* 7-mavzu. Chiziqli tenglamalar sistemasi.

n ta x_1 , x_2 , x_3 ,... , x_n noma”lumli, chiziqli, n ta

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{2n}x_n = b_n \end{cases}$$

a_{ij} , b_i , $\in R$, tenglamalar sistemasini echish usullari, echimi qanday bo’lishi masalalarini qaraymiz.

7. 1. Kramer formulasi.

* Noma"lumlar koeffisientlaridan tuzilgan determinant tenglamalar sisremasining asosiy determinanti, undagi j-ustun o'rniga ozod hadlardan iborat ustun qo'yilgan determinant esa *j*-yordamchi determinant deyiladi va ko'rinishida belgilanadi.
*
*

$$\Delta_j = \begin{vmatrix} a_{11} \ldots & a_{1j-1} & b_1 & a_{1j+1} \ldots a_{1n} \\ a_{21} \ldots & a_{2j-1} & b_2 & a_{2j+1} \ldots a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} \ldots & a_{nj-1} & b_n & a_{nj+1} \ldots a_{nn} \end{vmatrix}$$

*
*
* Dastlab, berilgan tenglamalar sistemasidan har bir i-tenglamani ko'paytiramiz va hosil bo'lgan tenglamalarni qo'shamiz:
* Determinantni yoyish haqidagi teoremaga ko'ra: .Endi sistamadagi har bir i-tenglama ga ko'paytirilib qo'shilsa, ,...., qo'shilsa, tenglik hosil bo'ladi.
* Demak, sistemadagi noma"lumlar formula yordamida xisoblanar ekan. Bu Kramer formulasidir.
* tenglikdan quyidagilar kelib chiqadi:
* da sistema yagona echimga ega, uni birgalikda deyiladi.
* bo'lsa, sistema cheksiz ko'p echimga ega.

Misol. Kram$\Delta = 0$,er formulasi yordamida yeching:

$$\begin{cases} x_1 + x_2 + x_3 - x_4 = 2 \\ x_1 + 2x_2 + 3x_3 - 4x_4 = -2 \\ 2x_1 + x_2 - x_3 + x_4 = 5 \\ 4x_1 + 3x_2 + 2x_3 - 4x_4 = 0 \end{cases} \qquad \Delta = \begin{vmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 3 & -4 \\ 2 & 1 & -1 & 1 \\ 4 & 3 & 2 & -4 \end{vmatrix} = 3 ,$$

$$\Delta_1 = \begin{vmatrix} 2 & 1 & 1 & -1 \\ -2 & 2 & 3 & -4 \\ 5 & 1 & -1 & 1 \\ 0 & 3 & 2 & -4 \end{vmatrix} = 3 , \ \Delta_2 = \begin{vmatrix} 1 & 2 & 1 & -1 \\ 1 & -2 & 3 & -4 \\ 2 & 2 & -1 & 1 \\ 4 & 0 & 2 & -4 \end{vmatrix} = 6 ,$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 2 & -1 \\ 1 & 2 & -2 & -4 \\ 2 & 1 & 5 & 1 \\ 4 & 3 & 0 & -4 \end{vmatrix} = 9 , \ \Delta_4 = \begin{vmatrix} 1 & 1 & 1 & 2 \\ 1 & 2 & 3 & -2 \\ 2 & 1 & -1 & 5 \\ 4 & 3 & 2 & 0 \end{vmatrix} = 12$$ bo’lganligi uchun

$x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$.

*7 2. Matrisaviy usulda echish.

Berilgan tenglamalar sistemasini matrisaviy

$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ . & . & \cdots & . \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ .. \\ b_n \end{pmatrix}$ yoki $A \cdot X = B$ ko'rinishida yozish mumkin.

Agar $|A| \neq 0$ bo'lsa, A^{-1}matrisa mavjud va yagona bo'lishidan $A^{-1} \cdot A \cdot X = A^{-1} \cdot B$ yoki $X = A^{-1} \cdot B$.

Nomalumlardan iborat X-ustun matrisani bunday topish matrisaviy usul deyiladi.

Misol.Yuqoridagi sistemani shu usulda qayta echamiz.

$|A| = \Delta = 3$ ekanligini hisoblaganmiz.

$A = \begin{vmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 3 & -4 \\ 2 & 1 & -1 & 1 \\ 4 & 3 & 2 & -4 \end{vmatrix}$ matrisaga teskari A^{-1} ni topamiz.

