Pre-Calculus

Day 38

Right Triangle Trigonometry

Today's Objective

Review right triangle trigonometry from Geometry and expand it to all the trigonometric functions

Begin learning some of the Trigonometric identities

What You Should Learn

- Evaluate trigonometric functions of acute angles.
- Use fundamental trigonometric identities.
- Use a calculator to evaluate trigonometric functions.
- Use trigonometric functions to model and solve real-life problems.

Plan

- Review of homework
- Look at the back of the book..
- 4.3 Right Triangle Trigonometry
  - Definitions of the 6 trig functions
  - Reciprocal functions
  - Co functions
  - Quotient Identities
- Homework

Review of Homework

Section 4.1

Page 269 -271 # 7, 9, 17, 20, 21, 23, 27, 31, 38, 39, 43, 47, 56, 57, 63, 66, 71, 77, 81, 85, 89, 91

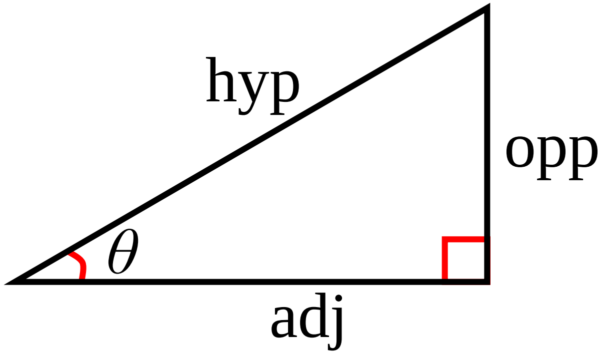
Trig handout

Right Triangle Trigonometry

Trigonometry is based upon ratios of the sides of right triangles.

The ratio of sides in triangles with the same angles is consistent. The size of the triangle does not matter because the triangles are similar (same shape different size).

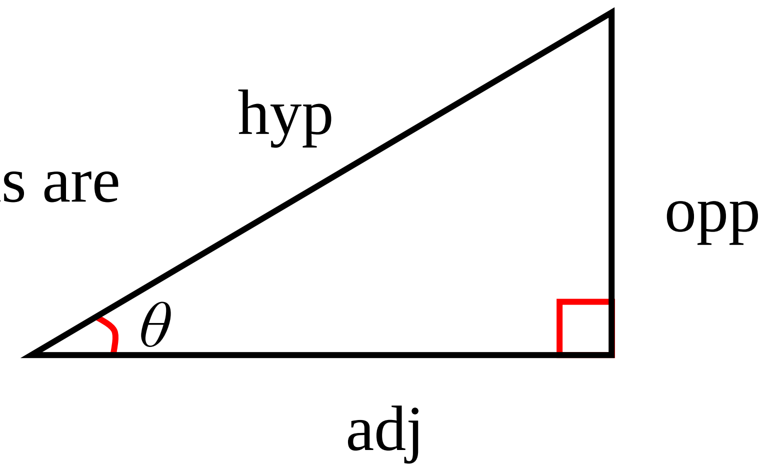
The six **trigonometric functions** of a right triangle, with an acute angle θ, are defined by **ratios** of two sides of the triangle.



The sides of the right triangle are:

- the side **opposite** the acute angle θ,
- the side **adjacent** to the acute angle θ,
- and the **hypotenuse** of the right triangle.

The trigonometric functions are



sine, cosine, tangent, cotangent, secant, and **cosecant**.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

Note: sine and cosecant are reciprocals, cosine and secant are reciprocals, and tangent and cotangent are reciprocals.

