TOSHKENT IRRIGATSIYA VA QISHLOQ XO'JALIGINI MEXANIZATSIYALASH MUHANDISLARI INSTITUTI

FAN: Oliy matematika

MAVZU: *Oliy matematika fanining maqsad va vazifalari. Determinant tushunchasi. Determinantni hisoblash usullari. Determinantlarning asosiy xossalari.*

Reja:

- *1. Oliy matematika fanining maqsad va vazifalari*
- *2. Determinant tushunchasi*
- *3. Determinantni hisoblash usullari*
- *4. Determinantlarning asosiy xossalari*

1.Ikkinchi tartibli determinant

<u>**1-Tarif**</u>

a_{11}, a_{12}, a_{21}, a_{22} sonlardan tuzilgan ushbu ko'rinishdagi ifodaga yoki jadvalga **ikkinchi tartibli determinant** deyiladi.

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

a_{11}, a_{12}, a_{21}, a_{22} sonlar determinantning elementlari deyiladi; bu sonlar ikkita satr va ikkita ustunga joylashtirilgan bo'ladi.

Ikkinchi tartibli determinant sondan iborat bo'lib, u bosh diagonal elementlari ko'paytmasidan ikkinchi diagonal elementlari ko'paytmasini ayirmasiga tengdir.

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

Bu yerda a_{ij} – larni xaqiqiy sonlar deb qaraladi. a_{ij}- ni determinantni i-chi satr j-ustun elementi deyiladi.

Umumiy xolda a_{ij}-lar funksiyalardan iborat bo'lishi ham mumkin.

Misollar:

1-misol

$$\begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix} = 2 \times 5 - 3 \times (-1) = 13$$

2-misol

$$\begin{vmatrix} sinx & cosx \\ cosx & sinx \end{vmatrix} = sin^2x - cos^2x = -cos2x$$

2.Uchinchi tartibli determinant

2-tarif

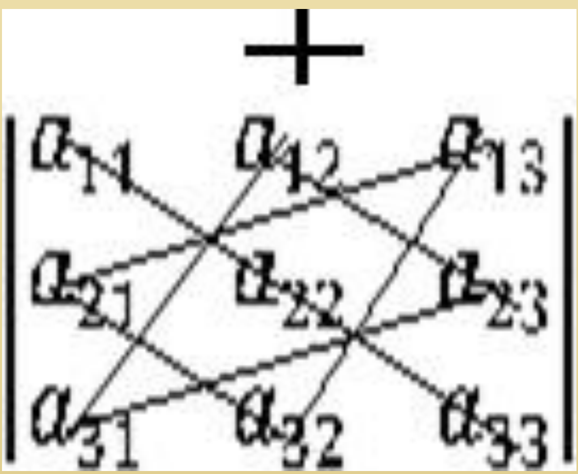
a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33} 9 ta sondan tuzilgan ushbu *ko'rinishdagi ifodaga yoki sonlardan tuzilgan jadvalga uchinchi tartibli determinant deyiladi.*

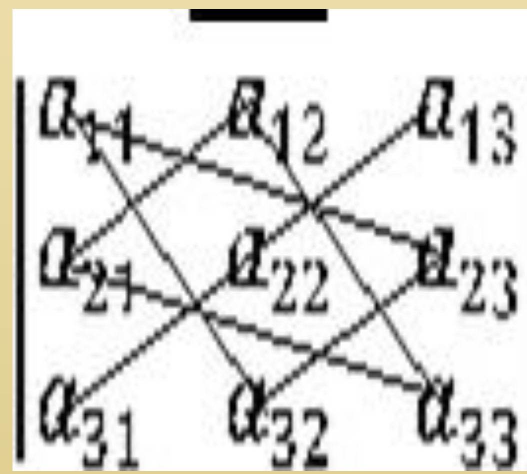
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Uchinchi tartibli determinantda 3 ta satr va 3 ta ustun bo'lib elementlari joylashtirilgan bo'ladi.

