# Mavzu:

**Irratsional funksiyalarni integrallash. Binomial differensiallarni integrallash.**

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# Reja:

1. $\int R\left(x^{\frac{m_1}{n_1}},\ x^{\frac{m_2}{n_2}},\dots,x^{\frac{m_k}{n_k}}\right)dx$ ko’rinishdagi integrallar
2. $\int R\left(z^{\frac{m_1}{n_1}},\ z^{\frac{m_2}{n_2}},\dots,z^{\frac{m_k}{n_k}}\right)dx,\ z=\frac{ax+b}{cx+d}$ ko’rinishdagi integrallar.
3. **Binomial differensiallarni integrallash**..
4. Chebishev teoremasi.

# 1. $\int R(x^{\frac{m_1}{n_1}},\ x^{\frac{m_2}{n_2}}, \dots, x^{\frac{m_k}{n_k}})dx$ ko'rinishdagi integrallar

- bu yerda *R* – uz argumentlarining ratsional funksiyasi. Irratsionallik argumentlarda namoyon buladi. Berilgan integralni ratsional funksiyani integrallashga keltiriladi. Buning uchun kuyidagi uzgaruvchini almashtirishni bajarish kifoya:

$$x = t^n, \qquad t = \sqrt[n]{x}, \qquad dx = nt^{n-1}dt.$$

- Bu yerda *n* soni $n_1, n_2, \dots, n_k$ sonlarining eng kichik umumiy karralisi:

$$n = \text{EKUK}(n_1, n_2, \dots, n_k),$$

## 1. $\int R\left(x^{\frac{m_1}{n_1}},\ x^{\frac{m_2}{n_2}}, \ldots, x^{\frac{m_k}{n_k}}\right)dx$ ko'rinishdagi integrallar

**Мисол 1.** $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$ integralни hisoblang.

Bu yerda $\sqrt{x} = x^{\frac{1}{2}},\ \sqrt[3]{x} = x^{\frac{1}{3}};\ \frac{1}{2}\ va\ \frac{1}{3}$ kasrlarning umumiy maxraji 6.
demak $x = t^6$ :

$$\int \frac{dx}{\sqrt{x}\ +\ \sqrt[3]{x}} = \left| \begin{matrix} x = t^6 \\ t = \sqrt[6]{x} \\ dx = 6t^5dt \end{matrix} \right| = \int \frac{6t^5dt}{t^3+t^2} = 6\int \frac{t^3dt}{t+1} =$$

$$= 6\int \left(t^2 - t + 1 - \frac{1}{t+1}\right) = 2t^3 - 3t^2 + 6t - 6\ln|t+1| + C =$$

$$= 2\sqrt{x}\ +\ 3\sqrt[3]{x}+6\sqrt[6]{x} - 6\ln(\sqrt[6]{x}+1) + C.$$

## 2. $\int R\left(z^{\frac{m_1}{n_1}}, z^{\frac{m_2}{n_2}}, \ldots, z^{\frac{m_k}{n_k}}\right) dx$ ko'rinishdagi integrallar

- bu yerda $z = \frac{ax+b}{cx+d}$.

$$\frac{ax+b}{cx+d} = t^n, \qquad t = \sqrt[n]{\frac{ax+b}{cx+d}},$$

$$x = \frac{dt^n - b}{a - ct^n}, \qquad dx = \frac{n(ad-bc)t^{n-1}}{(a-ct^n)^2}.$$

## $\boldsymbol{2+.\int R(z^{\frac{m_1}{n_1}},\ z^{\frac{m_2}{n_2}},\dots,z^{\frac{m_k}{n_k}})dx}$ ko'rinishdagi integrallar

- **Мисол 2.** $\int \frac{dx}{\sqrt[3]{(x-1)(x+1)^2}}$ integralни hisoblang.
- $\int \frac{dx}{\sqrt[3]{(x-1)(x+1)^2}} = \int \sqrt[3]{\frac{x+1}{x-1}}\frac{dx}{x+1}.$
- *x* ni almashtiramiz $t = \sqrt[3]{\frac{x+1}{x-1}}$ ,   $x = \frac{t^3+1}{t^3-1}$ ,   $dx = \frac{6t^2dt}{(t^3-1)^2}$; bundan