Reciprocal Functions

Another way to look at it…

$\sin\theta = 1/\csc\theta$

$\cos\theta = 1/\sec\theta$

$\tan\theta = 1/\cot\theta$

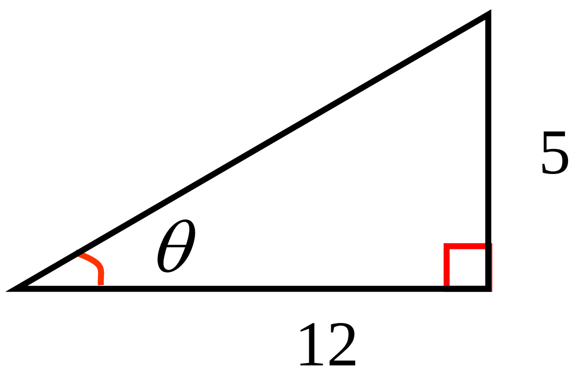
$\csc\theta = 1/\sin\theta$

$\sec\theta = 1/\cos\theta$

$\cot\theta = 1/\tan\theta$

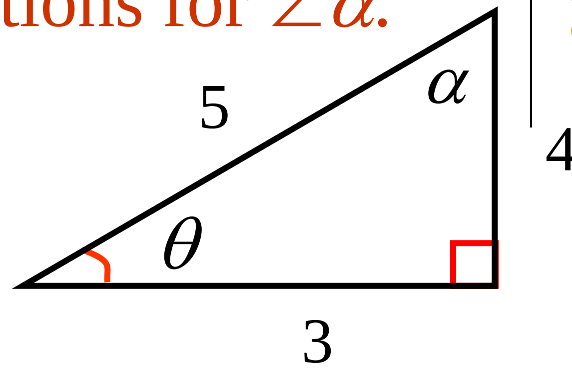
Given 2 sides of a right triangle you should be able to find the value of all 6 trigonometric functions.

Example:



Calculate the trigonometric functions for $\angle\theta$.

Calculate the trigonometric functions for $\angle\alpha$.



The six trig ratios are

$$\sin\theta = \frac{4}{5}$$

$$\cos\theta = \frac{3}{5}$$

$$\tan\theta = \frac{4}{3}$$

$$\cot\theta = \frac{3}{4}$$

$$\sec\theta = \frac{5}{3}$$

$$\csc\theta = \frac{5}{4}$$

$$\sin\alpha = \frac{3}{5}$$

$$\cos\alpha = \frac{4}{5}$$

$$\tan\alpha = \frac{3}{4}$$

$$\cot\alpha = \frac{4}{3}$$

$$\sec\alpha = \frac{5}{4}$$

$$\csc\alpha = \frac{5}{3}$$

What is the relationship of

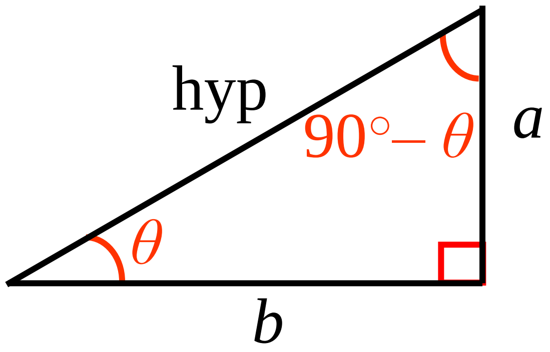
α and θ?

They are complementary
($\alpha = 90 - \theta$)

Note $\sin\theta = \cos(90° - \theta)$, for $0 < \theta < 90°$

Note that θ and $90° - \theta$ are complementary angles.

Side a is opposite θ and also adjacent to $90° - \theta$.



$$\sin\theta = \frac{a}{b} \text{ and } \cos(90° - \theta) = \frac{a}{b}.$$

So, $\sin\theta = \cos(90° - \theta)$.

Note : These functions of the complements are called cofunctions.

Cofunctions

$\sin\theta = \cos(90° - \theta)$ $\cos\theta = \sin(90° - \theta)$

$\sin\theta = \cos(\pi/2 - \theta)$ $\cos\theta = \sin(\pi/2 - \theta)$

$\tan\theta = \cot(90° - \theta)$ $\cot\theta = \tan(90° - \theta)$

$\tan\theta = \cot(\pi/2 - \theta)$ $\cot\theta = \tan(\pi/2 - \theta)$

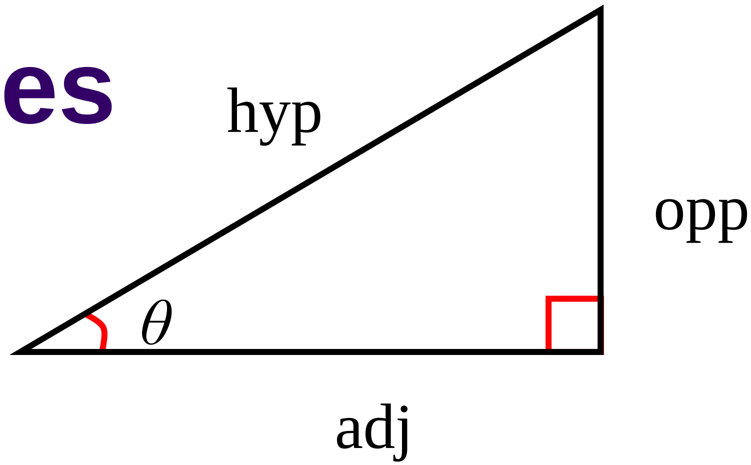
$\sec\theta = \csc(90° - \theta)$ $\csc\theta = \sec(90° - \theta)$