Uchinchi tartibli determinant ham sondan iborat bo'lib, uni quyidagi uchburchak yoki diagonal usulda hisoblanadi:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$





Uchburchak qoidasi

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{31}a_{23}a_{12})$$

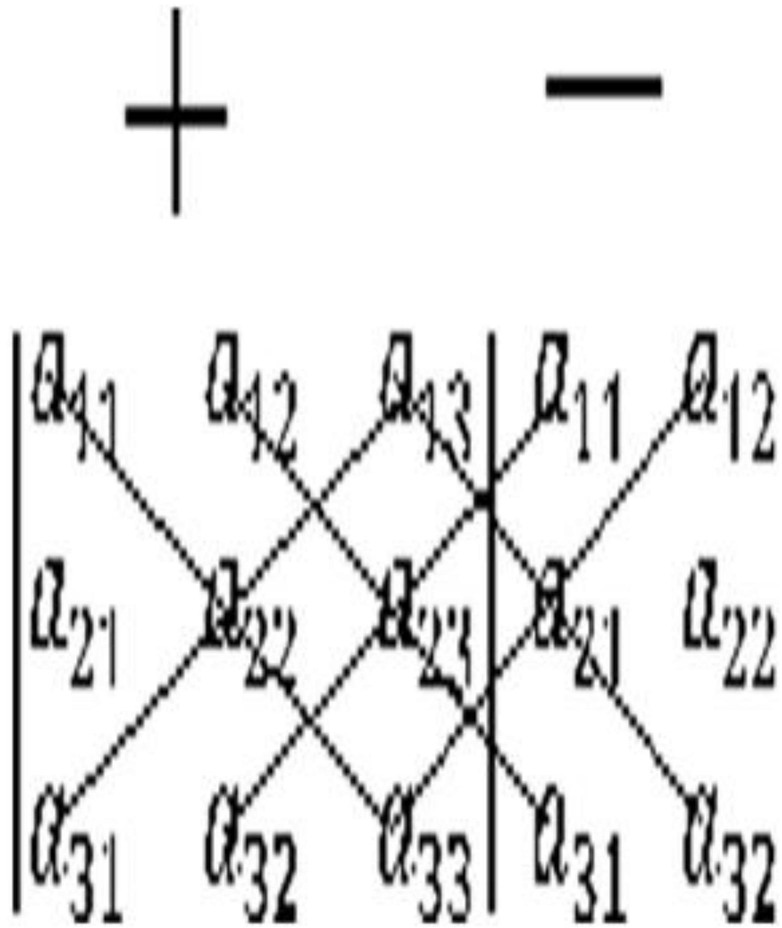
$$-(a_{13}a_{22}a_{31} + a_{23}a_{32}a_{11} + a_{33}a_{12}a_{21})$$

Uchburchak usuli orqali masalaning yechimi:

$$\begin{vmatrix} 2 & -2 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & 0 \end{vmatrix} = 2\cdot 2\cdot 0 + 1\cdot 1\cdot 3 + 3\cdot(-2)\cdot 1 - (3\cdot 2\cdot 3 + 2\cdot 1\cdot 1 + 1\cdot(-2)\cdot 0) =$$

$$= 0 + 3 - 6 - 18 - 2 - 0 = -23$$

b) Sarryus qoidasi



3 –тартибли детерминантни ***диагоналлар усули*** деб аталувчи ушбу усул билан ҳам ҳисоблаш мумкин:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} -$$

$$- a_{12}a_{21}a_{33} .$$

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1-мисолдаги детерминантни диагонал усулидан фойдаланиб ҳисобласак,

$$\begin{vmatrix} 2 & -1 & 0 \\ -1 & 3 & -2 \\ -3 & 0 & 4 \end{vmatrix} \begin{matrix} 2 & -1 \\ 1 & 3 \\ -3 & 0 \end{matrix} = 24 - 6 + 0 + 0 + 0 + 4 = 22$$

3.Determinantning asosiy xossalari

1.Determinantda xamma satrlar mos ustunlar qilib yozilsa uning qiymati o'zgarmaydi

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

4-misol $\begin{vmatrix} 2 & 3 \\ 5 & 8 \end{vmatrix} = \begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix} = 16-15=1$

2.Determinantning istalgan ikki ustuni (yoki ikki satri) almashtirilsa uning faqat ishorasi o'zgaradi.

5-misol $\begin{vmatrix} 3 & 5 \\ 4 & 2 \end{vmatrix} = 6-20=-14$ $\begin{vmatrix} 4 & 2 \\ 3 & 5 \end{vmatrix} = 20-6=14$

3. Determinantda biror ustun yoki satrninng xamma elementlari boshqa ustun yoki satrning mos elementlariga teng yoki proporsianal bo'lsa, bunday determinant nolga teng bo'ladi.

