerdan $x<-\dfrac{2}{3}$.

Demak, funksiyaning aniqlanish sohasi $\left(-\infty,-\frac{2}{3}\right)$ va $(1,+\infty)$ intervallar.

595. 1) $y=\ln\frac{5x-1}{3x-1}$; 2) $y=\lg(2x-3)$.

596. $y=\frac{1}{\sin x-\cos x}$

Yechilishi. Funksiya x ning kasrni nolga aylanadigan qiymatlaridan tashqari barcha qiymatlari uchun aniqlangan. $\sin x - \cos x = 0$ tenglamani yechamiz:

$$\sin x = \cos x; \ \frac{\sin x}{\cos x} = 1; \ tgx = 1; \ x = \frac{\pi}{4} + \pi n,$$

bu yerda n – istalgan butun son. Aniqlanish sohasi – $\left(-\infty; \frac{\pi}{4} + \pi n\right)$ va $\left(\frac{\pi}{4} + \pi n; +\infty\right)$ intervallar.

597. 1) $y = \dfrac{1}{\sin x \cos x}$; 2) $y = \dfrac{1}{\sin x + \cos x}$; 3) $y = \dfrac{1}{\sin^2 x - \sin x}$.

598. $y = \dfrac{ctgx}{\sin x - \cos x}$

Yechilishi. $ctgx$ funksiya πk uchun aniqlanmagan. Kasr $\frac{\pi}{4} + \pi n$ uchun aniqlanmagan(596-masalaga qarang). Demak, funksiya x ning $\frac{\pi}{4} + \pi n$ va πk (bu yerda n va k – istalgan butun sonlar) qiymatlaridan tashqari barcha qiymatlari uchun aniqlangan.

599. $y = \arcsin \dfrac{x-2}{3}$

Yechilishi. Agar $-1 \le \frac{x-2}{3} \le 1$ tengsizlik bajarilsa, bu funksiya aniqlangan bo‘ladi.

Bu tengsizlikni yechish uchun uning barcha hadlarini 3 ga ko‘paytiramiz: $-3 \le x - 2 \le 3$

Ushbu tengsizliklar sistemasini yechamiz.

$\begin{cases} -3 \le x-2 \\ x-2 \le 3 \end{cases}$ yoki $\begin{cases} -1 \le x \\ x \le 5 \end{cases}$ yoki $\begin{cases} x \ge -1 \\ x \le 5 \end{cases}$

Funksiyaning aniqlanish sohasi $[-1,5]$ kesma bo‘ladi.

600. 1) $y = \arcsin\frac{x}{2}$ 2) $y = \arccos\frac{x-1}{5}$

Funksiyalarning o‘zgarish sohasini toping.

601. $y = x^2 - 3x - 10$

Yechilishi. $x^2 - 3x - 10$ kvadrat uchhadda to‘liq kvadrat ajratib uni o‘zgartiramiz:

$$y = x^2 - 2\cdot\frac{3}{2}x + \left(\frac{3}{2}\right)^2 - 10 = \left(x - \frac{3}{2}\right)^2 - 12\frac{1}{4}$$

$\left(x - \frac{3}{2}\right)^2$ ifoda barcha manfiy bo‘lmagan qiymatlarni (chunki har qanday sonning kvadrati musbat sondir) qabul qiladi. Shu sababli berilgan funksiyaning o‘zgarish sohasi $-12\frac{1}{4}$ ga teng yoki undan katta bo‘lgan sonlar to‘plamidan iborat bo‘ladi, ya’ni $\left[-12\frac{1}{4};+\infty\right)$ yarim ochiq intervaldagi qiymatlarni qabul qiladi.

602. 1) $y = x^2 - 6x + 8$ 2) $y = 2x^2 - 7x + 3$.

603. $y = \sin x + \sqrt{3}\cos x$.

Yechilishi. Yordamchi burchak kiritish yo‘li bilan funksiyani qo‘yidagicha o‘zgartiramiz:

$$y = 3\sin x + \sqrt{3}\cos x = 3\left(\sin x + \frac{\sqrt{3}}{3}\cos x\right) = 3\left(\sin x + tg\frac{\pi}{6}\cos x = 3\left(\sin x + \frac{\sin\frac{\pi}{6}}{\cos\frac{\pi}{6}}\cos x\right)\right) =$$

$$= \frac{3(\sin x\cos\frac{\pi}{6} + \cos x\sin\frac{\pi}{6})}{\cos\frac{\pi}{6}} = \frac{3\sin(x+\frac{\pi}{6}}{\cos\frac{\pi}{6}} = \frac{\frac{3}{\sqrt{3}}}{2}\sin\left(x+\frac{\pi}{6}\right) = 2\sqrt{3}\sin\left(x+\frac{\pi}{6}\right)$$

$$\left|\sin\left(x+\frac{\pi}{6}\right)\right| \le 1$$

U holda $-2\sqrt{3} \le y \le 2\sqrt{3}$ ya’ni funksiyaning o‘zgarish sohasi $\left[-2\sqrt{3}, 2\sqrt{3}\right]$ kesmadan iborat.

604. 1) $y = \sin x + \cos x$ 2) $y = \sin x - \sqrt{3}\cos x$

29-§. Argumentning orttirmasi va funksiyaning orttirmasi

$y = f(x)$ funksiya uchun x argumentning ketma–ket keladigan ikkita qiymati (x_1 va x_2) ayirmasi argumentning ortirmasi deyiladi va Δx simvol bilan belgilanadi:

$$x_2 - x_1 = \Delta x .$$

$y = f(x)$ funksiyaning argumentning x_1 va x_2 qiymatlariga tegishli $y_1 = f(x_1)$ va $y_2 = f(x_2)$ qiymatlari ayirmasi funksiyaning orttirmasi deyladi va Δy simvol bilan belgilanadi, ya’ni

$$\Delta y = f(x_2) - f(x_1) = y_2 - y_1$$

Agar $x_2 > x_1$ bo‘lsa , u holda $\Delta x > 0$ agar $x_2 < x_1$ bo‘lsa u holda $\Delta x < 0$. Mos ravishda funksiya orttirmasi ham agar $y_2 > y_1$ bo‘lsa Δy va $y_2 < y_1$ bo‘lsa $\Delta y < 0$

$y = f(x)$ funksiyaning orttirmasi fo‘yidagi sxema bo‘yicha topiladi. Aytaylik x argument Δx orttirma olgan u holda argumentning orttirilgan yangi qiymati $x+\Delta x$ bo‘ladi, funksiyaning unga mos qiymati esa $y+\Delta y - f(x+\Delta x)$ bo‘ladi. Funksiyaning orttirmasini topish uchun funksiyaning orttirilgan yangi qiymatidan uning dastlabki qiymatini ayirish kerak:

$$y+\Delta y - f(x+\Delta x)$$

$$\frac{y = f(x)}{\Delta y = f(x+\Delta x) - f(x)}.$$

I. Argumentning berilgan ikkita qiymati bo‘yicha funksiyaning orttirmasini hisoblash

605. $y = x^2 + x + 1$ funksiya berilgan. Agar argument o‘z qiymatini $x_1 = 2$ dan $x_2 = 2{,}5$ gacha o‘zgartirgan bo‘lsa, argumentning orttirmasi va funksiyaning orttirmasini toping.

Yechilishi. 1. Argumentning orttirmasini topamiz:

$$\Delta x = x_2 - x_1 = 2{,}\text{–}2 = 0{,}5.$$

2. Funksiyaning argumentning $x_1 = 2$ va $x_2 = 2{,}5$ qiymatlariga mos bo‘lgan qiymatlarini topamiz:

$$y_1 = f(x_1) = f(2) = 2^2 + 2 + 1 = 7$$

$$y_2 = f(x_2) = f(2{,}5) = (2{,}5)^2 + 2{,}5 + 1 = 9{,}75$$

Funksiyaning orttirmasinn topamiz:

$$\Delta y = y_2 - y_1 = f(x_2) - f(x_1) = 9{,}75 - 7 = 2{,}75.$$

606. $y = x_2 - 2x + 4$ funksiya berilgan. Agar argument o‘z qiymatini $x_1 = 3$ dan $x_2 = 3{,}5$ gacha o‘zgartirgan bo‘lsa, funksiyaning orttirmasini toping.

II. Agar funksiyaning x argumenti Δx orttirma oladigan bo‘lsa, funksiyaning orttnrmasiny hisoblash

607. $y = x^2 + 2x - 4$ funksiya berilgan. $x = 2$ va $\Delta x = 0{,}5$ bo‘lganda Δy orttirmani toping.

Yechish isuli. 1. Agar x argument Δx orttirma olgan bo‘lsa, funksiyaning orttirilgan yangi qiymatini topamiz:

Yechilishi. 1. Agar x argument Δx orttirma olgan bo‘lsa, funksiyaning orttirilgan yangi qiymatini topamiz:

$$y+\Delta y=(x+\Delta x)-4=x^2+2x\Delta x+(\Delta x^2)+2x+2\Delta x-4.$$

2. Funksiyaning orttirmasini topamiz:

$$y+\Delta y=x^2+2x\Delta x+(\Delta x^2)+2x+2\Delta x-4$$

$$y=x^2+2x-4$$

$$\Delta y=2x\Delta x+2\Delta x+(\Delta x)^2$$

$$\Delta y=2\cdot 2\cdot 0,5+2\cdot 0,+(0,5)^2=3,25.$$

608. 1) $y=x^2+2x;$ $y=x^3-1;$ funksiyalar berilgan. $x=3$ va $\Delta x=0,1;$ bo‘lganda Δy orttirmani toping.

609. $y=\dfrac{1}{x}$ funksiya berilgan. $x=1$ va $\Delta x=0,2$ bo‘lganda Δy orttirmani toping.

Yechilishi.

1) $y+\Delta y=\dfrac{1}{x+\Delta x};$

2) $y=\Delta y=\dfrac{1}{x+\Delta x},$ $y=\dfrac{1}{x},$

3) $\Delta y=\dfrac{1}{x+\Delta x}-\dfrac{1}{x}=\dfrac{x-x-\Delta x}{x(x+\Delta x)}=-\dfrac{0,2}{1(1+0,2)}=-\dfrac{1}{6}.$

610. 1) $y=-\dfrac{3}{x}$ 2) $y=\dfrac{1}{2x}$ 3) $y=\dfrac{1}{x}-x$ orttirmani toping.

611. $y=\sqrt{x}$ funksiya berilgan. $x=1$ va $\Delta x=0,1$ bo‘lganda Δy orttirmani toping.

Yechilishi.

1) $y+\Delta y=\sqrt{x+\Delta x};$ 2) $y+\Delta y=\sqrt{x+\Delta x}$ $y=\sqrt{x}$

$$\Delta y=\sqrt{x+\Delta x}-\sqrt{x}=\sqrt{1+0,1}-\sqrt{1}=\sqrt{1,1}-1=0,049.$$

612. 1 *)* $y=\sqrt{2x}$ *;* 2) $y=\sqrt[3]{x}$ funksiyalar berilgan. $x=1$ va $\Delta x=0,2$ bo‘lganda Δy orttirmani toping.

30- §. Funksiyaning uzluksizligi

Funksiyaning nuqtada uzluksizligi

I ta’rif. Agar $y=f(x)$ funksiyaning $x\to a$ dagi limiti funksiyaning $x=a$ dagi qiymatiga teng, ya’ni

$$\lim_{x\to a} f(x)=f(x) \tag{5.1}$$

bo‘lsa, $f(x)$ funksiya $x=a$ nuqtada *uzluksiz* deyiladi.

Bunda quyidagi uchta shart bajarilishi kerak:

1) funksiya a nuqtada aniqlangan bo‘lishi kerak;

2) funksiyaning $\lim\limits_{x\to a} f(x)$ limiti mavjud bo‘lishi kerak;.

3) bu limit $f(x)$ funksiyaning $x=a$ dagi qiymatiga teng bo‘lishi kerak.

II ta’rif. Agar $f(x)$ funksiya $x=a$ nuqtada aniqlangan argumentning cheksiz kichik orttirmasiga funksiyaning cheksiz kichik orttirmasi mos kelsa, ya’ni

$$\lim_{\Delta x\to\infty} \Delta y=0 \tag{5.2}$$

bo‘lsa, $y=f(x)$ funksiya $x=a$ *nuqtada uzluksiz* deyiladi.

Agar funksiyaning $x=a$ nuqtadagi uzluksizlik sharti buzilgan bo‘lsa, u holda funksiya bu nuqtada uzilishga ega bo‘ladi va bu nuqtani funksiyaning uzilish nuqtasi deyiladi.

Elementar funksiyalar uchun qo‘yidagnlar o‘rinlidir:

1) elementar funksiyaning uzluksizlik sohasi uning aniqlanish sohasi bilan bir xil bo‘ladi, ya’ni elementar funksiya o‘zining butun aniqlanish sohasida uzluksizdir;

2) elementar funksiya biror intervalning barcha nuqtalarida emas, balki ayrim nuqtalaridagina uzilishga ega bo‘lishi mumkin;

3) elementar funksiya o‘zi aniqlanmagan nuqtadagina uzilishga ega bo‘lishi mumkin.

Funksiyaning intervalda yoki kesmada uzluksizligi

Agar funksiya interval yoki kesmaning barcha nuqtalarida uzluksiz bo‘lsa, u bu intervalda yoki kesmada uzluksiz deyiladi.

I. Funksiyaning butun aniqlanish sohasida uzluksizligini tekshirish

613. $y=3x$ funksiyaning uzluksizligini tekshiring.

Yechilishi. $y=3x$ funksiya x argumentning barcha haqiqiy qiymatlari uchun aniqlangan, ya’ni uning aniqlanish sohasi butun son uni $(-\infty+\infty)$ dan iborat. Uning uzluksizlik sohasi aniqlanish sohasi bilan bir xil ekanligini (5.2) ta’rifdan foydalanib osongina isbot qilish mumkin.

x argumentga Δx orttirma beramiz va funksiyaning Δy orttirmasini topamiz:

$$y+\Delta y=3(x+\Delta x)=3x+3\Delta x$$
$$y=3x$$

Ayirsak,

$$\Delta y=3\Delta x.$$

$\Delta x\to 0$da Δy ning limitini topamiz:

$$\lim_{\Delta x\to 0}\Delta y=\lim_{\Delta x\to 0}3\Delta x=3\lim_{\Delta x\to 0}\Delta x=3\cdot 0=0.$$

$\lim\limits_{\Delta x\to 0}\Delta y=0$ tenglik x ning istalgan chekli qiymatida o’rinli, shu sababli $y=3x$ funksiya x ning istalgan qiymatida uzluksizdir.

614. 1) $y=-5x$ 2) $y=4x-3$ funksiyalarning uzluksizligini tekshiring.

615. $y=3x^2-2x$ funksiyaning uzluksizligini tekshiring.

Yechilishi. Funksiya $(-\infty+\infty)$ intervalda aniqlangan, shu intervalning o‘zida u uzluksiz hamdir. (5.2) ta’rifga ko‘ra $\lim\limits_{\Delta x\to\infty}\Delta y$ ni topamiz:

$$y+\Delta y=3(x+\Delta x)^2-2(x+\Delta x)=3x^2+6x\Delta x+3(\Delta x)^2-2x-2\Delta x$$
$$y+\Delta y=3x^2+6x\Delta x+3(\Delta x)^2-2x-2\Delta x$$
$$y=3x^2-2x$$

$$\Delta y=6x\Delta x+3(\Delta x)^2-2\Delta x$$
$$\lim_{\Delta x\to 0}\Delta y=\left[6x\lim\Delta x+3(\lim\Delta x)^2-2\lim\Delta x\right]_{\Delta x\to 0}=$$
$$=6x\cdot 0+3\cdot 0^2-2\cdot 0=0$$

$y=3x^2-2x$ funksiya x ning istalgan chekli qiymatida uzluksizdir.

616. Quyidagi funksiyalarning uzluksizligini tekshiring:

1) $v=2t^2$; 2) $y=x^2+2$; 3) $s=t^2-t$; 4) $y=x-3x^2$;

5) $y=x^3$; 6) $y=-x^3-1$; 7) $y=2x^3$.

II. Funksiya berilgan nuqtada (argumentning berilgan qiymatida) uzluksizligini tekshirish

617. $y=x^2-2$ funksiyaning $x=3$ da uzluksizligini tekshiring

Yechilishi. Tekshirishda (5.1) ta'rifdan foydalanamiz:

$$\lim_{x\to 3}(x^2-2)=(\lim_{x\to 3}x)^2-2=3^2-2=7$$
$$f(3)=3^2-2=7,$$

ya'ni

$$\lim_{x\to 3}(x^2-2)=f(3),$$

funksiyaning $x\to 3$dagi limiti funksiyaning $x=3$ dagi qiymatiga teng, bunda (5.1) ta'rifni qo'llanish shartlari ham bajarilyapti. Demak, $y=x^2-2$ funksiya $x=3$ nuqtada uzluksizdir.

618. 1) $y=x^2+4x+3$ funksiyaning $x=3$ nuqtada; $y=x^3-5$ funksiyaning $x=1$ nuqtada uzluksizligini tekshiring

619. $y=\sin 2x$ funksiyaning $x=\dfrac{\pi}{2}$ nuqtada uzluksizligini tekshiring.

Yechilishi. $y=\sin 2x$ funksiya $(-\infty+\infty)$ intervalda aniqlangan. Tekshirish uchun (5.1) ta'rifdan foydalanamiz:

$$\lim_{x\to\frac{\pi}{2}}\sin 2x=\sin 2\cdot\frac{\pi}{2}=\sin\pi=0$$

$$f(\frac{\pi}{2})=\sin 2\cdot\frac{\pi}{2}=\sin\pi=0.$$

(5.1) ta'rifning qo'llanilishlik shartlari bajariladi, demak, $e=\sin 2x$ funksiya $x=\frac{\pi}{2}$ nuqtada uzluksiz.

620. 1) $y=\cos x$ va 2) $y=tg\frac{x}{2}$ funksiyalarning $x=\frac{\pi}{3}$ nuqtada uzluksizligini tekshiring.

III. Berilgan funksiyaning uzilish nuqtasini topish

621. $y=\dfrac{2}{2-x}$ funksiyaning uzilishini tekshiring.

Yechilishi. Berilgan funksiyaning aniqlanish sohasi $(-\infty;2)$ va $(2;+\infty)$ intervallardan iborat. Funksiya $x=2$ nuqtada uzilishga ega. Funksiyaning aniqlanish va uzluksizlik sohalari bir xil.

622. Quyidagi funksiyalarning uzilishini tekshiring:

1) $y=\dfrac{5}{2x-1}$ 2) $y=\dfrac{1}{x^2}$ 3) $y=\dfrac{1}{x^2-1}$ 4) $y=\dfrac{3}{x^2-2x+1}$.

6 BOB

HOSILA

31-§. Funksiyaning o‘zgarish tezligi

x va y o‘zgaruvchilar orasidagi bog‘lanish ko‘rinishida tasvirlanishi mumkin bo‘lgan turli xil fizik jarayonlar umumiy ko‘rinishda

$$y = f(x)$$

funksiya bilan yoziladi va bu munosabat o‘zgaruvchi miqdor y ning x o‘zgaruvchining o‘zgarishiga bog‘liq qolda o‘zgarish jarayonini ifodalaydi.

Funksiyaning o‘zgarish tezligini hisoblash quyidagi umumiy qoida buyicha bajariladi:

I. x argumentning biror Δx kattalikka o‘zgarishi y funksiyani Δy kattalikka o‘zgarishiga olib keladi, ya’ni

$$y + \Delta y = f(x + \Delta x)$$

II. Funksiya argumentining Δx orttirmasiga mos kelgan Δy orttirmasi topiladi:

$$y = \Delta y = f(x + \Delta x)$$
$$y = f(x)$$

Ayirsak,

$$\Delta y = f(x + \Delta x) - f(x)$$

III. y funksiyaning argument qiymatining x dan $x + \Delta x$ gacha o‘zgarishi oralig‘i uchun o‘zgarishining o‘rtacha tezligi

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

munosabat bilan ifodalanadi.

$\frac{\Delta y}{\Delta x}$ nisbat argument orttirmasi birligiga funksiya orttirmasining nechta birligi to‘g‘ri kelishini ko‘rsatadi.

IV. x ning berilgan qiymatida funksiya o‘zgarishining oniy yoki hahiqiy v tezligi x argumentning x dan $x+\Delta x$ gacha o‘zgarish oralig‘ida o‘rtacha tezlikning $\frac{\Delta y}{\Delta x}$ da intiladigan limitidir, ya’ni

$$v = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$y = kx + b$ chiziqli funksiya uchun o‘rtacha tezlik $v_{\text{ўр}} = \frac{\Delta y}{\Delta x} = k$ va haqiqiy tezlik $v = \lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = k$ kattalqgi bo‘yicha bir xil va haqiqiy tezlikning son qiymati k koeffidientga teng.

I. Funksiya o‘zgarishining o‘rtacha tezligini hisoblash

623. $y = 3x^2 - 6$ funksiya o‘zgarishining x argument $x_1 = 3$ dan $x_2 = 3,5$ gacha o‘zgargandagi o‘rtacha tezligini toping.

Yechilishi. 1- usul. 1. Argumentning orttirmasini topamiz:

$$\Delta x = x_2 - x_1 = 3,5 - 3 = 0,5$$

2. Funksiyaning x_1 va x_2 dagi qiymatlarini aniqlaymiz:

$$y_2 = 3 \cdot 3^2 - 6 = 21 \quad y_2 = 3 \cdot (3,5)^2 - 6 = 30,75$$

3. Funksiyaning orttirmasini hisoblaymiz:

$$\Delta y = y_2 - y_1 = 30,75 - 21 = 9,75$$

4. Funksiya o‘zgarishining o‘rtacha tezligini topamiz:

$$\frac{\Delta y}{\Delta x} = \frac{9,75}{0,5} = 19,5$$

2-usul. 1. Funksiya o‘zgarishining o‘rtacha tezligini argumentning istalgan qiymati uchun umumiy qoida buyicha hisoblaymiz:

1. $y+\Delta y=3(x+\Delta x)\ -6=3x^2+6x^2+6x\Delta x+3(\Delta x)^2-6$

II. $y=\Delta y=3x^2+6x\Delta x+3(\Delta x)^2-6$
$y-3x^2-6$

$\Delta y=6x\Delta x+3(\Delta x)^2$ $\qquad v_{ўр}=\dfrac{\Delta y}{\Delta x}=\dfrac{6x\Delta x+3(\Delta x)^2}{\Delta x}=6x+3\Delta x$

2. Argumentning orttirmasini topamiz:

$$\Delta x=x_2-x_1=3{,}5-3=0{,}5$$

3. $x=3$ va $\Delta x=0,5$ da $v_{ўр}$ ni aniqlaymiz

$$v_{ўр}=6\cdot 3+3\cdot 0{,}5=19{,}5$$

624. 1. $y=2x^2-5x$ funksiya o‘zgarishining x argument $x_1=2$ dan $x_2=3$ gacha o‘zgargandagi o‘rtacha tezligini toping.

2. Nuqtaning harakat qonuni $s=3t^2-2$ formula bilan berilgan. $t_1=4$ dan $t_2=6$ gacha bo‘lgan vaqt oralig‘ida nuqta harakatining o‘rtacha tezligini toping.

II. Nuqta to‘g‘ri chiziqli harakatining berilgan momentdagi tezligini bu nuqtaning harakat tenglamasi bo‘yicha hisoblash

625. Nuqtaning to‘g‘ri chiziqli harakati $s=3t^2-2t+5$ tenglama bilan berilgan, bu yerda t sekund hisobida va s metr hisobida berilgan.

Nuqta harakatining $t=5$ sek momentdagi tezligini toping.

Yechilishi. 1. Nuqta harakatining o‘rtacha tezligini topamiz:

I. $s=\Delta s=3(t+\Delta t)^2-2(t+\Delta t)+5=3t^2+$
$+6t\Delta t+3(\Delta t)^2-2t-2\Delta t+5$

II. $s+\Delta s=+3t^2+6t\Delta t+3(\Delta t)^2-2t-2\Delta t+5$
$s=3t^2-2t+5$

$\Delta s=6t\Delta t+3(\Delta t)^2-2\Delta t$

III $\dfrac{\Delta s}{\Delta t}=\dfrac{6t\Delta t+3(\Delta t)^2-2\Delta t}{\Delta t}=6t+3\Delta t-2$

2 . Nuqta harakatining vaqtning t momentidagi haqiqiy tezligini topamiz:

IV. $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \lim_{\Delta t \to 0}(6t + 3\Delta t - 2) = 6t - 2$

3. Nuqta harakatining 5 sek oxiridagi tezligini topamiz:

$$v_{t\to 5} = 6 \cdot 5 - 2 = 28(m / cek)$$

626. Nuqtaning to‘g‘ri chiziqli harakati $s = 5t^2$ tenglama bilan berilgan *(t* sek hisobida, s m hisobida). Nuqta harakatining 10 sek oxiridagi tezligini toping.

627. Nuqtaning to‘g‘ri chiziqli harakati $s = 2t^2 - 8t - 10$ tenglama bilan berilgan *(t* sek hisobida, s m hisobida). Nuqta harakatining 8 sek oxiridagi tezligini toping.

32- §. Hosila

$y = f(x)$ funksiyaning *hosilasi* deb funksiya orttirmasi Δy ni argumentning mos orttirmasi Δx ga nisbatining $\Delta x \to 0$ dagi limitiga aytiladi.

$y = f(x)$ funksiyaning hosilasini belgilash uchun bir qator belgilar mavjud:

$$y', \; y'_x, \; \frac{dy}{dx}, \; f'(x) \quad \text{yoki} \quad \frac{df(x)}{dx}.$$

$y = f(x)$ funksiyaning hosilasini hisoblash differensiallashning umumiy qoidasi bo‘yicha to‘rt bosqichda bajariladi.

I. x argumentga Δx orttirma beramiz va funksiyaga x argumentning o‘rniga $x + \Delta x$ orttirilgan qiymatni qo‘yib, funksiyaning orttirilgan qiymatini hosil qilamiz:

$$y + \Delta y = f(x + \Delta x)$$

II.Funksiyaning orttirilgan qiymatidan uning dastlabki qiymatini ayirib, funksiya orttirmasini hosil qilamiz:

$$\Delta y = f(x + \Delta x) - f(x)$$

III. Funksiyaning orttirmasi Δy ni argumentning orttirmasi Δx ga bo‘lamiz, ya’ni qo‘yidagi nisbatni tuzamiz:

$$\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{\Delta x}.$$

IV. Bu nisbatning $\Delta x\to 0$ dagi limitini topamiz:

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}.$$

Topilgan limit $y=f(x)$ funksiyaning hosilasidir. Hosilani topish *differensiallash* deyiladi.

Hosilalarni differensiallashning umumiy qoidasi bo‘yicha toping.

628. $y=2x^2-3x$. Hosilaning $x=3$ dagi xususiy qiymatini toping.

Yechilishi.

I. $y=\Delta y=2(x+\Delta x)^2-3(x+\Delta x)=2x^2+4x\Delta x+2(\Delta x)^2-3x-3\Delta x$

$$y=\Delta y=2x^2 4x\Delta x+2(\Delta x)^2-3x-3\Delta x$$

II. $y=2x^2-3x$

$$\Delta y=4x\Delta x+2(\Delta x)^2-3\Delta x$$

I. $$\frac{\Delta y}{\Delta x}=4x+2\Delta x-3$$

II. $$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}(4x+2\Delta x-3)=4x-3;\ y'=4x-3$$

Hosilaning $x=3$ dagi qiymatini topamiz: $y'_{x=3}=4\cdot 3-3=9$

629. 1) $y=x^2-x.$ $y'|_{x=0}$ ni toping; 2) $y=x^2-5x+4.$ $y'|_{x=1}$ ni toping; 3) $s=t^3.$ $s'|_{t=2}$ ni toping.

630. 1) $y=-\dfrac{3}{x}.$ $y'|_{x=3}$ ni toping; 2) $y=\dfrac{1}{x^2}.$ $y'|_{x=-1}$ ni toping.

631. 1) $y=\sqrt{x}.$ $y'|_{x=4}$ ni toping.

Yechilishi.

I. $y+\Delta y=\sqrt{x+\Delta x};$

II. $y+\Delta y$ dan y ni ayiramiz: $\Delta y=\sqrt{x+\Delta x}-\sqrt{x}$;

III. Nisbat tuzamiz: $\dfrac{\Delta y}{\Delta x}=\dfrac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x}$;

IV. $y'=\lim\limits_{\Delta x\to 0}\dfrac{\Delta y}{\Delta x}=\lim\limits_{\Delta x\to 0}\dfrac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x}=$

$$=\lim_{\Delta x\to 0}\frac{\left(\sqrt{x+\Delta x}-\sqrt{x}\right)\left(\sqrt{x+\Delta x}+\sqrt{x}\right)}{\Delta x\left(\sqrt{x+\Delta x}+\sqrt{x}\right)}=$$

$$=\lim_{\Delta x\to 0}\frac{x+\Delta x-x}{\Delta x\left(\sqrt{x+\Delta x}+\sqrt{x}\right)}=\lim_{\Delta x\to 0}\frac{\Delta x}{\Delta x\left(\sqrt{x+\Delta x}+\sqrt{x}\right)}=$$

$$=\lim_{\Delta x\to 0}\frac{1}{\sqrt{x+\Delta x}+\sqrt{x}}=\frac{1}{\sqrt{x+0}+\sqrt{x}}=\frac{1}{2\sqrt{x}};$$

$$y'\big|_{x=4}=\frac{1}{2\sqrt{4}}=\frac{1}{4}.$$

632. 1) $y=\sqrt{x-1}$. $y'\big|_{x=5}$ ni toping; 2) $y=\dfrac{1}{\sqrt{x}}$. $y'\big|_{x=4}$ ni toping; 3) $y=\sqrt[3]{x}$. $y'\big|_{x=2\sqrt{2}}$ ni toping.

633. 1) $y=\cos x$. $y'\big|_{x=\frac{\pi}{4}}$ ni toping.

Yechilishi.

I. $y=\Delta y=\cos(x+\Delta x)$

II.
$$y+\Delta y=\cos(x+\Delta x)$$
$$y=\cos x$$

$$\Delta y=\cos(x+\Delta x)-\cos x$$

$$\Delta y=-2\sin\frac{x+\Delta x+x}{2}\sin\frac{x+\Delta x-x}{2}=-2\sin(x+\frac{\Delta x}{2})\sin\frac{\Delta x}{2}$$

III. $$\frac{\Delta y}{\Delta x}=-2\sin\left(x+\frac{\Delta x}{2}\right)\frac{\sin\frac{\Delta x}{2}}{\Delta x}=-\sin\left(x+\frac{\Delta x}{2}\right)\frac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}$$

IV. $$y'=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=-\lim_{\Delta x\to 0}\sin\left(x+\frac{\Delta x}{2}\right)\lim_{\Delta x\to 0}\frac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}=-\sin x\cdot 1=-\sin x;$$

$$y'_{x=\frac{\pi}{4}}=-\sin\frac{\pi}{4}=-\frac{\sqrt{2}}{2}$$

634. 1) $y=\sin x$ $y'_{x=\frac{\pi}{4}}$ ni toping; 2) $y=\cos 2x$ $y'_{x=\frac{\pi}{2}}$ ni toping

635. $y=tgx.y'_{x=\frac{\pi}{4}}$ ni toping

Yechilishi.

I. $y+\Delta y=tg(x+\Delta x);$

II. $$\begin{array}{l} y+\Delta y=tg(x+\Delta x)\\ y=tgx \\ \hline \Delta y=tg(x+\Delta x)-tgx\end{array}$$

$$\Delta y=\frac{\sin(x+\Delta x-x)}{\cos(x+\Delta x)\cos x}=\frac{\sin\Delta x}{\cos(x+\Delta x)\cos x};$$

III. $$\frac{\Delta y}{\Delta x}=\frac{\sin\Delta x}{\Delta x\cos(x+\Delta x)\cos x}=\frac{1}{\cos(x+\Delta x)\cos x}\frac{\sin\Delta x}{\Delta x}$$

IV. $$y'=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{1}{\cos(x+\Delta x)\cos x}\lim_{\Delta x\to 0}\frac{\sin\Delta x}{\Delta x}=\frac{1}{\cos(x+0)\cos x}\cdot 1=\frac{1}{\cos^2 x};$$

$$y'_{x=\frac{\pi}{4}}=\frac{1}{\cos^2\frac{\pi}{4}}=\frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2}$$

636. $y=ctgx.y'_{x=\frac{\pi}{6}}$ ni toping.

33-§. Differensiallashning asosiy qoidalari. Darajaning va ildizning hosilalari

Belgilashlar: C — o‘zgarmas, x — argument, u, v va w- argument x ning hosilaga ega bo‘lgan funksiyalari.

Differensiallashning asosiy qoidalari

Funksiya algebraik yig‘indisining hosilasi:

$$(u+v-w)'=u'+v'-w' \qquad (6\ .1)$$

Ikkita funksiya ko‘paytmasining hosilasi:

$$(uv)'=u'v'+v'u \qquad (6\ .2)$$

Uchta funksiya ko‘paytmasining hosilasi:

$$(uvw)=u'vw+v'uw+w'uv \qquad (6.3)$$

O‘zgarmasning funksiyaga ko‘paytmasining hosilasi:

$$(Cu)'=Cu' \qquad (6.4)$$

Bo‘linmaning (kasrning) hosilasi:

$$\left(\frac{u}{v}\right)'=\frac{u'v-uv'}{v^2}. \qquad (6.5)$$

(6.5) formulaning xususiy hollari:

$$\left(\frac{u}{C}\right)'=\frac{u'}{C}. \qquad (6.6)$$

$$\left(\frac{C}{v}\right)'=-\frac{C}{v^2}. \qquad (6.7)$$

Agar y funksiya u ning funksiyasi: $y=f(u)$ va bu yerdagi u o‘z navbatida x ning funksiyasi: $u=\varphi(x)$ bo‘lsa , ya’ni y funksiya x ga oraliq argument u orqali bog‘liq bo‘lsa, u holda $y=f(u)$ funksiya x ning murakkab funksiyasi (funksiyaning funksiyasi) deyiladi: $y=f\left[\varphi(x)\right]$.

Murakkab funksiyaning hosilasi uning oraliq argument bo‘yicha hosilasini bu argumentning erkli o‘zgaruvchi bo‘yicha hosilasiga ko‘paytirilganiga teng:

$$\frac{du}{du}=\frac{du}{du}\frac{du}{du} \text{ yoki } \quad y'_x = y'_u \cdot u'_x .$$

Bu munosabatdan foydalanib, $u=\varphi(x)$ bo‘lgan murakkab funksiyalarni differensiallash uchun formulalar hosil qilingan.

Hosilalarni topishda quyidagilarni yodda tutish kerak (ta’rifga ko‘ra):

1. $a^0 = 1 (a \neq 0);$
2. $a^{-n} = \dfrac{1}{a^n} (a \neq 0)$
3. $\sqrt[n]{a^m} = a^{\frac{m}{n}} \cdot (a > 0)$

va darajalar hamda ildizlar bilan amallar bajarishda quyidagi qoidalarni bilish kerak:

4. $a^n a^m = a^{n+m};$
5. $\dfrac{a^n}{a^m} = a^{n-m};$
6. $(a^n)^m = a^{nm};$
7. $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b} = a^{\frac{1}{n}} b^{\frac{1}{n}} (a > 0, b > 0);$
8. $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \dfrac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}} (a > 0, b > 0)$

Bu yerda m va n – ixtiyoriy ratsional sonlar.

Differensiallash formulalari

$u=\varphi(x)$ shartda			$u=x$ shartda	
			$c'=0$	(6.8)
			$x'=0$	(6.9)
$(u^n)'=nu^{n-1}u'$ bu yerda n - ixtiyoriy haqiqiy son		(6.10)	$(x^n)'=nx^{n-1}$ bu yerda n - ixtiyoriy haqiqiy son	(6.10a)
$\left(\frac{1}{u}\right)'=\frac{1}{u^2}u'$		(6.11)	$\left(\frac{1}{x}\right)'=\frac{1}{x^2}$	(6.11a)
$\left(\sqrt{u}\right)'=\frac{1}{2\sqrt{u}}u'$		(6.12)	$\left(\frac{1}{\sqrt{x}}\right)'=\frac{1}{2\sqrt{x}}$	(6.12a)

1. Funksiyalarning hosilalarini $(x^n)'=nx^{n-1}$ formuladan foydalanib topish

Funksiyalarning hosilasini toping.

637. 1. $y=3x^4$; 2) $y=2x^{-5}$; 3) $y=4x^{\frac{1}{3}}$;

4) $y=5x^{-\frac{2}{3}}$ 5) $y=5\sqrt[5]{x^3}$

Yechilishi. 1) $y=3x^4$ (6.4) formulaga ko‘ra o‘zgarmas ko‘paytuvchi hosila belgisi tashqarisiga chiqadi va (6.10a) formula bo‘yicha

$$y'=3(x^4) \qquad y'=3\cdot 4x^{4-1}=12x^3;$$

2) $y=2x^{-5}$ (6.4) va (6.10a) formulaga ko‘ra topamiz:

$$y'=2(-5)x^{-5-1}=-10x^{-6}=-\frac{10}{x^5}$$

3) $y=4x^{\frac{1}{3}}$. (6.4) va (6.10a) formulaga ko‘ra topamiz:

$$y' = 4\cdot\frac{1}{3}x^{\frac{1}{3}-1} = \frac{4}{3}x^{-\frac{2}{3}} = \frac{4}{3x^{\frac{2}{3}}} = \frac{4}{3\sqrt[3]{x^2}}$$

4) $y = 5x^{-\frac{2}{5}}$ (6.4) va (6.10a) formulaga ko‘ra topamiz:

$$y = 5\cdot\left(-\frac{2}{5}\right)x^{-\frac{2}{5}-1} = -2x^{-\frac{7}{5}}$$

5) Ildizni kasr ko‘rsatkich bilan almashtiramiz va (6.4) va (6.10a) formulalardan foydalanamiz:

$$y = 5\sqrt[5]{x^3} = 5x^{\frac{3}{5}}$$

$$y' = 5\cdot\left(x^{-\frac{3}{5}}\right)' = 5\cdot\frac{3}{5}x^{\frac{3}{5}-1} = 3x^{-\frac{2}{5}}$$

638. 1) $y = x^4$; 2) $y = 5x^3$ 3) $y = 3x^{-5}$ 4) $y = -3x^{-2}$

5) $y = x^{-\frac{7}{5}}$; 6) $y = 4x^{-\frac{3}{2}}$; 7) $y = 5x^{-\frac{3}{5}}$; 8) $y = 2\sqrt{x^3}$;

9) $y = \sqrt[4]{x^{-3}}$; 10) $y = \sqrt[-3]{x^2}$.