$A_{11} = \begin{vmatrix} 2 & 3 & -4 \\ 1 & -1 & 1 \\ 3 & 2 & -4 \end{vmatrix} = 5$; $A_{21} = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ 3 & 2 & -4 \end{vmatrix} = -4$; $A_{31} = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 3 & -4 \\ 3 & 2 & -4 \end{vmatrix} = -3$; $A_{41} = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 3 & -4 \\ 2 & -1 & 1 \end{vmatrix} = 2$

$A_{12} = -\begin{vmatrix} 1 & 3 & -4 \\ 2 & -1 & 1 \\ 4 & 2 & -4 \end{vmatrix} = -6$; $A_{22} = \begin{vmatrix} 1 & 1 & -1 \\ 2 & -1 & 1 \\ 4 & 2 & -4 \end{vmatrix} = 6$; $A_{32} = -\begin{vmatrix} 1 & 1 & -1 \\ 1 & 3 & -4 \\ 4 & 2 & -4 \end{vmatrix} = 6$; $A_{32} = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 3 & -4 \\ 2 & -1 & 1 \end{vmatrix} = -3$

$A_{13} = \begin{vmatrix} 1 & 2 & -4 \\ 2 & 1 & 1 \\ 4 & 3 & -4 \end{vmatrix} = 9$; $A_{23} = -\begin{vmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ 4 & 3 & -4 \end{vmatrix} = -3$; $A_{33} = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & -4 \\ 4 & 3 & -4 \end{vmatrix} = -3$; $A_{43} = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & -4 \\ 2 & 1 & 1 \end{vmatrix} = -2$;

$$A_{14} = -\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ 4 & 3 & 2 \end{vmatrix} = 5\ ;\ A_{24} = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 4 & 3 & 2 \end{vmatrix} = -1\ ;\quad A_{34} = -\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 4 & 3 & 2 \end{vmatrix} = 0\ ;\ A_{44} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 1 & -1 \end{vmatrix} = -1$$

Demak, $A^{-1} = \frac{1}{3}\begin{vmatrix} 5 & -4 & -3 & 2 \\ -6 & 6 & 6 & -3 \\ 9 & -3 & -3 & -2 \\ 5 & -1 & 0 & -1 \end{vmatrix}$

$$X = A^{-1} \cdot B = \frac{1}{3}\begin{pmatrix} 5 & -4 & -3 & 2 \\ -6 & 6 & 6 & -3 \\ 9 & -3 & -3 & -2 \\ 5 & -1 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2 \\ 5 \\ 0 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 3 \\ 6 \\ 9 \\ 12 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}.$$

$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$, ya'ni $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$.

7.3. Noma’lumlarni ketma-ket yo’qotish (Gauss) usuli.

* Berilgan chiziqli tenglamalar sistemasi koeffisientlari orqali quyidagi jadvalni tuzib olamiz.

* Bu jadval berilgan sistema kengaytirilgan matrisasi deyiladi.

* Tushunarliki, har bir satrda bittadan tenglama turibdi, faqat tenglik o'rniga chiziqcha tortilgan.

* Bu matrisa ustida o'tkaziladigan har bir elementar almashtirish berilgan sistemaga ekvivalent sistema hosil qiladi. Shu sababli, elementar almashtirishlar yordamida kengaytirilgan matritsani uchburchak ko'rinishiga keltirib olamiz, buning uchun bo'lishi kifoya agar bo'lsa, birinchi tenglamani boshqa yo'ldagi tenglama bilan almashtirish orqali bunga erishish mumkin.

* Faraz qilaylik, elementar almashtirishlar yordamida kengaytirilgan matritsa ko'rinishga kelsin.

* Unga mos sistema

$$\left.\begin{matrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1 \\ C_{22}x_2 + C_{23}x_3 + \cdots + C_{2n}x_n = C_2 \\ C_{22}x_2 + \cdots + C_{3n}x_n = C_3 \\ \dots\dots\dots\dots\dots\dots\dots \\ C_{nn}x_n = C_n \end{matrix}\right)$$ ko'rinishida bo'ladi.

Bu sistemadan dastlab x_n , so'ngra x_{n-1} , ... , va nihoyat $x_1$topiladi.

Bu usulda 2-tenglamadan x_1, ni 3-tenglamadan x_1 ва x_2 , ... , n - tenglamadan $x_1, x_2, \dots, x_{n-1}$ ketma - ket yo'qotilayotganligi uchun noma'lumlarni ketma - ket yo'qotish usuli deyiladi. Bu usul Gauss nomi bilan bog'liq bo'lib, talabalarga elementar matematikadan ma'lum.

