KOMPLEKS SONLAR

ULAR USTIDA AMALLAR

REJA:

1. **Kompleks sonlar haqida dastlabki ta'riflar**

2. **Kompleks sonlar ustida asosiy amallar**

3. **Kompleks sonni darajaga ko’tarish va kompleks sondan ildiz chiqarish**

4. **Mustaqil yechish uchun misollar**

KOMPLEKS SONLAR TA'RIFI

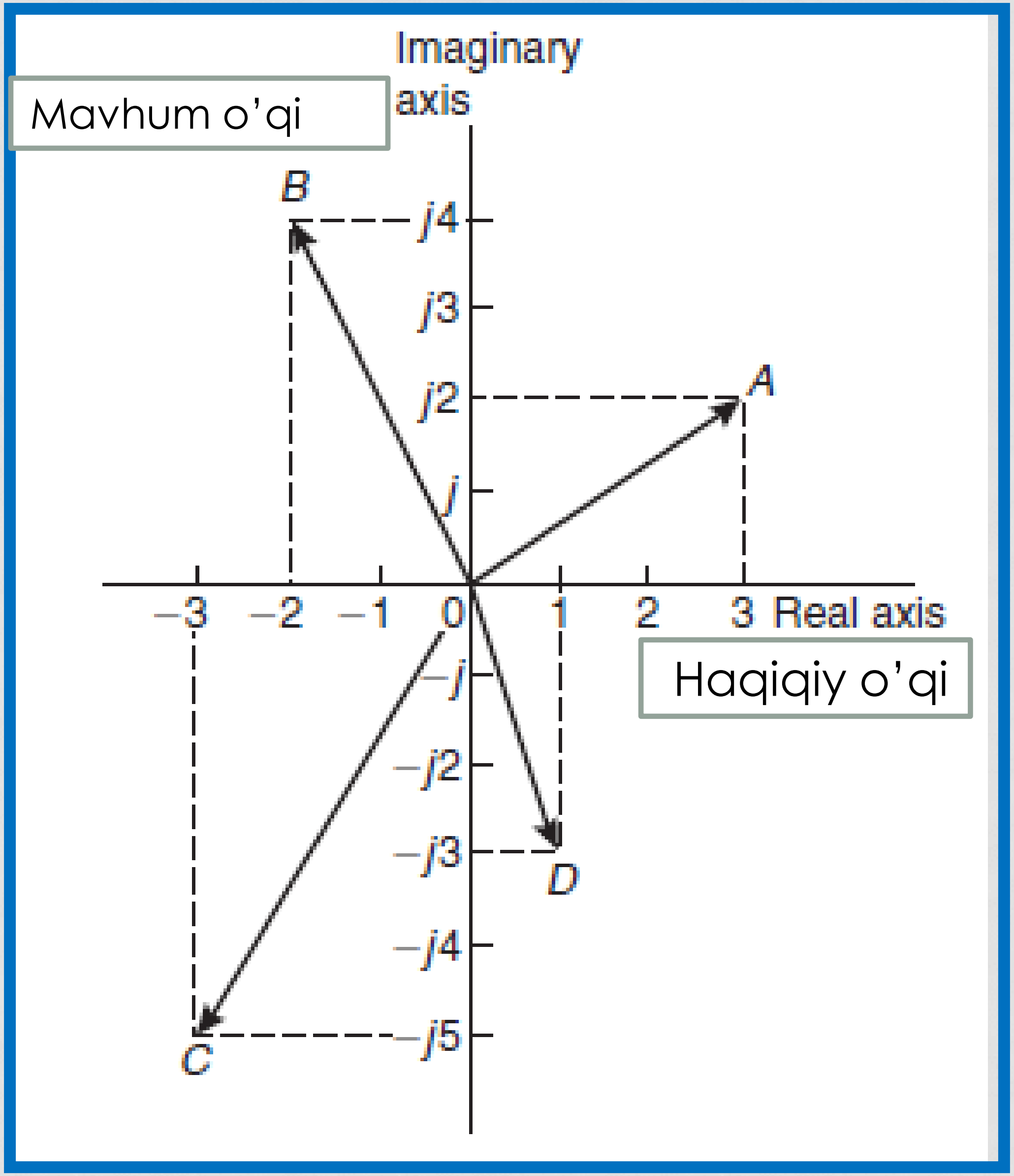
Kompleks son deb

$$z = a + ib \quad (1)$$

ifodaga aytiladi, bu erda *a* va *b* haqiqiy sonlar, *i* - mavhum birlik, ushbu tengliklar bilan aniqlanadi:

$$i = \sqrt{-1} \quad \text{yoki} \quad i^2 = -1 \quad (2)$$

a- kompleks son z ning haqiqiy qismi, *ib* - mavhum qismi deyiladi. Ular bunday belgilanadi: *a=Re z, b=Imz*. Agar *a=0* bo'lsa, *0+ib=ib* sof mavhum son deyiladi; *b=0* agar bo'lsa, haqiqiy son hosil bo'ladi: *a+I*0=a*. Faqat mavhum qismining ishorasi bilan farq qiladigan ikki kompleks son: *z=a+ib* va *z=a-ib* bir-biriga qo'shma deyiladi.



$Z=3+2i;$

$Z=1-3i;$

$Z=-2+4i;$

$Z=-3+5i$

KOMPLEKS SONNING TRIGONOMETRIK SHAKLI.

- Koordinatalar boshini qutb, *Ox* o'qining musbat yo'nalishini qutb o'qi deb olib, *A(a,b)* nuqtaning qutb koordinatalarini φ va $r(r \geq 0)$ bilan belgilaymiz. Unda ushbu tengliklarni yozish mumkin:
- $a = r\cos\varphi$ $\qquad$ $b = r\sin\varphi$
- demak, kompleks son z ni bunday tasvirlash mumkin:
- $a+ib=r\cos\varphi + ir\sin\varphi$

yoki

$$z= r(\cos\varphi + i\sin\varphi) \qquad (3)$$

- Bu tenglikning o'ng tomonidagi ifodada $z=a+ib$ kompleks son yozuvining trigonometrik shakli deb ataladi.

KOMPLEKS SONLARNI QO'SHISH.

- Ikki kompleks son $z_1=a_1+ib_1$ va $z_2=a_2+ib_2$ ning yig'indisi deb ushbu
- $z_1+z_2=(a_1+ib_1)+(a_2+ib_2)=(a_1+a_2)+i(b_1+b_2)$ (1)
- tenglik bilan aniqlangan kompleks songa aytiladi.
  - formuladan vektorlar bilan tasvirlangan kompleks sonlarni
- qo'shish-vektorlarni qo'shish qoidasiga muvofiq bajarilishi kelib chiqadi.

KOMPLEKS SONLARNI AYIRISH

- Ikki $z_1=a_1+ib_1$ **va** $z_2=a_2+ib_2$ kompleks sonlarni ayirmasi deb shunday kompleks songa aytiladiki, unga z_2 kompleks sonni qo'shganda z_1 kompleks son hosil bo'ladi:
- $z_1 - z_2=(a_1+ib_1)- (a_2+ib_2)=(a_1 - a_2)+i(b_1 - b_2)$
  Ikki kompleks son ayirmasining moduli shu sonlarni kompleks
- o'zgaruvchilar tekisligida tasvirlovchi nuqtalar orasidagi masofaga teng:
-