639. $y = \frac{1}{2x^{\frac{2}{3}}}$; 2) $y = 3x\sqrt[3]{x}$; 3) $y = \frac{5x^2}{\sqrt[3]{x}}$;

4) $y = 2\sqrt{\frac{2}{x^{-3}}}$; 5) $f(\varphi) = \varphi^{-1}\sqrt{\varphi^{-1}}\sqrt[3]{\varphi^2}$; 6) $f(v) = \frac{\sqrt{v}^3\sqrt{v}}{2\sqrt[3]{v^2}}$.

Yechilishi. Hosila olishdan avval har qaysi funksiyani $y = x^n$ (n - istalgan ratsional son) ko’rinishga keltiramiz:

$$y=\frac{1}{2x^{\frac{2}{3}}}=\frac{1}{2^{x}}{}^{-\frac{2}{3}}$$

(6.4) va (6. 10a) formulalarga ko‘ra topamiz:

$$y'=\frac{1}{2}=\left(x^{-\frac{2}{3}}\right)'=\frac{1}{2}\left(-\frac{2}{3}\right)x^{-\frac{2}{3}-1}=-\frac{1}{3}x^{-\frac{5}{3}}=-\frac{1}{3x^{\frac{5}{3}}}=-\frac{1}{3x\sqrt[3]{x^{2}}}$$

$$y=3x^{2}\sqrt[3]{x}=3x^{2}x^{\frac{1}{3}}=3x^{\frac{7}{3}}$$

(6.4) va (6.10a) forulalarga ko‘ra topamiz:

$$y'=3(x^{-\frac{7}{3}})'=3\cdot\frac{7}{3}x^{\frac{7}{3}-1}=7x^{\frac{4}{3}}=7x\sqrt[3]{x}$$

3) $y=\frac{2x}{\sqrt[3]{x}}=\frac{2x^{2}}{x^{\frac{1}{3}}}=2x^{\frac{5}{3}}$

(6.4) va (6.10a) formulalarga ko‘ra topamiz:

$$y'=2\left(x^{\frac{5}{3}}\right)'=2\cdot\frac{5}{3}x^{\frac{5}{3}-1}=\frac{10}{3}x^{\frac{2}{3}}=\frac{10}{3}\sqrt[3]{x^{2}}$$

4) $y=2\sqrt{\frac{2}{x^{-3}}}=2\cdot\frac{\sqrt{2}}{\sqrt{x^{-3}}}=\frac{2\sqrt{2}}{x^{-\frac{2}{3}}}=2\sqrt{2x^{\frac{3}{2}}}$

(6.4) va (6.10a) formulalarga ko‘ra:

$$y'=2\sqrt{2}\left(x^{\frac{3}{2}}\right)'=2\sqrt{2}\cdot\frac{3}{2}x^{\frac{3}{2}-1}=3\sqrt{2}x^{\frac{1}{2}}=3\sqrt{2}\sqrt{x}=3\sqrt{2x}$$

5) $f(\varphi)=\varphi^{-1}\sqrt{\varphi^{-1}}^{3}\sqrt{\varphi^{2}}=\varphi^{-1}\varphi^{-\frac{1}{2}\frac{2}{3}}=\varphi^{-1-\frac{1}{2}+\frac{2}{3}}=\varphi^{-\frac{5}{6}}$

6) $f(\varphi)=\dfrac{\sqrt{v}^{3}\sqrt[3]{v}}{2\sqrt[3]{v^{2}}}=\dfrac{v^{\frac{1}{2}}v^{\frac{1}{3}}}{2v^{\frac{2}{3}}}=\dfrac{1}{2}v^{\frac{1}{2}}v^{\frac{1}{3}}v^{-\frac{2}{3}}=\dfrac{1}{2}v^{\frac{1}{2}+\frac{1}{3}-\frac{2}{3}}=\dfrac{1}{2}v^{\frac{1}{6}}$

(6.4) va (6.10a) formulalarga ko‘ra topamiz:

$$f'\cdot(v)=\frac{1}{2}\left(v^{\frac{1}{5}}\right)'=\frac{1}{2}\cdot\frac{1}{6}v^{\frac{1}{6}-1}=\frac{1}{12}v^{-\frac{5}{6}}$$

640. 1) $y=-\dfrac{1}{x^{2}}$; 2) $y=\dfrac{3}{x^{\frac{4}{3}}}=2x^{\frac{5}{3}}$; 3) $y=\dfrac{3}{\sqrt{x^{3}}}$;

4) $y=2x^{3}\sqrt[3]{x}$; 5) $y=\dfrac{x^{4}}{\sqrt{x}}$; 6) $y=\dfrac{2x^{3}}{\sqrt{x}^{-3}}$;

7) $y=\dfrac{3\sqrt{x}}{x^{3}}$; 8) $y=\dfrac{6\sqrt[3]{x}}{\sqrt{x}}$; 9) $y=\dfrac{2x}{\sqrt[3]{x}}=\dfrac{2x^{2}}{x^{\frac{1}{3}}}=2x^{\frac{5}{3}}$;

10) $y=\sqrt[3]{\dfrac{1}{x^{-2}}}$; 11) $f(x)=x^{-2}\sqrt{x}^{3}\sqrt{x}$; 12) $s=\dfrac{\sqrt{t}^{3}\sqrt{t^{2}}}{t\sqrt{t}}$.

641. $f(x)=\dfrac{1}{x^{4}}$; $f'(-1)$ va $f'(2)$ ni toping.

Yechilishi. 1) $f(x)=\dfrac{1}{x^{4}}=x^{-4}$

(6.10a) formulaga ko‘ra topamiz

$$f'(x)=-4x^{4-1}=-4x^{-5}=-\frac{4}{x^{3}}$$

$f'(-1)$ va $f'(2)$ ni hisoblash uchun hosilada x ning o‘rniga — 1 va 2 qiymatlarni qo‘yish kerak:

$$f'(-1)=\frac{4}{(-1)^5}=-\frac{4}{-1}=4$$

$$f'(2)=-\frac{4}{2^5}=-\frac{4}{3^2}=-\frac{1}{8}$$

1) $\quad y=x^3\sqrt[2]{x}=x^3\cdot x^{\frac{1}{3}}=x^{\frac{10}{x^3}}$

(6.10 a) formulaga ko‘ra topamiz:

$$y'=\frac{10}{3}x^{\frac{10}{3}-1}=\frac{10}{3}x^{\frac{7}{3}};\quad y'_{x=1}=\frac{10}{3}\cdot 1^{\frac{7}{3}}=3\frac{1}{3}.$$

642. 1) $f'(x)=\frac{1}{x^3}\cdot f'(\frac{1}{2})$ ni toping; 2) $f'(x)=\sqrt[3]{x^4}\cdot f'(-8)$ ni toping; 3) $y=x\sqrt{x}^3\sqrt{x}\cdot y'_{x=1}$ ni toping.

II. Funksiyalar algebraik yig‘indisining hosilasi

Funksiyalarning hosilalarini toping.

643. $y=4x^3-2x^2+x-5$.

Yechilishi. (6.1), (6.4), (6.10a), (6.9), va (6.8) formulalarni birin ketin tatbiq qilib topamiz:

$$y'=\left(4x^3\right)'-\left(2x^2\right)'+x'-5';\qquad y'=4\left(x^3\right)'-2\left(x^2\right)'+x'-5';$$

$$y'=4\cdot 3x^2-2\cdot 2x+1;$$

Oxirida:

$$y'=12x^2-4x+1;$$

Differensiallashda malaka orta borgan sari oraliq amallar odatda hayolda bajariladi va shuning uchun ana shunga o‘xshash misollarda differensiallashning oxirgi natijasigina yoziladi.

644. 1) $f(x)=-x^2+9x^2+x-1$. $f'(-1)$ ni toping.

2) $f(x)=\frac{1}{4}x^4-\frac{1}{3}x^3+\frac{1}{2}x^2-1$. $f'(3)$ ni toping.

3) $f'(t)=0,5t^3+0,6t^2+0,8t+8$ $f'(1)$ ni toping.

645. $y=\sqrt[3]{x}-\frac{2}{\sqrt{x}}+\frac{2}{x^2}-\frac{1}{5x^3}+4$

Yechilishi. Radikallarni kasr ko‘rsatkichlar bilan almashtiramiz va manfiy ko‘rsatkichlar kiritamiz, so‘ngra (6.1), (6.4), (6.10a) va (6.8) formulalar bo‘yicha differensiallaymiz:

$$y=\sqrt[3]{x}=\frac{2}{\sqrt{x}}+\frac{3}{x^2}-\frac{1}{5x^3}+4=x^{\frac{1}{3}}-2x^{-\frac{1}{2}}+3x^{-2}-\frac{1}{5}x^{-3}+4$$

$$y'=\frac{1}{3}x^{\frac{1}{3}-1}-2\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}+3(-2)x^{-2-1}-\frac{1}{5}(-3)x^{-3-1}=$$

$$=\frac{1}{3}x^{-\frac{2}{3}}+x^{-\frac{2}{3}}-6x^3+\frac{3}{5}x^{-1}=\frac{1}{3\sqrt[3]{x^2}}+\frac{1}{x\sqrt{x}}-\frac{6}{x^3}+\frac{3}{5x^4}$$

646. 1) $y=-3x^5+15x^4-2x^3+x^1+2$

2) $y=4x^{\frac{3}{4}}+3x^{\frac{2}{3}}+4x^{\frac{1}{2}}+3x;$

3) $y=\sqrt[4]{x^3}-\frac{3}{\sqrt[3]{x}}+\frac{2}{x^2}+\frac{1}{x}+8$

4) $y=2\sqrt{x}+\frac{3}{2\sqrt[3]{x^2}}-\frac{2}{\sqrt{x}}-\frac{1}{x}+1$.

III. Funksiyalar ko‘paytmasining hosilasi

Funksiyalarning hosilalarini toping.

647. $f(x)=(x^3-1)(x^2+x+1)$.

Yechilishi. 1- usul. (6.2), (6.1), (6.10a), (6.8) va (6.9) formulalarga ko‘ra topamiz:

$$f'(x)=(x^3-1)'(x^2+x+1)+(x^2+x+1)'(x^3-1)$$

$$f'(x)=3x^2(x^2+x+1)+(2x^2+1)'(x^3-1)$$

$$f'(x)=3x^2(x^2+x+1)+(2x+1)(x-1)(x^2+x+1)=$$
$$=(x^2+x+1)\left[3x^2+(2x+1)(x-1)\right]=$$
$$=(x^2+x+1)(3x^2-2x+x-1)=(x^2+x+1)(5x^2-x-1)$$

yoki bu uchhadlarni ko‘paytirib,

$$f'(x)=5x^4+4x^3+3x^2-2x-1)$$

ni hosil qilamiz.

2-usul. Bu ko‘paytuvchilarning ko‘paytmasini topamiz:

$$f(x)=(x^3-1)(x^2+x+1)+x^5+x^4-x^3-x^2-1$$

(6.1), (6.10a), (6.9) va (6.8) formulalarga ko‘ra hosilani topamiz.

$$f'(x)=(x^5)'(x^4)'+(x^3)'-(x^2)-(x)'-1'=5x^4+4x^3+3x^2-2x-1)$$

Yana o‘sha natija hosil qilindi.

648. 1) $f(x)=(2x+1)'(x^2+3x-1);$

2) $f(x)=(3x^2+1)\times(2x^2+3)$,

3) $f(x)=(x^3+x^2+x+1)x-1)$.

IV. Bo‘linmaning hosilasi

Funksiyalarning hosilalarini toping.

649. $y = \dfrac{x^2+1}{x^2-1}$.

Yechilishi . (6.5), (6.1), (6.10a) va (6 .8) formulalarga ko‘ra topamiz:

$$y' = \frac{\left(x^2+1\right)'(x^2-1)-(x^2-1)'(x^2+1)}{(x^2-1)^2}$$

$$y' = \frac{2x(x^2-1)-2x(x^2+1)}{(x^2-1)^2};$$

$$y' = \frac{2x(x^2-1-x^2-1)}{(x^2-1)^2} = \frac{2x(-2)}{(x^2-1)^2} = -\frac{4x}{(x^2-1)^2}$$

650. 1) $y = \dfrac{x-a}{x+a}$; 2) $y = \dfrac{x^2}{2-x^2}$;

3) $y = \dfrac{x^2-x+1}{x^2+x+1}$; 4) $y = \dfrac{x^2-x+1}{x^2+1}$.

V.Funksiyalarning hosilalarini $\left(u^n\right)' = nu^{n-1}u'$ **(6.10) va** $\left(\dfrac{1}{u}\right)' = -\dfrac{1}{u^2}\cdot u'$ **(6.11) formulalardan foydalanib topish**

Funksiyalarning hosilalarnni toping.

651. $y = (x^2-5x+8)^6$

Yechilishi. $u = x^2-5x+8$ $y = u^6$ ni hosil qilamiz. (6.10) formulaga ko‘ra:

$$y' = 6u^5u' = 6(x^2-5x+8)^5(x^2-5x+8)' = 6(x^2-5x+8)^5(2x-5).$$

Bunday mufassal yozuv differensiallash texnikasini o‘zlashtirish protsessidagina yoziladi. Malaka hosil qilingandan so‘ng oraliq hisoblashlar xayolda bajariladi.

Shuni esda tutish kerakki, darajaning hosilasi ko‘rsatkichni asosning bitta birlikka kamaytirilgan darajasiga va asosning hosilasiga ko‘paytmasiga teng.

652. 1) $y=(x^2-2x^2+5)^5$ 2) $f(x)=(x^3-1)^6$;

3) $f(x)=(ax^2-bx+c)^n$; 4) $y=(r^2-x^2)^4$

653. $y=\dfrac{1}{(x^2-1)^4}$.

Yechilishi. 1-usul. (6.11) va (6.10) formulalarni ketma-ket tatbiq qilib topamiz:

$$y'=-\frac{1}{\left[(x^2-1)^4\right]^2}\left[(x^2-1)^4\right];$$

$$y'=-\frac{1}{(x^2-1)^{4=8}}\cdot 4(x^2-1)^3(x^2-1);$$

$$y'=-\frac{1}{(x^2-1)^4}\cdot 4(x^2-1)^3\cdot 2x=-\frac{8x(x^2-1)^3}{(x^2-1)^8}=-\frac{8}{(x^2-1)^5}.$$

2-usul. Manfiy ko‘rsatkich kiritamiz va (6.10) formulaii tatbiq qilamiz:

$$y=(x^2-1)^4;$$

$$y'=-4(x^2-1)^{-4-1}(x^2-1)'=-4(x^2-1)^{-5}\cdot 2x=-\frac{8}{(x^2-1)^5}$$

Yana o‘sha natijani hosil kildik.

654. 1) $y=\dfrac{1}{(1-x^3)^5}$; 2) $y=\dfrac{1}{(ax-b)^n}$

655. $y=\dfrac{\left(x^3-1\right)^4}{\left(x^2+1\right)^3}.$

Yechilishi. Bo‘linmani differensiallash qoidasini, so‘ngra murakkab funksiyaning hosilasi formulasini tatbiq qilib topamiz:

$$y'=\frac{\left[\left(x^3-1\right)^4\right]'\left(x^2+1\right)^3-\left[\left(x^2+1\right)^3\right]'\left(x^3-1\right)^4}{\left[\left(x^2+1\right)^3\right]^2}=$$

$$=\frac{4\left(x^3-1\right)^3\cdot 3x^2\cdot\left(x^2+1\right)^3-3\left(x^2+1\right)^2\cdot 2x\cdot\left(x^3-1\right)^4}{\left[\left(x^2+1\right)^3\right]^2}=$$

$$=\frac{2\cdot 3x\left(x^3-1\right)^3\left(x^2+1\right)^2\left[2x\left(x^2+1\right)-\left(x^3-1\right)\right]}{\left(x^2+1\right)^6}=$$

$$=\frac{6x\left(x^3-1\right)^3\left(x^2+1\right)^2\left(2x^3+2x-x^3+1\right)}{\left(x^2+1\right)^6}=$$

$$=\frac{6x\left(x^3-1\right)^3\left(x^2+1\right)^2\left(x^3+2x+1\right)}{\left(x^2+1\right)^6}=$$

$$=\frac{6x\left(x^3-1\right)^3\left(x^3+2x+1\right)}{\left(x^2+1\right)^4}.$$

656. 1) $y=\dfrac{\left(x^4+1\right)^3}{\left(x^3+1\right)^2}$; 2) $y=\left(\dfrac{a+x}{a-x}\right)^n.$

VI. Funksiyalarning hosilasini $\left(\sqrt{u}\right)'=\dfrac{1}{2\sqrt{u}}u'$ (6.12) formuladan foydalanib topish

Funksiyalarning xosilalarini toping:

657. $f(x)=\sqrt{4-x^2}$.

Yechilishi. $u=4-x^2$ deb, $f(x)=\sqrt{u}$ ni hosil qilamiz. (6.12) formulaga ko‘ra topamiz:

$$f'(x)=\frac{1}{2\sqrt{4-x^2}}(4-x^2)'=\frac{-2x}{2\sqrt{4-x^2}}=-\frac{x}{\sqrt{4-x^2}} .$$

Biror funksiyadan olingan kvadrat ildizning hosilasi birni ana shu funksiyadan olingan ildizning ikkilanganiga bo‘linganini ildiz ostidagi ifodaning hosilasiga ko‘paytirilganiga teng ekanligini esda saqlash kerak.

658. 1) $f(x)=\sqrt{x^2-4x+6}$; 2) $f(t)=\sqrt{t^2}-t+1$. $f'(2)$ ni hisoblang;

3) $y==\sqrt{r^2-x^2}$; 4) $y-\frac{a}{b}\sqrt{a^2-x^2}$

5) $y=\frac{a}{b}\sqrt{x^2-a^2}$ 6) $y=\sqrt{2px}$

659. $y=(x^2+6)\sqrt{x^2-3}$.

Yechilishi. Ko‘paytmaning hosilasi formulasiga ko‘ra topamiz:

$$y'=(x^2+6)'\sqrt{x^2-3}+(\sqrt{x^2-3})'(x^2+6) .$$

Har qaysi qo‘shiluvchining hosilasini topamiz va soddalashtiramiz:

660. 1) $y=x\sqrt{x^2-1}$; 2) $s=t^2\sqrt{t^2-1}$;

3) $s=(t^2+1)\sqrt{t^2-1}$; 4) $y=(2x-1)^2\sqrt{1-2x}$.

661. $y=\frac{1}{\sqrt{x^4-1}}$.

Yechilishi. *1-usul*. (6.11) va (6.12) formulalarni birin-ketin tatbiq qilib topamiz:

$$y'=-\frac{1}{\left(\sqrt{x^4-1}\right)^2}\left(\sqrt{x^4-1}\right)';$$

$$y' = -\frac{1}{x^4-1} \cdot \frac{1}{2\sqrt{x^4-1}} \left(x^4-1\right)' =$$

$$= -\frac{1}{x^4-1} \cdot \frac{1}{2\sqrt{x^4-1}} \cdot 4x^3.$$

Endi quyidagicha o‘zgartiramiz:

$$y' = -\frac{1}{x^4-1} \cdot \frac{2x^3}{\sqrt{x^4-1}} = -\frac{2x^3\sqrt{x^4-1}}{\left(x^4-1\right)^2}.$$

2-usul. Ildizni kasr ko‘rsatkich bilan almashtiramiz va hosilani (6.10) formula bo‘yicha topamiz:

$$y = \frac{1}{\sqrt{x^4-1}} = \frac{1}{\left(x^4-1\right)^{\frac{1}{2}}} = \left(x^4-1\right)^{-\frac{1}{2}};$$

$$y' = -\frac{1}{2}\left(x^4-1\right)^{-\frac{3}{2}}\left(x^4-1\right)' = -\frac{1}{2}\left(x^4-1\right)^{-\frac{3}{2}} \cdot 4x^3 =$$

$$= -\frac{2x^3}{\left(x^4-1\right)^{\frac{3}{2}}} = -\frac{2x^3}{\sqrt{\left(x^4-1\right)^3}} =$$

$$= -\frac{2x^3}{\left(x^4-1\right)\sqrt{x^4-1}} = -\frac{2x^3\sqrt{x^4-1}}{\left(x^4-1\right)^2}.$$

662. 1) $y = \frac{1}{\sqrt{ax+b}}$; 2) $y = \frac{1}{\sqrt{x^2+1}}$;

3) $y = \frac{1}{\sqrt{1-x^4}}$; 4) $y = \frac{1}{\sqrt{3x}} - \sqrt{3x}$.

663. $y = \frac{1+2x}{\sqrt{1-2x}}$.

Yechilishi. Bo‘linmaning hosilasi formulasiga ko‘ra:

$$y' = \frac{(1+2x)'\sqrt{1-2x} - \left(\sqrt{1-2x}\right)'(1+2x)}{\left(\sqrt{1-2x}\right)^2}.$$

Hosilalarni topamiz va almashtirishlar bajaramiz:

$$y' = \frac{2\sqrt{1-2x} - \dfrac{-2}{2\sqrt{1-2x}}(1+2x)}{1-2x} =$$

$$= \frac{2\sqrt{1-2x} + \dfrac{1+2x}{\sqrt{1-2x}}}{1-2x} = \frac{2\left(1-2x\right)+1+2x}{\left(1-2x\right)\sqrt{1-2x}} =$$

$$= \frac{2-4x+1+2x}{\left(1-2x\right)\sqrt{1-2x}} = \frac{3-2x}{\left(1-2x\right)\sqrt{1-2x}} = \frac{\left(3-2x\right)\sqrt{1-2x}}{\left(1-2x\right)^2}.$$

664. 1) $y = \dfrac{3x}{\sqrt{x^2-1}}$; 2) $y = \dfrac{x}{\sqrt{x^2+4}}$;

3) $y = \dfrac{\sqrt{9+x^2}}{x}$; 4) $y' = \sqrt[3]{(x^3+1)^2}$.

665. $y = \sqrt[3]{(x^3+1)^2}$.

Yechilishi. Kub ildizni kasr ko‘rsatkich bilan almashtiramiz va (6.10) formula buyicha darajaning hosilasini topamiz:

$$y = \sqrt[3]{(x^3+1)^2} = (x^3+1)^{\frac{2}{3}};$$

$$y' = \frac{2}{3}(x^3+1)^{-\frac{1}{3}}(x^3+1)' = \frac{2}{3}(x^3+1)^{-\frac{1}{3}} \cdot 3x^2 =$$

$$= \frac{2x^2}{(x^3+1)^{\frac{1}{3}}} = \frac{2x^2}{\sqrt[3]{x^3+1}}.$$

666. 1) $y = \sqrt[3]{x^3-1}$; 2) $y = \sqrt[4]{\left(ax+b\right)^3}$;

3) $y = \sqrt{\left(2x-1\right)^3}$; 4) $f(t) = \sqrt[3]{t^2+t-1}$. $f'(1)$ ni hisoblang.

34- §. Hosilaning fizik tatbiqlari

Nuqta to‘g‘ri chiziqli harakat qilayotganda uning berilgan $t = t_1$ momentdagi v tezligi s yuldan t vaqt bo’yicha olingan va berilgan $t = t_1$ moment uchun hisoblangan $\frac{ds}{dt}$ hisilaga teng.

Nuqtaning berilgan $t = t_1$ momentdagi a tezlanishi v tezlikdan t vaqt bo‘yicha olingan va berilgan $t = t_1$ moment uchun hisoblangan $\frac{ds}{dt}$ hosilaga teng.

Bu paragrafdagi masalalarda s yul metrlarda (m), t vaqt sekundlarda (sek), v tezlik sekundiga metr hisobida (m/sek) va a tezlanish sekundning kvadratiga metr hisobida (m/sek^2) ifodalangan.

667. Nuqta $s = 2t^3 + t^2 - 4$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilmoqda. $t = 4$ sek momentdagi tezlik va tezlanishni toping.

Yechilishi. 1. Nuqtaning istalgan t vaqtdagi harakat tezligini topamiz:

$$v = \frac{ds}{dt} = 6t^2 + 2t.$$

2. Nuqtaning $t = 4$ sek momentdagi harakat tezligini topamiz:

$$v\big|_{t=4} = 6 \cdot 4^2 + 2 \cdot 4 = 104 \text{ m/sek.}$$

3. Nuqta harakatining istalgan t vaqtdagi tezlanishini topamiz:

$$a = \frac{dv}{dt} = 12t + 2.$$

4. Nuqtaning $t = 4$ sek momentdagi harakat tezlanishini topamiz:

$$a\big|_{t=4} = 12 \cdot 4 + 2 = 50 \text{ m/sek}^2.$$

668. Agar to‘g‘ri chiziqli harakat qilayotgan nuqtaning harakati 1) $s = t^3 + 5t^2 + 4,\ t = 2;$ 2) $s = \sqrt{t},\ t = 1;$ 2) $s = t^2 + 11t + 30,\ t = 3$ tenglama bilan berilgan bo‘lsa, vaqtning ko‘rsatilgan momentlarida bu nuqtaning tezlik va tezlaninshni toping.

669. Agar to‘g‘ri chiziqli harakat qilayotgan nuqtaning tezligi 1) $v = t^2 + t - 1,\ t = 3;$ 2) $v = t^2 + 5t + 1,\ t = 3$ tenglama bilan berilgan bo‘lsa, nuqtaning ko‘rsatilgan momentdagi tezlanishini toping.

670. Nuqta $s=6t-t^2$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilmoqda. Vaqtning qaysi momentida nuqtaning tezligi nolga teng bo‘ladi?

Yechilishi. 1. Nuqtaning t vaqtning istalgan momentidagi harakat tezligini topamiz:

$$v=\frac{ds}{dt}=6-2t.$$

2. $v=0$ deb, t ni topamiz: $6-2t=0,\ t=3$ sek.

Uchinchi sekund oxirida nuqtaning tezligi nolga teng bo‘ladi.

671. Nuqta $s=t^2-8t+4$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilmoqda. Vaqtning qaysi momentida nuqtaning tezligi nolga teng bo‘ladi?

672. Tormozlanish paytida maxovik t sek davomida $\varphi=3+8t-t^2$ burchakka buriladi. 1) vaqtning t=3 sek momentida maxovik aylanishining burchak tezligini toping;

2) t momentdagi burchak tezlanishni toping; 3) maxovik to‘xtaydigan vaqt momenti t ni toping.

Yechilishi . 1. φ burchak tezlik deb, φ burchakning t vaqt davomida o‘zgarish tezligiga aytiladi. Burchak tezlik burilish burchagi φ dan t vaqt bo‘yicha olingan hosilaga teng:

$$\varphi=\frac{d\varphi}{dt}=8-2t\ .$$

$t=3$sekda burchak tezlikni topamiz:

$$\omega_{t=3}=8-2\cdot3=2 \text{ rad/sek}$$

2. ε burchak tezlanish ω burchak tezlikdan t vaqt bo‘yicha olingan hosilaga teng:

$$\varepsilon=\frac{d\omega}{dt}=-2 \text{ rad/sek}^2$$

3. $\omega=0$ deb t ni topamiz: $8-2t=0, t=4$ sek.

To‘rtinchi sekundning oxirida burchak tezlik nolga teng bo‘ladi.

673. Jism o‘q atrofida $\omega=10t-t^2$ qonun bo‘yicha aylanmoada. 1) aylanishning $t=2$ sek momentdagi burchak tezligini toping; 2) t momentdagi burchak tezlanishni toping;

3) aylanish tugaydigan momentni toping.

674. Jism temperaturasi T ning t vaqtga bog‘liq xolda o‘zgarishi $T = 0,2t^2$ tenglama bilan berilgan. Vaqtnnng $t = 10$sek momentida bu jism qanday tezlik bilan qiziydi?

Yechilishi. Jism qizdirilganda uning T temperaturasi t vaqtga bog‘liq holda o‘zgaradi, ya’ni T vaqtning funksiyasidnr: $T = f(t)$ Jismning qizish tezligi T temperaturaning t vaqt bo‘yicha hosilasi $\frac{dT}{dt}$ dan iboratdir:

$$\frac{dT}{dt} = 0,4t; \left(\frac{dT}{dt}\right)_{t=10сек} = 0,4 \cdot 10 = 4.$$

Vaqtning $t = 10$sek momentida jism sekundiga to‘rt gradus tezlik bilan qiziydi.

675. Jismning T temperaturasi t vaqtga bog‘liq holda $T = 0,5t^2 - 2t$ qonun bo‘yicha o‘zgaradi. Vaqtning $t = 5$ sek momentida bu jism qanday tezlik bilan qiziydi?

676. Massasi 10 kg bo‘lgan jism $s = 3t^2 + t + 4$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilmoqda. Jismning harakat boshlangandan 4 sek o‘tgandan keyingi kinetik energiyasi $\left(\frac{mv^2}{2}\right)$ ni toping.

Yechilishi. 1. Jism harakatining vaqtning t momentidagi tezligini topamiz:

$$v = \frac{ds}{dt} = 6t + 1.$$

2. Jismning $t = 4$ sek dagi tezligini hisoblaymiz:

$$v_{t=4сек} = 6 \cdot 4 + 1 = 25 м/сек.$$

3. 4 sek oxirida jismning kinetik energiyasini topamiz:

$$\frac{mv^2}{2} = \frac{10 \cdot 25^2}{2} = 3125(Ж)$$

677. Massasi 100 kg bo‘lgan jism $s = 5t^2 - 2$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilmoqda. Jismning harakat boshlangandan 2 sek o‘tgandagi kinetik energiyasi $\left(\frac{mv^2}{2}\right)$ ni toping.

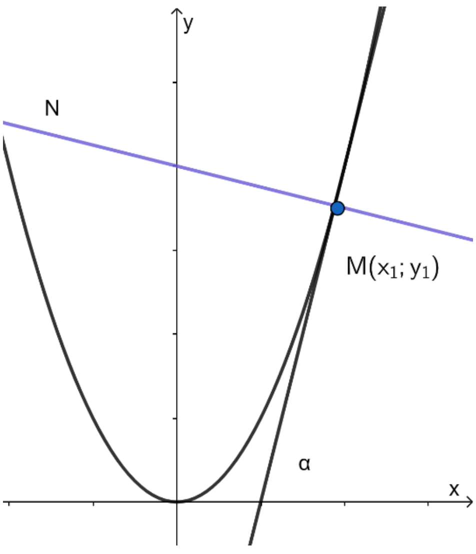
678. Tok kuchi I vaqt t ga bog‘lik, holda $I = 0,4t^2$ (I amperlarda, t sekundlarda) qonun bo‘yicha o‘zgaradi. Sakkizinchi sekund oxirida tok kuchi o‘zgarishining tezligini toping.

Yechilishi . Tok kuchi o‘zgarishining tezligi I tok dan t vaqt bo‘yicha olingan hosilaga teng:

$$\frac{dI}{dt} = 0,8t\left(\frac{dI}{dt}\right)_{t=8сек} = 0,8 \cdot 8 = 6,4A/сек$$

679. I tok kuchining t vaqtga bog‘liq holda o‘zgarishi $I = 2t^2 - 5t$ tenglama bilan berilgan (I amper hisobida, t sekund hisobida). 10 sek oxirida tok kuchining o‘zgarish tezligini toping.

35- §. Hosilaning geometriyaga tatbiqi



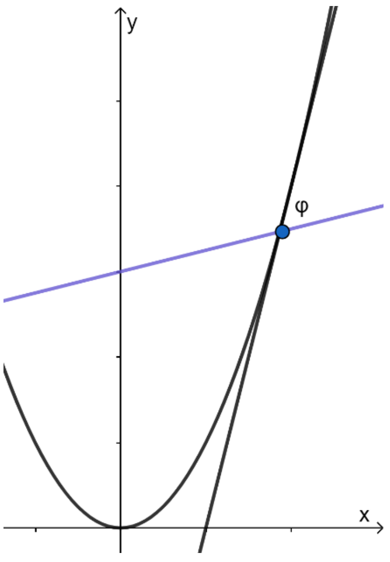
71-rasm

Tenglamasi $y = f(x)$ bo‘lgan egri chiziqda $M = (x_1, y_1)$ nuqta berilgan bo‘lib, uning uchun $y_1 = f(x_1)$ bo‘lsin (71-rasm).

$y = f(x)$ funksiyaning $x = x_1$ dagi hosilasi berilgan egri chiziqqa uning $x = x_1$ abssissali nuqtasiga o‘tkazilgan urinmaning burchak koeffitsienti $k_{x=x_1} = y'_{x=x_1} = f'(x_1) = tg\alpha$ ga teng. Bu yerda $\alpha -$ egri chiziqqa uning $M = (x_1, y_1)$ nuqtasida o‘tkazilgan urinmaning Ox o‘qning musbat yo‘nalishi bilan hosil qilgan burchagi.

Berilgan $y = f(x)$ egri chiziqning $M = (x_1, y_1)$ nuqtasiga o‘tkazilgan urinmaning tenglamasi quyidagi ko‘rinishga ega:

$$y - y_1 = f'(x_1)(x - x_1) \qquad (6.13)$$



72-rasm

Bu tenglama berilgan $M = (x_1, y_1)$ nuqtadan o‘tuvchi va berilgan k burchak koeffitsientli to‘g‘ri chiziqning tenglamasidan olingan, bu yerda

$$k_{x=x_1} = f'(x_1).$$

$y = f(x)$ egri chiziqqa uning $M(x_1, y_1)$ nuqtasida o‘tkazilgan *normal* deb, urinmaga uning egri chiziq bilan urinish nuqtasi $M(x_1, y_1)$ da o‘tkazilgan perpendikulyarga aytiladi.

MN normalning (71-rasm) tenglamasi quyidagicha bo‘ladi:

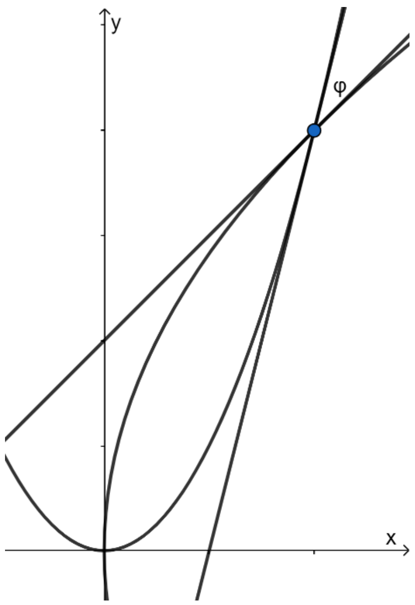
$$y = y_1 = -\frac{1}{f'(x_1)}(x - x_1) \qquad (6.14)$$

(6.14) tenglama (6.13) to‘g‘ri chiziqqa perpendikulyar to‘g‘ri chiziqning tenglamasidir; ikkita to‘g‘ri chiziqning perpendikulyarlik shartidan

$$k_2 = -\frac{1}{k_1}$$

ga egamiz, bu yerda $k_1 f'(x_1)$ va

$$k_2 = -\frac{1}{f'(x_1)}.$$



73-rasm

Egri chiziqning uning har bir nuqtasidagi yo‘nalishi unga ana shu nuqtada o‘tkazilgan urinmaning yo‘nalishi bilan aniqlanadi, shu sababli egri chiziqning berilgan nuqtasidagi og‘ish burchagini topish uchun nuqtaga o‘tkazilagan urinma bilan Ox o‘q orasidagi burchakni hisoblash kerak.

O‘zaro kesishuvchi to‘g‘ri chiziq va egri chiziq orasidagi burchak deb bu to‘g‘ri chiziq va ularning kesishgan nuqtasidan o‘tkazilgan urinma orasidagi burchakka aytiladi (72-rasm).

Kesishuvchi ikkita egri chiziq orasidagi burchak deb bu egri chiziqlarga ularning kesishish nuqasi orqali o‘tkazilgan urinmalar orasidagi burchakka aytiladi (73-rasm).

I. Berilgan egri chiziqqa berilgan nuqtada o‘tkazilgan urinmaning burchak koeffitsentini hisoblash

680. $y = 2x^2$ parbolaga abssissasi birga teng bo‘lgan nuqtada o‘tkazilgan urinmaning burchak koeffitsientini toping.

Yechilishi. $y = 2x^2$ parabolaga o‘tkazilgan urinmaning burchak koeffitsientini topish uchun $y = 2x^2$ funksiyaning hosilasini topamiz va hosilani $x = 1$dagi qiymatini hisoblaymiz:

$$y' = (2x^2)' = 4x; \quad y'_{x=1} = 4 \cdot 1 = 4; \quad k = tg\alpha = y'_{x=1} = 4;$$

681. $y = -x^2 + x$ parabolaga $x = -2$ nuqtada o‘tkazilgan urinmaning burchak koeffitsientini toping.

682. $y = x^2 - 3x + 2$parobolaga $x = 3$nuqtada o‘tkazilgan urinmaning burchak koeffitsientini toping.

II. Berilgan egri chiziqning berilgan nuqtadagi og‘ish burchagini (urinmaning Ox o‘qqa og‘ish burchagini) hisoblash

683. $y = x^2 - x + 1$ parabolaning $x = -1$ Ox o‘qqa og‘ish burchagini toping.

Yechilishi. Egri chiziqning uning berilgan nuqtasidagi og‘ish burchagini topish uchun shu nuqtaga o‘tkazilgan urinmaning Ox o‘q bilan hosil qilgan burchagini topamiz.

1. $y = x^2 - x + 1$ funksiyaning $x = -1$ nuqtadagi hosilasini topamiz:

$$y' = 2x - 1; \quad y'_{x=-1} = 2(-1) - 1 = -3$$

2. Urinmaning Ox o‘qqa og‘ish burchagini topamiz:

$$k = tg\alpha = y'_{x=-1} = -3; tg\alpha = -3; \quad \alpha = 108^\circ 26'$$

684. $y = x^2 - 2x$ parabolaning $x = 2$ dagi og‘ish burchagini toping.

685. $y = x^3$ egri chiziqqa $x = -2$ nuqtada o‘tkazilgan urinmaning Ox o‘qqa og‘ish burchagini toping.

III.Berilgan egri chiziqqa berilgan nuqtada o‘tkazilgan urinma va normalning tenglamalarini tuzish

686. $y = 3x^2 - x$ parabolaga $x = -1$ nuqtada urinma va normal o‘tkazilgan. Ularning tenglamalarini tuzing.

Yechilishi. Urinmaning tenglamasini tuzish uchun u o‘tadigan M nuqtaning ordinatasini va urinmaning burchak koeffitsientini topamiz.