Misol. Avvalgi usullarda yechilgan sistemani qaraylik. Uning kengaytirilgan matritsasi $\left(\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 1 \\ 1 & 2 & 3 & -4 & -2 \\ 2 & 1 & -1 & 1 & 5 \\ 4 & 3 & 2 & -4 & 0 \end{array}\right)$ ko'rinishda bo'ladi. 1- yo'l elementlarini (-1) ga ko'paytirib 2-yo'lga (-2) ga ko'paytirib 3-yo'lga, (-4) ga ko'paytirib 4- yo'lga qo'shamiz, natijada, kengaytirilgan matritsa. $\left(\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & -3 & -4 \\ 0 & -1 & -3 & 3 & 1 \\ 0 & -1 & -2 & 0 & -8 \end{array}\right)$ ko'rinishiga keladi. 2 yo'lni 3 , 4 – yo'l elementlariga qo'shamiz.

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & -3 & -4 \\ 0 & 0 & -1 & 0 & -3 \\ 0 & 0 & 0 & -3 & -12 \end{array}\right)$$

Bu matritsaga mos sistema. $\left.\begin{matrix} x_1 + x_2 + x_3 - x_4 = 2 \\ x_2 + 2x_3 - 3x_4 = -4 \\ -x_3 = -3 \\ -3x_4 = -12 \end{matrix}\right)$

Ko'rinishida bo'ladi. Ketma-ket $x_4 = 4$; $x_3 = 3$ larni topib 2-tenglamaga qo'yamiz.

$$x_2 + 2 \cdot 3 - 3 \cdot 4 = -4.$$

Bu erdan $x_2 = 2$ ekanligini topib, 1-tenglamaga o'tamiz.

$x_1 + 2 + 3 - 4 = 2$. Demak , $x_1 = 1$.

* 7.4. Bir jinsli sistemalar.

Agar qaralayotgan chiziqli tenglamalar sistemasida barcha ozod hadlar nol bo’lsa $b_i = 0\ ,(i = \overline{1,n})$, bunday sistema bir jinsli deyiladi.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = 0 \end{cases}$$

Bu holda $x_1 = x_2 = x_3 = \cdots = x_n = 0$ sonlar har bir tenglamani qanoatlantirib, sistemaning trivial yechimi deyiladi.

Bir jinsli sistemaning trivial bo’lmagan notrivial yechimlarini qidiramiz.

Kramer formulasiga ko’ra $\Delta_1 = \Delta_2 = \cdots = \Delta_n = 0$ Notrivial yechim mavjud bo’lishi uchun $\Delta = 0$ bo’lishi zarur. Unda sistema cheksiz ko’p yechimga ega bo’ladi.

Notrivial yechimlarni topish uchun sistema uchburchak ko’rinishga keltiriladi.

$\Delta = 0$ ekanligidan sistema oxirgi tenglamasida ikki noma'lum qoladi. Ulardan birini ozod parametr deb olib, qolgan noma'lumlarni u orqali yoziladi.

Parametr cheksiz ko’p qiymat qabul qilgani uchun notrivial cheksiz ko’p yechimlarni topamiz.

Misol .

$$\begin{cases} 2x_1 + x_2 - 4x_3 = 0 \\ 3x_1 + 5x_2 - 7x_3 = 0 \\ 4x_1 - 5x_2 - 6x_3 = 0 \end{cases}$$ sistema notrivial yechimlari topilsin

$$\Delta = \begin{vmatrix} 2 & 1 & -4 \\ 3 & 5 & -7 \\ 4 & -5 & -6 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 & 1 & -4 \\ 0 & 7 & -2 \\ 0 & -7 & 2 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 & 1 & -4 \\ 0 & 7 & -2 \\ 0 & 0 & 0 \end{vmatrix} = 0$$ bo’lgani uchun trivial bo’lmagan yechimlar mavjud.

Sistemaning oxirgi tengligi $-7x_2 + 2x_3 = 0$ ko’rinishda bo’ladi. Agar $x_3 = 7\lambda$ desak, $x_2 = 2\lambda$ bo’ladi. Ularni birinchi tenglamaga qo’’yib:

$$2x_1 + 2\lambda - 4 \cdot 7\lambda = 0 \quad va \quad x_2 = 13\lambda$$

Demak, ($13\lambda\,;\ 2\lambda\,;\ 7\lambda$), $\lambda \in R$ ko’rinishdagi uchlik sistemaning yechimidir. Bu yechimlar oilasi trivial yechim (0; 0; 0) ni o’zida saqlaydi.