## $\mathbf{2++.} \int \boldsymbol{R}\left(\boldsymbol{z}^{\frac{\boldsymbol{m_1}}{\boldsymbol{n_1}}},\ \boldsymbol{z}^{\frac{\boldsymbol{m_2}}{\boldsymbol{n_2}}}, \ldots, \boldsymbol{z}^{\frac{\boldsymbol{m_k}}{\boldsymbol{n_k}}}\right)\boldsymbol{dx}$ ko'rinishdagi integrallar

$$\int \sqrt[3]{\frac{x+1}{x-1}}\frac{dx}{x+1} = \int \frac{-3dt}{t^3-1} = \int \left(-\frac{1}{t-1} + \frac{t+2}{t^2+t+1}\right)dt =$$

$$= \frac{1}{2}\ln\frac{t^2+t+1}{(t-1)^2} + \sqrt{3}\,\mathrm{arctg}\,\frac{2t-1}{\sqrt{3}} + C,$$

bu yerda $t = \sqrt[3]{\frac{x+1}{x-1}}$.

# 3. Binomial differensiallarni integrallash.

- Binomal deb

$$x^m(a+bx^n)^p dx \qquad (3)$$

kurinishdagi differensiallarga aytiladi; bu yerda *a, b* – ixtiyoriy sonlar, *m, n, p* – ratsional sonlar. Bu ifodalarni kaysi xollarda integrallashini kurib chikamiz.

*1) r* - butun son (musbat, nol yoki manfiy). Bu xol (1) integralda kurilgan.Agar *m* va *n* kasrlahyb eng kichik umumiy maxrajini $\lambda$ deb belgilasak ifoda (3) $R\left(\sqrt[\lambda]{x}\right)dx$ boladi. bu yerda

$$t = \sqrt[\lambda]{x}$$

almashtirish masalani xal etadi.

# 3. Binomial differensiallarni integrallash.

Karalayotgan ifodada $z = x^n$ almashtirish bajaramiz. U xolda

$$x^m(a + bx^n)^p dx = \frac{1}{n}(a + bz)^p z^{\frac{m+1}{n}-1} dz$$

va, bu yerda $\frac{m+1}{n} - 1 = q$ deb olib

$$\int x^m(a + bx^n)^p dx = \frac{1}{n}\int (a + bz)^p z^q dz \,. \qquad (4)$$

# 3. Binomial differensiallarni integrallash.

2) Agar *q* butun son bulsa, avval kurilgan xol integral (2) vujudga keladi. bu yerda υ deb *r* ning maxrajini belgilasak

$$t = \sqrt[v]{a + bz} = \sqrt[v]{a + bx^n}$$

- almashtirish masalani xal etadi.
- 3) (4) dagi ikkinchi ifodani kuyidagicha yozamiz:

$$\frac{1}{n}\int \left(\frac{a+bz}{z}\right)^p z^{p+q} \ .$$

- Agar *p* + *q* butun son bulsa, avval kurilgan xol integral (2) vujudga keladi. bu yerda υ deb *r* ning maxrajini belgilasak

$$t = \sqrt[v]{\frac{a + bz}{z}} = \sqrt[v]{ax^{-n} + b}$$

- almashtirish masalani xal etadi.

# 4. Chebishev teoremasi.

(4) dagi ikkala integral fakat karalgan uchta xolda integrallanadi, kolgan xolatlar bundan mustusno.

**Мисол 3.** $\int \frac{dx}{\sqrt[4]{1+x^4}}$ integralни hisoblang.

$$\int \frac{dx}{\sqrt[4]{1+x^4}} = \int x^0(1+x^4)^{-\frac{1}{4}}\,dx.$$

bu yerda $m = 0$, $n = 4$, $p = -\frac{1}{4}$; uchinchi xolat $\frac{m+1}{n} + p = 0$, $υ = 4$.
ko'rsatilgan almashtirishni bajaramiz:

$$t = \sqrt[4]{x^{-4}+1} = \frac{\sqrt[4]{1+x^4}}{x}, \quad x = (t^4-1)^{-\frac{1}{4}}, \quad dx = -t^3(t^4-1)^{-\frac{5}{4}}dt,$$

bundan

$$\sqrt[4]{1+x^4} = tx = t(t^4-1)^{-\frac{1}{4}},$$

$$\int \frac{dx}{\sqrt[4]{1+x^4}} = -\int \frac{t^2dt}{t^4-1} = \frac{1}{4}\ln\left|\frac{t+1}{t-1}\right| - \frac{1}{2}\operatorname{arctg} t + C.$$