1. Urinish nuqtasining ordinatasini parabola tenglamasiga $x = -1$ qiymatini qo‘yib topamiz.

$$y_{x=1} = 3 \cdot (-1)^2 - (-1) = 4; \qquad M = (-1, 4)$$

2. k burchak koeffitsientni hisoblaymiz:

$$k_{x=-1} = y'_{x=-1} = (3x^2 - x)'_x = -1 = (6x - 1)_{x=-1} = 6 \cdot (-1) - 1 = -7$$

3. (6.13) tenglamaga $M(-1; 4)$ nuqtaning koordinatalarini va $k = -7$ qiymatni qo‘yib, urinmaning tenglamasini tuzamiz:

$$y - 4 = -7 \cdot (x + 1), \quad 7x + y + 3 = 0$$

4. (6.14) tenglamaga normal o‘tadigan $M(-1;4)$ urinish nuqtasining koordinatalarini va burchak koeffitsientningqiymati $k_{x=-1}=-7$ ni qo‘yib, normalning tenglamasini tuzamiz:

$$y-4=-\frac{1}{-7}(x+1) \quad \text{yoki} \quad x-7y+29=0$$

687. $y=x^2-7x+10$ parabolaga $x=4$ nuqtada o‘tkazilgan urinma va normalning tenglamalarini tuzing.

688. $y=2x^3$ egri chiziqqa $x=-1$ nuqtada o‘tkazilgan urinma va normalning tenglamalarini tuzing.

IV. Berilgan egri chiziqning biror nuqtasidan o‘tkazilgan urinma Ox o‘q bilan berilgan burchak tashkil etadi. Ana shu nuqtaning koordinatalarini hisoblash

689. Berilgan $y=x^2-x-12$ parabolada shunday nuqtaning koordinatalarini topingki, parabolaga bu nuqtada o‘tkazilgan urinma Ox o‘q bilan 45° li burchak tashkil etsin.

Yechilishi. 1. Izlanayotgan nuqtada o‘tkazilgan urinmaning Ox o‘qqa og‘ish burchagini topamiz:

$$tg\alpha = y' = (x^2-x-12)' = 2x-1$$

2. α burchak shartga ko‘ra 45^{o} ga teng, demak $tg45^\circ = 2x-1$ yoki $1=2x-1$ bu yerdan $x=1$.

3. Izlanayotgan nuqtaning ordinatasini topamiz:

$$y_{x=1}=(x^2-x-12)_{x=1}=1^2-1-12=-12 \quad M(1;-12).$$

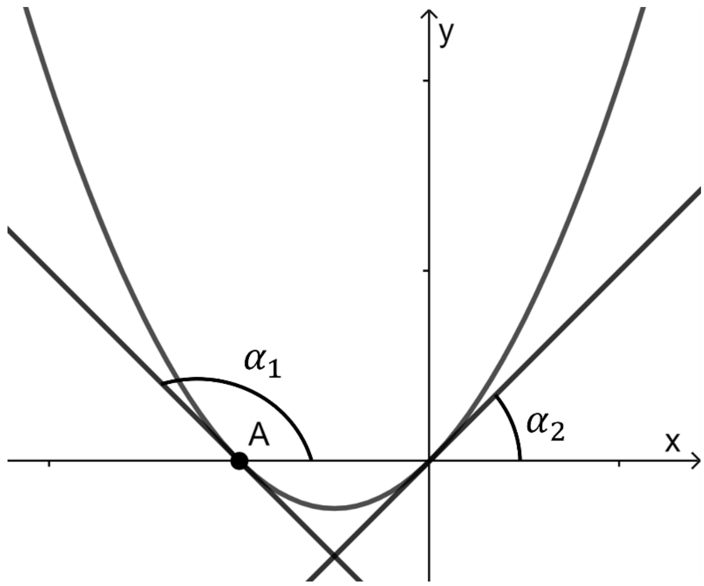
690. Berilgan $y=x^2+3x-10$ parabolada shunday nuqtaning koordinatalarini toping-ki, parabolaga bu nuqtada o‘tkazilgan urinma Ox o‘q bilan 135°li burchak tashkil etsin.

V. Berilgan egri chiziqning Ox bilan kesishishidan hosil bo‘lgan burchakni hisoblash

691. Ox o‘q $y=x^2+x$ parabolani qanday burchak ostida kesib o‘tishini aniqlang.

Yechilishi. 1. $y = x^2 + x$

parabolaning Ox o‘q bilan kesishish nuqtalarining koordinatalarini topamiz. Buning uchun quyidagi sistemani echamiz:



74-rasm

$$\begin{cases} y = x^2 + x \\ y = 0 \end{cases}$$

Bu sistemaning ildizlari: $x_1 = -1; x_2 = 0$. Parabola Ox o‘qni $A(-1;0)$ va $O(0;0)$ nuqtalarda kesib o‘tadi (74- rasm).

2. Parabolaga $A(-1;0)$ va $O(0;0)$ nuqtalarda o‘tkazilgan urinmalarning burchak koeffitsientlarini topamiz:

$$y' = (x^2 + x) = 2x + 1; k_{x=-1} = 2 \cdot (-1) + 1 = -1; k_{x=0} = 2 \cdot 0 + 1 = 1$$

4. Parabolaning Ox o‘q bilan kesishgan nuqtalarida urinmalar Ox o‘q bilan hosil qilgan α_1 va α_2 burchaklarni hisoblaymiz:

$$tg\alpha_1 = -1, \alpha_1 = 135^\circ; tg\alpha_2 = 1, \alpha_2 = 45^\circ.$$

692. $y = x^2 + 2x - 8$ parabola Ox o‘qni qanday burchak ostida kesib o‘tishini aniqlang.

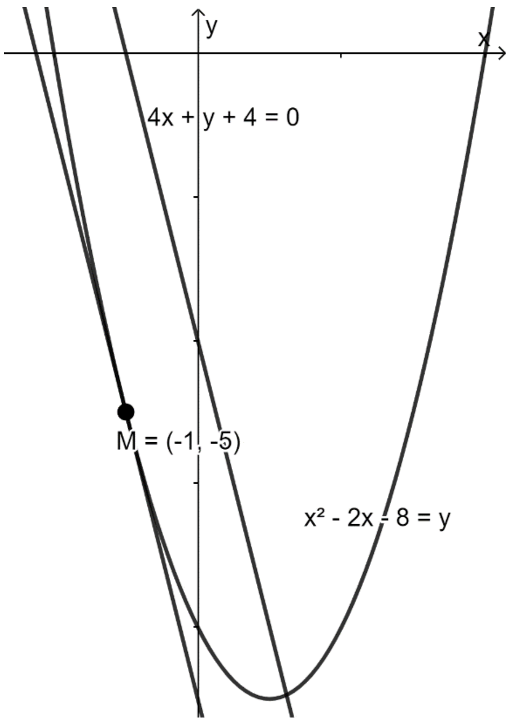
VI. Parabolaning biror nuqtasidan o‘tkazilgan urinma berilgan to‘g‘ri chiziqqa parallel (perpendikulyar). Ana shu nuqtani topish

693. $y = x^2 - 2x - 8$ parabolada shunday M nuqtani topingki, bu nuqtadan parabolaga o‘tkazilgan urinma $2x + y + 4 = 0$ to‘g‘ri chiziqqa parallel bo‘lsin.

Yechilishi. $y = x^2 - 2x - 8$ parabolaga o‘tkazilgan urinmaning burchak koeffitsientini topamiz:

$$k = y'(x^2 - 2x - 8)' = 2x - 2$$

$4x + y + 4 = 0$ to‘g‘ri chiziqning burchak koeffitstentini topamiz:



75-rasm

$$y = 4x - 4; k = -4$$

3. Parabolaga o‘tkazilgan urinma va $4x - y + 4 = 0$ to‘g‘ri chiziq o‘zaro parallel, demak, ularning burchak koeffitsientlari teng: $2x - 2 = -4$ bu yerdan urinish nuqtasining abssissasi: $x = -1$. Urinish nuqtasi M ning ordinatasini parabola tenglamasidan topamiz:

$$y_{x=-1} = (-1)^2 - 2\cdot(-1) - 8 = -5$$

$$M(-1; -5)$$

(75-rasm)

694. $y = x^2 + 7x - 10$ parabolada shunday nuqtani topingki, bu nuqtadan parabolaga o‘tkazilgan urinma $x + y - 1 = 0$ to‘g‘ri chiziqqa parallel bo‘lsin.

695. $y = -x^2 + 4$ parabolaning qaysi nuqtasidan o‘tkazilgan urinma $x - 2y + 2 = 0$ to‘g‘ri chiziqqa perpendikulyar bo‘ladi?

VII. Berilgan to‘g‘ri chiziq va berilgan egri chiziq kesishganda hosil bo‘ladigan burchaklarni hisoblash

696. $x^2 - 4y = 0$ parabola va $x - 2y + 4 = 0$ to‘g‘ri chiziq kesishganda hosil bo‘ladigan o‘tkir burchaklarni hisoblang.

Yechilishi. 1. Parabola va to‘g‘ri chiziqning kesishish nuqtalarini topamiz; buning uchun

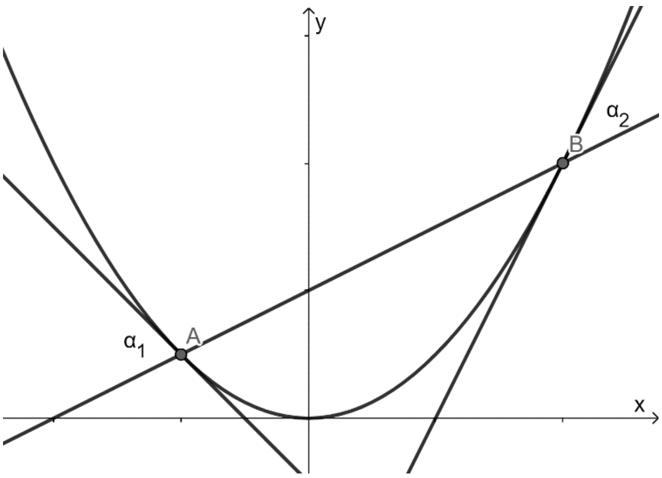
$$\begin{cases} x^2 - 4y = 0 \\ x - 2y + 4 = 0 \end{cases}$$

tenglamalar sistemasini echamiz.

Bu sistemaning ildizlari: $x_1 = -2$, $y_1 = 1$ va $x_2 = 4$, $y_2 = 4$ demak, parabola va to‘g‘ri chiziq $A(-2;1)$ va $B(4;4)$ nuqtalarda kesishadi (76-rasm).

2. To‘g‘ri chiziqning burchak koeffitsientini topamiz:

$$x-2y+4=0 \quad y=\frac{1}{2}x+2 \quad k=\frac{1}{2}$$



76-rasm

3. $A(-2;1)$ va $B(4;4)$ nuqtalardan o‘tkazilgan urinmalarning burchak koeffitsientlarini hisoblaymiz.

Parabolaning $x^2-4y=0$ tenglamasini $y=\frac{1}{4}x^2$ ko‘rinishda qayta yozib olamiz $k=y'=\frac{1}{2}x$

$A(-2;1)$ nuqtada o‘tkazilgan urinmaning burchak koeffitsienti

$$k_{x=-2}=\frac{1}{2}\cdot(-2)=-1;$$

$B(4;4)$ nuqtada o‘tkazilgan urinmaning burchak koeffitsienti

$$k_{x=4}=\frac{1}{2}\cdot 4=2;$$

4. Kesishuvchi to‘g‘ri chiziq va egri chiziq orasidagi burchak to‘g‘ri chiziq bilan uning egri chiziq bilan kesishgan nuqtasidan egri chiziqqa o‘tkazilgan urinma orasidagi burchak deb aniqlanadi.

Shu sababli, A nuqtadagi burchakni burchak koeffitsientlari $k_1=-1;$ (A nuqtadan o‘tkazilgan urinmaning burchak koeffitsienti) va $k_2=\frac{1}{2}$ (to‘g‘ri chiziqning burchak koeffitsienti) bo‘lgan to‘g‘ri chiziqlar orasidagi burchak kabi topamiz.

$$tg\varphi=\frac{k_2-k_1}{1+k_2k_1}=\frac{\frac{1}{2}-(-1)}{1+\frac{1}{2}\cdot(-1)}=3, \varphi_1=arctg3$$

Mos ravishda B nuqtadagi burchakni ham $k_1 = \frac{1}{2}$ va $k_2 = 2$ burchak koeffitsientlarga ko‘ra topamiz.

$$tg\varphi = \frac{2 - \frac{1}{2}}{1 + 2 \cdot \frac{1}{2}} = \frac{3}{4}, \varphi_2 = arctg\frac{3}{4}$$

697. $y^2 - x = 0$ parobolaning $x + y - 6 = 0$ to‘g‘ri chiziq bilan kesishishidan hosil bo‘lgan o‘tkir burchaklarni hisoblang.

VIII. Berilgan ikkita egri chiziq kesishganda hosil bo‘lgan burchaklarni hisoblash

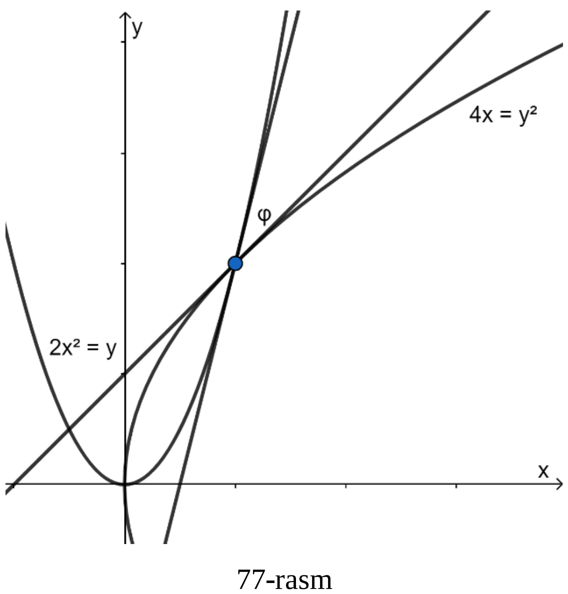
698. $y^2 = 4x$ va $x^2 = \frac{1}{2}y$ parobalaning kesishishidan hosil bo‘lgan o‘tkir burchaklarni hisoblang.

Yechilishi. 1. Parabolalarning kesishi nuqtalarini topamiz, buning uchun

$$\begin{cases} y^2 = 4x \\ x^2 = \frac{1}{2}y \end{cases}$$

tenglamalar sistemasini yechamiz.

Bu sistemaning ildizlari: $x_1 = 0, y_1 = 0$ va $x_2 = 1,\ y_2 = 2$ demak, parabolalar (0;0) va (1;2) nuqtalarda kesishadi (77-rasm).



77-rasm

2. O‘zaro kesishuvchi ikkita egri chiziq orasidagi burchak ularning kesishgan nuqtasi orqali bu egri chiziqlarga o‘tkazilgan urinmalar orasidagi burchak kabi aniqlanadi. SHu sababli egri chiziqlarning kesishgan nuqtasidan o‘tkazilagan urinmalarning burchak koeffitsientlarini topamiz. (0,0) nuqtada parabolalarga urinmalar Ox va Oy o‘qlardan iborat bo‘ladi, binobarin, bu nuqtada pabarolalar to‘g‘ri burchak ostida kesishadi. $y^2 = 4x$ parabolaga o‘tkazilgan urinmanng burchak koeffitsientini topamiz; tenglamani $y = 2\sqrt{x}$ ko‘rinishda qayta yozib olamiz (radikal oldida musbat ishora olamiz, chunki parabolalar birinchi chorakda kesishyapti)

$$k = y' = 2\cdot\frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{x}};$$

$$k_{x=1} = y'_{x=1} = \frac{1}{\sqrt{1}} = 1$$

$x^2 = \frac{1}{2}y$ parabolaga o‘tkazilgan urinmaning burchak koeffitsientini topamiz; parabola tenglamasini $y = 2x^2$ ko‘rinishda qayta yozib olamiz, so‘ngra

$$k = y^1 = 4x; \quad k_{x=1} = y'_{x=1} = 4\cdot 1 = 4$$

3.Urinmalar orasidagi φ burchakni ularning burchak koeffitsientlari $k_1 = 1$va $k_2 = 4$bo‘yicha

$$tg\varphi = \frac{k_2 - k_1}{1 + k_1 k_2}$$

formulaga ko‘ra topamiz;

$$tg\varphi = \frac{4-1}{1+4\cdot 1} = \frac{3}{5} = 0,6$$

$$\varphi = arctg\,0,6 = 30^{\circ}58'$$

699. $y = x^2$va $x = y^2$ parabolalarning kesishishidan hosil bo‘lgan o‘tkir burchaklarni hisoblang.

700. $y^2 = 4x$ va $x^2 = \frac{27}{2}y$ parabolalarning kesishishidan hosil

bo‘lgan o‘tkir burchaklarni hisoblang.

36-§. Aralash masalalar

Funksiyalarning hosilalarini toping.

701. 1) $y = \frac{x+1}{x-1}$; 2) $y = x^2 f(x) = \frac{x^3+1}{x} \cdot f'(1)$ ni toping;

3) $y = \frac{x^3+1}{x^2+1}$ 4) $f(u) = \frac{1}{u^2+3u+2} \cdot f'(0)$ ni toping.

702. 1) $s = \sqrt{t} + \sqrt[3]{t}$; 2) $y = \frac{2}{\sqrt{x}} - \frac{3}{\sqrt[3]{x}}$; 3) $f(x) = \frac{\sqrt{x+1}}{\sqrt{x}} \cdot f(4)$ ni

toping. 4) $f(x) = \frac{4+\sqrt{x}}{4-\sqrt{x}} \cdot f'(4)$ ni toping.

703. 1) $f(t) = (t^m + t^n)^3 \cdot f'(1)$ ni toping. 2) $y = \left(\frac{x-1}{x+1}\right)^2$;

3) $y = \frac{3x^3}{(3x-1)^3}$; 4) $f(x) = (x^2-1)^2\sqrt{x^2+1}.f'(\sqrt{3})$ ni toping.

704. 1) $f(u) = \sqrt{2+\sqrt{2u}}.f'(2)$ ni toping.

2) $f(u) = \sqrt{5x^2+2x+1}.f'(-1)$ ni toping. 3) $y = \sqrt{\frac{1+ax}{1-ax}}$;

4) $f(z) = \frac{\sqrt{4+z^2}}{z} \cdot f'(\sqrt{5})$.

705. 1) $f(x)=\frac{x^3}{\sqrt{6+x^3}}$ $f'(1)$ ni toping. 2) $f(x)=\sqrt{\frac{\sqrt{x}-1}{\sqrt{x}+1}}$.

$f'(4)$ ni toping. 3) $f(x)=\frac{x}{x+\sqrt{1+x^2}}\cdot f'(\sqrt{3})$ ni toping.

4) $z=\frac{u}{1-\sqrt{1-u^2}}$.

706. $y=x^2$ parabola va $x-y+6=0$ to‘g‘ri chiziqlarning kesishishidan hosil bo‘lgan o‘tkir burchaklarni hisoblang.

Yozma ish

I v a r i a n t

707. Funksiyalarning hosilalarini argumentining berilgan qiymatida toping (1 - 3):

1) $f(x)=\frac{2}{x}-\frac{8}{\sqrt{x}}+\frac{6}{\sqrt[3]{x^2}}+2x+6x^2\sqrt{x},\ f'(1).$

2) $f(x)=(x^2-2)\sqrt{x^2+1},\ f'(\sqrt{3}).$

3) $f(z)=\frac{9z}{\sqrt{z^2+1}},\ f'(2\sqrt{2}).$

4. Nuqta $s=2t^3-2t^2-4$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilmoqda (s m hisobida, t sek hisobida). Ikkinchi sekund oxirida nuqtaning tezlanishini topish .

5. $y=x^2-6x+8$ parabolaga abssissasi $x=4$ bo‘lgan nuqtada o‘tkazilgan normalning tenglamasini tuzing.

II v a r i a n t

708. Funksiyalarning hosilalarini argumentning berilgan qiymatida toping (1 - 3):

1). $f(x)=\frac{1}{x^2}+\frac{3}{2\sqrt[3]{x^2}}-\frac{4}{\sqrt{x}}+3x-2x^2\sqrt{x},\ f'(1).$

2) $f(u)=2(u^2+3)\sqrt{u^2-1},\ f'(\sqrt{2}).$

3) $f(x)=1-\frac{x}{\sqrt{x^2+1}}, \quad f'(\sqrt{3})$.

4. Nuqta $s=2t^3-3t^2+4$ (bu yerda s m hisobida, t sek hisobida) qonun bo‘yicha to‘g‘ri chiziqli harakat qilmoqda. Uchinchi sekund oxirida nuqtaning tezlanishini toping.

5. $y=\frac{1}{2}x^2$ va $x=\frac{1}{2}y^2$ parabolalarning kesishishidan hosil bo‘lgan o‘tkir burchakni hisoblang.

37-§. Trigonometrik funksiyalarning hosilalari.

Differensiallash formulalari

$u=c(x)$ shartda		$u=x$ shartda	
$(\sin u)'=\cos u\cdot u'$	(6.15)	$(\sin x)'=\cos x$	(6.15a)
$(\cos u)'=-\sin u\cdot u'$	(6.16)	$(\cos x)'=-\sin x$	(6.16a)
$(tgu)'=\frac{1}{\cos^2 u}u'$	(6.17)	$(tgx)'=\frac{1}{\cos^2 x}$	(6.17a)
$(ctgu)'=-\frac{1}{\sin^2 u}u'$	(6/18)	$(ctgx)'=-\frac{1}{\sin^2 x}$	(6.18a)

1. Sinusning hosilasi

Funksiyalarning hosilalarini toping.

709. $f(x)=\frac{1-\sin x}{1+\sin x}, \quad f'\left(\frac{\pi}{4}\right)$ ni toping.

Yechilishi. (6.5), (6.1), (6.8) va (6.15a) formulalar bo‘yicha topamiz:

$$f'(x)=\frac{(1-\sin x)'(1+\sin x)-(1+\sin x)'(1-\sin x)}{(1+\sin x)^2}=$$

$$=\frac{-\cos x(1+\sin x)-\cos x(1-\sin x)}{(1+\sin x)^2}=-\frac{2\cos x}{(1+\sin x)^2};$$

$$f'\left(\frac{\pi}{4}\right)=-\frac{2\cos\frac{\pi}{4}}{\left(1+\sin\frac{\pi}{4}\right)}=-\frac{\sqrt{2}}{\left(1+\frac{\sqrt{2}}{2}\right)}=8-6\sqrt{2}$$

710. 1). $f(x)=\frac{\sin x-1}{\sin x}$, $f'\left(\frac{\pi}{3}\right)$ ni toping.

2) $y=x^2+\sin x$; 3) $y=x\sin x$.

711. $y=\sin(2x^2+3)$

Yechilishi. $2x^2+3=u$ deb, $y=\sin u$ ni hosil qilamiz. (6.15) formula bo‘yicha

$$y'=\cos u\cdot u'=\cos(2x^2+3)(2x^2+3)'=$$
$$=\cos(2x^2+3)\cdot 4x=4x\cos(2x^2+3).$$

Sinusning hosilasi shu argument kosinusini argument hosilasiga ko‘paytmasiga teng ekanligini esda saqlash kerak.

712. 1) $y=\sin 3x;$ 2) $f(x)=\sin(4x-1);$ 3) $s=\sin t^2$; 4) $f(\theta)=\sin\frac{\theta}{2}$, $f'\left(\frac{\pi}{2}\right)$; 5) $s=\sin\frac{m}{t}$; 6) $y=\sin\sqrt{x}$.

713. $y=\sin^3 mx$.

Yechilishi . $mx=u$ deb, $y=\sin^3 u$ ni hosil qilamiz.

Bu yerda $\sin^3 u$ ifoda $y=(\sin u)^3$ ni bildiradi, ya’ni darajani differensiallash kerak. (6.10) va (6.15) formulalarni birin-ketin tatbiq qilib topamiz:

$$y'=3\sin^2 u(\sin u)';$$
$$y'=3\sin^2 u\cos u\cdot u=3\sin^2 mx\cos mx(mx)';$$
$$y'=3\sin^2 mx\cos mx\cdot m=3m\sin^2 mx\cos mx$$

714. 1) $y=\sin^2 x$; 2) $t=\sin^3 5\varphi^2$; 3) $y=\sin^2\frac{1}{x}$; 4) $y=\sin^3\sqrt{x}$.

715. $y=\dfrac{1}{\sin^2 3x}$.

Yechilishi . I -usul. (6.11), (6.10) va (6.15) formulalarni birin-ketin tatbiq qilib, topamiz:

$$y'=-\frac{1}{\left(\sin^2 3x\right)}\left(\sin^2 3x\right)';$$

$$y'=\frac{1}{\sin^4 3x}\cdot 2\sin 3x(\sin 3x)';$$

$$y'=\frac{1}{\sin^4 3x}\cdot 2\sin 3x\cos 3x\cdot 3-\frac{6\cos 3x}{\sin^3 3x}$$

2-usul. Manfiy ko‘rsatkich kiritamiz:

$$y=-\frac{1}{\sin^2 3x}\left(\sin^2 3x\right)^{-2};$$

(6.10) va (6.15) formulalar bo‘yicha topamiz:

$$y'=-2(\sin 3x)^{-3}(\sin 3x)';$$

$$y'=-2(\sin 3x)^{-3}\cos(3x)';$$

$$y'=-2(\sin 3x)^{-3}\cos 3x\cdot 3=-\frac{6\cos 3x}{\sin^3 3x}$$

716. 1) $y=\dfrac{1}{\sin x}$; 2) $y=\dfrac{1}{\sin 3x}$; 3) $y=\dfrac{1}{\sin(x^3-1)}$; 4) $y=\dfrac{1}{\sin\sqrt{x}}$; 5) $y=\dfrac{1}{\sin^2 x}$; 6) $y=\dfrac{1}{\sin^3 2x}$.

717. $y=\sqrt{\sin 2x}.$

Yechilishi . (6.12) za (6.15) formulalarga ko‘ra topamiz:

$$y'=\frac{1}{2\sqrt{\sin 2x}}(\sin 2x)'; \quad y'=\frac{1}{\sqrt{\sin 2x}}\cos 2x(2x)';$$

$$y'=\frac{1}{2\sqrt{\sin 2x}}\cos 2x\cdot 2=\frac{\cos 2x}{\sqrt{\sin 2x}}=ctg2x\sqrt{\sin 2x}.$$

718. 1) $f(t)=\sqrt{\sin t}$; 2) $y=\sqrt{\sin x^2}$.

719. $y=\sqrt[3]{\sin^5 5x}$.

Yechilishi. Radikalni kasr ko‘rsatkich bilan almashtiramiz:

$$y=\sqrt[3]{\sin^5 5x}=(\sin 5x)^{2/3}$$

(6.10) va (6.15) formulalar bo‘yicha:

$$y'=\frac{2}{3}(\sin 5x)^{-\frac{1}{3}}(\sin 5x)'; \quad y'=\frac{2}{3}(\sin 5x)^{-\frac{1}{3}}\cos 5x(5x)'$$

$$y'=\frac{2}{3}(\sin 5x)^{-\frac{1}{3}}\cos 5x\cdot 5=\frac{10\cos 5x}{3\sqrt[3]{\sin^2 5x}}.$$

720. 1) $y=\sqrt[3]{\sin 3x}$; 2) $y=\sqrt[3]{\sin\sqrt{x}}$.

721. 1) $y=\dfrac{1}{\sqrt{\sin 3x}}$; 2) $y=\dfrac{1}{\sqrt{\sin^3 x}}$; 3) $y=\dfrac{1}{\sqrt[3]{\sin^2 x}}$.

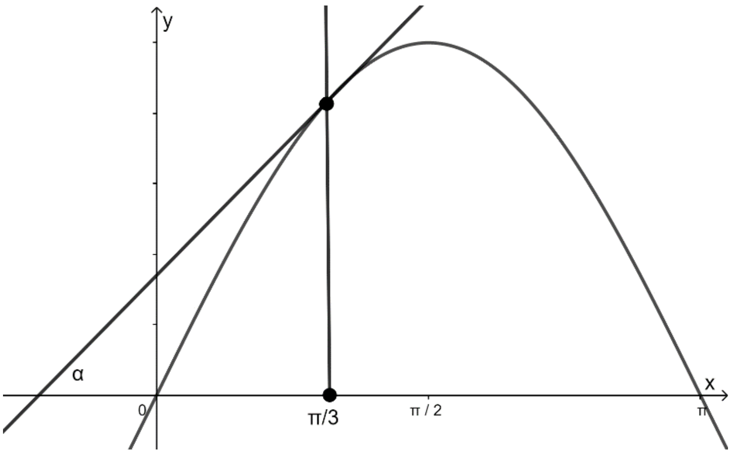
722. 1) $s=4\sin 3t$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilayotgan nuqtaning $t=\dfrac{\pi}{9}$ momentdagi tezligini toping (s m hisobida, t sek hisobida);

2) $s = \sin 2t$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilayotgan nuqtaning $t = \dfrac{\pi}{6}$ momentdagi tezligini toping (s m hisobida, t sek hisobida).

723. $y = \sin x$ egri chiziqda $x = \dfrac{\pi}{3}$ nuqtada o‘tkazilgan urinmaning Ox o‘qqa og‘ish burchagini toping.

Yechilishi. $y = \sin x$ funksiyaning $x = \dfrac{\pi}{3}$ dagi hosilasini topamiz:

$$y' = \cos x,\ \ y'_{x=\frac{\pi}{3}} = \cos\frac{\pi}{3} = \frac{1}{2}$$



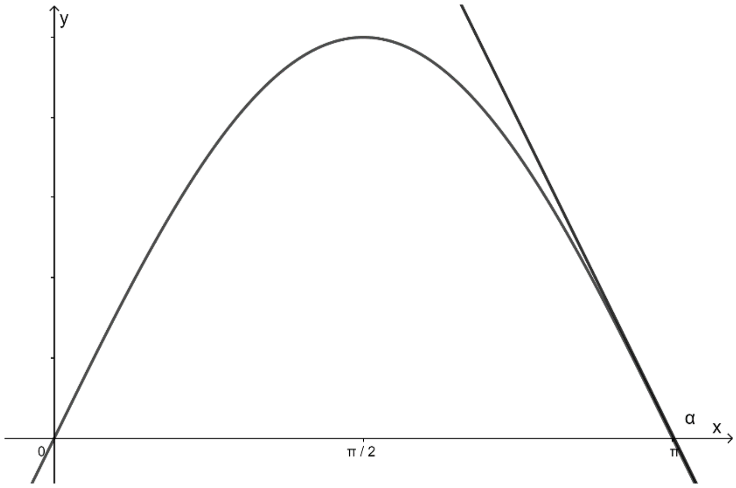
78-rasm

Urinmaning $x = \dfrac{\pi}{3}$ nuqtadagi og‘ish burchagining tangensi $\dfrac{1}{2}$ ga teng, ya’ni $k = tg\alpha = \dfrac{1}{2}$ bu yerda $\alpha = arctg\dfrac{1}{2}$ (78-rasm).

724. $y = \sin x$ egri chiziqqa $x = \dfrac{2\pi}{3}$ nuqtada o‘tkazilgan urinmaning Ox o‘qqa og‘ish burchagini toping.

725. $y = \sin x\left((0 < x < \dfrac{\pi}{2}\right)$ egri chiziqqa o‘tkazilgan urinma Ox o‘q bilan $arctg\dfrac{\sqrt{3}}{2}$ burchak tashkil etadi. Urinish nuqtasining koordinatalarini toping.

726. $y = \sin x$ egri chiziq Ox o‘qni $x = \pi$ nuqtada qanday burchak ostida kesib o‘tishini toping.



79-rasm

Yechilishi. 1. Sinusoidaga $x=\pi$ nuqtada o‘tkazilgan urinmaning burchak koeffitsientini topamiz:

$$k_{x=k}=(\sin x)_{x=\pi}=\cos\pi=-1$$

2. Urinma $x=\pi$ nuqtada hosil qiladigan burchakni topamiz:

$$tg\alpha=-1, \alpha=\frac{3\pi}{4} \qquad \text{(79- rasm).}$$

727. $y=\sin x$ egri chiziq Ox o‘qni $x=0$ nuqtada qanday burchak ostida kesib o‘tishini toping.

728. $y=\sin 3x$ egri chiziqqa $\left(\frac{\pi}{3};0\right)$ nuqtada o‘tkazilgan urinma va normalning tenglamasini tuzing.

Yechilishi . 1 $y=\sin 3x$ egri chiziqqa $\left(\frac{\pi}{3};0\right)$ nuqtada o‘tkazilgan urinmaning burchak koeffitsientini topamiz:

$$k=y'=3\cos 3x; \qquad k_{x=\frac{\pi}{3}}3\cos 3\cdot\frac{\pi}{3}=3\cos\pi=-3$$

2. Urinmaning tenglamasini tuzamiz:

$$y-0=-3\left(x-\frac{\pi}{3}\right); \qquad 3x+y-\pi=0$$

3.Normalning tenglamasini tuzamiz:

$$y-0=-\frac{1}{-3}\left(x-\frac{\pi}{3}\right); \qquad 3x-9y-\pi=0$$

729. $y=\sin\frac{1}{3}x$ egri chiziqqa $\left(\pi;\frac{\sqrt{3}}{2}\right)$ nuqtada o‘tkazilgan urinma va normalning tenglamasini tuzing.

II. Kosinusning hosilasi

Funksiyalarning hosilalarini toping:

730. $f(x)=\frac{\cos x+1}{\cos x-1}$, $f'\left(\frac{\pi}{3}\right)$ ni toping.

Yechilishi. (6.5), (6.1), (6 .8) va (6.16a) formulalar bo‘yicha topamiz:

$$f'(x)=\frac{(\cos x+1)'(\cos x-1)-(\cos x-1)'(\cos x+1)}{(\cos x-1)^2}=$$

$$\frac{-\sin x(\cos x-1)-(-\sin x)(\cos x+1)}{(\cos x-1)^2}=\frac{2\sin x^2}{(\cos x-1)^2};$$

$$f'\left(\frac{\pi}{3}\right)=\frac{2\sin^{\frac{\pi}{3}}}{\left(\cos\frac{\pi}{3}-1\right)^2}=\frac{\sqrt{3}}{\left(\frac{1}{2}-1\right)^2}=4\sqrt{3}$$

731. 1) $f(x)=\frac{1-\sin x}{1+\cos x}$, $f'\left(\frac{\pi}{4}\right)$; ni toping.
2) $y=2\sin x-\cos x+3$; 3) $y=3\sin x+\cos x-x$; 4) $f(x)=2\sin x-2\cos x\cdot f'\left(\frac{\pi}{6}\right)$;

732. 1) $f(t)=\sin t\cos t$; 2) $f(x)=\sin x(1-\cos x)$;
3) $y=x\cos x$; 4) $f(x)=\cos x(1+\sin x)$.

733. $y=\cos(x^2-3)$.

Yechilishi. (6.16) formulaga ko‘ra:

$$y'=-\sin(x^2-3)(x^2-3)';$$

$$y' = \sin(x^2 - 3)\cdot 2x = -2x\sin(x^2 - 3)$$

Kosinusning hosilasi o‘sha argumentning minus ishora bilan olingan sinusini bu argument hosilasiga ko‘paytirilganiga teng ekanligini yodda tutish kerak.

734. 1) $y = \cos x^3$; 2) $y = \cos\dfrac{1}{x^2}$; 3) $y = \cos\sqrt{2x}$;

735. 1) $y = \cos^3 x$; 2) $y = \cos^2\dfrac{1}{\sqrt{x}}$; 3) $y = \cos^2\sqrt[3]{x}$;

736. 1) $y = \dfrac{1}{\cos 2x}$; 2) $y = \dfrac{1}{\cos\sqrt{3x}}$; 3) $y = \dfrac{1}{\cos^2 x}$;

4) $y = \dfrac{1}{\cos^3(x^2 - 1)}$;

737. 1) $y = \sqrt{\cos 2x;}$ 2) $y = \sqrt{\cos x^3}$; 3) $y = \sqrt{\cos\sqrt{2x}}$.

738. 1) $y = \sqrt[3]{\cos x}$; 2) $y = \dfrac{1}{\sqrt{\cos x^2}}$; 3) $y = \dfrac{1}{\sqrt[3]{\cos^2 x^3}}$.

739. $y = (1 + \sin 2x)\cos 2x$.

Yechilishi. Ko‘paytmadan hosila olish qoidasi bo‘yicha topamiz:

$$y' = (1 + \sin 2x)'\cos 2x + (\cos 2x)'(1 + \sin 2x)$$

So‘ngra yig‘indini differensiallash qoidasidan hamda (6.15) va (6.16) formulalardan foydalanib, topamiz:

$$y' = 2\cos 2x\cos 2x - 2\sin 2x(1 + \sin 2x) = 2\cos^2 2x -$$

$$-2\sin 2x - 2\sin^2 2x = 2(\cos^2 2x - \sin^2 2x) - 2\sin 2x == 2(\cos 4x - \sin 2x)$$

740. 1) $y = (1 - \cos 2x)\sin 2x$; 2) $y = \dfrac{\cos^2 x}{1 + \cos^2 x}$;

3) $f(x) = \cos^3 x \cdot \sin x$, $f'\left(\dfrac{\pi}{3}\right)$ ni toping.

741. $s=-2\cos 2t$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilayotgan nuqtaning $t=\dfrac{\pi}{6}$ momentdagi tezligi va tezlanishini toping.

742. $y=\cos 3x$ egri chiziqqa $\left(\dfrac{\pi}{6};0\right)$ nuqtada o‘tkazilgan urinma va normalning tenglamasini tuzing.

III. Tangensning hosilasi

Funksiyalarning hosilalarini toping.

743. $f(x)=\dfrac{tgx-1}{tgx};\quad f'\left(\dfrac{\pi}{3}\right)$ ni toping.

Yechilishi. (6.5), (6.17a) va (6.8) formulalar bo‘yicha quyidagini hosil qilamiz:

$$f'(x)=\frac{\dfrac{1}{\cos^2 x}tgx-\dfrac{1}{\cos^2 x}(tfx-1)}{tg^2x}=\frac{\dfrac{1}{\cos^2 x}(tgx-tgx+1)}{tg^2x}=$$

$$=\frac{1}{\sin^2 x};\ f'\left(\frac{\pi}{3}\right)=\frac{1}{\sin^2\dfrac{\pi}{3}}=\frac{4}{3}$$

744. 1) $y=\dfrac{tgx}{1-tgx}$; 2) $y=tgx-x$; 3) $f(u)=utgu$;
4) $f(x)=\sin x+tgx,\ f'(\pi)$ ni toping.