Shu paytgacha qaralgan sistemalarda noma'lumlar soni tenglamalar soniga teng edi. Umuman,

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases} \quad m \neq n,$$

sistemalarni ham qarash mumkin. Bunday sistemalar birgalikda bo’lishi asosiy va kengaytirilgan quyidagi matritsalar rangiga bog’liq bo’ladi.

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots a_{1n} \\ a_{21} & a_{22} & \dots a_{2n} \\ . & . & \dots . \\ a_{m1} & a_{m2} & \dots a_{mn} \end{pmatrix}, \quad \overline{A} = \begin{pmatrix} a_{11} & a_{12} & \dots a_{1n} b_1 \\ a_{21} & a_{22} & \dots a_{2n} b_2 \\ . & . & \dots . \\ a_{m1} & a_{m2} & \dots a_{mn} b_m \end{pmatrix}$$

Teorema. (Kroneker-Kopelli): Tenglamalar sistemasi birgalikda bo'lishi uchun A va $\overline{A}$ matritsalar ranglari teng rang$A$=rang$\overline{A}$ bo'lishi zarur.

Mavzuga doir misol va masalalar.

1. Determinantlar hisoblansin.

1). $\begin{vmatrix} 5 & 4 \\ 3 & -2 \end{vmatrix}$ 2). $\begin{vmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{vmatrix}$ 3). $\begin{vmatrix} \sin^2\alpha & \cos^2\alpha \\ \sin^2\beta & \cos^2\beta \end{vmatrix}$ 4). $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

5). $\begin{vmatrix} 3 & 4 & -5 \\ 8 & 7 & -2 \\ 2 & -1 & 8 \end{vmatrix}$

Nollari ko'p qator elementlari bo'yicha yoyib hisoblang:

1). $\begin{vmatrix} 1 & b & 1 \\ 0 & b & 0 \\ b & 0 & -b \end{vmatrix}$ 2). $\begin{vmatrix} -x & 1 & x \\ 0 & -x & -1 \\ x & 1 & -x \end{vmatrix}$ 3). $\begin{vmatrix} 1 & 0 & 0 \\ -2 & -3 & 1 \\ 3 & 8 & -2 \end{vmatrix}$

Tenglamalarni yeching:

1). $\begin{vmatrix} 3 & x & -x \\ 2 & -1 & 3 \\ x+10 & 1 & 1 \end{vmatrix} = 0;$ 2). $\begin{vmatrix} x & x+1 & x+2 \\ x+3 & x+4 & x+5 \\ x+6 & x+7 & x+8 \end{vmatrix} = 0;$

3). $\begin{vmatrix} \cos 8x & -\sin 5x \\ \sin 8x & \cos 5x \end{vmatrix} = 0$

4. Determinant xossalaridan foydalanib hisoblang:

1). $\begin{vmatrix} \sin^2\alpha & 1 & \cos^2\alpha \\ \sin^2\beta & 1 & \cos^2\beta \\ \sin^2\gamma & 1 & \cos^2\gamma \end{vmatrix}$ 2). $\begin{vmatrix} \sin^2\alpha & \cos 2\alpha & \cos^2\alpha \\ \sin^2\beta & \cos 2\beta & \cos^2\beta \\ \sin^2\gamma & \cos 2\gamma & \cos^2\gamma \end{vmatrix}$

3). $\begin{vmatrix} (a_1+b_1)^2 & a_1^2+b_1^2 & a_1 b_1 \\ (a_2+b_2)^2 & a_2^2+b_2^2 & a_2 b_2 \\ (a_3+b_3)^2 & a_3^2+b_3^2 & a_3 b_3 \end{vmatrix}$

5. Determinant xossalaridan foydalanib hisoblang:

1). $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{vmatrix}$ 2). $\begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$

3). $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \\ 1 & 8 & 27 & 64 \\ 1 & 16 & 81 & 256 \end{vmatrix}$ 4). $\begin{vmatrix} 35 & 59 & 71 & 52 \\ 42 & 70 & 77 & 54 \\ 43 & 68 & 72 & 52 \\ 29 & 49 & 65 & 50 \end{vmatrix}$