6-misol $\begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 12 - 12 = 0$

4.Agar determinantda ayrim ustun va satr elementlari umumiy ko'paytuvchilarga ega bo'lsa,ularni determinant belgisi oldiga chiqarish mumkin.

7-misol

$$\begin{vmatrix} 6 & 18 \\ 5 & 15 \end{vmatrix} = \begin{vmatrix} 6 & 6x3 \\ 5 & 5x3 \end{vmatrix} = 6x3x5 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 90x0 = 0$$

5.Determinantni m≠0 songa ko'paytirish uchun uning biror satri yoki ustunidagi hamma elementlarini shu (m) soniga ko'paytirish lozim.

8-misol

$$2x\begin{vmatrix}3 & 4\\ 2 & 6\end{vmatrix} = \begin{vmatrix}2x3 & 2x4\\ 2 & 6\end{vmatrix} = \begin{vmatrix}6 & 8\\ 2 & 6\end{vmatrix} = 36-16=20$$

6.Agar determinantda biror ustun m ta ko'shiluvchilar yig'indisidan iborat bo'lsa,u holda D determinant m ta $D_1, D_2, \ldots, D_m$ determinantlar yig'indisiga yoyiladi.

$$\begin{vmatrix}2+3 & 4\\ 3+5 & 6\end{vmatrix} = \begin{vmatrix}2 & 4\\ 3 & 6\end{vmatrix} + \begin{vmatrix}3 & 4\\ 5 & 6\end{vmatrix} = -2$$

7.Determinantni biror ustun (satr)ga biror o'zgarmas m≠0 sonni ko'paytirib biror ustun (satr)ning mos elementriga qo'shilsa determinantni qiymati o'zgarmaydi.

9-misol

$$\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 5+2x2 \\ 3 & 6+3x2 \end{vmatrix} = \begin{vmatrix} 2 & 9 \\ 3 & 12 \end{vmatrix} =$$

=24-27=-3

Детерминантнинг кейинги хоссаларини келтиришдан олдин, детерминант бирор элементининг минори ва алгебраик тўлдирувчиси тушунчалари билан танишиб чиқамиз.

Mustaqil yechish uchun misollar:

1. Determinantlarni hisoblang.

$$a)\begin{vmatrix} 3 & 1 & 2 & 3 \\ 4 & -1 & 2 & 4 \\ 1 & -1 & 1 & 1 \\ 4 & -1 & 2 & 5 \end{vmatrix}; \qquad b)\begin{vmatrix} 2 & -1 & 2 & 0 \\ 3 & 4 & 1 & 2 \\ 2 & -1 & 0 & 1 \\ 1 & 2 & 3 & -2 \end{vmatrix};$$

$$c)\begin{vmatrix} -1 & -2 & 4 & 1 \\ 2 & 3 & 0 & 6 \\ 2 & -2 & 1 & 4 \\ 3 & 1 & -2 & -1 \end{vmatrix}; \qquad d)\begin{vmatrix} 0 & 4 & 1 & 1 \\ -4 & 2 & 1 & 3 \\ 0 & 1 & 2 & -2 \\ 1 & 3 & 4 & -3 \end{vmatrix}.$$

Mustaqil yechish uchun misollar:

1) $$\begin{vmatrix} 8 & 7 & 2 & 10 \\ -8 & 2 & 7 & 10 \\ 4 & 4 & 4 & 5 \\ 0 & 4 & -3 & 2 \end{vmatrix}$$

2) $$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & 2 \\ 1 & 1 & -1 & 3 \\ 1 & 1 & 1 & -1 \end{vmatrix}$$

3) $$\begin{vmatrix} 2 & -3 & 4 & 1 \\ 4 & -2 & 3 & 2 \\ a & b & c & d \\ 3 & -1 & 4 & 3 \end{vmatrix}$$

4) $$\begin{vmatrix} 5 & a & 2 & -1 \\ 4 & b & 4 & -3 \\ 2 & c & 3 & -2 \\ 4 & d & 5 & -4 \end{vmatrix}$$

Berilgan tenglamalar dan x ni toping va ildizlarni determinantga qo‘yib tekshiring:

1) $$\begin{vmatrix} x^2 & 3 & 2 \\ x & -1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0$$

2) $$\begin{vmatrix} x^2 & 4 & 0 \\ x & 2 & 3 \\ 1 & 1 & 0 \end{vmatrix} = 0$$

Adabiyotlar ro'yhati:

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