745. $y=tg(2x^2+1)$.

Yechilishi. (6.17) formula bo‘yicha quyidagini hosil qilamiz:

$$y'=\frac{1}{\cos^2(2x^2+1)}(2x^2+1);$$

$$y'=\frac{1}{\cos^2(2x^2+1)}\cdot 4x=\frac{4x}{\cos^2(2x^2+1)}$$

746. 1) $y = tg(ax+b)$;   2) $y = tg\frac{x}{3}$;   3) $y = tgx^2$;
4) $y = tg\sqrt{2x}$.

747. 1) $y = tg^2 3x$; **2)** $y = tg^2\sqrt{x}$.

748. $y = tgx\sin^2 x$.

Yechilishi. Ko‘paytmaning hosilasini topish formulalariga ko‘ra:

$$y' = (tgx)'\sin^2 x + (\sin^2 x)'tgx$$

(6,15) va (6.17) formulalar bo‘yicha:

$$y' = t\frac{1}{\cos^2 x}\sin^2 x + 2\sin x\cos xtgx$$

Soddalashtirsak,

$$y' = tg^2 x + 2\sin x\cos x\frac{\sin x}{\cos x} = tg^2 x + 2\sin^2 x$$

749. 1) $y = 3x - tg3x$;   2) $f(x) = tg^2 x\sin x$, $f'\left(\frac{\pi}{3}\right)$ ni toping; 3) $y = tg\frac{x}{2} + \frac{1}{3}tg^3\frac{x}{2}$.

750. 1) $f'(x) = \dfrac{tg\frac{x}{2}}{1 + tg\frac{x}{3}}$, $f'\left(\frac{\pi}{2}\right)$ ni toping;

2) $y = \dfrac{1 - tg2x}{tg2x}$.

751. $y = tgx$ egri chiziqda $x = \frac{\pi}{3}$ nuqtada o‘tkazilgan urinmaning Ox o‘qqa og‘ish burchagini toping.

752. $y = tgx$ egri chiziq Ox o‘qni $x = \frac{\pi}{4}$ nuqtada qanday burchak ostida kesib o‘tishini toping.

IV. Kotangensning hosilasi

Funksiyalarning hosilalarini toping.

753. $y = ctgx + x$.

Yechilishi. (6.1), (6.18a) va (6.9) formulalarga binoan topamiz:

$$y' = \frac{1}{\sin^2 x} + 1 = \frac{-1+\sin^2 x}{\sin^2 x} = -\frac{1-\sin^2 x}{\sin^2 x} = -ctg^3 x$$

754. 1) $f(x) = \frac{1+ctgx}{ctgx}$; 2) $f(x) = ctgx - tgx$, $f'\left(\frac{\pi}{4}\right)$ ni toping;

755. $y = ctg(ax+b)$.

Yechilishi. (6.18) formula bo‘yicha:

$$y' = \frac{1}{\sin^2(ax+b)}(ax+b)'; \qquad y' = -\frac{1}{\sin^2(ax+b)}.$$

756. 1) $y = ctgx^3$; 2) $y = ctg\frac{x^2}{2}$; 3) $y = ctg\frac{1}{x^2}$

4) $y = ctg\sqrt{2x}$

757. 1) $y = ctg^3 x$; 2) $y = \sqrt{ctg 2x}$;

3) $y = -ctg\frac{x}{2} - \frac{1}{3} \times ctg^3\frac{x}{2}$

758. 1) $y = \frac{1}{ctg^2 2x}$; 2) $y = \frac{1}{\sqrt[3]{ctgx}}$.

38-§. Logarifmik funksiyalarning hosilalari

Differensiallash formulalari.

$u = \varphi(x)$ shartda		$u = x$ shartda	
$(\ln u)' = \frac{1}{u}u'$	(6.19)	$(\ln x)' = \frac{1}{x}$	(6.19a)
$(\ln u)'(0,4343\ln u)' = \frac{0,4343}{u}u'$	(6.20)	$(\lg x)' = \frac{0,4343}{x}$	(6.20a)

Funksiyaning hosilalarini toping.

759. $y = x + \ln x;\qquad y = 5\lg x.$

Yechilishi. 1. (6.1), (6.9) va (6.19a) formulalarga ko‘ra topamiz:

$$y' = 1 + \frac{1}{x} = \frac{x+1}{x}.$$

2). (6.4) va (6.20a) formulalarga muvofiq differensiallaymiz:

$$y' = 5 \cdot \frac{0,4343}{x} = \frac{2,1715}{x}.$$

760. 1) $f(x) = 3\ln x - x^2;\ f'(1)$ ni toping;
$f(x) = \lg x + x^3;\ f'(-1)$ ni toping;

3) $y = x^2 \ln x;$ 4) $y = (1 - \ln x);$ 5) $f(z) = z^3 - \ln z;\ f'(3)$ ni toping;

761. $f(x) = \dfrac{\ln x}{1 - \ln x}.$

Yechilishi. 1. (6.5), (6.19) va (6.8) formulalarga ko‘ra topamiz:

$$f'(x) = \frac{(\ln x)'(1-\ln x) - (1-\ln x)'\ln x}{\left(1-\ln x\right)^2} =$$

$$\frac{\frac{1}{x}(1-\ln x) + \frac{1}{x}\ln x}{(1-\ln x)} = \frac{1}{x(1-\ln x)^2}$$

762. 1) $y = \dfrac{\ln x - 2}{\ln x};$ 2) $y = \dfrac{\ln x + 1}{\ln x}.$

763. $y = \ln(ax^2 + b).$

Yechilishi. (6.19) formulalarga ko‘ra topamiz:

$$y' = \frac{1}{ax^2+b}(ax^2+b)'$$

$$y' = \frac{1}{ax^2+b}\cdot 2ax = \frac{2ax}{ax^2+b}.$$

Natural logarfmning hosilasi logarifm belgisi ostida turgan ifodaga teskari ifodani bu ifodaning hosilasiga ko‘paytirilganiga teng ekanligini yodda saqlash kerak.

764. 1) $y = \ln 3x;$ 2) $y = \ln(2x^2-3).$

765. $f(x) = \ln\frac{u-x}{u+x},\quad f'(2a)$ ni hisoblang.

Yechilishi. 1-usul. (6.19) formulaga ko‘ra topamiz:

$$f'(x) = \frac{1}{\frac{u-x}{a+x}}\left(\frac{a-x}{a+x}\right)'$$

Bo‘linmani differensiallash qoidasiga ko‘ra:

$$f'(x) = \frac{a+x}{a-x}\frac{(a-x)'(a+x)-(a+x)'(a-x)}{(a+x)^2} =$$

$$= \frac{1}{a-x}\frac{-(a+x)-(a-x)}{a+x} = \frac{1}{a-x}\frac{-a-x-a+x}{a+x} =$$

$$= \frac{-2a}{a^2-x^2} = \frac{2a}{x^2-a^2}$$

$$f'(2a) = \frac{2a}{(2a)^2-a^2} = \frac{2a}{4a^2-a^2} = \frac{2a}{3a^2} = \frac{2}{3a}$$

2- usul Kasrni lagorifmlaymiz:

$$f(x) = \ln(a-x) - \ln(a+x)$$

(6.1), (6.19), (6.8) va (6.9) formulalar bo‘yicha:

$$f'(x)=\frac{1}{a-x}(a-x)'-\frac{1}{a+x}(a+x)'=$$
$$=-\frac{1}{a-x}-\frac{1}{a+x}=\frac{2a}{x^2-a^2}$$

Hosila topishning ikkinchi usuli birinchi usulga qaraganda ancha soddadir, chunki kasrning hosilasi formulasini tatbiq etishga hojat qolmaydi.

Logarifmik funksiyaning hosilasini topishni osonlashtirish uchun dastlab logarifm belgisi ostida turgan ifodani logarifmlanadi.

766. 1) $y=\ln\frac{x+1}{x-1}$; 2) $y=\frac{2}{2+x}$.

767. $y=\lg(5x^2+1)$.

Yechilishi. (6.20) formulaga ko‘ra topamiz:

$$y'=\frac{0,4343}{5x^2+1}\left(5x^2+1\right)'$$

$$y'=\frac{0,4343}{5x^2+1}\cdot 10x=\frac{4,343x}{5x^2+1}.$$

768. 1) $y=\lg 10x$; 2) $y=\lg(2x+1)$.

769. $y=\ln\sqrt{2x}$.

Yechilishi . Kvadrat ildizni logarifmlaymiz:

$$y=\frac{1}{2}\ln(2x)=\frac{1}{2}\ln 2+\frac{1}{2}\ln x$$

(6.1), (6 ,8) va (6.19 a) formulalarga ko‘ra

$$y'=\frac{1}{2}\cdot\frac{1}{x}=\frac{1}{2x}$$

ni hosil qilamiz.

770. 1) $y=\ln\sqrt{2x-1}$; 2) $y=\ln\sqrt{x^2-a^2}$; 3) $y=\lg\sqrt{x^2+4}$; 4) $y=\ln\sqrt{\frac{1-x}{1+x}}$.

771. 1) $y=\ln\sin x$; 2) $y=\ln\cos x$; 3) $y=\ln tgx$; 4) $y=\ln ctg\, x$; 5) $f(x)=\ln\sin\frac{x}{3}$; $f'\left(\frac{\pi}{2}\right)$ni hisoblang. 6) $y=\ln cos^2 x$.

772. 1) $y=\ln\sqrt{\frac{1-4x}{1+4x}}$; 2) $y=ln(x-\sqrt{1}+x^2)$;

773. $y=\ln^2(x^2-1)$;

Yechilishi . (6.10) va (6.19) formulalarga ko‘ra:

$$y'=2\ln(x^2-1)\left[\ln(x^2-1)\right]';$$

$$y'=2\ln(x^2-1)\frac{1}{x^2-1}(x^2-1)';$$

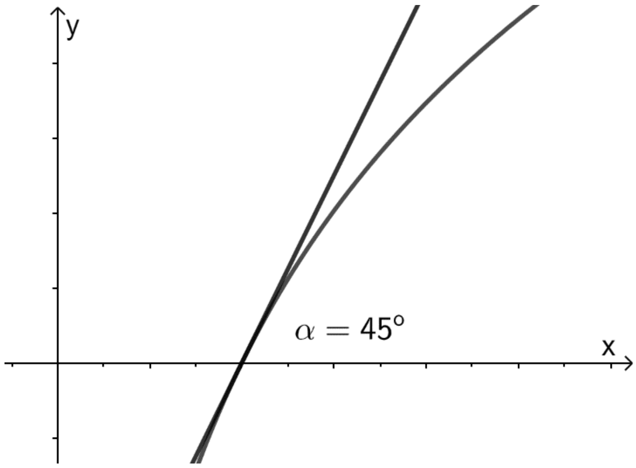
$$y'=2\ln^2(x^2-1)\cdot\frac{1}{x^2-1}\cdot 2x=\frac{4x\ln(x^2-1)}{x^2-1};$$

774. 1) $y=\ln^3 3x$; 2) $y=\ln^2(2x+1)$; 3) $y=\ln^2\sqrt{\sin x}$;

775. $y=\ln x$ egri chiziq Ox o‘qni kanday burchak ostida kesib o‘tishini toping.

Yechilishi. 1. $y=\ln x$ egri chizikning Ox o‘q bilan kesisish nuktasini topamiz. Bu nuqtada $\ln x=0$, bu yerdan $x=1$ (80- rasm).

2. Urinmaning, $x=1$nuqtadagn burchak koeffitsientini topamiz:



80-rasm

$$(\ln x)'_{x=1} = \left(\frac{1}{x}\right)_{x=1} = \frac{1}{1} = 1$$

3. $y = \lg x$ egri chizik Ox o‘qni qanday burchak ostida kesib o‘tishini aniqlang.

777. $y = \lg x$ egri chiziq bilan $y = 1$ to‘g‘ri chiziqning kesishishidan hosil bo‘lgan o‘tkir burchakni hisoblang.

39-§ . Ko‘rsatkichli funksiyalarning hosilalari

Differemsiallash formulalari.

$u = \varphi(x)$ shartda		$u = x$ shartda	
$(a^n) = a^n \ln a \cdot u'$	(6.21)	$(a^x) = a^2 \ln a$	(6.21a)
$(e^u)' = e^n u'$	(6.22)	$(e^x)' = e^x$	(6.22a)

Funksiyalarning hosilalarini toping.

778. $y = 2 \cdot 5^x + 3e^x$

Yechilishi . (6.1), (6.21a), (6.22a) va (6.4) formulalarga ko‘ra topamiz:

$$y' = 2 \cdot 5^x \ln 5 + 3e^x = 2\ln 5 \cdot 5^2 + 3e^x$$

779. 1) $f(x) = \ln x \cdot e^x$; 2) $f(x) = x^2 e^x$; 3) $f(x) = e^x - xe^x$

4) $y = 3^x e^x$; 5) $y = \dfrac{e^x}{2^x}$; 6) $f(x) = 5\ln x + e^x$, $f'(1)$ ni toping.

780. $f(x) = \dfrac{e^x + 1}{e^x - 1}$, $f'(-1)$ ni toping.

Yechilishi. 1-usul. (6.5), (6.1), (6.22 a) va (6.9) formulalarga binoan quyidagini hosil qilamiz:

$$f'(-1)=\frac{(e^x+1)'(e^x-1)'(e^x+1)}{(e^x-1)^2}=$$

$$=\frac{e^x(e^x-1)-e^x(e^x+1)}{(e^x-1)^2}=-\frac{2e^x}{(e^x-1)^2}$$

$$f'(-1)=-\frac{2e^{-1}}{(e^{-1}-1)^2}=-\frac{2e}{(1-e)^2}.$$

2-usul. Funksiyani logarifmlab logarifmning hosilasini topamiz:

$$\ln f(x)=\ln(e^x+1)-\ln(e^x-1);$$

$$\frac{1}{f(x)}f'(x)=\frac{e^x}{e^x+1}-\frac{e^x}{e^x-1}=\frac{-2e^x}{(e^x+1)(e^x-1)},$$ $f'(x)$ ni topamiz:

$$f'(x)=f(x)\frac{-2e^x}{(e^x+1)(e^x-1)}=\frac{e^x+1}{e^x-1}\cdot\frac{-2e^x}{(e^x+1)(e^x-1)}=-\frac{2e^x}{(e^x-1)^2}.$$

781. 1) $y=\frac{5-e^x}{e^x+2}$; 2) $y=\frac{1-e^x}{e^x}$.

782. $y=3^{2x^2}$.

Yechilishi . (6.21) formulaga ko‘ra:

$$y'=e^{2x^2}\ln 3\cdot(2x^2)'; \quad y'=3^{2x^2}\ln 3\cdot 4x=4x\cdot 3^{2x^2}\ln 3.$$

783. 1) $y=5^{x^2}$; 2) $y=2^{\sqrt{x}}$; 3) $y=3^{\ln x}$; 4) $y=2^{\cos x}$; $f'(\pi)$ ni toping.

786. $y=\frac{e^x-e^{-x}}{e^x+e^{-x}}$.

Yechilishi . Bo‘linmaning hosilasini topish qoidasiga ko‘ra topamiz:

$$y'=\frac{(e^x-e^{-x})'(e^x+e^{-x})-(e^x+e^{-x})'(e^x-e^{-x})}{(e^x+e^{-x})^2}$$

Yig‘indidan hosila olish qoidasiga va (6.22) formulaga ko‘ra:

$$y' = \frac{\left[e^{x} - e^{-x}(x)'\right](e^{x} + e^{-x}) - \left[e^{x} + e^{-x}(-x)'\right](e^{x} - e^{-x})}{(e^{x} + e^{-x})^{2}}$$

$$y' = \frac{(e^{x} + e^{-x})(e^{x} + e^{-x}) - (e^{x} - e^{-x})(e^{x} - e^{-x})}{(e^{x} + e^{-x})^{2}} =$$

$$= \frac{(e^{x} + e^{-x})^{2} - (e^{x} - e^{-x})^{2}}{(e^{x} + e^{-x})^{2}} = \frac{e^{2x} + 2e^{0} + e^{-2x} - 2e^{0} - e^{-2x}}{(e^{x} + e^{-x})^{2}} =$$

$$= \frac{4e^{0}}{(e^{x} + e^{-x})^{2}} = \frac{4}{(e^{x} + e^{-x})^{2}}.$$

787. 1) $y = 3\left(e^{\frac{x}{e^{3}}} - e^{\frac{x}{3}}\right)$; 2) $y = \dfrac{e^{-x} + e^{x}}{e^{-x} - e^{x}}$.

788. $y = e^{\sqrt{3x}}$ egri chiziq Oy o‘qni qanday burchak ostida kesib o‘tishini aniqlang.

40-§. Teskari trigonometrii funksiyalarning xosilalari

Differensiallash formulalari.

$u = \varphi(x)$ shartda		$u = x$ shartda	
$(\arcsin u)' = \dfrac{1}{\sqrt{1-u^{2}}}u'$ $\lvert u \rvert < 1$	(6.23)	$(\arcsin x)' = \dfrac{1}{\sqrt{1-x^{2}}}$ $\lvert x \rvert < 1$	(6.23a)
$(\arccos u)' = -\dfrac{1}{\sqrt{1-u^{2}}}u'$ $\lvert u \rvert < 1$	(6.24)	$(\arccos x)' = -\dfrac{1}{\sqrt{1-x^{2}}}$	(6.24a)
$(arctgu)' = \dfrac{1}{1+u^{2}}u'$	(6.25)	$(\mathrm{arc}\,tgx)' = \dfrac{1}{1+x^{2}}$	(6.25a)
$(\mathrm{arc}\,ctgu)' = -\dfrac{1}{1-u^{2}}u'$	(6.26)	$(\mathrm{arc}\,ctgx)' = -\dfrac{1}{1+x^{2}}$	(6.26a)

I. Arksinus va arkkosinusning hosilalari

Funksiyalarning hosilalarini toping.

789. $f'(x)=5\arcsin x-3\arccos x.$ $f'\left(\frac{\sqrt{3}}{2}\right)$ ni toping.

Yechilishi. (6.1), (6.23a) va (6.24a) formulalarni qo‘llab, topamiz:

$$f'(x)=\frac{5}{\sqrt{1-x^2}}+\frac{3}{\sqrt{1-x^2}},$$

$$f'\left(\frac{\sqrt{3}}{2}\right)=\frac{8}{\sqrt{1-\frac{3}{4}}}=16.$$

790. 1) $f'(x)=2\arcsin x+\arccos x,$ $f'\left(\frac{\sqrt{2}}{2}\right)$ ni toping:

2) $f(x)=5\arcsin x+2\arccos x,$ $f'\left(\frac{1}{2}\right)$ ni toping:

3) $y=x\times(\arcsin x+2\arccos x).$

791. $y=\arcsin 2x.$

Yechilishi. (6.23) formulaga kura:

$$y'=\frac{1}{\sqrt{1-(2x)^2}}(2x)'; \quad y'=\frac{1}{\sqrt{1-4x^2}}\cdot 2=\frac{2}{\sqrt{1-4x^2}}.$$

792. 1) $y=\arcsin 3x;$ 2) $y=\arccos\frac{x}{a};$ 3) $y=\arcsin x^2;$ 4) $y=\arccos ax.$

793. $y=\arccos\sqrt{2x}.$

Yechilishi. (6.24) va (6.12) formulalarga binoan quyidagini hosil qilamiz:

$$y'=-\frac{1}{\sqrt{1-\left(\sqrt{2x}\right)^2}}(\sqrt{2x})'$$

$$y' = -\frac{1}{\sqrt{1-2x}}\frac{1}{2\sqrt{2x}}(2x)';$$

$$y' = -\frac{1}{\sqrt{1-2x}}\frac{1}{2\sqrt{2x}}\cdot 2 = -\frac{1}{\sqrt{1-2x}}\frac{1}{\sqrt{2x}} = -\frac{1}{\sqrt{2x(1-2x)}}.$$

794. 1) $y = \arcsin\sqrt{3x}$; 2) $y = \arccos\sqrt{x-1}$;

3) $y = \arcsin\dfrac{x^2-a^2}{x^2+a^2}.$

II. Arktangens va arkkotangensning hosilalari

Funksiyalarning hosilalarini toping.

795. $f(x) = 3arctgx - 2arcctgx,\quad f'(2)$ ni toping.

Yechilishi. (6.1), (6.25a) va (6.26a) formulalarga binoan topamiz:

796. 1) $f(x) = \mathrm{arc}\,tgx,\quad f'(\sqrt{3})$ ni toping;

2) $y = x(\mathrm{arc}\,tgx + arcctgx)$.

797. $y = arctg2x.$

Yechilishi. (6.25) formulaga ko‘ra:

$$y' = \frac{1}{1+(2x)^2}(2x)';\qquad y' = \frac{1}{1+4x^2}\cdot 2 = \frac{2}{1+4x^2}.$$

798. 1) $y = arctgx^2$; 2) $y = arcctg3x$; 3) $y = arctg\dfrac{a}{x}$;

4) $y = arcctg\dfrac{x}{a}$.

799. $y = arcctg\sqrt{2x}.$

Yechilishi.

$$y' = -\frac{1}{1+(\sqrt{2x})^2}\left(\sqrt{2x}\right)' = -\frac{1}{1+2x}\times$$

$$\times\frac{1}{2\sqrt{2x}}(2x) = -\frac{1}{1+2x}\frac{1}{2\sqrt{2x}}\cdot 2 = -\frac{1}{(1+2x)\cdot\sqrt{2x}}.$$

800. 1) $y = arctg\sqrt{x}$; 2) $y = arctg\,\dfrac{1}{\sqrt{x}}$.

801. 1) $y = arctg\,\dfrac{1+x}{1-x}$; 2) $y = arctg\,\dfrac{x-1}{x+1}$.

41- §. Oshkormas funksiyaning hosilasi

x va y o‘zgaruvchilarni o‘z ichiga olgan tenglama bilan berilgan $F(x,y)=0$ funksiya x ning *oshkormas funksiyasi* deyiladi.

Masalan, $xy + x - 1 = 0$, $xy + 1 = \cos y u$ va $\ln y = xy + x^2$ funksiyalar oshkormas funkniyalardir.

Ba’zi hollarda $F(x,y)=0$ tenglamani y ga nisbatan echish mumkin, u holda y funksiya x orqali oshkor ifodalangan bo‘ladi. Masalan, birinchi misolda $y = \dfrac{1-x}{x}$, ya’ni funksiyaning $F(x,y)=0$ oshkormas holda berilishidan $y = f(x)$ oshkor holda berilishiga o‘tdik.

Ayrim hollarda (ikkinchi va uchinchi misollardagiga o‘xshash) bunday o‘tishni amalga oshirib bo‘lmaydi.

Funksiya oshkormas usulda berilganida y dan x bo‘yicha hosila quyidagi qoida bo‘yicha topiladi:

1) y ni x ning funksiyasi deb qarab, $F(x,y)=0$ funksiyaning hosilasini topamiz;

2) hosil qilingan tenglamani y' ga nisbatan echib, oshkormas funksiyaning hosilasini $y' = f(x,y)$ ko‘rinishida hosil qilamiz.

Funksiyalarning hosilalarini toping.

802. $3y + 5x^3 - 2 = 0.$

Yechilishi. Algebraik yig‘indini differensiallash qoidasiga binoan:

$$(3y) + (5x^2)' - (2)' = 0$$

y ni x ning funksiyasi deb qarab, x bo‘yicha hosilani topamiz:

$$3y' + 15x^2 = 0$$

So‘nggi tenglamani y' ga nisbatan echib,

$$y' = -5x^2$$

ni hosil qilamiz.

803. 1) $2x^2 - 5y + x = 0$; 2) $2y + x - x^2 + 1 = 0.$

804. $y^2 + 5x + x^2 = 0.$

Yechilishi. y ni x ning funksiyasi deb qarab, x o‘zgaruvchi bo‘yicha hosilani topamiz.

$$(y^2)' - (5x)' + (x^2)' = 0; \quad 2yy' - 5 + 2x = 0$$

bu yerda

$$y' = \frac{5 - 2x}{2y}.$$

805. 1) $y^2 - x^2 + 4x - 5 = 0$; 2) $2y^2 - 3x^2 + x = 0.$

806. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

Yechilishi. y ni x ning funksiyasi deb qarab, x o‘zgaruvchi bo‘yicha diffferensiallaymiz.

$$\frac{2x}{a^2} + \frac{2y}{b^2} = 0; \quad b^2x + a^2yy' = 0; \quad y' = -\frac{b^2x}{a^2y}.$$

807. 1) $x^2 + y^2 = a^2$; 2) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; 3) $y^2 = 2px.$

808. $\frac{x^2}{27} + \frac{y^2}{24} = 1$ ellipsga (-3;-4) nuqtada o‘tkazilgan urinma va normalning tenglamasini tuzing.

Yechilishi. y ni x ning funksiyasi deb qarab, ellips tenglamasini x o‘zgaruvchi bo‘yicha differensiallaymiz:

$$\frac{2x}{27}+\frac{2yy'}{24}=0,\quad 8x+9yy'=0 \quad \text{bu yerdan} \quad y'=-\frac{8x}{9y},$$

(-3;-4) nuqtada o‘tkazilgan urinmaningburchak koeffitsienti topamiz:

$$k_{x=-3}=y'_{=-3}=-\frac{8(-3)}{9(-4)}=-\frac{2}{3}$$

Bu nuqtada ellipsga o‘tkazilgan urinmaning teglamasini tuzamiz (686-masalaga qarang):

$$y+4=-\frac{2}{3}(x+3),\ 2x+3y+18=0$$

(-3;-4) nuqtada o‘tkazilgan normalning tenglamasini tuzamiz:

$$\kappa_{норм}=-\frac{1}{\kappa_{ур}}=-\frac{3}{2}$$

Bu nuqtada ellipsga o‘tkazilgan normalning teglamasini tuzamiz:

$$y+4=\frac{3}{2}(x+3),\qquad 3x-2y+1=0.$$

809. 1) $x^2+y^2=25$ aylanaga (-3;4) nuqtada; 2) $x^2+y^2=169$ aylanaga (12;-5) nuqtada; 3) $\frac{x^2}{100}+\frac{y}{25}=1$ elipsga (-8;3) nuqtada; 4) $\frac{x^2}{16}-\frac{y^2}{64}=1$ giperbolaga (-5;6) nuqtada; 5) $y^2=8x$ parabolaga (2;-4) nuqtada; 6) $y^2=\left(\frac{3}{2};3\right)$ nuqtada o‘tkazilgan urinma va normalning tenglamalarini tuzing.

810. $x^2-3xy-4=0$

Yechilishi. $(x^2)'-(3xy)'-(4)'=0$ xy - o‘zgaruvchi miqdorlarning ko‘paytmasi, shu sababli $(xy')=x'y+y'x;\ 2x-3(x'y+y'x)=0;\ 2x-3y-3y'x=0;$

$$y'=\frac{2x-3y}{3x}$$

811. 1) $x^2+xy+1=0;\ 2) xy=1$

812. 1) $y^2+y-x=0;\quad 2) y^2+x^2-x-y=0; 3) xy^2-x^2y=2$

813. 1) $(y+1)^2-5x=0;\ \ 2)(y-1)^2+x^2=0;\ \ 3) y^3-x^2=0$

814. $\sin y=xy^2$

Yechilishi. $(\sin y)'=\left(xy^2\right)';\ \cos y\cdot y'=x'y'+(y^2)'x;$

$\cos y\cdot y^2=y^2+2yy'x;\ \ \cos y\cdot y'-2xyy'=y^2;\quad y'=\dfrac{y^2}{\cos y-2xy};$

815. 1) $\cos^2 y=x^2;\quad 2)\ tgy=xy$

816. 1) $\arcsin y=x;\ \ 2)\ arcctgy=x^2$

817. 1) $e^{2x+y}=xy$

Yechilishi. $\left(e^{2x+y}\right)'=\left(xy\right)';\ \ e^{2x+y}=(2x+y)'=y+xy';$

$2e^{2x+y}=y'e^{2x+y}=y+xy';\ \ y'e^{2x+y}-xy=y-2e^{2x+y};$

$y'(e^{2x+y}-x)=y-2e^{2x+y};\ \ y'=\dfrac{y-2e^{2x+y}}{e^{2x+y}-x}$

818. 1) $e^y=x^3;\ \ 2)\ln y=x^2;\quad 3)\ln e^y=x$

42- §. Ikkinchi tartibli hosila va uning mexanikadagi tatbiqlari

Agar $y = f(x)$ funksiya $y' = f'(x)$ hosilaga ega bo‘lsa, u holda $f'(x)$ hosiladan x bo‘yicha olingan hosila (agar u mavjud bo‘lsa) ikkinchi hosila yoki ikkinchi tartibli hosila deyiladi.
Ikkinchi tartibli hosila uchun quyidagicha belgilashlar mavjud:

$$y''; \; y''_{xx}; \; \frac{d^2y}{dx^2} \quad \text{yoki} \quad f''(x); \; \frac{d^2 f(x)}{dx^2}$$

Nuqta to‘g‘ri chiziqli harakat qilganda berilgan $t = t_1$ momentdagi a tezlanish s yo‘ldan t vaqt bo‘yicha olingan va (berilgan $t = t_1$ moment uchun hisoblangan ikkinchi tartibli $\frac{d^2s}{dt^2}$ hosilaga tengdir).

Funksiyalarning ikkinchi tartibli hosilalarini toping.

819. $y = x^3 - 2x^2 + 4x - 5$

Yechilishi. $y' = 3x^2 - 4x + 4$. Birinchi tartibli hosilani funksiya deb qarab, ikkinchi tartibli hosilani topamiz:

$$y'' = 6x - 4$$

820. 1) $y = 5x^3 + 3x^2 - 7x + 1$; 2) $s = 4t^2 - 3t + 1$

821. $y = \sqrt{x}$

Yechilishi. $y' = \frac{1}{2\sqrt{x}}$; $y'' = -\frac{1}{\left(2\sqrt{x}\right)^2}\left(2\sqrt{x}\right)'$;

$$y'' = \frac{1}{4x} \cdot 2 \cdot \frac{1}{2\sqrt{x}} = -\frac{1}{4x\sqrt{x}}$$

822. 1) $y = \sqrt{2x}$; 2) $y = \frac{1}{\sqrt{x}}$

823. 1) $s = -\frac{1}{t}$; 2) $s = (2t^2 - 1)^2$; 3) $s = \frac{3t + 1}{t}$

824. $y = \sin^2 x$

Yechilishi. $y' = 2\sin x \cos x = \sin 2x$

$$y''=\cos 2x(2x)'=\cos 2x\cdot 2=2\cos 2x$$

825. 1) $y=\cos x$; 2) $y=tgx$

826. 1) $s=e^{\cos t}$; 2) $s=e^{-\sin t}$

827. $y=\ln\sqrt{x}$

Yechilishi.

$$y'=\frac{1}{\sqrt{x}}\frac{1}{2\sqrt{x}}=\frac{1}{2x}$$

$$y''=-\frac{1}{(2x)^2}(2x)'=-\frac{1}{4x^2}\cdot 2=-\frac{1}{2x^2}$$

828. 1) $y=\ln x^2$; 2) $y=\ln\frac{1}{x}$

829. 1) $v=t^3-2t, t=2$; 2) $v=2\sin\frac{t}{2}, t=\frac{2\pi}{3}$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilayotgan nuqtaning berilgan t momentdagi tezlanishini toping.

Yechilishi.

1) $a=\frac{fv}{dt}=3t^2-2;$ $\qquad a_{t=2}=3\cdot 2^2-2=10;$

2) $a=\frac{dv}{dt}=2\cos\frac{t}{2}\left(\frac{t}{2}\right)'=2\cos\frac{t}{2}\cdot\frac{1}{2}=\cos\frac{t}{2};$

$$a_{t=\frac{2\pi}{3}}=\cos\frac{2\pi}{3\cdot 2}=\cos\frac{\pi}{3}=\frac{1}{2}$$

830. Qo‘yidagi qonun bo‘yicha chiziqli harakat qilayotgan nuqtaning t vaqtning momentidagi tezlanishini toping.

1) 1)$v=6\sin\frac{t}{3}; t=\pi$ $\qquad$ 2) $v=4\cos\frac{t}{4}; t=\frac{2\pi}{3}$

3) $v=t^3-t^2+1, t=3;$ $\qquad$ 4) $v=t^3-2t^2+t, t=2$

831. $s=2\sin\frac{\pi t}{3}, t=1$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilayotgan nuqtaning t vaqtning ko‘rsatilgan momentidagi tezligi va tezlanishini toping.

Yechilishi.

$$v=\frac{ds}{dt}-2\cos\frac{\pi t}{3}\left(\frac{\pi t}{3}\right)'=2\cos\frac{\pi t}{3}\frac{\pi}{3}=\frac{2\pi}{3}\cos\frac{\pi t}{3};$$

$$v_{t=1}=\frac{2\pi}{3}\cos\frac{\pi}{3}=\frac{2\pi}{3}\cdot\frac{1}{2}=\frac{\pi}{3}$$

$$a=\frac{d^2s}{dt^2}=\frac{du}{dt}=\frac{2\pi}{3}\left(-\sin\frac{\pi t}{3}\right)\left(\frac{\pi t}{3}\right)'=$$

$$-\frac{2\pi}{3}\sin\frac{\pi t}{3}\frac{\pi}{3}=-\frac{2\pi^2}{9}\sin\frac{\pi t}{3}$$

$$a_{t=1}=-\frac{2\pi}{9}\sin\frac{\pi}{3}=-\frac{2\pi}{9}\frac{\sqrt{3}}{2}=-\frac{\pi\sqrt{3}}{9}$$

832. 1)$s=t^3-t^2-t, t=1$; 2) $s=t^2-6t+8, t=3$; 3) $s=\sin\frac{\pi t}{4}, t=1$; 4) $s=-\cos\frac{\pi t}{3}, t=1$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilayotgan nuqtaning t vaqtning ko‘rsatilgan momentidagi tezligi va tezlanishini toping.

833. Nuqta $s=\frac{1}{3}t^3+2t^2-1$qonun bo‘yicha to‘g‘ri chiziqli harakat qilmoqda. Tezlanish nolga teng bo‘lgan momentni va bu momentda jismning tezligini toping.

Yechilishi. $v=\frac{ds}{dt}=-t^2+4t;\quad a=\frac{d^2s}{dt}=\frac{dv}{dt}=-2t^2+4$

Tezlanishni nolga teng deb, t ni topamiz.

$$-2t+4=0.t=2$$

ya'ni $t=2$ bo'lgan momentda tezlanish nolga teng bo'lar ekan.
$t=2$ momentda jismning tezligini topamiz:

$$v_{t=2}=-2^2+4\cdot 2=4$$

834. Jism $s=-t^2+3t^2-8$ qonun bo'yicha to'g'ri chiziqli harakat qilmoqda. Tezlanish nolga teng bo'lgan t momentni va bu momentda jismning tezligini toping.

835. Tik yuqoriga otilgan jismning uchish balandligi $s=v_0t-4,9t^2$ tenglamadan topiladi, bu yerda t - jism s (metr hisobida) balandlikka erishishi uchun ketgan vaqt (sekund xisobida), v_0 - boshlang'ich tezlik (m/sek). Agar $v_0=100$ m/sek bo'lsa, jismning $t=5$ sek momentdagi tezligi va tezlanishini toping (havoning qarshiligini hisobga olmang). Necha sekunddan so'ng jism eng yuqori nuqtaga erishadi va bu yerdan qancha masofada ro'y beradi?

Yechilishi. $s=100-4,9t^2;\quad v=\dfrac{ds}{dt}=100-9,8t;$

$v_{t-5}=100-9,8\cdot 5=51$ m/sek; $a=\dfrac{dt}{dt}=-9,8$ m/sek^2.

Jism eng yuqori nuqtaga tezligi nolga teng bo'lganda erishadi, shuning uchun $v=0$ deb, t ni topamiz:

$$100-9,8t=0, t=10,2cek.$$

t ning topilgan $t=10,2$ sek qiymatini harakat tenglamasiga qo'yib, jism harakatining eng yuqori nuqtasini topamiz:

$$s=100\cdot 10,2-4,9(10,2)^2=510m$$

836. Jism er sirtidan $v_0=50$ m/sek boshlang'ich tezlik bilan yuqoriga tik otilgan: 1) $t=3$ sek momentdagi ko'tarilish balandligini; 2) $t=3$ sek momentdagi tezlik va tezlanishni; 3) jism ko'tarilgan eng yuqori nuqtani va bu nuqtaga ko'tarilish uchun ketgan vaqtni toping.

837. Jism $s=3e^{-2t}$ qonun bo'yicha to'g'ri chiziqli harakat qilmoqda. $t=0$ momentda jism harakatining tezligi va tezlanishini toping.

Yechilishi .

$$v_0=\frac{ds}{dt}3e^{-2t}\left(-2t\right)'=3e^{-2t}(-2)=-6e^{-2t}$$

$$v_{t=0}=-6e^{-2\cdot 0}=-6e^0=-6;\ a=\frac{dv}{dt}=-6e^{-2}(-2)=12e^{-2t};$$

$$a_{t=0}12e^{-2\cdot 0}=12e^{0}=12$$

838. Jism $s=3e^{-2t}$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilmoqda. $t=0$ momentda jism harakatining tezligi va tezlanishini toping.

839. Massasi m bo‘lgan moddiy nuqta F kuch ta’siri ostida $s=t^3+3t^2$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilmokda. $t=3$ momentda ana shu kuchni toping.

Ech i l ishi . Ma’lumki, massasi m bo‘lgan moddiy nuqtaga ta’sir etadigan kuch nuqtaning massasini uning tezlanishiga ko‘paytirilganiga teng, ya’ni $F=ma$.

Nuqta harakatining tezlanishini topamiz:

$$s=3t^2+6t;\ a=s''=6t+6$$

$$a_{t-3}=6\cdot 3+6=24$$

$a=24$ qiymatni kuch tenglamasiga qo‘yib,

$$F=m\cdot 24=24m$$

ni hosil qilamiz.

840. Massasi m bo‘lgan moddiy nuqta F kuch ta’siri ostida $s=-\sin 3t$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilmoqda. $t=\frac{\pi}{6}$ momentda ana shu kuchni toping.

841. Massasi m bo‘lgan moddiy nukta F kuch ta’siri ostida $s=5\sin\left(\frac{\pi}{3}t+\frac{\pi}{6}\right)$ qonun bo‘yicha sodda garmonik tebranadi. $T=0$ momentda ana shu kuchni toping.