6. Uchburchak ko'rinishiga keltirib hisoblang

1). $\begin{vmatrix} 1 & 2 & 3 & \cdots & n \\ -1 & 0 & 3 & \cdots & n \\ -1 & -2 & 0 & \cdots & n \\ \cdots & \cdots\cdots & \cdots & \cdots\cdots & \cdots \\ -1 & -2 & -3 & & 0 \end{vmatrix}$ 2). $\begin{vmatrix} 1 & 2 & 2 & \cdots & 2 \\ 2 & 2 & 2 & \cdots & 2 \\ 2 & 2 & 3 & \cdots & 2 \\ \cdots & \cdots\cdots & \cdots & \cdots\cdots & \cdots \\ 2 & 2 & 2 & \cdots & n \end{vmatrix}$ 3). $\begin{vmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 1 & \cdots & 1 \\ 1 & 1 & 0 & \cdots & 1 \\ \cdots & \cdots\cdots & \cdots & \cdots\cdots & \cdots \\ 1 & 1 & 1 & \cdots & 0 \end{vmatrix}$ 4). $\begin{vmatrix} n & 1 & 1 & \cdots & 1 \\ 1 & n & 1 & \cdots & 1 \\ 1 & 1 & n & \cdots & 1 \\ \cdots & \cdots\cdots & \cdots & \cdots\cdots & \cdots \\ 1 & 1 & 1 & \cdots & n \end{vmatrix}$

7. $A = \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix}$, $C = \begin{pmatrix} 4 & 2 \\ 7 & -1 \end{pmatrix}$ bo'lsa, $A \cdot B - 2C$ ni hisoblang.

8. $A = \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}$, $B = \begin{pmatrix} i & 1 \\ -i & 1 \end{pmatrix}$ bo'lsa, $(i+1)\cdot A + (i-1)\cdot B$ ni hisoblang.

9. Hisoblang.

1) $\begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix}^2$, 2) $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}^n$, 3) $\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}^n$, 4) $\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}^n$, 5) $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}^n$

10. Kvadrati no'l matritsa bo'lgan barcha kvadrat matritsalarni toping.

11. Kvadrati birlik matritsa bo'lgan barcha kvadrat matritsalarni toping.

12. Quyidagi matritsalarga teskari matritsani toping.

1) $\begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$ 2) $\begin{pmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$ 3) $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$

13. Matritsaviy tenglamalarni yeching.

1) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot x = \begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix}$ 2) $x \cdot \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -5 & 6 \end{pmatrix}$ 3) $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \cdot x \cdot \begin{pmatrix} -3 & 2 \\ 5 & -3 \end{pmatrix} = \begin{pmatrix} -2 & 4 \\ 3 & -1 \end{pmatrix}$

1) $\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix} \cdot x = \begin{pmatrix} 1 & -3 & 0 \\ 10 & 2 & 7 \\ 10 & 7 & 8 \end{pmatrix}$

14. Matritsa rangini hisoblang.

1) $\begin{pmatrix} 0 & 4 & 10 & 1 \\ 4 & 8 & 18 & 7 \\ 10 & 18 & 40 & 17 \\ 1 & 7 & 17 & 3 \end{pmatrix}$ 2) $\begin{pmatrix} 1 & -1 & 2 & 3 & 4 \\ 2 & 1 & -1 & 2 & 0 \\ -1 & 2 & 1 & 1 & 3 \\ 1 & 5 & -8 & -5 & -12 \\ 3 & -7 & 8 & 9 & 13 \end{pmatrix}$

15. Tenglamalar sistemalarini 1) Kramer formulasi 2) Matritsaviy 3) Gauss usullarida yeching.

1) $\begin{cases} 3x + 2y = 7 \\ 4x - 5y = 40 \end{cases}$ 2) $\begin{cases} 5x + 2y = 4 \\ 7x + 4y = 8 \end{cases}$ 3) $\begin{cases} x_1 + x_2 + 2x_3 = -1 \\ 2x_1 - x_2 + 2x_3 = -4 \\ 4x_1 + x_2 + 4x_3 = -2 \end{cases}$

$$4)\ \begin{cases} x_1 + 2x_2 + 4x_3 = 31 \\ 5x_1 + x_2 + 2x_3 = 29 \\ 3x_1 - x_2 + x_3 = 10 \end{cases} \quad 5)$$