$\sec\theta = \csc(\pi/2 - \theta)$ $\csc\theta = \sec(\pi/2 - \theta)$

Trigonometric Identities are trigonometric equations that hold for all values of the variables.

We will learn many Trigonometric Identities and use them to simplify and solve problems.

Quotient Identities



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{opp}{hyp}}{\frac{adj}{hyp}} = \frac{opp}{hyp} \bullet \frac{hyp}{adj} = \frac{opp}{adj} = \tan\theta$$

The same argument can be made for cot… since it is the reciprocal function of tan.

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

Three additional identities that we will use are those related to the Pythagorean Theorem:

Pythagorean Identities

$\sin^2 \theta + \cos^2 \theta = 1$

$\tan^2 \theta + 1 = \sec^2 \theta$

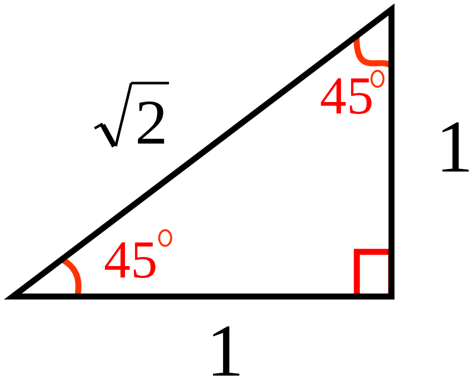
$\cot^2 \theta + 1 = \csc^2 \theta$

Some old geometry favorites…

Let's look at the trigonometric functions of a few familiar triangles…

Geometry of the 45-45-90 triangle

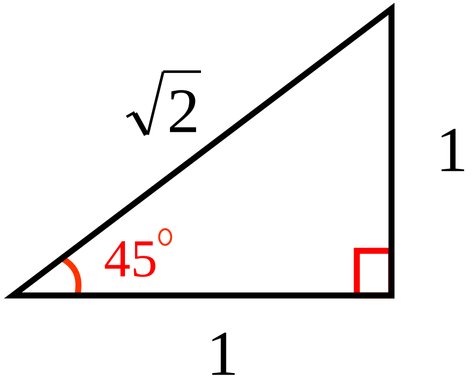
Consider an isosceles right triangle with two sides of length 1.



$$\sqrt{1^2 + 1^2} = \sqrt{2}$$

The *Pythagorean Theorem* implies that the hypotenuse is of length $\sqrt{2}$.

Calculate the trigonometric functions for a 45° angle.



$$\sin 45° = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45° = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45° = \frac{\text{opp}}{\text{adj}} = \frac{1}{1} = 1$$

$$\cot 45° = \frac{\text{adj}}{\text{opp}} = \frac{1}{1} = 1$$

$$\sec 45° = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\csc 45° = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