43- §. Aralash masalalar

842. Funksiyalarning hosilalarini toping: 1) $y=\frac{1}{3}\sin^3 x-\sin x$;

2) $y=\cos(x+a)\sin(x-a)$; 3) $y=2\sin^2 x\cos 2x$

843. 1) $y=\sin 2x$ egri chizikda $\left(\frac{\pi}{6};\frac{\sqrt{3}}{2}\right)$ nuqtada; 2) $y=\cos 2x$ egri chiziqda $\left(\frac{\pi}{12};\frac{\sqrt{3}}{2}\right)$ nuqtada o‘tkazilgan urinma va normalning tenglamasini tuzing.

844. $y=\sin x$ va $y=\cos x$ egri chiziqlarning $\left(\frac{\pi}{2}\right)$ intervaldagi kesishish nuqtasida ular hosil qilgan o‘tkir burchakni toping.

845. Funksiyalarning hosilalarini toping: 1) $y=\frac{tg3x}{tg3x-1}$; 2) $y=tg2x-ctg2x$; $y=tg^2 2x+ctg^2 2x$

846. $y=tgx$ egri chiziq Ox o‘qni qanday burchak ostida kesib o‘tishini toping.

847. $y=tgx$ va $y=ctgx$ egri chiziqlarning $\left(0;\frac{\pi}{2}\right)$ intervaldagi kesishish iuqtasida ular hosil qilgan o‘tkir burchakni toping.

Funksiyalarning hosilalarini toping:

848. 1) $y=\ln(x-\sqrt{x^2-1})$; 2) $y=\ln\frac{x+\sqrt{x^2-1}}{\sqrt{x^2-1}}$;

3) $y=\ln\sqrt{\frac{1+ax}{1-ax}}$

849. 1) $y=\ln\sqrt{\frac{1+\cos x}{1-\cos x}}$; 2) $y=\ln\sin^2(x-)$; 3) $u=\ln tg^2 z^2$

850. 1) $y=\ln\sqrt{x}\ln x^2$; 2) $y=\ln\left(\ln\frac{1}{x}\right)$;

3) $y=\ln\left(\ln\sqrt{x^2-1}\right)$

851. 1) $s=\ln e^{\sin 2t}$; 2) $y=e^{\sin x}\cos x$; 3) $y=e^{tgx}\cos^2 x$

852. 1) $y = \arccos\sqrt{1-x^2};$ 2) $u = \operatorname{arc}ctg\dfrac{1+z}{1-z};$
3) $y = \operatorname{arc}tg\sqrt{x} + arcctg\sqrt{x}$

853. 1) $y = \arccos\sqrt{1-e^{2x}};$ 2) $y = \arcsin\dfrac{e^x - e^{-x}}{e^x + e^{-x}};$
3) $y = \operatorname{arc}tg\sqrt{e^x - 1}$

Yozma ish

I-variant

854. Hosilalarni argumentning berilgan qiymatida toping:
1) $f(x) = \sin^2 \ln e^x,\ f'(0);$ 2) $f(x)\ln\sqrt{\dfrac{x+3}{x-3}},\ f'(\sqrt{6});$
3) $f(x) = \arccos\sqrt{x},\ f'\left(\dfrac{1}{2}\right);$ 4) $\dfrac{x^2}{6} - \dfrac{y^2}{8} = 1$ giperbolaga (- 3;2) nuqtada o‘tkazilgan urinmaning tenglamasini tuzing; 5) nuqta $s = -t^2 + 6t^2 + 5$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilmoqda. Nuqtaning tezlanishi nolga teng bo‘lgan t momentni toping.

II-variant

855. Hosilalarni argumentning berilgan qiymatida toping:
1) $f(x) = \ln tg^2 2x,\ f'\left(\dfrac{\pi}{8}\right);$ 2) $f(x) = 2\ln\sqrt{\sin 2x}\ f'\left(\dfrac{\pi}{8}\right);$ 3) $f(x) = arctg\dfrac{1}{e^x},\ f'(0);$ 4) $\dfrac{x^2}{32} + \dfrac{\pi^2}{18} = 1$ellipsga (-4; 3) nuqtada o‘tkazilgan normalning tenglamasini tuzing; 5) nuqta
$s = -\dfrac{1}{3}t^3 + 4t^2 - 8t + 1$ qonun bo‘yicha to‘g‘ri chiziqli harakat qilmoqda. Nuqtaning tezlanishi 6 m/sek^2 bo‘ladigan t vaqt momentini toping.

7 BOB

HOSILANI FUNKSIYALARNI TEKSHIRISHGA TATBIQ ETISH

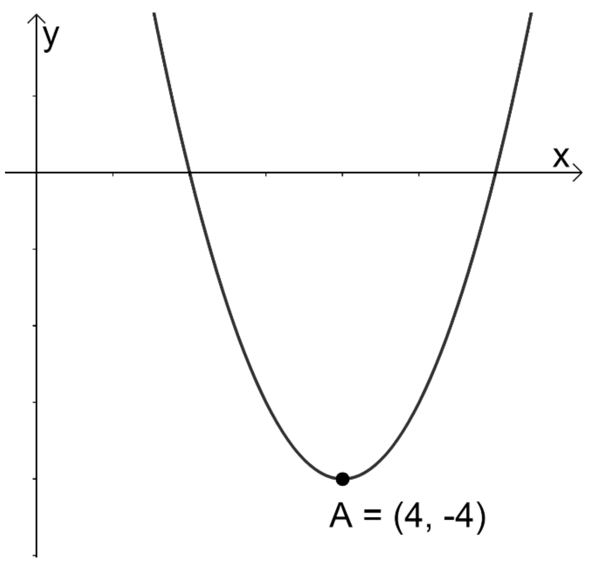
44- §. Funksiyaning o‘sishi va kamayishi

Agar $y = f(x)$ funksiyaning qiymatlari (a,b) intervalda x o‘sishi bilan o‘sib borsa, u holda bu funksiya x ning o‘zgarish intervali (a,b) da o‘suvchi funksiya deyiladi. Agar $y = f(x)$ funksiyaning qiymatlari (a,b) intervalda x o‘sishi bilan kamayib borsa, u holda bu funksiya x ning o‘zgarish intervali (a,b) da kamayuvchi deyiladi. Bu intervallar monoton o‘zgarish intervallari deyiladi.

Funksiyaning o‘sish va kamayish alomatlari.

Agar berilgan funksiyaning hosilasi x ning (a,b) intervaldagi barcha qiymatlari uchun musbat bo‘lsa, u holda funksiya bu intervalda o‘sadi.

Agar berilgan funksiyaning hosilasi x ning (a,b) intervaldagi barcha qiymatlari uchun manfiy bo‘lsa, u holda funksiya bu intervalda kamayuvchi bo‘ladi.



81-rasm

Funksiyalarning o‘sish va kamayish intervallarini toping.

856. $y = x^2 - 8x + 12$

Yechilishi. Berilgan funksiyaning hosilasini topamiz:

$$y' = 2x - 8$$

Berilgan funksiyaning hosilasi kamayish intervalida manfiy, o‘sish intervalida musbat, shuning uchun quyidagi tengsizliklarni yechamiz:

1) $2x-8<0;\quad 2x<8, x<4$ ya'ni x ushbu (-∞; 4) intervalda o'zgaradi; bu intervalda funksiya kamayadi;
2) $2x-8>0;$, $x>4$, ya'ni x ushbu (4; + ∞) intervalda o'zgaradi; bu intervalda funksiya o'sadi (81- rasm).

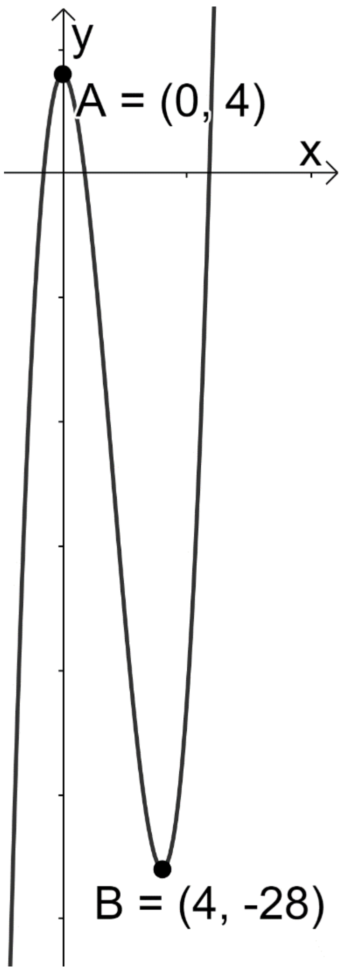
857. 1) $y=x^2-6x+5;\quad 2) y=2x^2-4x+5;$
$3) y=-x^2+4x+1$

858. $y=x^3-6x^2+4$

Yechilishi . Berilgan funksiyaning hosilasini topamiz:

$$y'=3x^2-12x$$

Kamayish intervalini topish uchun $3x^2-12x<0$ tengsizlikni echamiz:



82-rasm

$$x^2-4x<0$$

$D=16>0$ (168- betdagi 2- jadval, III xol). $x^2-4x=0$ tenglamaning ildizlari $x_1=0,$ $x_2=0,$ Tengsizlik x ning (0,4) intervaldagi barcha qiymatlari uchun to'g'ri. Demak, (0,4) intervalda funksiya kamayadi.

O'sish intervalini topamiz: $3x^2-12x>0,$
$x^2-4x>0$ tengsizlik (-∞; 4) va (4; + ∞) intervallardagi barcha haqiqiy qiymatlar uchun o'rinlidir. Bu intervallarda funksiya o'sadi (82- rasm).

859. 1) $y=x^2-3x^2+1,$

$2) y=-\frac{1}{3}x^2+\frac{1}{2}x^2+2,$

860. $y=x^4-4x+3$

Yechilishi. Berilgan funksiyaning hosilasini topamiz: $y'=4x^3-4$

Funksiyaning kamayish intervalini topamiz: $4x^3-4<0$, $x^3-1<0,$ $x^3<1,$ $x<1$demak, kamayish intervali$(-\infty;1)$
.

Funksiyaning o‘sish intervalini topamiz: $4x^3 - 4 > 0$ $x^3 - 1 > 0$, $x^3 > 1$, $x > 1$ o‘sish intervali $(1; +\infty)$.

861. 1) $y = x^2 - 32x + 40$; 2) $y = \frac{1}{4}x^4 + x - 1$.

862. $y = 2x^3 - 9x^2 + 12x - 15$.

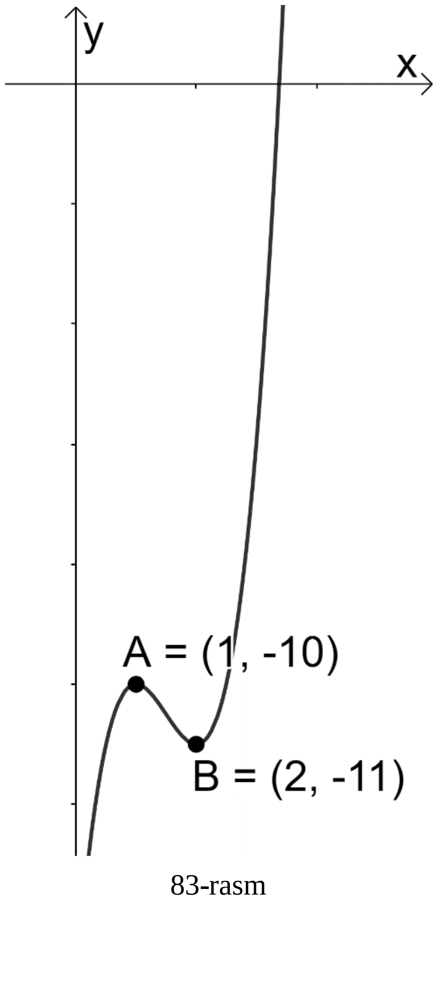
Yechilishi. Berilgan funksiyaning hosilasini topamiz:

$$y' = 6x^2 - 18x + 12$$

Funksiyaning kamayish intervalini topamiz:

$$6x^2 - 18x + 12 < 0, \qquad x^2 - 3x + 2 < 0.$$

$D = 9 - 8 = 1 > 0$ (168-betdagi 2- jadval, III hol). $x^2 - 3x + 2 = 0$ tenglamaning ildizlari: $x_1 = 1$, $x_2 = 2$.



83-rasm

Tengsizlik x ning (1, 2) intervaldagi barcha haqiqiy qiymatlari uchun o‘rinlidir. Demak, funksiyaning kamayish intervali (1, 2) ekan.

Funksiyaning o‘sish intervalini topamiz:

$$x^2 - 3x + 2 > 0.$$

Tengsizlik $(-\infty; 1)$ va $(2; +\infty)$ intervallardagi barcha haqiqiy qiymatlarda o‘rinli, demak, bu intervallarda funksiya o‘sadi (83- rasm).

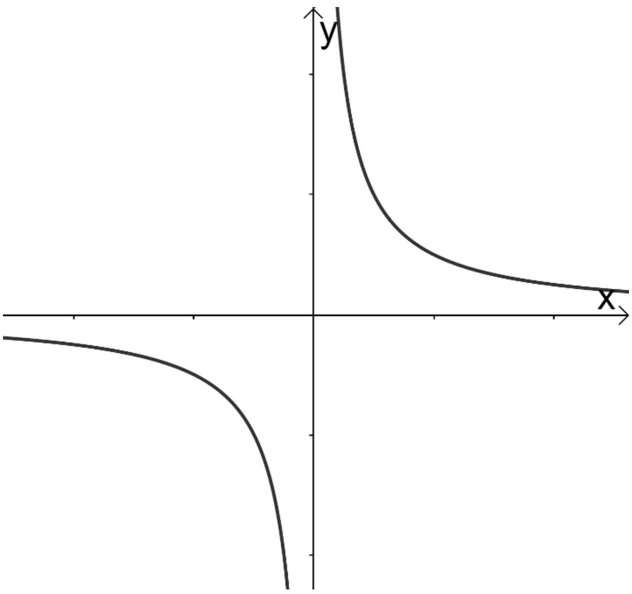
863. $y = 2x^2 - 15x + 36x - 20$.

864. $y = \frac{1}{2x}$.

Yechilishi. Berilgan funksiyaning hosilasini topamiz:

$y' = -\frac{1}{2x^2}$. $y = \frac{1}{2x}$ funksiyaning aniqlanish sohasi: $(-\infty; 0)$ va $(0; +\infty)$, $y' = -\frac{1}{2x^2}$ hosila funksiya aniqlanish sohasining hamma nuqtalarida manfiy bo‘ladi, chunki x argument kvadratda aniqlangan. Demak, funksiya $(-\infty; 0)$ va $(0; +\infty)$ intervallarda kamayadi (84- rasm).

865. $y = -\frac{1}{x}.$



84-rasm

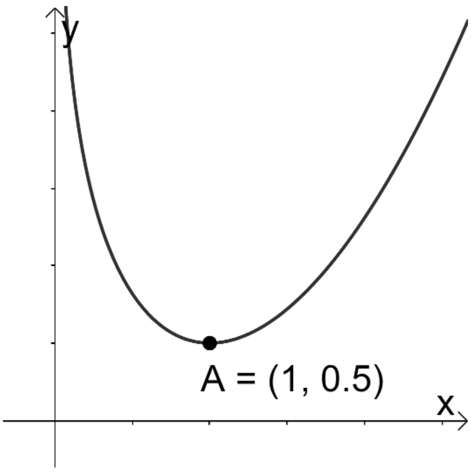
856. $y = \ln x.$

Yechilishi. Berilgan funksiyaning hosilasini topamiz: $y' = \frac{1}{x}$. $y = \ln x$ funksiyaning aniqlanish sohasi: (0; $+\infty$); bu sohada hosila musbat: $\frac{1}{x} > 0$, demak, (0; $+\infty$) intervalda funksiya o‘sadi.

867. 1) $y = \ln x^2$; 2) $y = \ln \frac{1}{x}$.

868. $y = \frac{1}{2}x^2 - \ln x.$

Yechilishi. $\ln x$ hadning aniqlanish sohasi (0; $+\infty$) intervaldan iborat, demak, $\frac{1}{2}x^2$ hadning argumenti faqat musbat qiymatlar qabul qilishi mumkin.



85-rasm

Berilgan funksiyaning hosilasini topamiz: $y' = x - \frac{1}{x} = \frac{x^2 - 1}{x}$; berilgan funksiyaning aniqlanish sohasi barcha musbat sonlardan iborat bo‘lgani uchun $y' = \frac{x^2 - 1}{x}$ hosila $x > 1$ bo‘lganda - musbat, $0 < x < 1$bo‘lganda esa manfiy bo‘ladi. Demak, funksiya (0;1) intervalda kamayadi, (1; $+\infty$) intervalda esa o‘sadi (85-rasm).

869. $y = \ln x - \frac{1}{3}x^3.$

870. $y = e^{-x}$.

Yechilishi. Berilgan funksiyaning hosilasini topamiz:
$y' = -e^{-x}$ hosila istalgan x da manfiydir, demak, funksiya $(-\infty, +\infty)$ intervalda kamayadi.

871. 1) $y = e^{x^2}$; 2) $y = e^{\frac{1}{x}}$.

872. $y = \sqrt{x - x^2}$.

Yechilishi. Funksiyaning aniqlanish sohasini topamiz:
$x - x^2 \ge 0$ yoki

$$x^2 - x \le 0.$$

$D > 0$ (168- betdagi 2- jadval, III hol). $x^2 - x = 0$ tenglamaning ildizlari: $x_1 = 0$ va $x_2 = 1$. Tengsizlik (nolga tenglik) x ning [0; 1] yopiq intervaldagi barcha haqiqiy qiymatlari uchun o‘rinlidir.

Demak, funksiya [0; 1] yopiq intervalda aniqlangan.

Berilgan funksiyaning hosilasini topamiz:

$$y' = \frac{1 - 2x}{2\sqrt{x - x^2}}.$$

Funksiyaning o‘sish intervalida hosila musbat:

$$\frac{1 - 2x}{2\sqrt{x - x^2}} > 0.$$

Kasr musbat bo‘lishi uchun surat va maxraj bir xil ishoraga ega bo‘lishi kerak. $2\sqrt{x - x^2} > 0$, demak, surat ham $1 - 2x > 0$, quyidagiga egamiz:

$$\begin{cases} 1 - 2x > 0 \\ 2\sqrt{x - x^2} > 0, \end{cases}, \qquad \text{bu yerdan} \quad -2x > -1, \quad x < \frac{1}{2}.$$

Funksiyaning aniqlanish sohasi [0; 1] ekanligini nazarga olsak:
$0 \le x \le \frac{1}{2}$.

Maxraj $2\sqrt{x - x^2} > 0$, biroq $x = 0$ va $x = 1$ da u nolga aylanadi, funksiya [0; 1] intervalda aniqlangani uchun esa x faqat $0 < x < 1$ qiymatlarni qabul qilishi mumkin.

Shunday qilib, $0 \le x < \frac{1}{2}$ va $0 < x < 1$, bu yerdan $0 < x < \frac{1}{2}$.

Demak, $\left(0; \frac{1}{2}\right)$ intervalda funksiya o‘suvchi ekan.

Funksiyaning kamayish intervalida

$$\frac{1-2x}{2\sqrt{x-x^2}} < 0.$$

Kasr manfiy bo‘lishi uchun surat va maxrajning ishoralari turlicha bo‘lishi kerak. $2\sqrt{x-x^2} > 0$, demak, surat ham $1-2x < 0$ bo‘lishi kerak, bu yerdan $-2x < -1$ yoki $x > \frac{1}{2}$. Funksiya $[0;1]$ intervalda aniqlanganligini nazarda tutsak, funksiya $\left(\frac{1}{2};1\right)$ intervalda kamayuvchi ekanligini hosil qilamiz.

873. $y = \sqrt{x^2 - 2x}$

45-§. Funksiyaning maksimum va minimumini birinchi tartibli hosila yordamida tekshirish

Argumentning funksiya eng katta qiymatga ega bo‘ladigan qiymati funksiyaning *maksimum nuqtasi* deyiladi.

Argumentning funksiya eng kichik qiymatga ega bo‘ladigan qiymati funksiyaning *minimum nuqtasi* deyiladi.

Funksiyaning maksimum nuqtasi funksiyaning o‘sishdan kamayishga o‘tishida chegaraviy nuqta hisoblanadi, mos ravishda funksiyaning minimum nuqtasi uning kamayishdan o‘sishga o‘tishida *chegaraviy nuqta* hisoblanadi.

Funksiyaning maksimumi va minimumi terminlari bitta terminga birlashtirilib, funksiyaning *ekstremumi* deyiladi.

Funksiya bir nechta ekstremumga ega bo‘lishi mumkin, shu sababli ekstremum nuqtalar unga qo‘shni bo‘lgan nuqtalarga nisbatan qaraladi.

Agar a ga etarlicha yakun bo‘lgan barcha x nuqtalarda $f(a) > f(x)$ tengsizlik bajarilsa, $y = f(x)$ funksiya $x = a$ nuqtada maksimumga ega bo‘ladi.

Agar a ga etarlicha yaqin bo‘lgan barcha x nuqtalarda $f(a) < f(x)$ tengsizlik bajarilsa, $y = f(x)$ funksiya $x = a$ da minimumga ega bo‘ladi.

$y = f(x)$ **funksiya maksimumining yetarli sharti**

Agar

1) $f'(a) = 0$;

2) $x < a$ da $f'(x) > 0$;

3) $x > a$ da $f'(x) < 0$ bo’lsa,

$y = f(x)$ funksiya $x = a$ da maksimumga ega bo‘ladi.

$y = f(x)$ **funksiya minimumining yetarli sharti**

Agar

1) $f'(a) = 0$;

2) $x < a$ da $f'(x) < 0$;

3) $x > a$ da $f'(x) > 0$ bo’lsa,

$y = f(x)$ funksiya $x = a$ da minimumga ega bo‘ladi.

$f'(a) = 0$ bo‘ladigan $x = a$ nuqta $f(x)$ funksiyaning *statsionar nuqtasi* deyiladi.

Agar funksiya hosilaga ega bo‘lsa uning ekstremumini statsionar nuqtalarda izlash kerak.

$y = f(x)$ funksiyaning maksimum va minimumini birinchi tartibli hosila yordamida tekshirish qoidasi.

I. Berilgan funksiyaning $y' = f'(x)$ hosilasi topiladi.

II. Topilgan hosila nolga tenglanadi: $f'(x) = 0$. $f'(x) = 0$ tenglama yechiladi, ya’ni uning haqiqiy ildizlari (statsionar nuqtalar) topiladi: $x_1, x_2, x_3....., x_n$.

III. Topilgan $x_1, x_2, x_3....., x_n$ ildizlarni o’sib borish tartibida joylashtiriladi. $f'(x)$ hosilani ko‘paytuvchilarga yoyiladi va unga x_1 ildiz o’rniga x_1 dan kichik son qo‘yilib, hosilaning ishorasi topiladi, so‘ngra x_1 o‘rniga x_1 dan kattaroq (lekin albatta x_2 dan kichik) son qo‘yilib, yana hosilaning ishorasi topiladi.

Agar bunda:

1) hosila ishorasini (+) dan (-) ga o‘zgartirsa, $y = f(x)$ funksiya $x = x_1$ da maksimumga ega bo‘ladi;

2) hosila ishorasini (-) dan (+) ga o‘zgartirsa, $y = f(x)$ funksiya $x = x_1$ da minimumga ega bo‘ladi;

3) hosila ishorasi o‘zgarmasa, funksiya $x = x_1$ da minimumga ham, maksimumga ham ega bo‘lmaydi.

So‘ngra $f(x)$ hosilaning ishoralarini $x < x_2$ va $x > x_2$ uchun va shu tartibda har bir ildiz uchun topiladi.

IV. Funksiyaning maksimal va minimal qiymatlari topiladi. Buning uchun funksiyaning qiymatlari statsionar nuqtalarda (maksimum va minimum nuqtalarda) hisoblanadi.

V. Egri chiziqning grafigi nuqtalar (funksiyaning maksimum va minimum nuqtalari, egri chiziqning Ox va Oy o‘qlar bilan kesishish nuqtalari) bo‘yicha yasaladi.

Funksiyaning maksimum va minimumini tekshiring.

874. $y = x^2 - 4x.$

Yechilishi. 1. Berilgan funksiyaning hosilasini topamiz:

$$y' = 2x - 4.$$

2. Hosilani nolga tenglaymiz: $2x - 4 = 0$ va bu tenglamani yechib, $x = 2$ statsionar nuqtani topamiz.

3.Hosilani ko‘paytuvchilarga ajratamiz: $y' = 2x - 4 = 2(x - 2).$ $x < 2$ ni (2 dan kichikroq) olamiz va x ning bu qiymatini (masalan, 1,9 ni) $y' = 2(x - 2)$ hosilaga hayolda (og‘zaki) qo‘yamiz va $x < 2$ da hosila ishorasini topamiz. Hosila manfiy ishoraga ega, uni qisqacha bunday yozamiz: $y_{x<2} = (-)$.

Endi $x > 2$ ni (2 dan kattaroq) olamiz va yana hayolan x ning bu qiymatini (masalan, 2,1 ni) $y' = 2(x - 2)$ hosilaga qo‘yib, hosilaning $x > 2$ dagi ishorasini topamiz, hosila musbat ishoraga ega, uni bunday yozamiz: $y_{x>2} = (+)$. Hosila ishorasini (-) dan (+) ga o‘zgartiryapti, demak, funksiya $x = 2$ da minimumga ega.

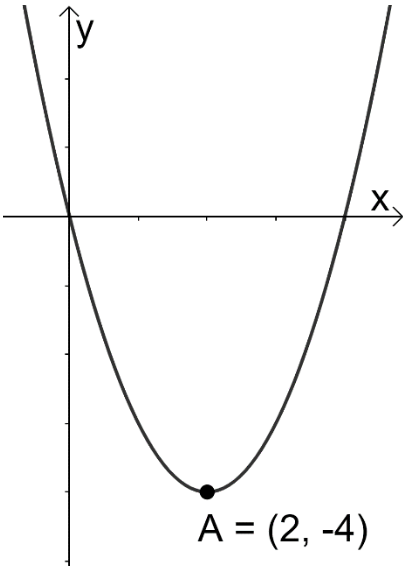
4. Funksiyaning minimal qiymatini topamiz, buning uchun berilgan funksiya ifodasiga $x = 2$ qiymatni qo‘yamiz:

$$y_{x=2} = 2^2 - 4 \cdot 2 = -4.$$

5. $y = x^2 - 4x$ funksiyaning grafigini yasaymiz. Argumentning qiymatlari va funksiyaning unga mos qiymatlari jadvalini tuzamiz:

x	0	2	4
y	0	-4	0
		Funksiya minimumi	Ox o‘q bilan kesishish nuqtasi

Bu nuqtalarini yasab, $y = x^2 - 4x$ parabolani hosil qilamiz (86-rasm).



86-rasm

Funksiyaning minimum nuqtasi (2;-4) parabolaning uchidir. Kelgusida parabolaning uchini kvadrat funksiyaning maksimumi yoki minimumi sifatida topishimiz mumkin.

875. 1**)** $y = x^2 - x$; 2) $y = x^2 + 3x$.

876. $y = -x^2 + 2x$.

Yechilishi. I. Hosilani topamiz: $y' = -2x + 2$.

2. Hosilani nolga tenglaymiz: $-2x + 2 = 0$ va statsionar nuqtani topamiz: $x = 1$.

3. Hosilani ko‘paytuvchilarga ajratamiz: $y' = -2(x-1)$. $x < 1$ da hosila ishorasi: $y'_{x<1} = (-)(-) = (+)$ [birinchi (-) ishora qavs oldidagi ishora, ikkinchi (-) ishora $(x-1)$ qavsning ishorasi].

$x > 1$ da hosila ishorasi: $y_{x>1} = (-)(+) = (-)$

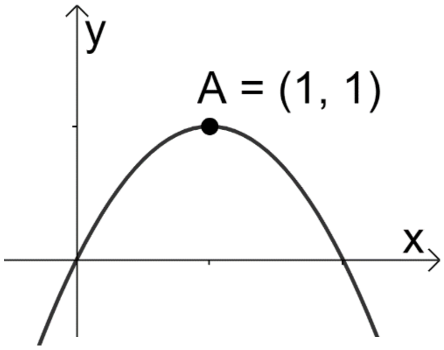
Hosila ishorasini (+) dan (-) ga o‘zgartiradi, demak, $x = 1$ da funksiya maksimumga ega ekan.

4. Funksiyaning $x = 1$dagi maksimal qiymatini topamiz:

$$y_{x=1} = -1^2 + 2 \cdot 1 = 1$$

5) Jadval tuzamiz:

x	0	1	2
y	0	1	0
		Funksiyaning maksimumi	Ox o‘q bilan kesishish nuqtasi



87-rasm

$y=-x^2+2x$ parabolani yasaymiz (87-rasm).

877. 1) $y=-x^2-x$; 2) $y=-x^2+4x$.

878. $y=x^2-8x+12$.

Yechilishi . 1) $y'=2x-8$;

2) $2x-8=0$; $x=4$;

3) $y'=2(x-4)$, $y'_{x<4}=(-)$; $y'_{x>4}=(+)$.

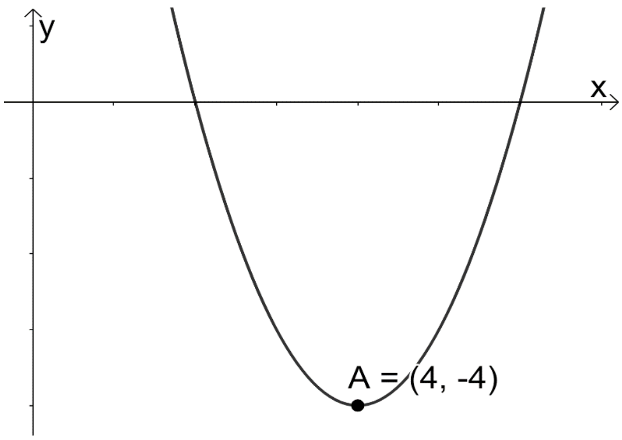
Hosila ishorasini (-) dan (+) ga o‘zgartiryapti. Demak, funksiya $x=4$ da minimumga ega;

4) Funksiyaning minimal qiymatini topamiz;

$y_{x=4}=4^2-8\cdot4+12=-4$;

5) Jadval tuzamiz:

x	0	2	4	6
y	0	0	-4	0
	Ou o‘q bilan kesishish nuqtasi	*Ox* o‘q bilan kesishish nuqtasi	Funksiyaning minimumi	*Ox* o‘q bilan kesishish nuqtasi



88-rasm

Endi $y=x^2-8x+12$ parabolani yasaymiz (88- rasm).

879. 1) $y=x^2-4x+3$; 2) $y=x^2-10x+9$

880. $y=-x^2+5x-6$.

Yechilishi. 1) $y'=-2x+5$;

2) $-2x+5=0$; $x=\frac{5}{2}=2,5$;

3) $y'=-2(x-2,5)$; $y'_{x<2,5}=(-)(-)=(+)$, $y'_{x>2,5}=(-)(+)=(-)$.

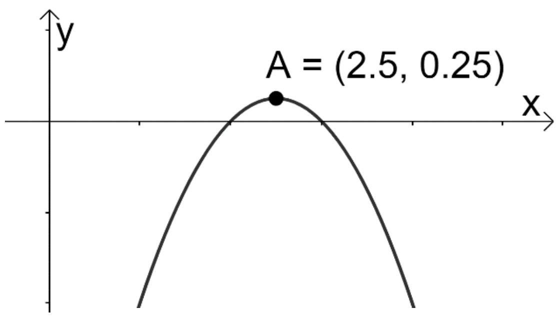
Hosila ishorasini (+) dan (-) ga o‘zgartiryapti, demak, funksiya $x=2,5$ da maksimumga ega;

4) funksiyaning maksimal qiymatini topamiz:

$$y'_{x=2,5} = -(2,5)^2 + 5\cdot 2,5 - 6 = 0,25$$

5) Jadval tuzamiz:

x	0	2	2,5	3
y	-6	0	0,25	0
	Oy o‘q bilan kesishish nuqtasi	Ox o‘q bilan kesishish nuqtasi	Funksiyaning maksimumi	Ox o‘q bilan kesishish nuqtasi



89-rasm

$y = -x^2 + 5x - 6$ parabolani yasaymiz (89- rasm).

881. 1) $y = -x^2 + 2x + 3$; 2) $y = -x^2 - x + 6$.

882. $s = 2t^2 - 8t + 6$

Yechilishi. 1) $s' = 4t - 8$;

2) $4t - 8 = 0,\ \ t = 2$.

3) $s = 4(-2);\ \ s_{t<2} = (-);.\,s_{t>2} = (+)$.

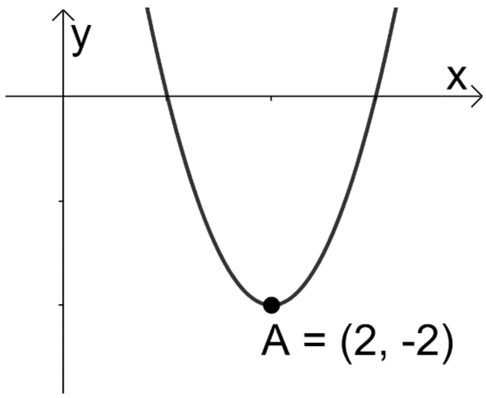
Hosila ishorasini (—) dan (+) ga o‘zgartiryapti, demak, funksiya $t = 2$da minimumga ega;

4) $s_{t=2} = 2^2 - 8\cdot 2 + 6 = -2$;

5) Jadval tuzamiz:

t	0	1	2	3
s	-6	0	-2	0
	Os o‘q bilan kesishish nuqtasi	Ot o‘q bilan kesishish nuqtasi	Funksiyaning minunimumi	Ot o‘q bilan kesishish nuqtasi

t argumentning son qiymatlarini Ot o‘q bo‘ylab, funksiyaning mos qiymatlarini Oso‘q bo‘ylab qo‘yib chiqib, $s = 2t^2 - 8t + 6$ funksiyaning grafigini hosil qilamiz (90- rasm).



90-rasm

883. 1) $s = 2t^2 - t - 1$; 2) $s = 2t^2 - 4t + 2$.

884. $y = \frac{1}{2}x^4$.

Yechilishi. 1) $y' = 2x^3$;

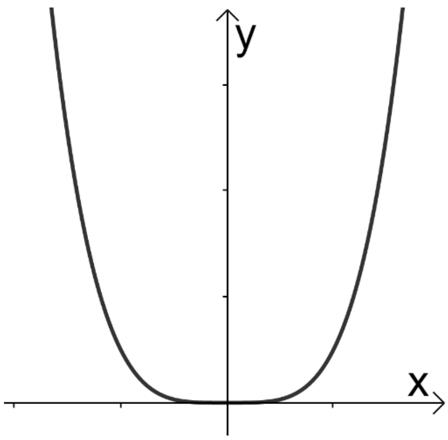
2) $2x^3 = 0, \quad x = 0$;

3) $y'_{x<0} = (-), y'_{x>0} = (+)$

hosila ishorasini (-) dan (+) ga o'zgartiryapti, demak, funksiya $x = 0$ da minimumga ega;

4) $y_{x=0} = \frac{1}{2} \cdot 0^4 = 0$

5) grafikni yasash uchun (0; 0) nuqtadan tashqari yana $\left(\pm 1; \frac{1}{2} \right)$ va $(\pm 2; 8)$ qo'shimcha nuqtalarni hosil qilamiz.



91-rasm

Ana shu nuqtalar bo'yicha grafikni yasaymiz (91- rasm).

885. 1) $y = 2x^4 - x$; 2) $y = \frac{1}{4}x^4 + 8x$

886. $y = x^3 - 3x^2$

Yechilishi. 1) $y' = 3x^2 - 6x$; 2) $3x^2 - 6x = 0$; $x^2 - 2x = 0, x(x-2) = 0$; $x_1 = 0$, $x_2 = 2$;

3) $y' = 3x(x-2)$

$a) x_1 = 0$ kritik nuqtani tekshiramiz.

$$y'_{x<0} = (-)(-) = (+); y'_{x>0} = (+)(-) = (-)$$

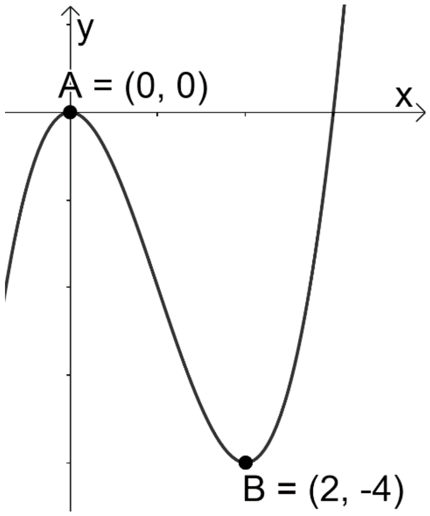
Birinchi (—) ishora x ko'paytuvchiga tegishli, ikkinchi (—) ishora $x - 2$ ko'paytuvchiga tegishli. Hosila ishorasini (+) dan (—) ga o'zgartiryapti. Demak, funksiya $x = 0$ da maksimumga ega.

b) $x_2 = 2$; kritik nuqtani tekshiramiz:

$$y_{x<2} = (+)(-) = (-); y'_{x>2} = (+)(+) = (+)$$

Hosila ishorasini (—) dan (+) ga o'zgartiryapti, demak, $x = 2$ da funksiya minimumga ega;

4) $y_{x=0} = 0^3 - 3 \cdot 0^2 = 0; \quad y_{x=2} = 2^3 - 3 \cdot 2^2 = -4$



92-rasm

5) grafikni (92- rasm) yasash uchun ba’zi bir nuqtalarning koordinatalarini hisoblaymiz.
Grafikning koordinata uchlari bilan kesishish nuqtalarini topamiz. $y=0$ deb, $x^3-3x^2=0;\ x^2(x-3)=0$ ni hosil qilamiz, bu yerdan $x=0$ va $x=3$, ya’ni (0; 0) va (3; 0),nuqtalarga ega bo‘ldik.
Ushbu jadvalni tuzamiz:

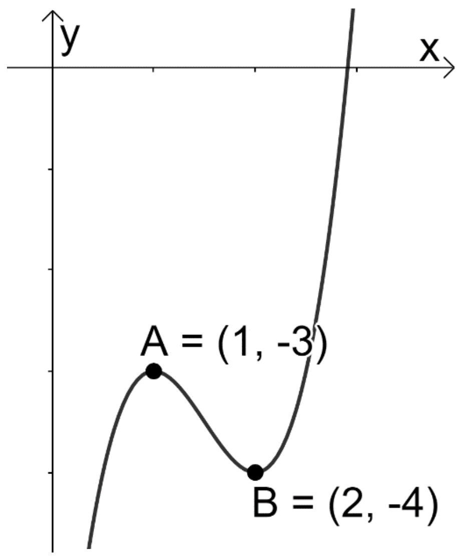
x	0	2	3
y	0	-4	0
	Funksiyaning maksimumi	Funksiyaning minumumi	Ox o‘q bilan kesishish nuqtasi

887. 1) $y=\frac{1}{3}x^3-4x;$ 2) $y=\frac{1}{3}x^3-x^2.$

888. $y=2x^3-9x^3+12x-8$

Yechilishi. 1) $y'=6x^2-18x+12;\quad 2)6x^3-18x+12=0\ x^2-3x+2=0;$
$x_1=1,\ x_2=2;\ 3).y'=6(x^2-3x+2)=6(x-1)(x-2)$

a) $x_1=1,$ kritik nuqtani tekshiramiz:
$y'_{x>1}=(-)(-)=(+);$
$y'_{x>1}=(+)(-)=(-);$
Hosila ishorasini (+) dan (-) ga o‘zgartiryapti, demak, funksiya $x_1=1$ da maksimumga ega.
b) $x_2=2$ kritik nuqtani tekshiramiz:
$y'_{x<2}=(+)(-)=(-);\qquad y'_{x>2}=(+)(+)=(+);$
Hosila ishorasini (-)dan (+) ga o‘zgartiryapti, funksiya $x=2$ da minimumga ega;



93-rasm

4) $y'_{x=1} = 2\cdot 1^3 - 9\cdot 1^2 + 12\cdot 1 - 8 = -3;$

$y'_{x=2} = 2\cdot 2^3 - 9\cdot 2^2 + 12\cdot 2 - 8 = -4;$

5) Ushbu nuqtalar egri chiziq grafigining nuqtalari bo‘ladi (93-rasm);

x	0	1	2
y	-8	-3	-4
	Oy o‘q bilan kesishish nuqtasi	Funksiyaning maksimumi	Funksiyaning minumumi

889. 1) $y = 2x^3 - 3x^2 - 12x + 8,$ 2) $y = 2x^3 + 9x^2 + 12x - 2$.

