Funktsiyaning hosilasi

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Hosilaning ta'rifi

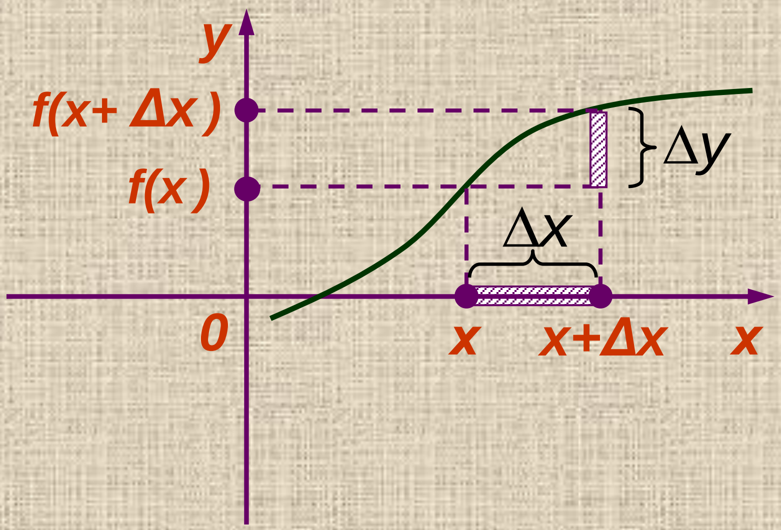
y = f(x) funktsiya ***(a; b)*** intervalda aniqlangan bo'lsin.

Argument x *ga* qandaydir Δx orttirma beramiz:

$$x + \Delta x \in (a;b)$$

Argument orttirmasiga mos funktsiya orttirmasini topamiz:

$$\Delta y = f(x + \Delta x) - f(x)$$



Agar ushbu limit $\lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ mavjud bo'lsa

u holda uni ***y = f(x)*** funktsiyaning hosilasi deyiladi va quyidagicha belgilanadi:

$$y'; \quad f'(x); \quad \frac{dy}{dx}$$

Hosilaning ta'rifi

Shunday qilib, ta'rifga ko'ra:

$$y' = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

y = f(x) funktsiya **(a; b)** intervalning har bir nuqtasida hosilaga ega bo'lsa shu intervalda ***differentsiallanuvchi*** deyiladi; Funktsiya hosilasini topish amaliga ***differentsiallash*** deyiladi.

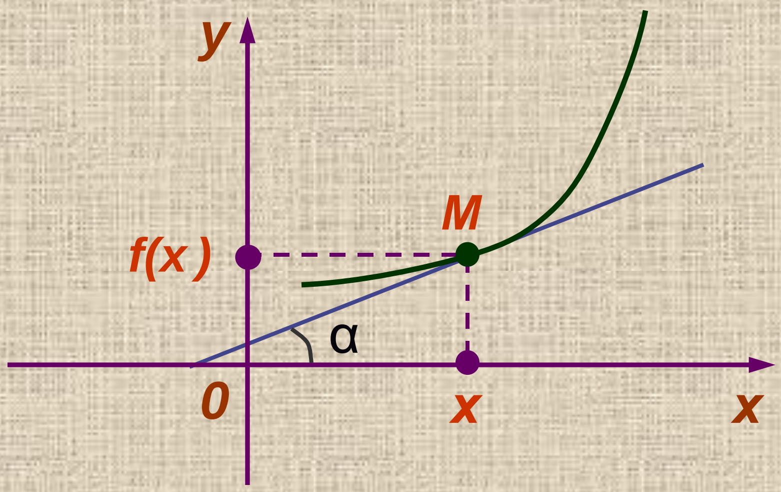
y = f(x) funktsiyaning x_0 nuqtadagi qiymati quyidagi simvollar orqali belgilanadi:

$$y'(x_0); \quad f'(x_0); \quad y'\big|_{x_0}$$

*Agar **y = f (x)** funktsiya fizik jarayonni tavsiflasa, unda **f '(x)** bu jarayonning tezligi - hosilaning fizik ma'nosi.*

Hosilaning geometrik ma'nosi

Uzluksiz ***L*** egri chiziqda ikkita ***M*** va M_1 nuqtalarni olamiz



M va M_1 nuqtalar orqali kesuvchi o'tkazamiz va ***φ*** kesuvchining og'ish burchagi.

$$tg\varphi = \frac{\Delta y}{\Delta x} =$$

$$= \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$\Delta x \to 0$ funktsiya uzluksizligiga ko'ra Δy ham nolga intiladi. Shuning uchun M_1 nuqta egri chiziq bo'ylab ***M*** nuqtaga yaqinlashadi, MM_1 kesuvchi urinmaga aylanadi.

$$\varphi \to \alpha \quad \Rightarrow \lim_{\Delta x \to 0} \varphi = \alpha \quad \Rightarrow \lim_{\Delta x \to 0} tg\varphi = tg\alpha$$

Hosilaning geometrik ma'nosi

$$\Rightarrow \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = tg\alpha = k \ = y'$$

f '(x) hosila ***y = f(x)*** funktsiyaning ***x*** abtsissa nuqtasiga o'tkazilgan urinmaning burchak koeffitsientiga teng

y'

Agar ***M*** urinish nuqtasi ***(x_0; y_0),*** koordinataga ega bo'lsa urinmaning burchak koeffitsiyenti ***k = f '(x_0)*** ga teng.

To'g'ri chiziqning burchak koeffitsiyentli tenglamasi

$$y - y_0 = f'(x_0)(x-x_0)$$

Urinma tenglamasi

Normal tenglamasi

Urinmaga perpendikulyar to'g'ri chiziqqa ***normal*** deyiladi.

