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Exploring Analytic Geometry with *Mathematica*®

by Donald L. Vossler

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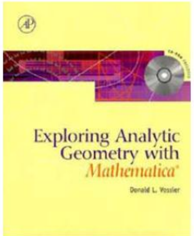

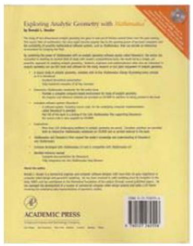

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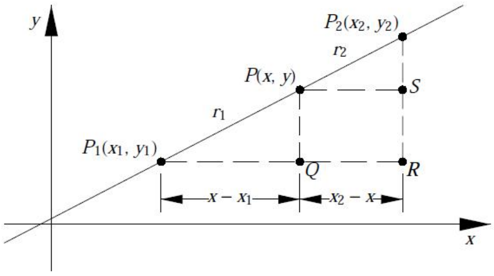

Figure 3.5: Point of division.

coordinates (x, y) which are to be determined. Sense is important here and P must be located so that $P_1P/PP_2 = r_1/r_2$.

Since $\triangle P_1PQ$ and $\triangle PSP_2$ are similar, it follows that $(x - x_1)/r_1 = (x_2 - x)/r_2$. Solving this equation for x yields

$$x = \frac{x_1 r_2 + x_2 r_1}{r_1 + r_2}. \tag{3.1}$$

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$$x = \frac{x_1 r_2 + x_2 r_1}{r_1 + r_2}. \tag{3.1}$$

Similarly,

$$y = \frac{y_1 r_2 + y_2 r_1}{r_1 + r_2}. \tag{3.2}$$

To find the midpoint of the segment P_1P_2 the ratio r_1/r_2 must be unity; hence $r_1 = r_2$ and Equations (3.1) and (3.2) specialize to

$$x = \frac{x_1 + x_2}{2} \quad \text{and} \quad y = \frac{y_1 + y_2}{2}. \tag{3.3}$$

Equations (3.1), (3.2) and (3.3) also have useful physical interpretations. In (3.1) and (3.2), let x and y be the coordinates of the center of gravity of masses r_1 and r_2 placed at P_1 and P_2, respectively. If the masses are equal, the center of gravity lies halfway between them as indicated by (3.3).

It is of further interest to note the positions of P for various values of the ratio r_1/r_2. If this ratio is zero, then P coincides with P_1, and if this ratio is a positive number, P is an internal point of division. As $r_1/r_2 \to +\infty$, $P \to P_1$. For $-\infty < r_1/r_2 < -1$, P is an external point of division (in the direction of P_1P_2). For $-1 < r_1/r_2 < 0$, P is an external point in the opposite direction with P_1P negative and P_2P positive.

Example. Find the point that divides the line segment between the points $P_1(-2, 5)$ and $P_2(4, -1)$ into the ratio $r_1/r_2 = -2$.

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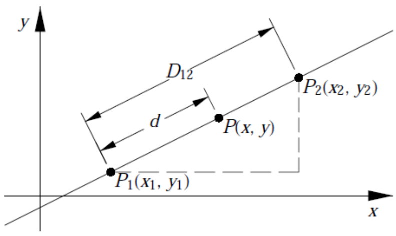

Figure 3.6: Point offset a distance towards a point.

Solution. The *Descarta2D* function `Point2D[`*point*, *point*, r_1, r_2`]` returns the point that divides the line segment between the points into the ratio r_1/r_2.

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3.6 Collinear Points

Three distinct points $P_1(x_1, y_1)$, $P_2(x_2, y_2)$ and $P_3(x_3, y_3)$ are said to be *collinear* if they lie on the same straight line. We can construct any point, P_3, on the line P_1P_2 by selecting an appropriate value for d and applying Equation (3.4). All such points P_1, P_2 and P_3 are obviously *collinear* by construction. Now consider the value of the determinant

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}.$$

Mathematica provides the `Det` command for expanding the value of such a determinant.

```
In[10]:= Clear[x1, y1, x2, y2, x3, y3, d, D12];
         Det[{{x1, y1, 1}, {x2, y2, 1}, {x3, y3, 1}}] /.
         {x3 -> x1 + (x2 - x1) * d / D12, y3 -> y1 + (y2 - y1) * d / D12} // Simplify

Out[10]  0
```

We see from *Mathematica* that for any value of d, the determinant given is zero. Therefore, the necessary and sufficient condition that three points lie on the same line is given by the determinant equation

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0,$$

where the coordinates of the points are $P_1(x_1, y_1)$, $P_2(x_2, y_2)$ and $P_3(x_3, y_3)$.

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