46- §. Funksiyaning maksimum va minimumini ikkinchi tartibli hosila yordamida tekshirish

$y = f(x)$ funksiyaning maksimum va minimumini ikkinchi tartibli hosila yordamida tekshirish qoidasi

I. Berilgan funksiyaning $y' = f'(x)$ hosilasi topiladi.

II. Topilgan hosila nolga tenglanadi: $f'(x) = 0;$ $f'(x) = 0$ tenglama echiladi, ya’ni haqiqiy ildizlar (statsionar nuqtalar) topiladi.

III. Berilgan funksiyaning ikkinchi tartibli hosilasi topiladi.

IV. Ikkinchi tartibli hosilaning ishorasi har qaysi statsionar nuqtada topiladi.

Agar bu nuqtada ikkinchi tartibli hosila manfiy bo‘lsa, u holda funksiya maksimumga ega bo‘ladi, agar hosila musbat bo‘lsa, funksiya minimumga ega bo‘ladi.

Agar ikkinchi tartibli hosila nolga teng bo‘lsa, u holda tekshirishni birinchi tartibli hosila yordamida o‘tkazish kerak.

V. Funksiyaning maksimal va minimal qiymatlari topiladi. Buning uchun statsionar nuqtalarda (maksimum va minimum nuqtalarda) funksiyaning qiymatlari hisoblanadi.

VI. Egri chiziqning topilgan nuqtalari (funksiyaning maksimum va minimum nuqtalari, egri chiziqning Ox va Oy o‘qlari bilan kesishish nuqtalari) bo‘yicha funksiyaning grafigi chiziladi (agar egri chiziq ikkinchi darajadan yuqori darajali tenglama bilan berilgan bo‘lsa, egri chiziqning

Ox o‘q bilan kesishish nuqtalarini topish qiyin, chunki elementar algebra kursida yuqori tartibli tenglamalarni echishning xususiy hollarigina ko‘riladi).

Ikkinchi tartibli hosila yordamida maksimum va minimumni tekshiring.

890. $y = x^2 - 2x - 3$.

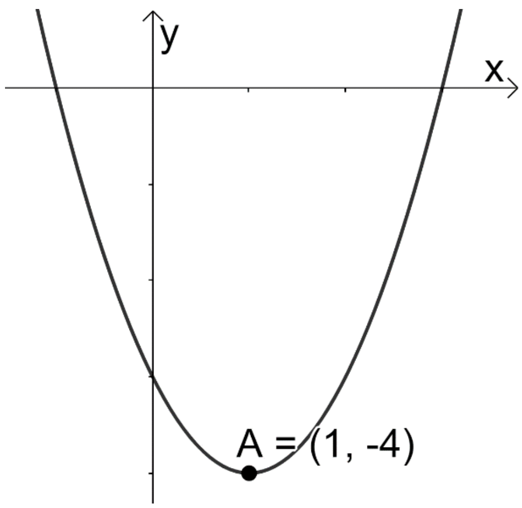
Yechilishi. 1. Birinchi tartibli hosilani topamiz.

$$y' = 2x - 2.$$

2. Birinchi tartibli hosilani nolga tenglab, statsionar nuqtani topamiz: $2x - 2 = 0, x = 1$

3. Ikkinchi tartibli hosilani topamiz: $y'' = 2$.

4. Ikkinchi tartibli hosila musbat, demak, funksiya $x = 1$ statsionar nuqtada minimumga ega (94- rasm).



94-rasm

891. 1) $y = 2x^2$; 2) $y = 2x^2 - 2$; 3) $y = x^2 - 2x$; 4) $y = -x^2 + 4x$; 5) $y = 2x^2 - 5x + 2$; 6) $y = -x^2 + x + 6$

892. $y = x^3 - 9x^2 + 24x - 12$

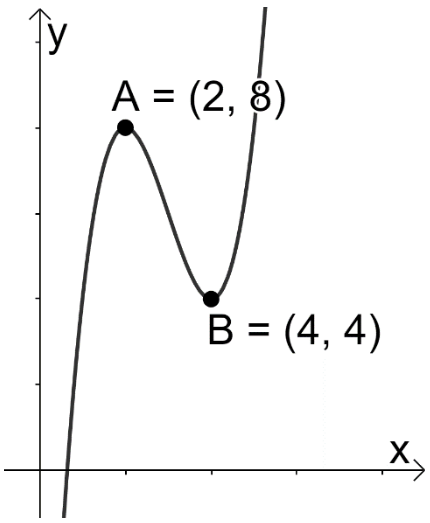
Yechilishi. 1) $y' = 3x^2 - 18x + 24$; 2) $3x^2 - 18x + 24 = 0$; $x^2 - 6x + 8 = 0$; $x_1 = 2$; $x_2 = 4$; 3) $y'' = 6x - 18$; 4) ikkinchi tartibli hosilaning statsionar nuktalardagi ishorasini topamiz: $y''_{x=1} = 6 \cdot 2 - 18 < 0; x = 2$ da funksiya maksimumga ega. $y''_{x=4} = 6 \cdot 4 - 18 > 0; x = 4$ da funksiya minimumga ega; 5) funksiyaning maksimal va minimal qiymatlarini topamiz:

$$y_{x=2} = 2^3 - 9 \cdot 2^2 + 24 \cdot 2 - 12 = 8$$

$$y_{x=4} = 4^3 - 9 \cdot 4^2 + 24 \cdot 4 - 12 = 4$$

6) jadval tuzamiz:

x	0	2	4
y	-12	8	4
	Oy o‘q bilan kesishish nuqtasi	Funksiyaning maksimumi	Funksiyaning minumumi



95-rasm

Funksiyaning grafigini yasaymiz (95-rasm).

893. 1) $y=\frac{1}{3}x^3-2x^2+3x+4;$

2) $y=\frac{1}{3}x^3-3x^2+5x+5;$

3) $y=x^3-\frac{9}{2}x^2+6x-2$

894. $y=3x^4-16x^3+30x^2-24x+9$

Yechilishi.

1) $y'=12x^3-48x^3+60x-24;$ 2) $12x^3-48x^2+60x-24=0;$

$x^3-4x^2+5x-2=0;$

Uchinchi tartibli bu tenglamani yechish uchun chap tomonni chiziqli ko‘paytuvchilarga ajratamiz, buning uchun ikkinchi va uchinchi hadni ikkita qo‘shiluvchining yig‘indisi shaklida quyidagicha ifodalab, tenglama hadlarini gruppalaymiz:

$$x^3-x^2+3x^2 3x+2x-2=0; \quad (x^3-x^2)+-(3x^3-3x)+(2x-2)=0;$$

$$x^2(x-1)-3x(x-1)+2(x-1)=(x-1)(x^2-3x+2)=0;$$

Har bir ko‘paytuvchini nolga tenglab, statsionar nuqtalarni topamiz:

$x-1=0;$ $x_1=0;$ $x^2+3x+2=0;$ $x_2=1;$ $x_3=2;$

3) $y''=36x^2-96x+60=0;$

4) $x=1;$ statsionar nuqtada ikkinchi tartibli hosilaning ishorasini aniqlaymiz:

$$y_{x=1}=36\cdot 1-96\cdot 1+60=0;$$

Ikkinchi tartibli hosila nolga teng, shuning uchun funksiya maksimumga yoki minimumga ega ekanligini aniqlash mumkin emas. $x=1$statsionar nuqtani birinchi tartibli hosila yordamida tekshiramiz; birinchi tartibli hosilani ko‘paytma ko‘rinishida ifodalaymiz:

$$y'=(x-1)(x-1)(x-2);$$

Birinchi tartibli hosilani argumentning birdan kichikroq va kattarok, qiymatlari uchun tekshirib, quyidagiga ega bo‘lamiz:

$$y_{x<1}=(-)(-)=(-);$$

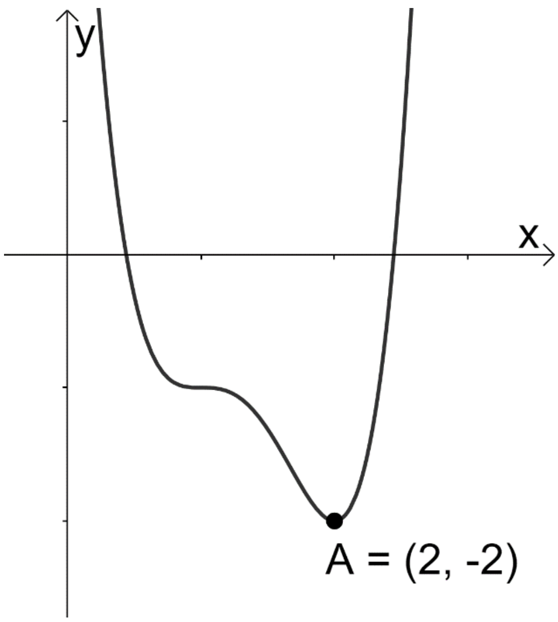
$$y_{x>1}=(+)(+)(-)=(-);$$

Hosila ishorasini o‘zgartirmayapti, demak, $x=1$ da funksiya maksimumga ham, minimumga ham ega emas.

$x=2$ statsionar nuqtani tekshiramiz;

$$y''_{x=2}=36\cdot 2^2-96\cdot 2+60>0;$$

$x=2;$ da funksiya minimumga ega;



96-rasm

5) funksiyaning minimal qiymatini topamiz:

$$y_{x=2}=3\cdot 2^4-16\cdot 2^3+30\cdot 2^2-24\cdot 2+6=-2;$$

6) jadval tuzamiz:

x	0	1	2	3
y	6	-1	-2	15
	Oy o‘q bilan kesishish nuqtasi		Funksiyaning minumumi	

Funksiya grafigini tuzamiz (96-rasm).

895. $y = x^4 + 3x^2 - 4;$

896. $y = \dfrac{x^3+1}{x}$

Yechilishi .

1) $y' = \dfrac{(x^2+1)'x - x'(x^2+1)}{x^2} = \dfrac{2x \cdot x - x^2 - 1}{x^2} =$
$= \dfrac{x^2-1}{x^2} = 1 - \dfrac{1}{x^2}$

2) $\dfrac{x^2-1}{x^2} = 0$. Kasrning surati nolga teng bo‘lsa (maxraj nolga teng emas), kasr ham nolga teng bo‘ladi:

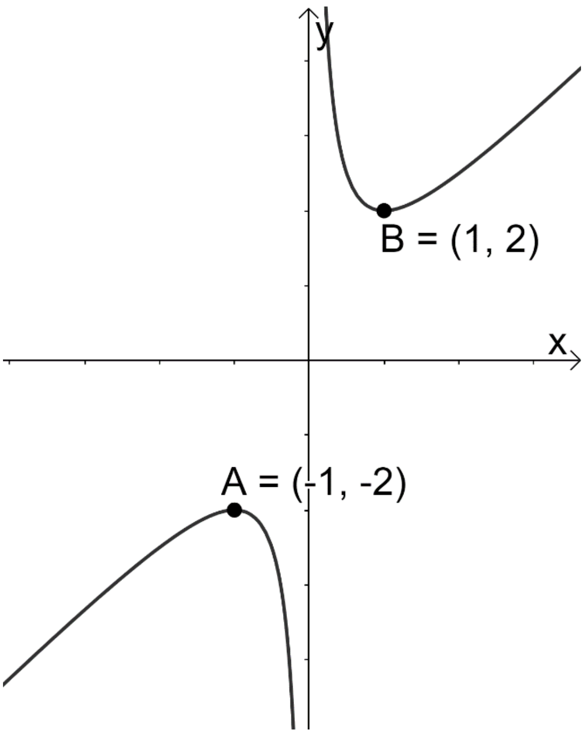
$x^2 - 1 = 0;\ \ x_1 = -1;\ \ \ x_2 = 1$

3) $y'' = \dfrac{1}{x^4} \cdot 2x = \dfrac{2}{x^3};$

4) $y''_{x=-1} = -2 < 0$, demak, funksiya $x = -1$da maksimumga ega;
$y''_{x=1} = 2 > 0$. Demak; funksiya $x = 1$da minimumga ega;

5) funksiyaning maksimal va minimal qiymatlarini topamiz:

$y_{x=-1} = -2$; $\ \ \ y_{x=1} = 2$



97-rasm

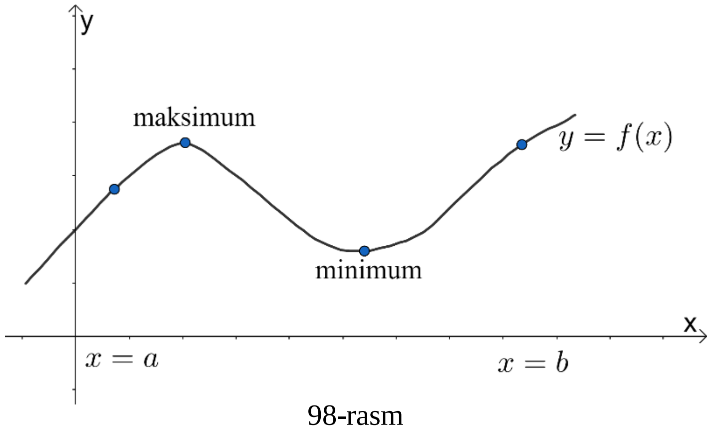
6) funksiyaning grafigini yasaymiz (97- rasm) $x = 0$ nuqtada funksiya uzilishga ega.

897. $y = \dfrac{3x}{x^2+1}$

47- §. Funksiyaning eng katta va eng kichik kiymatlari

Nazariy masalalarda va tatbiqlarda ko‘pincha x argumentning shunday qiymatlarini topishga to‘g‘ri keladiki, bu kiymatlarga $[a,b]$ kesmada uzluksiz bo‘lgan $y = f(x)$ funksiyaning eng katta va eng kichik qiymatlari mos keladi.

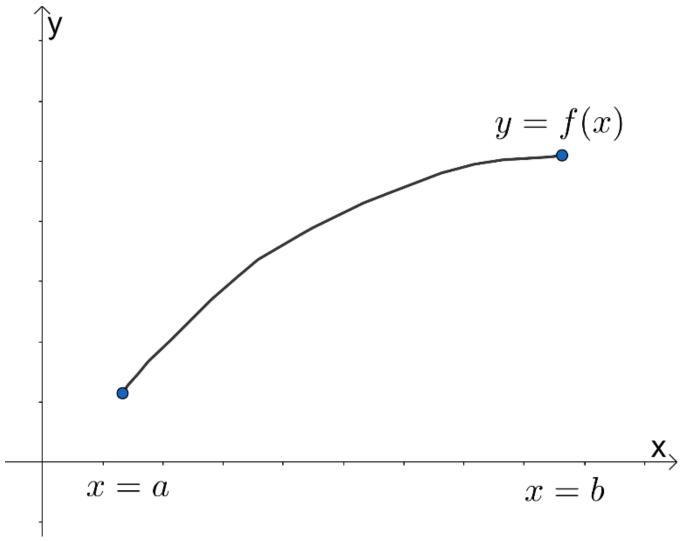
Eng katta va eng kichik qiymatlar mos ravishda maksimum va minimum bo‘lishi ham (98- rasm), bo‘lmasligi ham mumkin (99- rasm).



98-rasm

Funksiyaning eng katta va eng kichik qiymatlarini topishda quyidagi qoidalar bo‘yicha ish tutamiz:

1) Statsionar nuqtalar topiladi;
2) Funksiyaning statsionar nuqtalardagi va kesma uchlaridagi qiymatlari topiladi. Bu sonlarning eng kattasi va eng kichigi mos ravishda funksiyaning kesimadagi eng katta va eng kichik qiymati bo‘ladi.



99-rasm

Funksiyaning kesmadagi eng katta va eng kichik qiymatini toping.

898. $f(x) = x^2 - 4x + 3$ $[0,3]$ kesmada.

Yechilishi. $f'(x) = 2x - 4$.

Statsionar nuqtani topamiz.

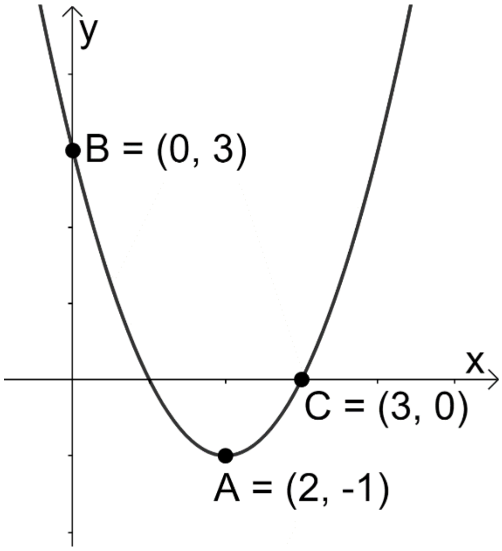
$$2x - 4 = 0, x = 2, f''(x) = 2, f(2) = -1$$

Demak, minimum $(2;-1)$. Bu nuqta $[0,3]$ kesmaga tegishli.

Kesmaning uchlarini tekshiramiz: $f(0) = 3, \quad f(2) = 0$.

Funksiyaning eng katta qiymati 3ga, eng kichik qiymati (-1) ga teng (100-rasm).

899. 1) $f(x) = x^2 - 6x + 13, \quad [0,6]$



100-rasm

2) $f(x)=8-\frac{1}{2}x^2,\ \ [-2,2]$

900. 1) $f(x)=\frac{1}{2}x^2-\frac{1}{3}x^3,\ \ [1,3]$

2) $f(x)=6x^2-x^3,\ \ [-1,6]$

901. $f(x)=2\sin x-\cos 2x,\ \left[0,\frac{\pi}{2}\right]$

Yechilishi.

Statsionar nuqtalarni topamiz:

$$f'(x)=2\cos x-2\sin 2x=0,$$

$$2\cos x+4\sin x\cos x=0,$$

$$\cos x(1-2\sin x)=0,$$

$$\cos x=0, x=\frac{\pi}{2}+\pi k$$

$k=0,\pm1;2;...,1+2\sin x=0,\sin x=-\frac{1}{2}$

x ning qiymatlari $\left[0,\frac{\pi}{2}\right]$ kesmadan tashqarida yotadi va shuning uchun ularni hisoblab o‘tirmaymiz.

Ikkinchi tartibli hosilani topamiz:

$$f''(x)=-2\sin x+4\cos 2x;$$

$$f''\left(\frac{\pi}{2}\right)=-2\sin\frac{\pi}{2}+4\cos\pi=-2-4=-6;$$

ikkinchi tartibli hosila $x=\frac{\pi}{2}$ da manfiy va

$$f\left(\frac{\pi}{2}\right)=-2\sin\frac{\pi}{2}-\cos\pi=-2+1=3;$$

demak, maksimum $\left(\frac{\pi}{2};3\right)$.

Fuknsiyaning qiymatini $x=0$ nuqtada hisoblaymiz.

$$f(0)=2\sin 0-\cos 0=-1.$$

Funksiyaning qiymati 3ga, eng kichik kiymati (-1)ga teng.

902. $f(x)=\sin 2x\left|-\frac{\pi}{2},\frac{\pi}{2}\right|$ kesmada.

48-§. Miqdorlarning eng katta va eng kichik qiymatlariga doir masalalar

903. Ikkita musbat sonning yig‘indisi a ga teng. Ularning ko‘paytmasi eng katta bo‘lganda bu sonlarni topamiz.

Yechilishi. Qo‘shiluvchilarning biri x bo‘lsin; u holda ikkinchi son $a-x$ bo‘ladi. Bu qo‘shiluvchilarining ko‘paytmasi o‘zgaruvchi miqdor; uni y orqali belgilab, topamiz:

$$y = x(a-x) \text{ yoki } y = ax - x^2 \left(0 < x < a\right).$$

Bu funksiyaning maksimum va minimumini ikkinchi tartibli hosila yordamida tekshiramiz:

$$1)\ \ y' = a - 2x;\ 2)\ a - 2x = 0,\ x = \frac{a}{2};\quad 3)\ y'' = -2.$$

Ikkinchi tartibli hosila manfiy, demak, $x = \dfrac{a}{2}$ da fuksiya maksimumga ega, a sonni teng ikkiga bo‘lish kerak, shunda bu qo‘shiluvchilarning ko‘paytmasi eng katta bo‘ladi.

904. 24 sonini ko‘paytmasi eng katta bo‘lgan ikkiga qo‘shiluvchiga ajrating.

905. Ikkita musbat sonning yig‘indisi a ga teng. Agar ularning kublari yig‘indisi eng kichik bo‘lsa, bu sonlarni toping.

Yechilishi. Qo‘shiluvchilardan biri x bo‘lsin, u holda ikkinchi qo‘shiluvchi $a-x$ bo‘ladi. Bu qo‘shiluvchilarning kublari yig‘indisi o‘zgaruvchi miqdor; uni y orqali belgilab, quyidagiga ega bo‘lamiz:

$$y = x^3 + (a-x)^3 \ \ \left(0 < x < a\right)$$

yoki

$$y = a^3 - 3a^2x + 3ax^2.$$

Bu funksiyaning maksimum va minimumini ikkinchi tartibli hosila yordamida tekshiramiz: 1) $y' = -3a^2 + 6ax$;

$$2)\ -3a^2 + 6ax = 0,\ x = \frac{a}{2};\quad 3)\ y'' = 6a.$$

Ikkinchi tartibli hosila musbat, demak, $x = \dfrac{a}{2}$ da funksiya minimumga ega. a sonni teng ikkiga bo‘lish kerak, shunda bu qo‘shiluvchilarning kublari yig‘indisi eng kichik bo‘ladi.

906. 6 sonini shunday ikkita qo‘shiluvchiga ajratingki, ularning kvadratlari yig‘indisi eng kichik bo‘lsin.

907. Ikkita musbat sonning ko‘paytmasi a ga teng. Ularning yig‘indsi eng kichik bo‘lganda bu sonlarni toping.

Yechilishi. Ko‘paytuvchilarning biri x ga teng bo‘lsin, u holda ikkinchi ko‘paytuvchi $\frac{a}{x}$ ga teng bo‘ladi. Bu qo‘shiluvchilarning yig‘indisi – o‘zgaruvchi miqdor; uni y bilan belgilaymiz, u holda

$$y = x + \frac{a}{x} \quad (x > 0).$$

Bu funksiyaning maksimum va minimumini ikkinchi tartibli hosila yordamida tekshiramiz: 1) $y' = 1 - \frac{a}{x^2}$; 2) $1 - \frac{a}{x^2} = 0,\ x^2 = a,$ $x = \sqrt{a}$ (masala shartiga ko‘ra $x > 0$), 3) $y'' = \frac{a}{x^4} \cdot 2x = \frac{2a}{x^3}$; 4) $y''\big|_{x=\sqrt{a}} = \frac{2a}{\left(\sqrt{a}\right)^3} > 0,$ demak, funksiya $x = \sqrt{a}$ da minimumga ega.

Qo‘shiluvchilar o‘zaro teng bo‘lganda yig‘indi eng kichik bo‘ladi.

908. 9 sonini shunday ikkita musbat ko‘paytuvchiga ajratinki, ularning yig‘indisi eng kichik bo‘lsin.

909. Berilgan perimetrli to‘g‘ri to‘rtburchaklar ichidan yuzi eng katta bo‘lganini toping.

Yechilishi. To‘g‘ri to‘rtburchakning perimetri p ga teng bo‘lsin. To‘g‘ri to‘rtburchakning tomonlaridan birini x bilan belgilaymiz, u holda ikkinchi tomon

$$\frac{p-2x}{2} = \frac{p}{2} - x$$

ga teng bo‘ladi.

To‘g‘ri to‘rtburchakning yuzi - o‘zgaruvchi miqdor. Uni y bilan belgilaymiz, u holda

$$y = x\left(\frac{p}{2} - x\right) = \frac{p}{2}x - x^2 \left(0 < x < \frac{p}{2}\right)$$

Funksiyaning maksimum va minimumini ikkinchi tartibli hosila yordamida tekshiramiz: 1) $y' = \frac{p}{2} - 2x;$

2) $\frac{p}{2} - 2x = 0,\ x = \frac{p}{4};$ 3) $y'' = -2.$

Ikkinchi tartibli hosila manfiy, demak, fuksiya $x = \frac{p}{4}$ da maksimumga ega. Berilgan perimetrli barcha to‘g‘ri to‘rtburchaklar ichida kvadrat eng katta yuzga ega bo‘ladi.

910. Uzunligi 50 sm bo‘lgan sim bo‘lagidan eng katta yuzga ega bo‘lgan to‘g‘ri to‘rtburchak yasang.

911. Berilgan perimetrli barcha to‘g‘ri to‘rtburchaklar ichidan diagonali eng kichik bo‘lganini toping.

Yechilishi. To‘g‘ri to‘rtburchakning perimetri $2p$ ga va bir tomoni x ga teng bo‘lsin, u holda uning ikkinchi tomoni $\frac{2p-2x}{2}$ ga teng bo‘ladi. To‘g‘ri to‘rtburchakning diagonali – o‘zgaruvchi miqdor. Uni y bilan belgilaymiz, u holda Pifagor teoremasiga ko‘ra:

$$y^2 = x^2 + (p-x)^2; \qquad \text{yoki} \quad y^2 = 2x^2 - 2px + p^2;$$

ni hosil qilamiz, bu yerdan

$$y = \sqrt{2x^2 - 2px + p^2} \quad (0 < x < p)$$

Funksiyani birinchi tartibli hosila yordamida tekshiramiz:

1) $y' = \dfrac{4x - 2p}{2\sqrt{2x^2 - 2px + p^2}} = \dfrac{2x - p}{\sqrt{2x^2 - 2px + p^2}};$

2) $\dfrac{2x - p}{\sqrt{2x^2 - 2px + p^2}} = 0;\ 2x - p = 0;\ \ x = \dfrac{p}{2}$ (kvadrat);

3) $y' = \dfrac{2\left(x - \frac{p}{2}\right)}{\sqrt{2x^2 - 2px + p^2}}.$

Hosilaning maxraji musbat, shu sababli hosilaning faqat suratini tekshiramiz:

$$y'\big|_{x<\frac{p}{2}} < 0 \qquad \text{va} \qquad y'\big|_{x>\frac{p}{2}} > 0.$$

Hosila ishorasini (-) dan (+) ga o‘zgartiryapti. demak, funksiya $x = \dfrac{p}{2}$ da minimumga ega.

Berilgan perimetrli barcha to‘g‘ri to‘rtburchaklar ichidan eng kichik diagonalga ega bo‘lgani kvadratdir.

912. Perimetri 16sm bo‘lgan to‘g‘ri to‘rtburchaklarning qaysi biri eng kichik diaganalga ega bo‘ladi?

913. Berilgan yuzaga bo‘lgan barcha to‘g‘ri to‘rtburchaklar ichida eng kichik perimetrga ega bo‘lganini toping.

Yechilishi. To‘g‘ri to‘rtburchakning yuzi S , tomonlaridan biri x bo‘lsin, u holda to‘g‘ri to‘rtburchakning ikkinchi tomoni $\dfrac{S}{2}$ bo‘ladi.To‘g‘ri to‘rtburchakning barcha tomonlari yig‘indisi – o‘zgaruvchi miqdor; uni p bilan belgilab,

$$p = 2x + \frac{2S}{x}$$

ni hosil qilamiz.

Bu funksiyani ikkinchi tartibli hosila yordamida tekshiramiz:

1) $p' = 2 - \dfrac{2S}{x^2};$ 2) $2 - \dfrac{2S}{x^2} = 0,\ x = \sqrt{S};$

3) $p'' = \dfrac{4S}{x^3};$ 4) $p''\big|_{x=\sqrt{S}} = \dfrac{4S}{\left(\sqrt{S}\right)^3} > 0.$

Ikkinchi tartibli hosila musbat, demak, funksiya $x = \sqrt{S}$ da minumumga ega. Berilgan yuzga ega bo‘lgan barcha to‘g‘ri to‘rtburchaklar ichida kvadrat eng kichik perimetrga ega bo‘ladi.

914. Qog‘oz varag‘idan yuzi 100 sm^2 bo‘lgan shunday to‘g‘ri to‘rtburchak qirqib olingki, bu to‘g‘ri to‘rtburchakning perimetri eng kichik bo‘lsin.

915. R radiusli doiraga ichki chizilgan barcha to‘g‘ri to‘rtburchaklar ichidan eng katta yuzga ega bo‘lganini toping.

Yechilishi. Doiraga ichki chizilgan to‘g‘ri to‘rtburchakning diagonali $2R$ ga teng ; to‘g‘ri to‘rtburchakning tomonlaridan birini x bilan belgilaymiz, u holda ikkinchi tomon $\sqrt{(2R)^2 - x^2}$ bo‘ladi.

To‘g‘ri to‘rtburchakning tomoni – o‘zgaruvchi miqdor; uni y bilan belgilab,

$$y = x\sqrt{4R^2 - x^2} \quad (0 < x < 2R)$$

ni hosil qilamiz.

Bu funksiyani birinchi tartibli hosila yordamida tekshiramiz:

1) $y' = \sqrt{4R^2 - x^2} - \dfrac{2x \cdot x}{2\sqrt{4R^2 - x^2}} = \dfrac{4R^2 - 2x^2}{\sqrt{4R^2 - x^2}};$

2) $\dfrac{4R^2 - 2x^2}{\sqrt{4R^2 - x^2}} = 0, \ 4R^2 - 2x^2 = 0, \ x = R\sqrt{2};$

3) $y' = \dfrac{2\left(2R\sqrt{2} - x\right)\left(2R\sqrt{2} + x\right)}{\sqrt{4R^2 - x^2}}; \ y'\big|_{x<R\sqrt{2}} > 0, \quad y'\big|_{x>R\sqrt{2}} < 0.$

Hosila ishorasini (+) dan (-) ga o‘zgartirayapti, demak funksi $x = R\sqrt{2}$ da maksimumga ega.

To‘g‘ri to‘rtburchakning tomonlari $x = R\sqrt{2}$ va

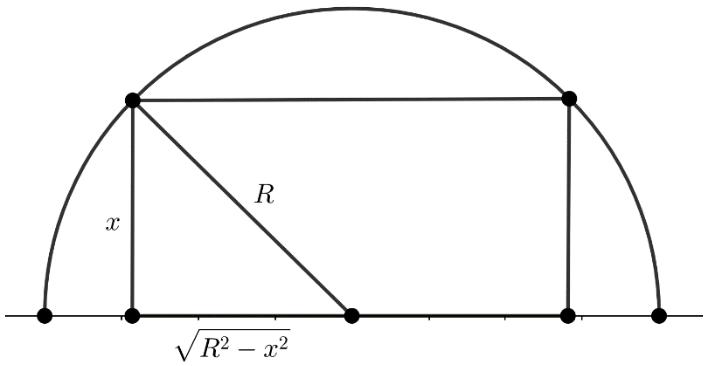
$$\sqrt{4R^2 - \left(R\sqrt{2}\right)^2} = \sqrt{4R^2 - 2R^2} = R\sqrt{2}$$

ga teng.

To‘g‘ri to‘rtburchakning tomonlari teng, demak doiraga ichki chizilgan to‘g‘ri to‘rtburchaklar ichida yuzi eng katta bo‘lgani kvadratdir.

916. R radiusli doiraga ichki chizilgan barcha to‘g‘ri to‘rburchaklar ichidan eng katta perimetrga ega bo‘lganini toping

917. R radiusli yarim doiraga eng katta yuzga ega bo‘lgan to‘g‘ri trtburchakni ichki chizing.



101-rasm

Yechilishi . To‘g‘ri to‘rtburchakning tomonlaridan birini x bilan belgilaymiz (101-rasm). Ikkinchi tomonni x tomon va R radius orqali Pifagor teoremasiga ko‘ra ifodalaymiz: $\sqrt{R^2 - x^2}$

Tomonlari x va $2\sqrt{R^2-x^2}$ bo‘lgan to‘g‘ri turtburchaknipg yuzi — o‘zgaruvchi mikdor; uni y bilan belgilab,

$$y = x\cdot 2\sqrt{R^2-x^2} = 2x\sqrt{R^2-x^2} \quad (0<x<R)$$

ni hosil qilamiz.
Bu funksiyani birinchi tartibli hosila yordamida tekshiramiz:

$$1) y' = 2\left[x'\sqrt{R^2-x^2}+\left(\sqrt{R^2-x^2}\right)'x\right] = 2\left(\sqrt{R^2-x^2}+\frac{-2x\cdot x}{2\sqrt{R^2-x^2}}\right) =$$

$$= 2\left(\frac{R^2-x^2-x^2}{\sqrt{R^2-x^2}}\right) = \frac{2\sqrt{R^2-2x^2}}{\sqrt{R^2-x^2}}$$

$$2)\ y' = \frac{\left(\sqrt{R^2-2x^2}\right)}{\sqrt{R^2-x^2}} == 0; \quad R^2-2x^2=0; \quad x=\frac{R}{\sqrt{2}};$$

$$3)\ y' = \frac{4\left(\frac{R^2}{2}-x^2\right)}{\sqrt{R^2+x^2}}\,\frac{4\left(\frac{R}{\sqrt{2}}-x\right)\left(\frac{R}{\sqrt{2}}+x\right)}{\sqrt{R^2-x^2}};$$

$$y'_{x<\frac{R}{\sqrt{2}}} = (+)(+) = (+); \qquad y'_{x>\frac{R}{\sqrt{2}}} = (-)(+) = (-)\ .$$

Hosila ishorasini (+) dan (-) ga o‘zgartiryapti, demak, funksiya $x=\frac{R}{\sqrt{2}}$ da maksimumga ega. To‘g‘ri to‘rtburchakning tomonlari:

$$x=\frac{R}{\sqrt{2}} \quad \text{va} \quad 2\sqrt{R^2-\left(\frac{R}{\sqrt{2}}\right)^2} = 2\sqrt{R^2-\frac{R^2}{2}} = 2\sqrt{\frac{R^2}{2}} = \frac{2R}{2}$$

To‘g‘ri to‘rtburchak tomonlarining nisbati: $\frac{R}{\sqrt{2}} : \frac{2R}{\sqrt{2}} = 1:2$.

918. R radiusli yarim doiraga eng katta perimetrga ega bo‘lgan to‘g‘ri turtburchakni ichki chizing.

919. Ma’lumki, to‘sinning siqishga bo‘lgan qarshiligi kesim yuziga proporsional. d diametrli dumaloq xodadan kesim yuzi to‘g‘ri to‘rtburchak bo‘lgan shunday to‘sin qirqib olish kerakki, uning siqishga bo‘lgan qarshiligi eng katta bo‘lsin.

Yechilishi . Agar to‘g‘ri to‘rtburchakning tomonlaridan birini x bilan belgilasak, uning ikkinchi tomoni $\sqrt{d^2-x^2}$ bo‘ladi. Kesim yuzi - o‘zgaruvchi miqdor: $x\sqrt{d^2-x^2}$.

To‘sinning siqishga bo‘lgan qarshiligini p bilan, o‘zgarmas bo‘lgan proporsionallik koeffitsientini k bilan belgilab,

$$p = kx\sqrt{d^2-x^2} \qquad (0<x<d)$$

ni hosil qilamiz.

Funksiyani soddalashtirish uchun $k=1$deb olamiz, u holda $p = x\sqrt{d^2-x^2}$. Bu funksiyani birinchi tartibli hosila yordamida tekshiramiz.

$$p' = x'\sqrt{d^2-x^2} + \left(\sqrt{d^2-x^2}\right)'x = \sqrt{d^2-x^2} +$$

1) $$+\frac{(-2x)x}{2\sqrt{d^2-x^2}} = \frac{d^2-x^2-x^2}{\sqrt{d^2-x^2}} = \frac{d^2-2x^2}{\sqrt{d^2-x^2}};$$

2) $$p' = \frac{d^2-x^2}{\sqrt{d^2-x^2}} = 0 \qquad x = \frac{d}{\sqrt{2}};$$

3) $$p' = \frac{\left(\frac{d^2}{2}-x^2\right)}{\sqrt{d^2-x^2}} = \frac{2\left(\frac{d}{\sqrt{2}}-x\right)\left(\frac{d}{\sqrt{2}}+x\right)}{\sqrt{d^2-x^2}};$$

$$p'_{x<\frac{d}{\sqrt{2}}} = (+)(+) = (+); \qquad p'_{x>\frac{d}{\sqrt{2}}} = (-)(+) = (-);$$

Hosila ishorasini (+) dan (-) ga o‘zgartiryapti, demak, funksiya $x = \frac{d}{\sqrt{2}}$ da maksimumga ega.