$$\begin{cases} x_1 + 2x_2 + 3x_3 - 2x_4 = 6 \\ 2x_1 - x_2 - 2x_3 - 3x_4 = 8 \\ 3x_1 + 2x_2 - x_3 + 2x_4 = 4 \\ 2x_1 - 3x_2 + 2x_3 + x_4 = -8 \end{cases} \quad 6)\ \begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 5 \\ 2x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 3x_1 + 2x_2 + x_3 + 2x_4 = 1 \\ 4x_1 + 3x_2 + 2x_3 + x_4 = -5 \end{cases}$$

$$7)\ \begin{cases} \lambda x + y + z = 1 \\ x + \lambda y + z = \lambda \\ x + y + \lambda z = \lambda^2 \end{cases} \quad 8)\ \begin{cases} x + ay + a^2 z = a^3 \\ x + by + b^2 z = b^3 \\ x + cy + c^2 z = c^3 \end{cases} \quad 9)\ \begin{cases} ax + by + z = 1 \\ x + aby + z = b \\ x + by + az = 1 \end{cases}$$

16. Bir jinsli tenglamalar sistemasini yeching:

$$1)\ \begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 0 \\ x_1 + x_2 + 2x_3 + 3x_4 = 0 \\ x_1 + 5x_2 + x_3 + 2x_4 = 0 \\ x_1 + 5x_2 + 5x_3 + 2x_4 = 0 \end{cases} \quad 2)\ \begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ x_2 + x_3 + x_4 + x_5 = 0 \\ x_1 + 2x_2 + 3x_3 = 0 \\ x_2 + 2x_3 + 3x_4 = 0 \\ x_3 + 2x_4 + 3x_5 = 0 \end{cases}$$

Oliy algebra elementlariga doir joriy nazorat uchun uy vazifalari.

I. $z = \frac{1}{(-1)^N + \sqrt{(3)^k} \cdot i}$ kompleks son berilgan, bunda k= $\begin{cases} 1, agar\ n = 3k-2 \\ 0, agar\ n = 3k-1 \\ -1, agar\ n = 3k \end{cases}$

a) z ni algebraik formada yozing va z^2 ni hisoblang.

b) z ni tirgonometrik formada yozing va z^{N+20} , $\sqrt[4]{z}$ larni hisoblang.

c) z ni ko'rsatkichli formada yozing.

2. $x^3 + Nx + 1 = 0$ tenglama yechimlari x_1, x_2, x_3 bo'lsa, quyidagilarni hisoblang

1) $x_1^2 + x_2^2 + x_3^2$; 2) $x_1^2 x_2 + x_1 x_2^2 + x_2^2 x_3 + x_2 x_3^2 + x_3^2 x_1 + x_3 x_1^2$;

3) $x_1^4 x_2^2 + x_1^2 \cdot x_2^4 + x_2^4 x_3^2 + x_2^2 x_3^4 + x_3^4 x_1^2 + x_3^2 \cdot x_1^4$

4) $\frac{x_1^2}{(x_1+1)^2} + \frac{x_2^2}{(x_2+1)^2} + \frac{x_3^2}{(x_3+1)^2}$

3. Ferrari usuli bilan yeching;

$x^4 - x^3 + \left(4 - \frac{N}{2} - \frac{N^2}{4}\right)x^2 + 2Nx + \left(\frac{N^2}{16} - 1\right) = 0$,

yordamchi kubik tenglama bitta yechimi $-\frac{N}{4}$

4. Gorner sxemasi yordamida

$P_6(x) = x^6 + (1-N)x^5 - Nx^4 - x^2 + (N-1)x + N$ va

$P_4(x) = x^4 - (N+1)x^3 + (N+x)x^2 - (N+1)x + N$ ko'phadlar EKUB va EKUK lari topilsin.

5. $\frac{x+N}{x^5+2x^4+(N-1)x^3+2(N-1)x^2-Nx-2N}$ kasrni sodda kasrlarga yoying.

6. Nollari ko'p qator elementlari bo'yicha yoyib hisoblang.

$$\begin{vmatrix} 1 & 2 & 3 & -5 \\ 2 & 0 & -1 & 0 \\ 3 & 0 & 2 & N \\ 4 & 0 & 5 & 0 \end{vmatrix}$$

7. $\begin{cases} x_1 - Nx_2 - x_3 - x_4 = 3 \\ x_1 + x_2 + Nx_3 + 2x_4 = 0 \\ 2x_1 + Nx_2 + x_3 + x_4 = 0 \\ N \cdot x_1 - 3x_2 + 2x_3 - x_4 = -1 \end{cases}$ sistemani 1) Kramer qoidasi, 2)Matritsaviy,

3) Gauss usuli da yeching.