$$|z_1 - z_2| = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$$

MISOL

$Z_1 = 2 + j4$; $Z_2 = 3 - j$ sonlar berilgan.Quyidagi amallarni bajaring

(a) $Z_1 + Z_2 = (2 + j4) + (3 - j)$

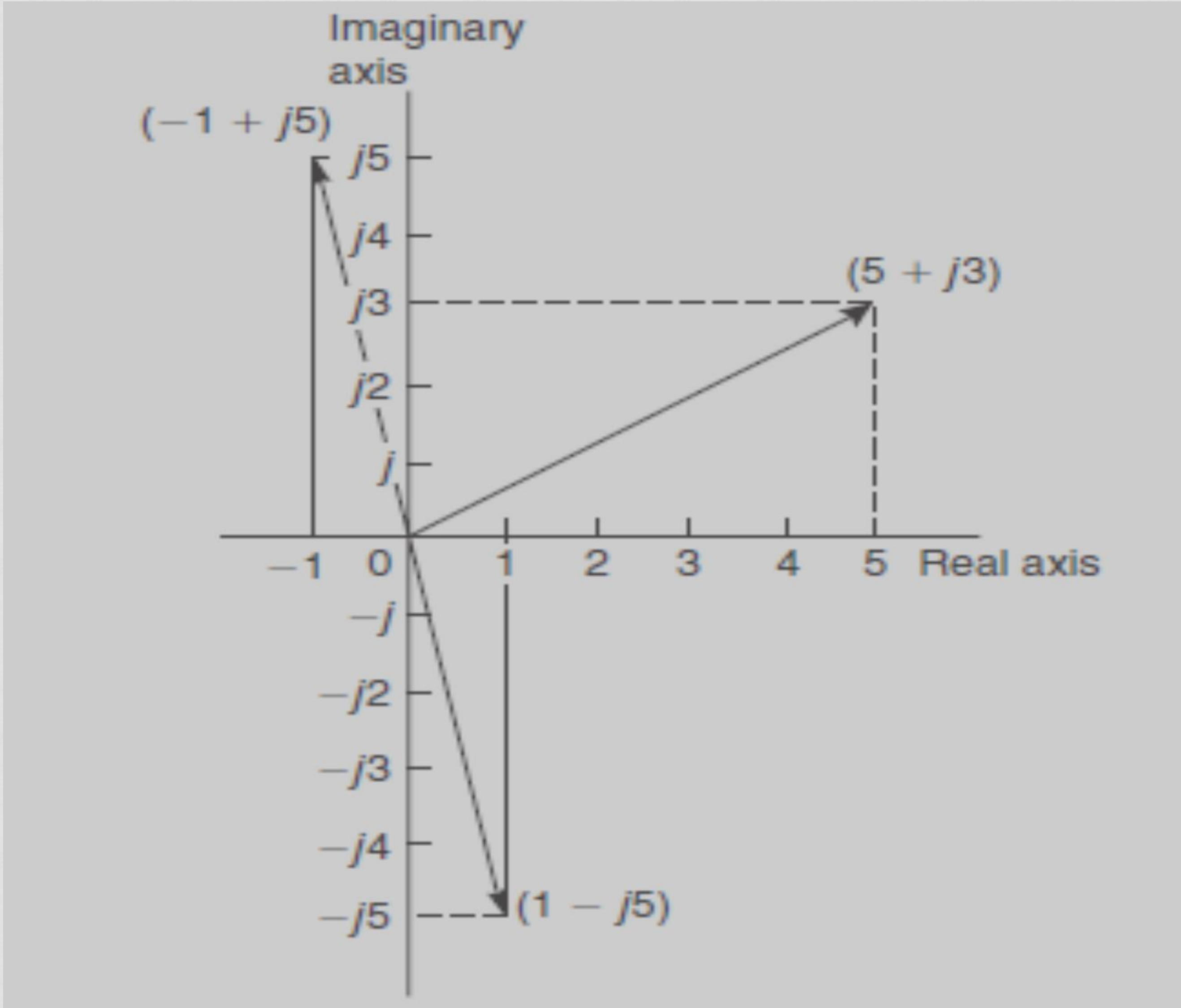
$$= (2 + 3) + j(4 - 1) = \mathbf{5 + j3}$$

(b) $Z_1 - Z_2 = (2 + j4) - (3 - j)$

$$= (2 - 3) + j(4 - (-1)) = \mathbf{-1 + j5}$$

(c) $Z_2 - Z_1 = (3 - j) - (2 + j4)$

$$= (3 - 2) + j(-1 - 4) = \mathbf{1 - j5}$$



Imaginary
axis
(−1 + j5)
j5
j4
(5 + j3)
j3
j2
j
−1
0
1
2
3
4
5
Real axis
−j
−j2
−j3
−j4
−j5
(1 − j5)

KOMPLEKS SONLARNI KO'PAYTIRISH.

- $z_1=a_1+ib_1$ v**a** $z_2=a_2+ib_2$ kompleks sonlar ko'paytmasi deb, ularni ikki xadlar singari algebra qoidasiga muvofiq, lekin

$i^2=-1$, , $i^3=-i$, $i^4=(-i)i=-i^2=1$, $i^5=i$ va hokazo.

umuman *k* butun bo'lganda:

$i^{4\kappa}=-1$, ,$i^{4\kappa+1}=i$, $i^{4\kappa+2}=-1$, $i^{4\kappa+3}=-i$

ekanligini e'tiborga olib ko'paytirganda hosil bo'lgan kompleks songa aytiladi.