To‘sin kesimining o‘lchamlari $x = \frac{d}{\sqrt{2}}$ ga teng.

To‘sinning kesimi tomoni $\frac{d\sqrt{2}}{2} = 0,707d$ bo‘lgan kvadratdan iborat.

920. Ma’lumki, gorizontal to‘sinning egilishga qarshiligi kesim enini balandlikning kvadratiga ko‘paytmasiga proporiional. d

diametrli xodadan kesimida tug‘ri to‘rtburchak bo‘lgan shunday to‘sin qirqib olish kerakki, uning
egilishga qarshiligi gorizontal xolatda eng katta bo‘lsin.

Yechilishi . To‘sinning eni x bo‘lsin, u holda uning balandligi $\sqrt{d^2-x^2}$ bo‘ladi. Egilishga qarshilikni p bilan, proporsionallik koeffitsientini k bilan belgilab,

$$p=kx\left(\sqrt{d^2-x^2}\right)^2=kx\left(d^2-x^2\right)$$

ni hosil qilamiz.

O‘zgarmas koeffitsientni $k=1$ deb olamiz, u holda
$p=x\times\left(d^2-x^2\right)$ yoki $p=d^2x-x^3$ $(0<x<d)$.

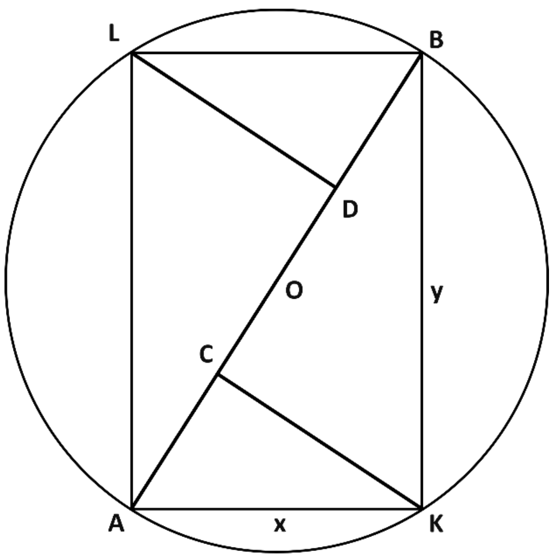
Funksiyani ikkinchi tartibli hosila yordamida tekshiramiz:

1) $p'=d^2-3x^2$; 2) $d^2-3x^2=0, x=\dfrac{d}{\sqrt{3}}$; 3) $p''=-6x$.

Ikkinchi tartibli hosila manfiy, demak, funksiya $x=\dfrac{d}{\sqrt{3}}$ da maksimumga ega.

Kesimning o‘lchamlari: $x=\dfrac{d}{\sqrt{3}}$ va

$$\sqrt{d^2-\left(\frac{d}{\sqrt{3}}\right)^2}=\sqrt{d^2-\frac{d^2}{3}}=\sqrt{\frac{2d^2}{3}}=d\sqrt{\frac{2}{3}}.$$



102-rasm

Tomonlarning nisbati:

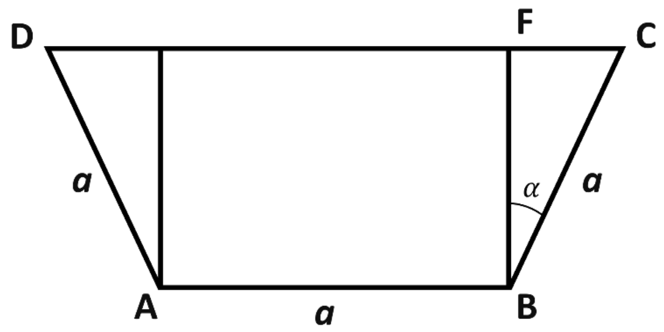
$$d\sqrt{\frac{2}{3}}:\frac{d}{\sqrt{3}}=\sqrt{2}$$

Tomonlari $\dfrac{d}{\sqrt{3}}$ va $d\sqrt{\dfrac{2}{3}}$ bo‘lgan to‘g‘ri turtburchakni yasalishi. Doiraning AB diametrini uchta teng bo‘lakka bo‘lamiz (102- rasm). Bo‘linish nuqtalari C va D dan AB ga (uning turli tomonlariga) aylana bilan K va L nuqtalarda kesishguncha perpendikulyarlar chiqaramiz $AKBL$ to‘g‘ri to‘rtburchak izlanayotgan to‘g‘ri to‘rtburchak ekanligini isbot qilamiz. To‘g‘ri burchakli uchburchakdagi metrik munosabatlar haqidagi teoremaga ko‘ra quyidagiga egamiz:

$$AK^2 = AC \cdot AB = \frac{1}{3} d \cdot d = \frac{1}{3} d^2, AK = \frac{d}{\sqrt{3}}$$

$$BK^2 = BC \cdot BA = \frac{2}{3} d \cdot d = \frac{2}{3} d^2; BK = d\sqrt{\frac{2}{3}}.$$

921. Ochiq tarnovning kesimi asosi va yon tomonlari a ga teng bo‘lgan teng yonli trapetsiya shaklida (103-rasm). Tarnovning o‘tkazish imkoniyati eng katta bo‘lganda tarnov devorining o‘tmas burchak uchidan o‘tkazilgan balandlikka og‘ish burchagi α nimaga teng?



103-rasm

Yechilishi. Tarnovning kesim yuzi S eng katta bo‘lganda uning o‘tkazish qobiliyati ham eng katta bo‘ladi deb hisoblaymiz:

$$S = \frac{AB + CD}{2} BF, BF = a\cos\alpha . FC = a\sin\alpha,$$

$CD = a + 2a\sin\alpha,$ u holda

$$S = \frac{a + a + 2a\sin\alpha}{2} a\cos\alpha = a^2\left(\cos\alpha + \frac{1}{2}\sin 2\alpha\right)$$

bu yerda $0 < \alpha < \frac{\pi}{2}$.

Bu funksiyaning maksimum va minimumini ikkinchi tartibli hosila yordamida tekshiramiz: 1) $S' = a^2(-\sin\alpha + \cos 2\alpha);$

$$S=a^2\left[\cos 2\alpha-\cos\left(2\frac{\pi}{2}-\alpha\right)\right]=a^2 2\sin\frac{2\alpha+\frac{\pi}{2}-a}{2}\times$$

$$\times\sin\frac{\frac{\pi}{2}-\alpha-2\alpha}{2}=2a^2\sin\left(\frac{\alpha}{2}+\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}-\frac{3a}{2}\right);$$

2) $\sin\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)=0; \frac{\pi}{4}+\frac{\alpha}{2}=0$ bu yerda $\alpha=-\frac{\pi}{2}$ biroq $0<\alpha<\frac{\pi}{2}$;

$\sin\left(\frac{\pi}{2}-\frac{3a}{2}\right)=0;$ $\frac{\pi}{4}-\frac{3a}{2}=0;$ bu yerda $\alpha=\frac{\pi}{6}$;

3) $S''=a^2(-\cos\alpha-2\sin 2\alpha);$ $S''_{\alpha=\frac{\pi}{2}}<0$ demak, funksiya $\alpha<\frac{\pi}{2}$ da maksimumga ega.

922. Ochiq tarnovnnng kesimi perimetrn a ga teng bo‘lgan to‘g‘ri to‘rtburchakdan iborat. Tarnovnnng eni va balandligi qanday nisbatda bo‘lganda uning kesimi eng katta yuzga ega bo‘ladi?

923. Asosi va balandligining yigindisi a ga teng bo‘lgan barcha uchburchaklar ichidan yuzi eng katta bo‘lganini toping.

Yechilishi. Uchburchakning asosi x bo‘lsin, u holda balandligi $a-x$ ga teng bo‘ladi. Uchburchakning yuzi - o‘zgaruvchi mikdor, uni y bilan belgilab,

$$y=\frac{1}{2}x(a-x) \text{ yoki } y=\frac{1}{2}ax-\frac{1}{2}x^2 \quad (0<x<a)$$

ni hosil qilamiz.

Bu funksiyani ikkinchi tartibli hosila yordamida tekshiramiz:

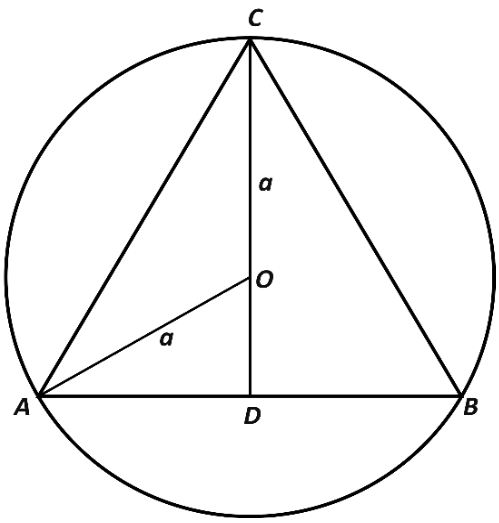
1) $y'=\frac{1}{2}a-x;$ 2) $\frac{1}{2}a-x=0,$ $x=\frac{a}{2};$ 3) $y''=-1$

Ikkinchi tartibli hosila manfiy, demak, funksiya $x=\frac{a}{2}$ da maksimumga ega. Uchburchakning asosi $\frac{a}{2}$ ga, balandligi

$a-\frac{a}{2}=\frac{a}{2}$ ga teng.

924. Gipotenuzasni C bo‘lgan barcha to‘g‘ri burchakli uchburchaklar ichidan yuzi eng katta bo‘lganini toping.

925. a radiusli doiraga teng yonli uchburchak ichki chizilgan. Tomonlarning nisbati qanday bo‘lganda uchburchak eng katta yuzga ega bo‘ladi?



104-rasm

Yechilishi. Quyidagicha belgilashlar kiritamiz (104-rasm):

$$OC = OA = a, \quad OD = x, \quad S_{\Delta ABC} = y,$$

u holda $AD = \sqrt{a^2 - x^2}$

$AB = 2\sqrt{a^2 - x^2}$ va $DC = x + a,$

$y = \frac{1}{2} \cdot 2\sqrt{a^2 - x^2}(x+a)$ yoki

$y = (x+a) \cdot \sqrt{a^2 - x^2}(0 < x < \alpha)$.

Hosil qilingan funksiyani birinchi tartibli hosila yordamida tekshiramiz:

$$y' = (x+a)' = \sqrt{a^2 + x^2} + (\sqrt{a^2 + x^2})'(x+a) =$$

1) $$= \sqrt{a^2 + x^2} + \frac{-2x(x+a)}{2\sqrt{a^2 - x^2}} = \frac{a^2 - x^2 - x^2 - ax}{\sqrt{a^2 - x^2}} - \frac{2x^2 + ax - a^2}{\sqrt{a^2 + x^2}};$$

2) $$y' = -\frac{2x^2 + ax - a^2}{\sqrt{a^2 + x^2}} = 0; \quad 2x^2 + ax - a^2 = 0$$

Tenglamaning ildizlari: $x_1 = -a;\ x_2 = \frac{a}{2};\quad y' = -\frac{2(x+a)(x - \frac{a}{2})}{\sqrt{a^2 - x^2}}$

Faqat $x = \frac{a}{2}$ kritik nuqtani tekshiramiz, chunki $0 < x < a$:

$$y'_{x<\frac{a}{2}} = (-)(+)(-) = (+); \quad y'_{x>\frac{a}{2}} = (-)(+)(+) = (-);$$

Birinchi (-) ishora kasr oldidagi ishoradir.

$x = \frac{a}{2}$ da funksiya maksimumga ega. $x = \frac{a}{2}$ bo‘lganda uchburchakning tomonlarini topamiz.

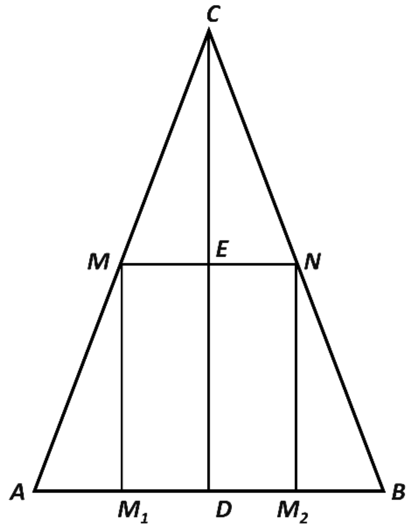
$$AB = 2\sqrt{a^2 - \left(\frac{a}{2}\right)^2} = a\sqrt{3};$$

$$AC = BC = \sqrt{(AD)^2 + (DC)^2} = \sqrt{\left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{3a}{2}\right)^2} = a\sqrt{3}$$

Uchburchak teng tomonli.

926. a radiusli doiraga to‘g‘ri burchakli uchburchak ichki chizilgan. Katetlarning munosabati qanday bulganda uchburchak eng katta yuzga ega bo‘ladi?

927. Asosi a va balandligi h bo‘lgan uchburchakka eng katta yuzga ega bo‘lgan to‘g‘ri to‘rtburchak ichki chizilgan(to‘g‘ri to‘rtburchakning asosi uchburchakning asosida yotadi.).
Bu to‘g‘ri to‘rtburchakning tomonlarini toping.



105-rasm

Yechilishi. Quyidagicha belgilashlar kiritamiz (105-rasm): $AB = a, CD = h, De = x, EC = h - x$

ABC va MNC uchburchaklarning o‘xshashligidan: $\frac{MN}{AB} = \frac{EC}{DC}$ yoki $\frac{MN}{a} = \frac{h-x}{h}$ bu yerda $MN = \frac{a}{h}(h-x).M_1MNN_1$ to‘g‘ri to‘rtburchakning yuzi (uni y bilan belgilaymiz):

$$y = MN \cdot DE = \frac{a}{h}(h-x)x \quad \text{yoki}$$

$$y = ax - \frac{a}{h}x^2 (0 < x < h)$$

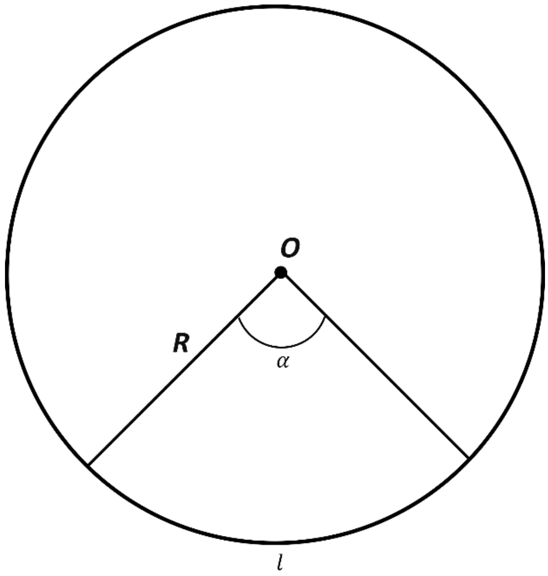
Bu funksiyani ikkinchi tartibli hosila yordamida tekshiramiz:

1) $y' = a - \frac{2a}{h}x;$ 2) $y' = a - \frac{2a}{h}x = 0 \quad x = \frac{h}{2};$ 3) $y'' = \frac{2a}{h}; \quad x = \frac{h}{2}$ da funksiya maksimumga ega, chunki $y'' < 0.$ Eng katta to‘g‘ri to‘rtburchakning balandligi $\frac{h}{2}$ va asosi $MN = \frac{a}{h}\left(h - \frac{h}{2}\right) = \frac{a}{h}\frac{h}{2} = \frac{a}{2}.$

To‘g‘ri to‘rtburchakning balandligi va asosi mos ravishda uchburchak balandligining va asosining yarmiga teng.

928. Teng yonli uchburchakka eng katta yuzga ega bo‘lgan to‘g‘ri to‘rtburchakni ichki chizing.

929. Perimetrlari p ga teng bo‘lgan barcha doiraviy sektorlar ichidan eng katta yuzga ega bo‘lganini toping.



106-rasm

Yechilishi. $p = 2R + l$ (106-rasm), biroq $l = \alpha R$ bu yerda α -yoy l ning radian o‘lchovi, u holda $p = 2R + \alpha R$.

Sektorning yuzi: $S = l\frac{R}{2} S = \alpha$ biroq $R\frac{R}{2} = \frac{1}{2}R^2\alpha$ u holda

$$S = \frac{1}{2}R^2\frac{p-2R}{R} = \frac{1}{2}R(p-2R) = \frac{1}{2}pR - R^2$$

$$(0 < 2R < p ёки 0 < R < \frac{p}{2})$$

Funksiyani ikkinchi tartibli hosila yordamida tekshiramiz:

1) $S' = \frac{1}{2}p - 2R;$ 2) $\frac{1}{2}p - 2R = 0, \quad R = \frac{p}{4};$ 3) $S'' = -2$, $R = \frac{p}{4}$

da funksiya maksimumga ega. α yoyni va S yuzni hisoblaymiz:

$$\alpha = \frac{4R-2R}{R} = 2 \text{ rad } S = \frac{1}{2}R^2\alpha = \frac{1}{2}\frac{n^2}{16}\cdot 2 = \frac{p^2}{16} = R^2$$

930. Berilgan yuzga ega bulgan barcha doiraviy sektorlar ichidan eng kichik perimetrga ega bo‘lganini toping.

931. Berilgan S to‘la sirtga ega bo‘lgan va asosi kvadrat bo‘lgan barcha to‘g‘ri parallelepipedlar ichidan eng katta hajmga ega bo‘lganini toping.

Yechilishi. Parallelepiped asosining tomoni x va balandligi y bo‘lsin, u holda uning to‘la sirti:

$$S = 2x^2 + 4xy$$

bu yerdan

$$y = \frac{S - 2x^2}{4x}$$

Parallelepipedning hajmi: $V = x^2 y$ yoki $V = x^2 \frac{2x^3}{4x} = \frac{x(S - 2x^2)}{4}$;

$$V = \frac{1}{4}Sx - \frac{1}{2}x^3 \left(0 < 2x^2 < S, 0 < x < \sqrt{\frac{S}{2}}\right)$$

Funksiyani ikkinchi tartibli xosila yordamida tekshiramiz:

$$1) V' = \frac{1}{4}S - \frac{3}{2}x^2; \quad 2) \frac{1}{4}S - \frac{3}{2}x^2 = 0,\ x^2 = 0,\ x = \sqrt{\frac{S}{6}} = \frac{1}{6}\sqrt{6S};$$

$$3) V'' = -3x \ 4) V''_{x=\frac{1}{6}\sqrt{6S}<0}$$

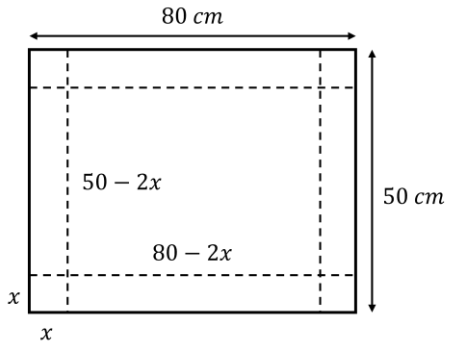
Ikkinchi tartibli hosila $x = \frac{1}{6}\sqrt{S}$ da manfiy, demak, argumentning bu qiymatida funksiya maksimumga ega.

Parallelepiped asosining tomoni: $x = \frac{1}{6}\sqrt{S}$. Parallelepipedniig balandligi:

$$y = \frac{S - 2\left(\frac{1}{6}\sqrt{6S}\right)^2}{4 \cdot \frac{1}{6}\sqrt{6S}} = \frac{1}{6}\sqrt{6S}$$

Demak, eng katta hajmga kub ega bo‘ladi.

932. Tomonlari 80 ***sm*** va 50 ***sm*** bo‘lgan to‘g‘ri to‘rtburchak tunukaning uchlaridan teng kvadratlar qirqib olib tashlab, so‘ngra uning chetlarini bukib, eng katta hajmga ega bo‘lgan usti ochiq yashik yasash kerak. Qirqib olib tashlanadigan kvadratlarnnng tomoni qanday bo‘lishi kerak (107-racm)?



107-rasm

933. Hajmi V ga teng bo‘lgan va asosida kvadrat yotgan barcha to‘g‘ri parallelepipedlar ichidan eng kichik to‘la sirtga ega bo‘lganini toping.

Yechilishi. Parallelepiped asosining tomoni x va balandligi y bo‘lsin, u holda uning hajmi:

$$V = x^2 y, \qquad \text{bu yerda } y = \frac{V}{x^2}$$

Parallelepipedning to‘la sirti: $S = 2x^2 + 4xy$ yoki

$$S = 2x^2 + 4x\frac{V}{x^2} \quad \text{yoki } S = 2x^2 + \frac{4V}{x}(x > 0).$$

Funksiyani birinchi tartibli hosila yordamida tekshiramiz:

1) $S' = 4x - \dfrac{4V}{x^2};$

2) $4x - \dfrac{4V}{x^2} = 0; \quad x^3 = V, \quad x = \sqrt[3]{V}$

3)

$$S' = \frac{4x^3 - 4V}{x^2} = \frac{4(x^3 - V)}{x^2} = \frac{4\left(x - \sqrt[3]{V}\right)\left(x^2 + x\sqrt[3]{V} + \sqrt[3]{V^2}\right)}{x^2}$$

$$S'_{x<\sqrt[4]{V}} = (-)(+) = (-); \; S'_{x>\sqrt[3]{V}} = (+)(-) = (+)$$

Hosila ishorasini (-) dan (+) ga o‘zgartiryapti, demak, funksiya $x = \sqrt[3]{V}$ da minimumga ega.

Asosning tomoni $x = \sqrt[3]{V}$. Balandlik:

$$y = \frac{V}{\left(\sqrt[3]{V}\right)^2} = \frac{V}{\sqrt[3]{V^2}} = \frac{V\sqrt[3]{V}}{V} = \sqrt[3]{V}$$

Eng kichik sirtga kub ega bo‘ladi.

934. Hajmi V berilgan, to‘la sirti esa eng kichik bo‘lgan, tubi kvadrat shaklidagi usti ochik (qopqoqsiz) yashikning o‘lchamlarini toping.

935. Berilgan V hajmga ega bo‘lgan barcha silindrlar ichidan to‘la sirti eng kichik bo‘lganini toping.

Yechilishi. Silindrning hajmi: $V=\pi R^2 H$, bu yerdan

$$H=\frac{V}{\pi R^2}$$

To‘la sirt: $S=2\pi R^2+2\pi RH;\ S=2\pi R^2+2\pi R\frac{V}{\pi R^2};$

$$S=2\pi R^2+\frac{2V}{R}\left(R>0\right)$$

Funksiyani birinchi tartibli hosila yordamida tekshiramiz:

1) $S'=4\pi R-\frac{2V}{R^2};$

2) $4\pi R-\frac{2V}{R^2}=0 \quad 4\pi R^3-2V=0; \quad 2\pi R^2-V=0; \quad R=\sqrt[3]{\frac{V}{2\pi}};$

3)$S'=\frac{4\pi R^2-2V}{R^2}=\frac{4\pi\left(R^3-\frac{V}{2\pi}\right)}{R^2}=\frac{4\pi\left(R-\sqrt[3]{\frac{V}{2\pi}}\right)\left(R^3+R\sqrt[3]{\frac{V}{2\pi}}+\sqrt[3]{\frac{V^2}{4\pi^2}}\right)}{R^2};$

$S'_{R<\sqrt[3]{\frac{V}{2\pi}}}=(-)(+)=(-);\quad S'_{R>\sqrt[3]{\frac{V}{2\pi}}}=(+)(+)=(+);$

Hosila $\sqrt[3]{\frac{V}{2\pi}}$ ishorasini (—) dan (+) ga o‘zgartiryapti, demak,

funksiya $R=\sqrt[3]{\frac{V}{2\pi}}$ da minimumga ega.

Silindrning radiusi: $R=\sqrt[3]{\frac{V}{2\pi}};$ balandligi:

$$H=\frac{V}{\pi\sqrt[3]{\frac{V^2}{4\pi^2}}}=2\sqrt[3]{\frac{V}{2\pi}}=2R.$$

936. Berilgan V hajmga ega bo‘lgan (qopqoqsiz) silindr baknnng sirti eng kichik bo‘lganda uning asosi radiusini va balandligini toping.

937. Yon sirti S bo‘lgan barcha konuslar ichidan hajmi eng katta bo‘lganini toping.

Yechilishi . Konusning hajmi: $V=\frac{1}{3}\pi R^2 H;,$ biroq $H=\sqrt{l^2-R^2}$. Konusning yon sirti formulasi $S=\pi Rl$ dan $l=\frac{S}{\pi R}$ ni topamiz, u holda

$$H=\sqrt{\frac{S^2}{\pi^2R^2}-R^2}=\frac{1}{\pi R}\sqrt{S^2-\pi^2R^4}$$

$$\left(S^2-\pi^2R^4>0, R<\sqrt{\frac{S}{\pi}}\right)$$

H ning qiymatini konus hajmi formulasiga qo‘yib,

$$V=\frac{1}{3}\pi R^2\frac{1}{\pi R}\sqrt{S^2-\pi^2R^4}=\frac{1}{3}R\sqrt{S^2-\pi^2R^4}$$

ni hosil qilamiz.

Bu funksiyani birinchi tartibli hosila yordamida tekshiramiz:

1) $V'=\frac{1}{3}\left(\sqrt{S^2-\pi^2R^4}-\frac{2\pi^2R^4}{\sqrt{S^2-\pi^2R^4}}\right)=\frac{S^2-3\pi^2R^4}{\sqrt[3]{S^2-\pi^2R^4}};$

2) $S^2-3\pi^2R^4=0,$ $R^4=\frac{S^2}{3\pi^2},$ $R=\sqrt{\frac{S}{\pi\sqrt{3}}};$ $V'_{R<\sqrt{\frac{S}{\pi\sqrt{3}}}>0}$ $V'_{R>\sqrt{\frac{S}{\pi\sqrt{3}}}<0}$ va

$V'_{R>\sqrt{\frac{S}{\pi\sqrt{3}}}<0}$

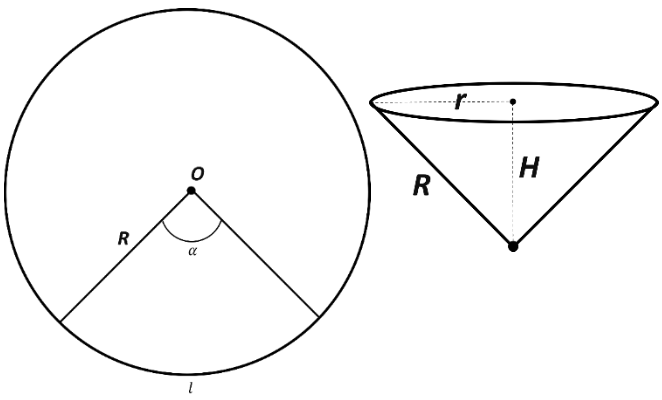
Demak, funksiya $R=\sqrt{\frac{S}{\pi\sqrt{3}}}$ da maksimumga ega. Quyidagilarni topamiz.

$$H_{макс}=\frac{1}{\pi\sqrt{\frac{S}{\pi\sqrt{3}}}}\sqrt{S^2-\pi^2\frac{S^2}{\pi\sqrt{2}}}=\frac{1}{\pi\sqrt{\frac{S}{\pi\sqrt{3}}}}S\sqrt{\frac{2}{3}}=\sqrt{\frac{2S}{\pi\sqrt{3}}}$$

$$\frac{H_{макс}}{R_{макс}}=\frac{\sqrt{\frac{2S}{\pi\sqrt{3}}}}{\sqrt{\frac{S}{\pi\sqrt{3}}}}=\sqrt{2};\ V_{макс}=\frac{S}{3}\sqrt{\frac{2S}{3\pi\sqrt{3}}};$$

938. Yasovchisi l berilgan barcha konuslar ichida hajmi eng katta bo‘lganini toping.

939. R radiusli qog‘oz doiradan sektor qirqib olingan va doiraning qolgan bo‘lagidan konus shaklida voronka yasalgan (108-rasm).



108-rasm

Voronkaning hajmi eng katta bo‘lishi uchun sektor qanday burchakka ega bo‘lishi kerak? Voronkaning asosi radiusini va balandligini toping.

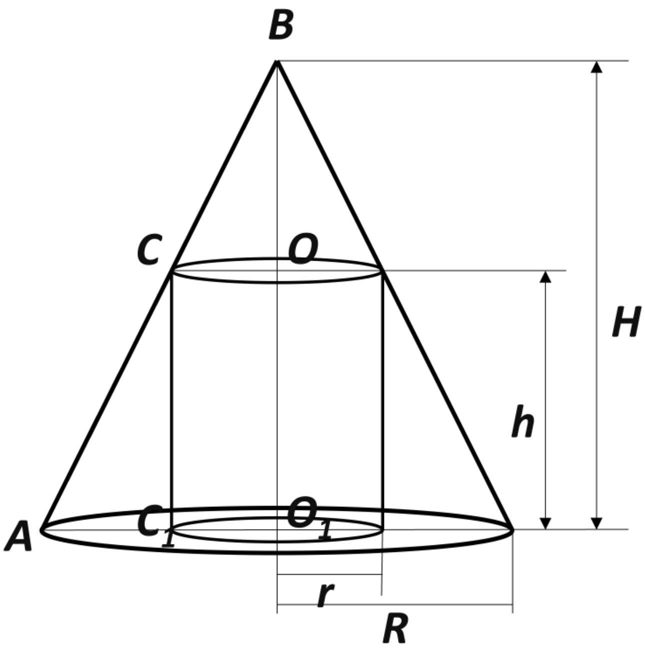
Yechilishi. Voronkaning balandligini H bilan belgilaymiz, u holda voronka asosining radiusi r quyidagiga teng bo‘ladi: $r=\sqrt{R^2-H^2}$ Voronkaning hajmi:

$$V=\frac{1}{3}\pi^2 rH=\frac{1}{3}\pi(R^2-H^2)H;$$

$V=\frac{1}{3}\pi\left(R^2H-H^2\right)$ funksiyaga egamiz, i bu yerda H argument $0<H<R$ da o‘zgaradi.

Funksiyaning maksimum va minimumini ikkinchi tartibli hosila yordamida tekshiramiz: 1) $V'=\frac{1}{3}\pi\left(R^2-3H^2\right)$ 2) $\frac{1}{3}\pi\left(R^2-3H^2\right)=0$ $R^2-3H^2=0;$ $H=\frac{R}{\sqrt{3}};$ 3) $V''=\frac{1}{3}\pi(-6H)=-2\pi H;$ $V'_{H=\frac{R}{\sqrt{3}}}<0$ demak, $H=\frac{R}{3}$ da funksiya maksimumga ega. Voronka asosining radiusi:

$$r=\sqrt{R^2-\frac{R^2}{3}}=\sqrt{\frac{3R^2-R^2}{3}}=R\sqrt{\frac{2}{3}}$$



109-rasm

940. Berilgan konusga ichki chizilgan barcha silindrlar ichidan yon sirti eng katta bo‘lganini toping (konusda R va H berilgan).

Yechilishi. Izlanayotgan silindrda r- asosning radiusi, h—balandlik bo‘lsin (109- rasm). ABO_1 va CBO uchburchaklarnnng o‘xshashligidan: funksiya maksimumga ega. Voronka asosining radiusi:

$$\frac{R}{r}=\frac{H}{H-h};$$

bu yerdan

$$r=\frac{R(H-h)}{H}.$$

r ning qiymatini silindrning yon sirti formulasi $S=2\pi rh$ ga qo‘yib quyidagini hosil qilamiz:

$$S=2\pi h\frac{R(H-h)}{H};$$

yoki

$$S=\frac{2\pi R}{H}R(H-h^2)(0<h<H)$$

Funksiyani ikkinchi tartibli hosila yordamida tekshiramiz:

1) $S'=\frac{2\pi R}{H}(H-2h);$ 2) $\frac{2\pi R}{H}(H-2h)=0$ $H-2h=0$

$$h=\frac{H}{2};\qquad S''=-\frac{4\pi R}{H}$$

Ikkinchi tartibli hosila manfiy, demak, funksiya $h=\frac{H}{2}$ da maksimumga ega.

Silindrning radiusi r ni topamiz:

$$r=\frac{R\left(H-\frac{H}{2}\right)}{H}=\frac{RH}{H\cdot 2}=\frac{R}{2}.$$

941. Berilgan konusga ichki chizilgan barcha silindrlar ichidan to‘la sirti eng katta bo‘lganini toping (konusda R va H berilgan).

942. Berilgan konusga ichki chizilgan barcha silindrlar ichidan hajmi eng katta bo‘lganini toping (konusda R va H berilgan).

Yechilishi. Izlanayotgan silindrda r — asosning radiusi, h — balandlik bo‘lsin (109-rasm). ABO_1 va ACC_1 uchburchaklarning o‘xshashligidan:

$$\frac{R}{H}=\frac{R-r}{h};\ \text{bu yerda}\ h=\frac{H(R-r)}{R};$$

h ning qiymatini silindr hajmi formulasi $V=2\pi r^2 h$
ga qo‘yib,

$$V=\pi r^2\,\frac{H(R-r)}{R}=\frac{\pi H}{R}(Rr^2-r^3)(0<r<R)$$

ni hosil qilamiz.
Funksiyani ikkinchi tartibli hosila yordamida tekshiramiz.

$$1)V'==\frac{\pi H}{R}(2Rr-3r^2);\quad 2)\frac{\pi H}{R}(2Rr-3r^3)=0$$

$$2Rr-3r^3=0;\ r(2R-3r)=0;\ r_1=0;\ r_2=\frac{2}{3}R;$$

$$3)V''=\frac{\pi H}{R}(2R-6r);\ V''_{r=\frac{2}{3}R}=-2\pi H$$

Ikkinchi tartibli hosila manfiy, demak, funksiya $r=\frac{2}{3}R$da maksimumga ega.
Silindrning balandligini topamiz:

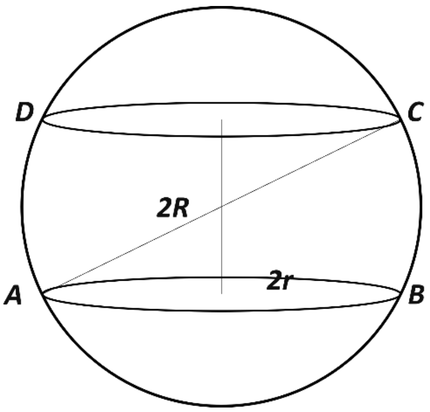
$$h=\frac{H\left(R-\frac{2}{3}R\right)}{R}=\frac{1}{3}H.$$

943. R radiusli sharga ichki chizilgan barcha silindrlar ichidan hajmi eng katta bo‘lganini toping.

Yechilishi. Izlanayotgan silindrda r - asosning radiusi, h — balandlik bo‘lsin (110-rasm). Silindrning hajmi: $V = \pi r^2 h$

Δ ABC dan: $4r^2 + h^2 = 4R^2$

Bu yerdan 110- racm.



110-rasm

$$r^2 = \frac{4R^2 - h^2}{4} = R^2 + \frac{1}{4}h^2$$

r^2 ning bu qiymatini silindr hajmi formulasiga qo‘yib , qo‘yidagiga ega bo‘lamiz.

$$V = \pi h\left(R^2 - \frac{1}{4}h^2\right)$$

yoki

$$V = \pi\left(R^2 h - \frac{1}{4}h^2\right)(0 < h < 2R)$$

Funksiyani ikkinchi tartibli hosila yordamida tekshiramiz:

1) $V' = \pi\left(R^2 - \frac{3}{4}h^2\right)$; 2) $\pi\left(R^2 - \frac{3}{4}h^2\right) = 0$; $R^2 - \frac{3}{4}h^2 = 0$

$$3) V'' = -\frac{3}{2}\pi h$$

Ikkinchi tartibli hosila $h = \frac{2R}{\sqrt{3}}$ da mnfiy, demak, funksiya maksimumga ega.

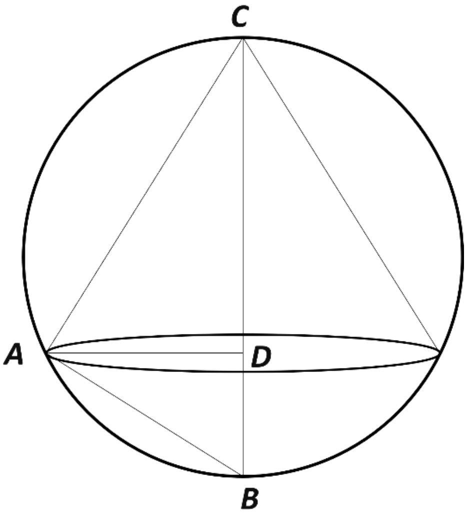
Silindrning radiusi r ni topamiz.

$$r^2 = R^2 - \frac{1}{4}\cdot\frac{4R^2}{3} = R^2 - \frac{1}{3}R^2 = \frac{2}{3}R^2; r = R\sqrt{\frac{2}{3}}$$

944. R radiusli sharga ichki chizilgan barcha silindrlar ichidan yoy sirti eng katta bo‘lganini toping.

945. R radiusli sharga ichki chizilgan barcha konuslar ichidan hajmi eng katta bo‘lganini toping.

Yechilishi. Izlanayotgan konusning radiusi $AD = r$ va balandligi $DC = h$ bo‘lsin. (111-rasm).



111-rasm

Konusining hajmi: $V = \frac{1}{3}\pi r^2 h.$ ΔBAC dan to‘g‘ri burchakli uchburchakdagi metrik munosabatlar haqidagi teoremaga ko‘ra topamiz:

$AD^2 = BD \cdot DC$ yoki $r^2 = (2R-h)h$

r^2 ning qiymatini konus hajmi formulasiga qo‘yib,

$$V = \frac{1}{3}\pi\left(2R-h\right)\cdot h\cdot h = \frac{1}{3}\pi\times\left(2Rh^2 - h^3\right), 0 < h < 2R$$

ni hosil qilamiz.

Bu funksiyani ikkinchi tartibli hosila yordamida tekshiramiz.

