MAVZU : Bir o'zgaruvchili funksiya hosilasi. Defferensiallanuvchi funksiya, hosilaning geometrik ma'nosi. Elementar funksiyalarning hosilasi. Hosilalar jadvali.

Reja:

1.Bir o'zgaruvchili funksiya hosilasi

2.Differensiallanuvchi funksiya

3.Hosilaning geometrik ma'nosi

4.Elementar funksiyalarning hosilasi

5.Hosilalar jadvali

1.Bir o’zgaruvchili funksiya hosilasi

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Demak, berilgan y=f(x) funksiyaning argument x bo’yicha **hosilasi deb** argument orttirmasi Δx iytiyoriy ravishda nolga intilgan holda funksiya orttirmasi Δy ning argument orttirmasi Δx ga nisbatining limitiga aytiladi.

Berilgan f(x) funksiyadan hosila topish amali shu funksiyani differentsiallash deyiladi.

Misol: $y=x^2$ hosilasi topilsin.

Yechish: Berilgan funksiya uchun orttirma quyidagicha bo'ladi.

$$\Delta y = y(x + \Delta x) - y(x)$$

Natijada $\Delta y = (x + \Delta x)^2 - x^2 = x^2 + 2x\Delta x + (\Delta x)^2 - x^2 = 2x\Delta x + (\Delta x)^2$

bundan hosila ta'rifiga ko'ra

$$y' = \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} = 2x$$

Demak, $(x^2)' = 2x$

2.Differensiallanuvchi funksiya

Agar y=f(x) funksiya $x=x_0$ nuqtada hosilaga ega bo'lsa, ya'ni

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

mavjud bo'lsa, u holda berilgan $x=x_0$ qiymatda funksiya differensiallanuvchi yoki (bari bir) hosilaga ega deyiladi.

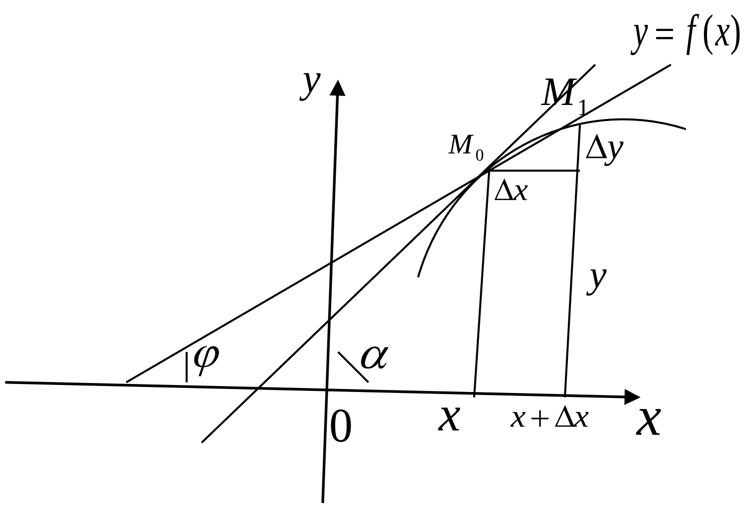
Agar y=f(x) funksiya biror $x=x_0$ nuqtada differensilalanuvchi bo'lsa, u holda funksiya shu nuqtada uzluksizdir.

3.Hosilaning geometrik ma’nosi

Agar M_1 nuqta egri chiziq bo’yicha istalgan tomondan M_0 nuqtaga cheksiz yaqinlasha borganda kesuvchi ma’lum M_0T to’g’ri chiziq vaziyatini egallashga intilsa, г holda bu to’g’ri chiziq M_0 nuqtada egri chiziqqa urinma deyiladi.

$$f'(x)=tg\alpha$$

Ya’ni argument x ning berilgan qiymatida f’(x) hosilaning qiymati f(x) funksiyaning grafigiga uning $M_0(x,y)$ nuqtasidagi urinmaning Ox o’qning musbat yo’nalishi bilan hosil qilganburchak tangenisiga teng.



4.Elementar funksiyalarning hosilasi

y=sinx ning hosilasi **cosx**, ya’ni agar **y=sinx** bo’lsa, **y`=cosx** bo’ladi.

y=cosx ning hosilasi **-sinx**, ya’ni agar **y=cosx** bo’lsa, **y`=-sinx** bo’ladi.

log$_a$x fynksiyaning hosilasi

$$\frac{1}{x}\log_a x$$

tgx funksiyaning hosilasi $\frac{1}{\cos^2 x}$ ga teng, ya’ni agar **y=tgx** bo’lsa u holda $y' = \frac{1}{\cos^2 x}$ bo’ladi.

ctgx funksiyaning hosilasi

$-\dfrac{1}{\sin^2 x}$ ga teng,

ya'ni agar **y=ctgx** bo'lsa

u holda $y' = -\dfrac{1}{\sin^2 x}$ bo'ladi.

ln|x| funksiyaning hosilasi $\dfrac{1}{x}$ ga teng ya'ni agar **y=ln|x|** bo'lsa, u holda $y' = \dfrac{1}{x}$ teng.

5.Hosilalar jadvali

$$y = x^{a} \qquad y' = ax^{a-1}$$

$$y = \sqrt{x} \qquad y' \frac{1}{2\sqrt{x}}$$

$$y = \frac{1}{x} \qquad y' = -\frac{1}{x^{2}}$$

$$y = \sin x \qquad y' = \cos x$$

$$y = \cos x \qquad y' = -\sin x$$

$$y = tgx \qquad y' = \frac{1}{\cos^2 x}$$

$$y = ctgx \qquad y' = -\frac{1}{\sin^2 x}$$

$$y = a^x \qquad y' = a^x \ln a$$

$$y = e^x \qquad y' = e^x$$

$$y = \log_a x \qquad y' = \frac{1}{x \ln a}$$

$$y = \ln x \qquad y' = \frac{1}{x}$$

$$y = \arcsin x \qquad y' = \frac{1}{\sqrt{1 - x^2}}$$

$$y = \arccos x \qquad y' = -\frac{1}{\sqrt{1 - x^2}}$$

$$y = arctgx \qquad y' = \frac{1}{1 + x^2}$$

$$y = arcctgx \qquad y' = -\frac{1}{1 + x^2}$$

O’z bilimini tekshirish uchun misollar

1. $y = \dfrac{3x + \sqrt{x}}{\sqrt{x^2 + 2}}$

2. $y = (2^{\arcsin x} - \sqrt{1 - x^2})^5$

3. $y = 2^{\sqrt{x}} + x^3 tgx$

4. $y = \ln arcctg \dfrac{1}{x}$

5. $y = (2^{\arccos\sqrt{x}} - \sqrt{1 - x})^4$

6. $y = (3^{ctg^2 x} + \ln\sin x)^3$

Misol: $y = \log_3 \sqrt[3]{x}$ funksiya hosilasi topilsin.

Yechish: Yuqorida keltirilgan hosila olish formulasiga asosan,

$$y' = (\log_3 \sqrt[3]{x})' = (\log_3 x^{1/3})' = \frac{1}{3}(\log_3 x)';$$

Bu erda $y' = \frac{1}{3}(\log_3 x)' = \frac{1}{3xln3}$