- Shu qoidaga asosan quyidagi ko'paytmani hosil qilamiz: $z_1z_2=(a_1+ib_1)(a_2+ib_2)=a_1a_2+ib_1a_2+ia_1b_2+i^2b_1b_2$
- yoki
- . $z_1z_2=(a_1a_2-b_1b_2)+i(b_1a_2+a_1b_2)$

MISOL. *Z=(3+J2) VA Y=(4-J5)* KOMPLEKS SONLAR KO'PAYTMASINI TOPING.

$$(3+j2)(4-j5)$$
$$= 12 - j15 + j8 - j^2 10$$
$$= (12 - (-10)) + j(-15 + 8)$$
$$= 22 - j7$$

KOMPLEKS SONLARNI BO'LISH.

- Bo'linmani topish uchun surat va maxrajini maxrajga qo'shma bo'lagan songa ko'paytiramiz:

- $$\frac{a_1+ib_1}{a_2+ib_2} = \frac{a_1+ib_1}{a_2+ib_2} * \frac{a_1-ib_1}{a_2-ib_2} = \frac{a_1a_2+b_1b_2}{{a_2}^2+{b_2}^2} + i\,\frac{a_2b_1-a_1b_2}{{a_2}^2+{b_2}^2}$$

MISOL. *Z=2-J5 VA Y=3+J4* BERILGAN, BO'LISH AMALINI BAJARING.

$$\frac{2-j5}{3+j4} = \frac{2-j5}{3+j4} \times \frac{(3-j4)}{(3-j4)}$$

$$= \frac{6-j8-j15+j^2 20}{3^2+4^2}$$

$$= \frac{-14-j23}{25} = \frac{-14}{25} - j\frac{23}{25}$$

KOMPLEKS SONLAR TRIGONOMETRIK SHAKLDA

$$z_1 = r_1(\cos\varphi_1 + i\sin\varphi_1),$$
$$z_2 = r_2(\cos\varphi_2 + i\sin\varphi_2)$$

- berilgan bo'lsa, ushbuni hosil qilamiz:

$$\frac{z_1}{z_2} = \frac{r_1(\cos\varphi_1 + i\sin\varphi_1)}{r_2(\cos\varphi_2 - i\sin\varphi_2)} = \frac{r_1}{r_2}[\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2) \qquad (5)$$

- Bu tenglikni tekshirish uchun bo'luvchini bo'linmaga ko'paytirish kifoya:

- $r_2(\cos\varphi_2 + i\sin\varphi_2)\,\frac{r_1}{r_2}[\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2) =$
- $= r_2\,\frac{r_1}{r_2}[\cos(\varphi_2 + \varphi_1 - \varphi_2 + i\sin(\varphi_2 + \varphi_1 - \varphi_2)] = r_1(\cos\varphi_1 + i\sin\varphi_1).$

KOMPLEKS SONNI DARAJAGA KO'TARISH.

- Bundan oldingi paragrafdagi (3) formuladan, agar *n* butun musbat son bo'lsa, ushbu formula kelib chiqadi:

$$[r(\cos\varphi + i\sin\varphi)]^n = r^n(\cos n\varphi + i\sin n\varphi). \qquad (1)$$

- Bu Muavr formulasi deb ataladi. Bundan ko'rinadiki, kompleks sonni butun musbat darajaga ko'tarishda modul shu darajaga ko'tariladi, argument esa daraja ko'rsatkichiga ko'paytiriladi.
- Endi Muavr formulasining yana bir tadbiqini qaraymiz.
- Bu formulada r=1 deb faraz qilib, $(\cos\varphi + i\sin\varphi)^n = \cos n\varphi + i\sin n\varphi$
- tenglikni hosil qilamiz. Chap tomonni Nyuton binomi formulasi bo'yicha yoyib, haqiqiy va mavhum qismlarini tenglab $\sin n\varphi$ va $\cos n\varphi$ ni $\sin\varphi$ va $\cos\varphi$ning darajalari orqali ifoda qila olamiz.

MUSTAQIL YECHISH UCHUN MISOLLAR

1 $z_1 = 1 + i\sqrt{3}$ $z_2 = 1 - i\sqrt{3}$

$z_1 \cdot z_2 = ?$ $z_1 + z_2 = ?$ $z_1 - z_2 = ?$ $\frac{z_1}{z_2} = ?$

2 $z = \frac{1}{(1 - i\sqrt{3})^6}$

4 $(-1)^{\sqrt{3}}$

3 $z = (1 + i\sqrt{3})^{15}$

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