1) $V' = \frac{1}{3}\pi(4R-3h^2);$ 2) $\frac{1}{3}\pi\left(4Rh-3h^2\right)=0$ $4Rh-3h^2=0$

$h(4R-3h)=0;$ $h_1=0;$ $4R-3h=0;$ $h_2=\frac{4}{3}R;$

$3) V'' = \frac{1}{3}\pi\left(4R-6h\right) = \frac{2}{3}\pi\left(2R-3h\right);$

$$V''_{h=\frac{3}{4}R} = \frac{2}{3}\pi\left(2R-3\cdot\frac{4}{3}R\right) = \frac{2}{3}\pi\left(-2R\right) = -\frac{4\pi R}{3};$$

Ikkinchi tartibli hosila argumentning $h=\frac{4}{3}R$ qiymatida manfiy, demak, argumentning bu qiymatida y maksimumga ega. $h=\frac{4}{3}R$ bo‘lganda r ning qiymatini topamiz.

$$r^2 = \left(2R-\frac{4}{3}R\right)\cdot\frac{4}{3}R = \frac{2R}{3}\cdot\frac{4}{3}R = \frac{8}{9}R^2$$

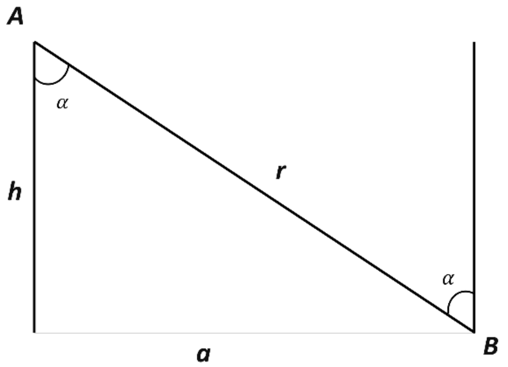
Bu yerdan

$$r = \frac{2\sqrt{2R}}{3}$$

Eng katta hajmga radiusi $\frac{2\sqrt{2}R}{3}$ va balandligi $h=\frac{4}{3}R$ bo‘lgan konus ega bo‘ladi.

946. R radiusli sharga ichki chizilgan barcha konuslar ichidan yon sirti eng katta bo‘lganini toping.

947. Radiusi a ga teng bo‘lgan doiraviy maydonchaning chegarasi maksimal yoritilgan bo‘lishi uchun fonarni maydon o‘rtasida qanday h balandlikka o‘rnatish kerak?



112-rasm

Yechilishi. Fizika kursidan ma’lumki E yoritilganlik yorug‘lik manbaigacha bo‘lgan masofa kvadratiga teskari proporsional va tushish burchagining (sirtga o‘tkazilgan normal oqimi yo‘nalishi orasidagi burchak) konusiga to‘g‘ri proporsional 112-rasm):

$$E=k\frac{\cos\alpha}{r^2}$$

bu yerda k koeffitsient A nuqtaga joylashtirilgan yorug‘lik manbaiga bog‘liq OAB uchburchakdan:

$$\cos\alpha=\frac{h}{r}r=\sqrt{h^2+a^2}$$

h ni erkli o‘zgaruvchi deb hisoblab,

$$E=k\frac{h}{\sqrt{h^2+a^2}(h^2+a^2)}=k\frac{h}{(h^2+a^2)\frac{3}{2}}(h>0)$$

ni hosil qilamiz.

Funksiyani birinchi tartibli hosila yordamida tekshiramiz:

1) $$E'=k\frac{(h^2+a^2)^{\frac{3}{2}}-\frac{3}{2}(h^2+a^2)^{\frac{1}{2}}\cdot 2h\cdot h}{(h^2+a^2)^3}=k\frac{(h^2+a^2)^{\frac{1}{2}}(h^2+a^2-3h^2)}{(h^2+a^2)^3}=k\frac{a^2-2h^2}{(h^2+a^2)^{\frac{5}{2}}};$$

2) $a^2+2h^2=0$; $h=\frac{a}{\sqrt{2}}$; 3) $E'=k\frac{\left(\frac{a}{\sqrt{2}}-h\right)\left(\frac{a}{\sqrt{2}+h}\right)}{(h^2+a^2)^{\frac{3}{2}}}$

$$E'_{h<\frac{a}{\sqrt{2}}} = (+)(+) = (+); \qquad E'_{h>\frac{a}{\sqrt{2}}} = (-)(+) = (-);$$

Hosila ishorasini (+) dan (-) ga o‘zgartiryapti, demak,funksiya $h = \dfrac{a}{\sqrt{2}}$ da maksimumga ega, ya’ni $h = \dfrac{a}{\sqrt{2}} = 0,7a$ da B nuqtadagi yoritilganlik eng katta bo‘ladi.

948. Jismning to‘g‘ri chiziqli harakati $s = t^3 + 9t^2 - 24t - 24.$ $v = -3t^2 + 18t - 24$ tenglama bilan berilgan. Jism hapakatining maksimal tezligini toping (s m hisobida, t sek hisobida berilgan).

Yechilishi. Jism harakatining tezligi yo‘ldan vaqt bo‘yicha olingan birinchi tartibli hosilaga teng: $v = s' = -3t^2 + 18t - 24$ $v = -3t^2 + 18t - 24$ funksiyaga egamiz.

Uning maksimum va minimumini ikkinchi tartibli hosila yordamida tekshiramiz:

1) $v' = -6t + 18;$ 2) $-6t + 18 = 0,$ $t = 3;$ 3) $v'' = -6$

Ikkinchi tartibli hosila manfiy, demak, eng katta tezlikka $t = 3$ sek bo‘lganda erishiladi.

$t = 3$ sek momentdagi tezlikni topamiz:

$$v_{t=3} = -3\cdot 3^2 + 18\cdot 3 - 24 = 3(m/cek)$$

949. Jismning to‘g‘ri chiziqli harakat qonuni $s = -t^3 + 3t^2 + 9t + 3$ tenglama bilan berilgan. Jism hapakatining maksimal tezligini toping (s m xisobida, t sek hisobida berilgan).

950. Yuqoriga tik otilgan jismning harakat qonuni $s = v_0 t - \dfrac{1}{2} g t^2$ tenglama bilan berilgan. Jism ko‘tarilgan eng yuqori balandlikni toping.

Yechilishi. Yuqoriga tik otilgan jismning eng yuqoriga ko‘tarilgan nuqtasidagi tezligi nolga teng bo‘ladi,

demak, $v = s' = v_0 - gt = 0$ bu yerdan $t = \dfrac{v_0}{g}$

Berilgan funksiyani tekshiramiz: 1) $s' = v_0 - gt$ 2) $v_0 - gt = 0,$ $t - \dfrac{v_0}{g};$

3) $s' = g\left(\dfrac{v_0}{g} - t\right);$ $s_{t<\frac{v_0}{g}} = (+)$ $s'_{t>\frac{v_0}{g}} = (-)$

Funksiya ishorasini (+) dan (—) ga o‘zgartiryapti, demak , y

$t=\frac{v_0}{g};$ da maksimal qiymatga ega bo‘ladi.

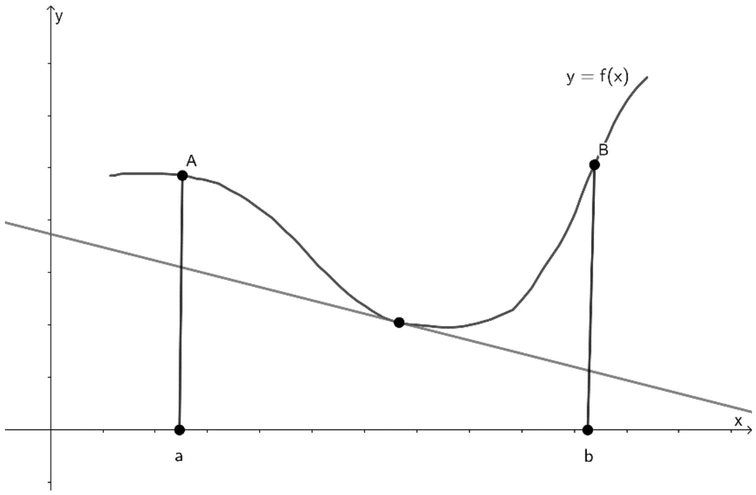
$t=\frac{v_0}{g};$ bo‘lganda s ni topamiz:

$$s=v_0\frac{v_0}{g}-\frac{1}{2}g\frac{v_0^2}{g^2}=\frac{v_0^2}{2g};$$

951. Yuqoriga tik otilgan jismning harakat qonuni $s=19,6t-4,9t^2;$ tenglama bilan berilgan. Jism ko‘tarilgan eng yuqori balandlikni toping (s m hisobida, t sek hisobida berilgan).

49-§. Egri chiziqning qavariqligi va botiqligi

Agar $y=f(x)$ egri chiziqning $(a;b)$ intervaldagi yoyi bu intervalning istalgan nuqtasidagi urinmasidan yuqorida yotsa, bu yoyga *botiq yoy* deyiladi (113-rasm).



113-rasm

Agar $y=f(x)$ egri chiziqning $(a;b)$ intervaldagi yoyi bu intervalning istalgan nuqtasidagi urinmasidan pastda yotsa, bu yoyga *qavariq yoy* deyiladi.

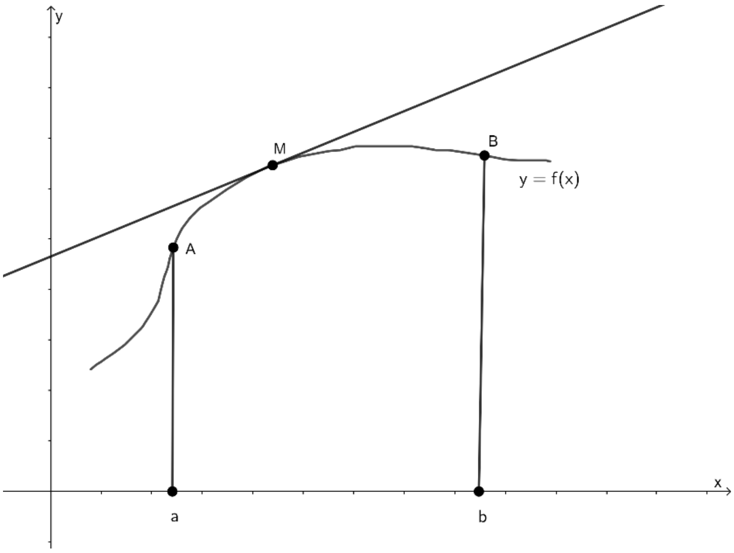
Egri chiziqning qavariqlik va botiqlik alomatlari.

Agar $y=f(x)$ funksiyaning ikkinchi tartibli hosilasi x argumentning $(a;b)$ intervaldagi qiymatlari uchun musbat bo‘lsa, egri chiziq bu intervalda botiq, manfiy bo‘lsa, qavariq bo‘ladi.

$y = f(x)$ egri chiziqning qavariqlik va botiqligini tekshirish qoidasi

1. Berilgan $y = f(x)$ funksiyaning ikkinchi tartibli hosilasini topiladi:

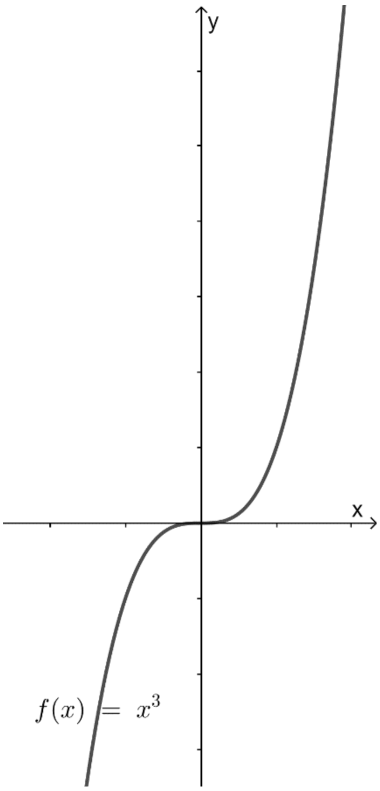
$$y'' = f''(x)$$



114-rasm

II. Ikkinchi tartibli hosilani noldan kichik deb faraz qilinadi: $f''(x) < 0$; $f''(x) < 0$ tengsizlikni x ga nisbatan yechib, $y = f(x)$ egri chiziq, qavariq bo‘lgan intervallarni topiladi.

III. Ikkinchi tartibli hosilani noldan katta deb faraz qilinadi $f''(x) > 0$; $f''(x) > 0$ tengsizlikni x ga nisbatan echib, egri chiziq botiq bo‘lgan intervallarni topiladi.



115-rasm

952. $y = x^3$ egri chiziqning qavariqligi va botiqligini tekshiring.

Yechilishi. 1) $y' = 3x^2$; $y'' = 6x$; 2) $6x < 0$; $x < 0$. $(-\infty; 0)$ bu intervalda $y = x^3$ egri chiziq qavariq. 3) $6x > 0$; $x > 0$. $(0; +\infty)$ bu intervalda $y = x^3$ egri chiziq botiq (115-rasm).

953. Quyidagi egri chiziqlarning qavariqligi va botiqligini tekshiring:

1) $y = 2x^3$; 2) $y = x^2$; 3) $y = -x^2 - 1$; 4) $y = x^3 + 3x - 1$.

954. $y = \frac{1}{x}$ egri chiziqning $y = -2$ va $x = 1$ nuqtalarda qavariqligi va botiqligini tekshiring.

Yechilishi. 1) $y' = -\frac{1}{x^2}$; $y'' = \frac{1}{x^4}$, $2x = \frac{2}{x^3}$

2) ikkinchi tartibli hosilaga argumentning berilgan qiymatlarini qo‘yib, ikkinchi tartibli hosilaning ishorasini topamiz:

$$y''_{x=-2} = \frac{2}{(-2)^3} = -\frac{2}{8} < 0$$

demak, $x = -2$ nuqtada egri chiziq qavariq;

$$y''_{x=1} = \frac{2}{13} > 0$$

u holda $x = 1$nuqtada egri chiziq botiq.

955. 1) $y = -\frac{1}{x}$ egri chiziqni $x_1 = -1$ va $x_2 = 1$ nuqtalarda; 2) $y = \frac{1}{x^2}$ egri chiziqni $x_1 = -2$ va $x_2 = 1$ nuqtalarda qavariqlik va botiqligini tekshiring.

956. $y = x^2 - 2x^3 + 6x - 4$ egri chiziqning qavariqlik va botiqlik intervallarini toping.

Yechilishi. Ikkinchi tartibli hosilani topamiz:

$$y' = 4x^3 - 6x^2 + 6, y'' = 12x^2 - 12x$$

$12x^2 - 12x < 0$ tengsizlikni yechamiz: $x^2 - x < 0$; $D = 1 > 0$.

$x^2 - x = 0$ tenglamaning ildizlari: $x_1 = 0, x_2 = 1$ Tengsizlik x ning (0, 1) intervaldagi barcha haqiqiy qiymatlari uchun o‘rinli.

Egri chiziq (0, 1) intervalda qavariq, $x^2 - x > 0$ tengsizlikni echamiz. Tengsizlik x ning $(-\infty, 0)$ va $(1, +\infty)$ intervallardagi barcha haqiqiy qiymatlarida o‘rinli. Bu intervallarda egri chiziq botiq.

957. Quyidagi egri chiziqlarning qavariqlik va botiqlik intervallarini toping:

1) $y = x^3 - 6x^2 + 2x - 6$; 2) $y = x^4 - 2x^3 - 12x^2 + 24x + 8$.

50-§. Egilish nuqtalari

$y = f(x)$ egri chiziqlarning qavariqlik botiqlikdan ajralgan nuqtasi egri chiziqning egilish nuqtasi deyiladi.

$y = f(x)$ egri chiziqning egilish nuqtalarini topish qoidasi

I. $y = f(x)$ funksiyaning ikkinchi tartibli hosilasini topiiladi.

$$y'' = f''(x).$$

II. Ikkinchi tartibli hosilani nolga tenglanadi $f'(x) = 0$ va $f'(x) = 0$ tenglamani yechiladi.

III. Ikkinchi tartibli hosilaning topilgan ildizlar bilan chegaralangan har qaysi intervaldagi ishorasi topiladi.

IV. Agar ikkinchi tartibli hosilaning berilgan ildiz bilan ajralgan ikkita qo‘shni intervaldagi ishoralari turlicha bo‘lsa, u holda ildizning berilgan qiymatida egilish nuqtasi mavjud; agar ishoralar bir xil bo‘lsa, u holda egilish nuqtasi mavjud emas.

V. Egilish nuqtalarining ordinatalarini topiladi, ya’ni ildizning egilish mavjud bo‘lgan qiymatlari uchun funksiya hisoblanadi.

958. $y = \frac{1}{3}x^3$ egri chiziqning egilish nuqtalarini toping.

Yechilishi. Egilish nuqtalarini topish qoidasiga ko‘ra:

1) $y' = x^2; y'' = 2x$

2) $2x = 0; x = 0;$

3) $y''_{x<0} = (-); y''_{x>0} = (+)$

4) $x = 0$ nuqtada egilish nuqtasi mavjud, chunki ikkinchi tartibli hosila $x = 0$ ildiz bilan ajratilgan qo‘shni intervallarda turli ishoralarga ega;

5) $y_{x=0} = \frac{1}{3} \cdot 0^3 = 0$

Egilish nuqtasi: (0;0).

959. Quyidagi egri chiziqlarning egilish nuqtalarini toping.

1) $y = x^3 - x$; 2) $y = 6x^2 - x^3$; 3) $y = \frac{1}{3}x^3 - 3x^2 + 8x - 4$

960. $y = x^4 - 10x^3 + 36x^2 - 100$ egri chiziqning egilish nuqtalarini toping.

Yechilishi. Egilish nuqtalarini topish qoidasiga ko‘ra:

1) $y' = 4x^3 - 30x^2 + 72x$; $y'' = 12x^2 - 60x + 72$

2) $12x^2 - 60x + 72 = 0$ $x^2 - 5x + 6 = 0$; $x_1 = 2, x_2 = 3$

3) $y'' = 12x^2(x-2)(x-3)$

Ikkinchi tartibli hosilaning ishoralari $x = 2$ va $x = 3$ ildizlar bilan ajratilgan $(-\infty;2),(2;3)$ va $(3;+\infty)$ intervallarda tekshiriladi:

$y''_{x<2} = (-)(-) = (+)$; $y''_{2<x<3} = (+)(-) = (-)$; $y''_{x>3} = (+)(+) = (+)$

4) Ikkinchi tartibli hosilaning ishorasi $(-\infty;2)$ intervalda – musbat, (2;3) intervalda – manfiy va $(3;+\infty)$ intervalda – musbat, ya’ni ikkinchi tartibli hosilaning $x = 2$ va $x = 3$ ildizlar bilan ajratilgan qo‘shni intervallardagi ishoralari turlicha, demak, $x = 2$ va $x = 3$ nuqtalarda egilish nuqtasi mavjud;

5) $y_{x=2} = 2^4 - 10 \cdot 2^3 + 36 \cdot 2^2 - 100 = -20$

$y_{x=3} = 3^4 - 10 \cdot 3^3 + 36 \cdot 3^2 - 100 = 35$

Egilish nuqtalari: (2;-20) va (3;35)

961. Qo‘yidagi egri chiziqlarning egilish nuqtalarini toping:

1) $y = x^4 - 8x^3 - 48x + 31$;

2) $y = x^4 - 6x^3 + 12x^2 - 10$

51-§. Funksiyalarning grafiklarini yasash

To'g'ri chiziqli koorditalar sistemasida egri chiziqlarni chizish

1. Funksiyaning maksimum va minimumini birinchi yoki ikkinchi tartibli hosila yoramida tekshiriladi. Maksimum va minimum nuqtalarning ordinatalari topiladi.
2. Funksiyaning egilish nuqtasi tekshiriladi. Egilish nuqtasining ordinatalari topiladi.
3. Egri chiziqning koordinata o'qlari bilan kesishish nuqtalari (agar bu qiyinchilik tug'dirmasa) topiladi yoki bir necha qo'shimcha nuqtaning koordinatalari topiladi.
4. Topilgan nuqtalar (ularning argumentlari ortib borish tartibida) jadvalga yoziladi va bu nuqtalar yasalib, ular orqali silliq egri chiziq o'tkaziladi. Agar ordinatalarning qiymatlari juda ham katta bo'lsa, u holda masshtabni egri chiziq tanlab olingan koordinata o'qlari sistemasiga joylasha oladigan qilib kichiraytirish kerak.
5. Funksiyaning grafiklarini yasang.

962. $y = x^2 + 4$

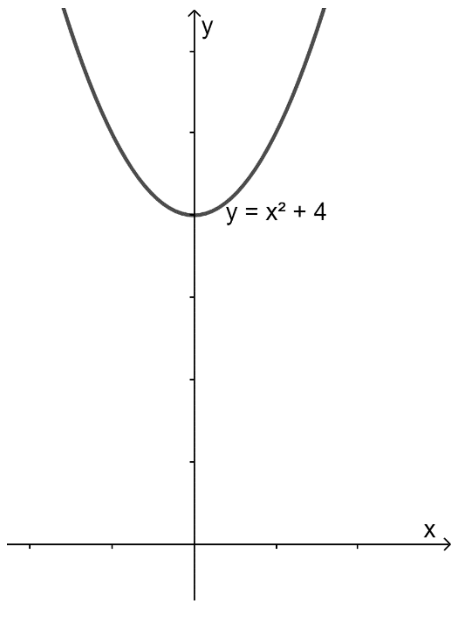
Tekshirish va yasash. Funksiyani yuqorida keltirilgan qoida asosida tekshiramiz:

1) $y' = 2x, 2x = 0, x = 0; y'_{x<0} = (-); y'_{x>0} = (+)$

Funksiya $x = 0$ da minimumga ega $y_{x=0} = 4$;

2) egri chiziqning egilish nuqtalari yo'q, chunki ikkinchi tartibli hosila istalgan x da musbat;

3) egri chiziq Oy o'q bilan $A(0;4)$ nuqtada kesishadi, Ox o'q bilan kesishmaydi;



116-rasm

4) parabola nuqtalar bo'yicha parabolani yasaymiz (116-rasm).

963. 1) $y = x^2 + 2$ 2) $y = x^2 - 4$

964. $y=-2x^2+4x$

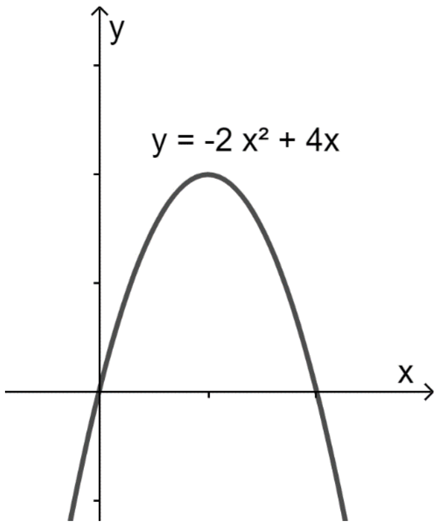
Tekshirish va yasash. Funksiyani tekshiramiz:

1) $y'=-4x^2+4;\ -4x+4=0;\ x=1;\quad y''=-4$

$y_{x=1}=-2\cdot1^2+4\cdot1=2;$

2) egri chiziqning egilish nuqtalari yo‘q, chunki ikkinchi tartibli hosila istalgan x da manfiy;

3) egri chiziq Oy o‘q bilan (0; 0) nuqtada kesishadi. Ox o‘q bilan kesishish nuqtasini topamiz: $y=0.\ x(-2x+4)=0,$ bu yerdan $x_1=0$ va $-2x+4=0,\ x_2=2.$ (0; 0) va (2; 0) nuqtalarga egamiz (117-rasm).



117-rasm

965. 1) $y=2x^2-8x;$ 2) $y=-3x^2+12x.$

966. $y=2x^2-12x+10.$

Tekshirish va yasash. 1) $y'=4x^2-12;$ $4x-12=0;\ x=3, y''=4$

Funksiya $x=3$ da minimumga ega:

$y_{x=3}=2\cdot3^2-12\cdot3+10=-8;$

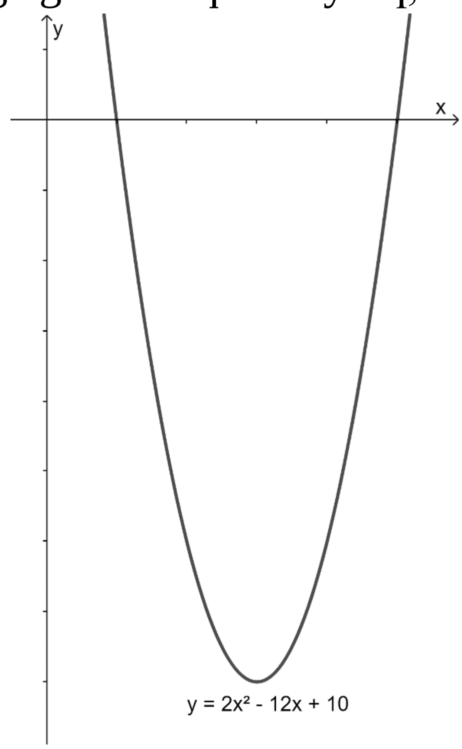
2) egri chiziqning egilish nuqtalari yo‘q, chunki ikkinchi tartibli hosila istalgan x da musbat;

3) Ox o‘q bilan kesishish nuqtalarini topamiz:

$y=0, 2x^2-12x+10=0; x_1=1, x_2=5$

Oy o‘q bilan kesishish nuqtalarini topamiz: $x=0, y=10$. Quyidagi nuqtalarga egamiz: (1; 0), (5; 0) va (0; 10);

4) parabola nuqtalari jadvalini tuzamiz:



118-rasm

Bu nuqtalar bo‘yicha parabolani yasaymiz (118-rasm).

967. 1) $y = -x^2 + 2x + 15$; 2) $y = x^2 + 5x + 4$

968. $y = \frac{1}{4}x^4$

Tekshirish va yasash. Funksiya juft, demak, egri chiziq nuqtalari Oy o‘qqa nisbatan simmetrik joylashgan;

1) $y' = x^3; x^3 = 0, \ x = 0; \ \ y'_{x<0} = (-), \ \ y'_{x>0} = (+)$

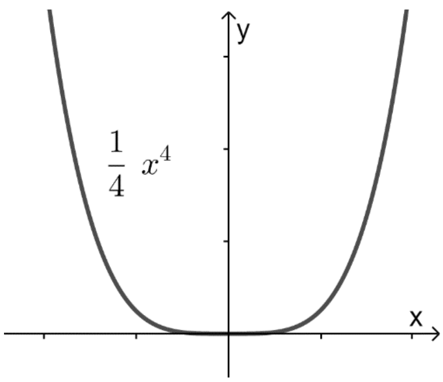
Funksiya $x = 0$ da minimumga ega: $y_{x=0} = 0$

2) $y'' = 3x^2$. Egri chiziqning egilish nuqtalari yo‘q, chunki ikkinchi tartibli hosila istalgan x da musbat;

3) egri chiziq koordinata uchlarini (0; 0) nuqtada kesib o‘tadi;

4) egri chiziq nuqtalarining jadvalini tuzamiz:

Bu nuqtalar bo‘yicha egri chiziqni yasaymiz (119-rasm).



119-rasm

969. $y = \frac{1}{16}x^4 - 1$

970. $y = x^2 - 3x$

Tekshirish va yasash. 1) $y' = 3x^2 - 3; \ \ 3x^2 - 3 = 0$

$x_1 = -1; x_2 = 1; \quad y'' = 6x; \quad y''_{x=-1} = -6$

Funksiya $x = -1$da maksimumga ega:

$$y_{x=-1} = (-1)^3 - 3(-1) = 2, y''_{x=1} = 6$$

Funksiya $x = 1$da minimumga ega: $y_{x=1} = 1^3 - 3 \cdot 1 = -2;$

2) Egilish nuqtasini topamiz:

$$y'' = 6x; 6x = 0, x = 0; y''_{x<0} = (-); y''_{x>0} = (+)$$

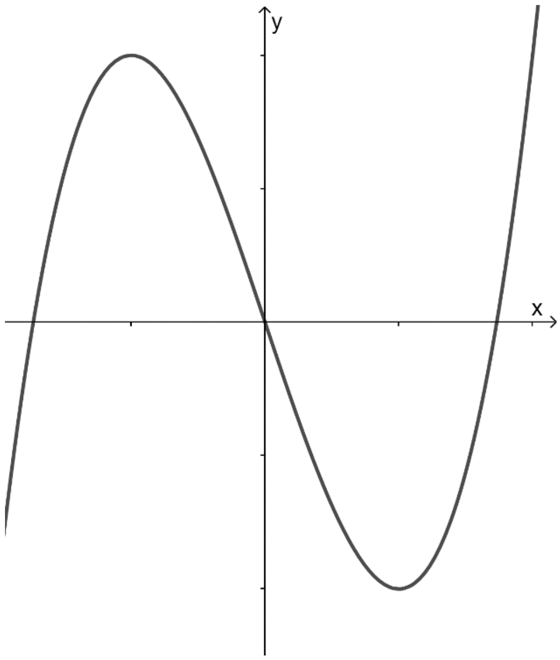
Egri chiziq (0; 0) nuqtada egilishga ega;

3) Egri chiziqning Ox o‘q bilan kesishish nuqtasini topamiz:

1) $y = 0, x^3 - 3x = 0; x(x^2 - 3) = 0; x_1 = 0$

$x^2 - \sqrt{3} \approx -1,7, x_3 = \sqrt{3} \approx 1,7$

3) egri chiziq nuqtalarining jadvalini tuzamiz:



120-rasm

Bu nuqtalar bo‘yicha egri chiziqni yasaymiz (120- rasm).

971. 1) $y = 3x^3 - x;$ 2) $y = -x^3 + x;$ 3) $y = \frac{1}{3}x^3 - 9$

972. $y = \frac{1}{5}x^5$

Tekshirish va yasash. Funksiya toq, demak, egri chiziqning nuqtalari koordinatalar boshiga nisbatan simmetrik joylashgan.

1) $y' = x^4$, $x^4 = 0;$ $x = 0$;

$y'_{x<0} = (+),\ y'_{x>0} = (+);$

Birinchi tartibli hosila ishorasini o‘zgartirmayapti, demak, funksiya maksimumga ham, minimumga ham ega emas;

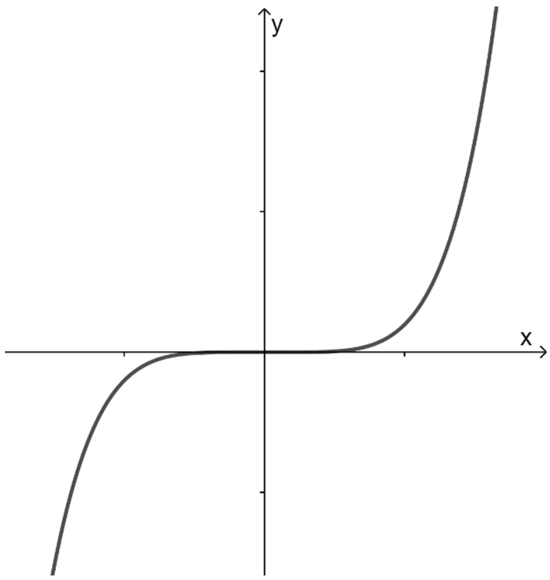
2) egilish nuqtasini topamiz:

$y'' = 4x^3;$ $4x^3 = 0;$ $x = 0;$ $y''_{x<0} = (-);$ $y''_{x>0} = (+);$

Egilish nuqtasi: (0; 0);

3) egri chiziq Oy o‘qni (0; 0) nuqtada kesib o‘tadi;

4) egri chiziq nuqtalari jadvalini tuzamiz:



121-rasm

Bu nuqtalar bo‘yicha egri chiziqni yasaymiz (121- rasm).

973. $y = \frac{1}{7}x^7$

974. $y = x^3 - 6x^2 + 9x - 3$

*Tekshirish va yasash.*1)

$y' = 3x^3 - 12x + 9;$

$3x^2 - 12x^2 + 9 = 0;$ $x^2 - 4x + 3 = 0$

$x_1 = 1,\ x_2 = 3;$ $y'' = 6x - 12;$

$y''_{x=1} = (-)$

Funksiya $x = 1$da maksimumga ega:

$$y_{x=1} = 1^3 - 6\cdot1^2 + 9\cdot1 - 3 = 1,$$
$$y''_{x=3} = (+)$$

Funksiya $x = 3$da minimumga ega:

$y_{x=3} = 3^3 - 6\cdot3^2 + 9\cdot3 - 3 = -3;$

2) egilish nuqtasini topamiz:

$$y'' = 6x - 12;\ 6x - 12 = 0; x = 2$$

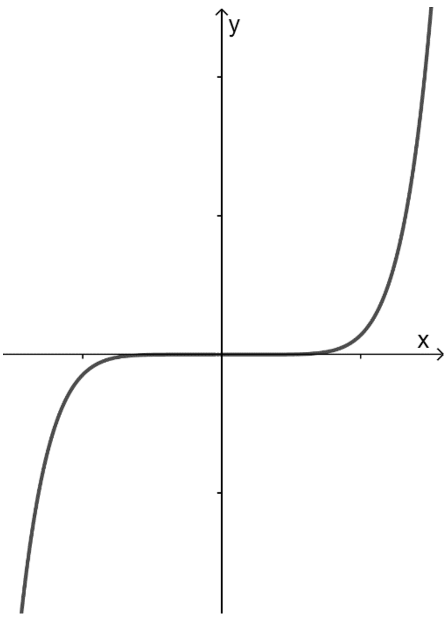
$$y''_{x<2} = (-); y''_{x>2} = (+); y_{x=2} = 2^3 - 6\cdot2^2 + 9\cdot2 - 3 = -1$$

Egilish nuqtasi: (2; -1),

3) egri chiziq Oy o‘qni (0; -3) nuqtada kesib o‘tadi;

4) egri chiziqning topilgan nuqtalari jadvalini tuzamiz:

Bu nuqtalar bo‘yicha egri chiziqni yasaymiz (122-rasm).



122-rasm

975. 1) $y'' = x^3 - x + 1$; 2) $y = 2x^3 - 9x^2 + 18x - 6$; 3) $y = 2x^3 + x^2 + 2$; 4) $y = 3x^4 - 16x^3 + 18x^2$

52- §. Aralash masalalar

976. 5 ni shunday ikkita qo‘shiluvchiga ajratingki ularning kublari yig‘indisi eng kichik bo‘lsin.

977. Ikkita sonning ayirmasi a ga teng. Agar bu sonlarning ko‘paytmasi eng kichik bo‘lsa, shu sonlarni toping.

978. Yuzi 12 ga, asosi 6 ga teng bo‘lgan uchburchakning eng kichik perimetrini hisoblang.

979. Agar tubi kvadrat, usti ochiq (qopkoqsiz) yashikning yon devorchalari va tubi yuzlarining umumiy sirti S ga teng bo‘lsa, eng katta hajmga ega bo‘lgan bu yashikning o‘lchamlarini toping.

980. Sirti S berilgan holda eng katta hajmga ega bo‘lgan silindrik bakning (qopqoqsiz) asosi radiusini va balandligini toping.

981. To‘la sirti S berilgan barcha siliidrlar ichidan hajmi eng katta bo‘lganini toping.

982. Eni 27 *m* va 64 *m* bo‘lgan ikkita kanal o‘zaro to‘g‘ri burchak hosil qiladi. Bir kanaldan ikkinchisiga o‘ta oladigan kema eng ko‘pi bilan qanday uzunlikka ega bo‘lishi mumkin?

983. Ochiq doiraviy silindrik tarnov eni a santimetr bo‘lgan tunukadan taYorlanmoqda α markaziy burchak qanday bo‘lganda tarnovnnng hajmi eng katta bo‘ladi?

984. Daryoda A va B pristanlar orasidagi masofa 144 km ga teng. C pristan A va B pristanlar orasida bo‘lib, B dan 81 *km* narida joylashgan. Kater oqim bo‘yicha A pristandan B gacha suzib borib, C pristanga qaytib keldi. Agar kater ABC yo‘lni o‘rtacha 35 km/soat tezlik bilan eng qisqa vaqt davomida o‘tgan bo‘lsa, daryo oqimining tezligi qanday?

985. 1) $y = x^3 + 6x^2 + 9x + 8$; 2) $y = 2x^3 - 3x^2 - 12x - 1$;

3) $y = x^3 - 6x^2 + 16$;

4) $y = 2x^3 + 3x^2 - 12x - 10$ funksiyalar uchun a) o‘sish va kamayish intervallarini; b) maksimum va minimumni; v) qavariqlik va botiqlik intervallarini; g) egilish nuqtasini toping.

Yozma ish

I variant

986. 1) $y = -\frac{1}{3}x^3 + \frac{1}{2}x^2 + 1$ funksiyaning o‘sish va kamayish intervallarini toping; 2) $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x - \frac{1}{3}$ funksiyaning $[-2;2]$ kesmadagi eng katta va eng kichik qiymatini toping.

3) $y = x^3 - 3x^2$ egri chiziqning qavariqlik va botiqligini tekshiring;

4) $y = \frac{1}{3}x^3 - 4x$ egri chiziqning egilish nuqtasini tekshiring.

5) Nuqtaning to‘g‘ri chiziqli harakat qonuni

$S=-\frac{1}{6}t^3+\frac{1}{2}t^2+\frac{1}{2}t+1$

berilgan. Bu nuqta harakatining maksimal tezligini toping (t sek hisobida, s m hisobida).

II variant

987. 1) $y=x^4-4x+4$ funksiyaning o‘sish va kamayish intervallarini toping;

2) $y=\frac{1}{3}x^3+x^2-3x-4$ funksiyaning $[-4;2]$ kesmadagi eng katta va eng kichik qiymatini toping.

3) $y=x^3-12x^2+1$egri chizisning qavariqlik va botiqligini tekshiring; 4) $y=\frac{1}{3}x^3+x^2+\frac{1}{3}$egri chiziqning egilish nuqtasini tekshiring.

5) Nuqtaning to‘g‘ri chiziqli harakat qonuni $S=-\frac{1}{3}t^3+3t^2+5t+3$ berilgan. Bu nuqta harakatining maksimal tezligini topish (t sek hisobida, s m hisobida berilgan).

Mundarija

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2-bob. To'g'ri chiziq

3-bob. Tekislikda nuqtalarning geometrik o'rni. Ikkinchi tartibli egri chiziqlar

DIFFERENSIAL HISOB ELEMENTLARI

4-bob. Limitlar

5-bob.Funksiya tushunchasi

6-bob.Hosila

7-bob. Hosilani funksiyalarni tekshirishga tatbiq etish

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Chapter 5. The concept of a function

Chapter 6. Derivative

Chapter 7. Applications of the derivative in the study of a function

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