$f'(x_0)$

$$k_{norm} = -\frac{1}{k_{urin}} = -\frac{1}{f'(x_0)} \Rightarrow y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)$$

Asosiy elementar funktsiyalarning hosilalari

1 *Chiziqli funktsiya*: $y = x$

Argument *x ga* Δx orttirma beramiz va funktsiya orrtirmasini topamiz :

$$\Delta y = (x + \Delta x) - x = \Delta x$$

$$\frac{\Delta y}{\Delta x} = \frac{\Delta x}{\Delta x} = 1$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \left(\frac{\Delta x}{\Delta x} \right) = \lim_{\Delta x \to 0} 1 = 1$$

Differentsiyallash qoidalari

Faraz qilaylik ***u(x)*** , ***v(x)*** va ***w(x)*** – funktsiyalar ***(a; b)*** intervalda differentsiallanuvchi bo'lsin, ***C*** –o'zgarmas son.

- $(C)' = 0$
- $(u \pm v)' = u' \pm v'$
- $(u \cdot v)' = u' \cdot v + u \cdot v' \quad \Rightarrow \quad (C \cdot u)' = C \cdot u'$

$$(u \cdot v \cdot w)' = u' \cdot v \cdot w + u \cdot v' \cdot w + u \cdot v \cdot w'$$

- $\left(\dfrac{u}{v}\right)' = \dfrac{u' \cdot v - u \cdot v'}{v^2} \quad \Rightarrow \quad \left(\dfrac{C}{v}\right)' = \dfrac{-C \cdot v'}{v^2}$

Murakkab funktsiyaning hosilasi

Faraz qilaylik ***y = f(u)*** va ***u = φ(x)*** bo'lsin, u holda ***y = f(φ(x))*** – ***x*** argumentning murakkab funktsiyasi bo'ladi.

Teorema

Agar ***u = φ(x)*** funktsiya ***x*** nuqtada u'_x hosilaga ega , ***y = f(u)*** funktsiya esa ***u*** nuqtada, y'_u hosilaga ega bo'lsin, : U holda murakkab funksiyaning hosilasi y'_x :

$$y'_x = y'_u \cdot u'_x$$

Bu qoida funksiya oraliq argumentlari bir nechta bo'lganda ham o'rinli:

$$y = f(u); \quad u = \varphi(v); \quad v = g(x) \quad \Rightarrow y = f(\varphi(g(x)))$$

$$y'_x = y'_u \cdot u'_v \cdot v'_x$$

1-Misol

Quyidagi funktsiyaning hosilasini toping

$$y = \frac{1 + \sin x}{x^3 \cdot \ln x}$$

$$y' = \left(\frac{1 + \sin x}{x^3 \cdot \ln x} \right)'$$

$$= \frac{(1 + \sin x)' \cdot (x^3 \cdot \ln x) - (1 + \sin x) \cdot (x^3 \cdot \ln x)'}{\left(x^3 \cdot \ln x\right)^2}$$

$$= \frac{(1' + (\sin x)') \cdot (x^3 \cdot \ln x) - (1 + \sin x) \cdot ((x^3)' \cdot \ln x + x^3 (\ln x)')}{\left(x^3 \cdot \ln x\right)^2}$$

$$= \frac{\cos x \cdot x^3 \cdot \ln x - (1 + \sin x) \cdot (3x^2 \cdot \ln x + x^2)}{\left(x^3 \cdot \ln x\right)^2}$$

2-Misol

Quyidagi funktsiya hosilasini toping $y = \cos(\ln^{12} x)$

Berilgan funktsiyani quyidagicha ifodalab olamiz

$$y = \cos u; \quad u = v^{12}; \quad v = \ln x$$

$$y'_x = y'_u \cdot u'_v \cdot v'_x$$

$$y'_u = -\sin u = -\sin v^{12} = -\sin\left(\ln^{12} x\right)$$

$$u' = 12v^{11} = 12\ln^{11} x$$

$$v' = \frac{1}{x}$$

$$y' = -\sin\left(\ln^{12} x\right) \cdot 12\ln^{11} x \cdot \frac{1}{x}$$

qisqacha

$$y' = (\cos(\ln^{12} x))' = -\sin(\ln^{12} x) \cdot (\ln^{12} x)'$$

$$= -\sin(\ln^{12} x) \cdot 12\ln^{11} x \cdot (\ln x)' =$$

Oshkormas funktsiyaning hosilasi

Agar ***y = f(x)*** quyidagi tenglama bilan berilgan bo'lib, ***y*** ga nisbatan yechilgan bo'lsa u holda funktsiya *oshkor* ko'rinishda berilgan deyiladi.

Oshkormas ko'rinishdagi funktsiya ***y*** ga nisbatan yechilmagan bo'lib quyidagi tenglama bilan aniqlanadi

$$F(x;y)=0$$

Oshkormas funktsiyaning hosilasini topish uchun oshkormas ko'rinishdagi tenglamani ***x*** o'zgaruvchi bo'yicha differentsiallash zarur. Bu yerda ***y*** ni ***x*** ning funktsiyasi deb qaraymiz va olingan natijani hosilaga nisbatan yechamiz.

$$\left[x^3+y^3-3xy\right]'=[0]' \Longrightarrow (x^3)'+(y^3)'-3(xy)'=0 \Longrightarrow$$

$$3x^2+3y^2\cdot y'-3(x'y+xy')=0 \Longrightarrow$$

$$x^2+y^2\cdot y'-y-xy'=0 \Longrightarrow \boxed{y'=\frac{y-x^2}{y^2-x}}$$