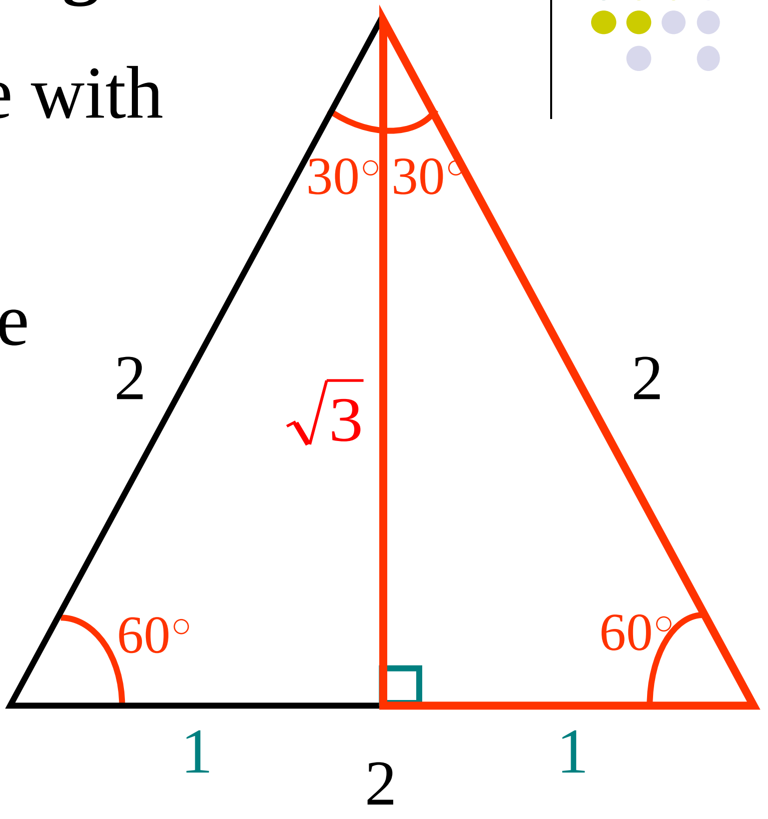
Geometry of the 30-60-90 triangle

Consider an equilateral triangle with each side of length 2.

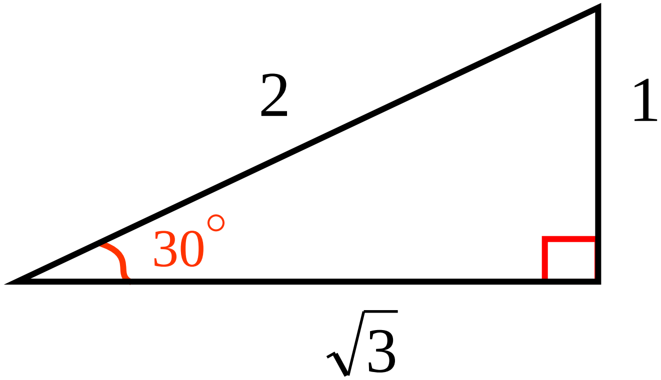
The three sides are equal, so the angles are equal; each is 60°.

The perpendicular bisector of the base bisects the opposite angle.

Use the Pythagorean Theorem to find the length of the altitude, $\sqrt{3}$.



Calculate the trigonometric functions for a 30° angle.

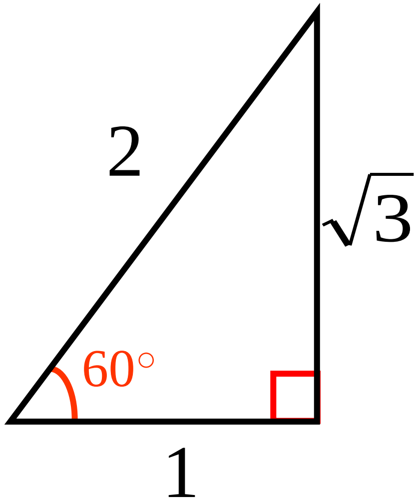


$$\sin 30^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{1}{2} \qquad \cos 30^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \cot 30^\circ = \frac{\text{adj}}{\text{opp}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\sec 30^\circ = \frac{\text{hyp}}{\text{adj}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \qquad \csc 30^\circ = \frac{\text{hyp}}{\text{opp}} = \frac{2}{1} = 2$$

Calculate the trigonometric functions for a 60° angle.



$$\sin 60° = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{3}}{2}$$

$$\cos 60° = \frac{\text{adj}}{\text{hyp}} = \frac{1}{2}$$

$$\tan 60° = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\cot 60° = \frac{\text{adj}}{\text{opp}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\sec 60° = \frac{\text{hyp}}{\text{adj}} = \frac{2}{1} = 2$$

$$\csc 60° = \frac{\text{hyp}}{\text{opp}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Some basic trig values

	Sine	Cosine	Tangent
30^0 $\pi/6$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45^0 $\pi/4$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60^0 $\pi/3$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

IDENTITIES WE HAVE REVIEWED SO FAR…

Fundamental Trigonometric Identities

Reciprocal Identities

$\sin\theta = 1/\csc\theta$ $\quad$ $\cos\theta = 1/\sec\theta$ $\quad$ $\tan\theta = 1/\cot\theta$

$\cot\theta = 1/\tan\theta$ $\quad$ $\sec\theta = 1/\cos\theta$ $\quad$ $\csc\theta = 1/\sin\theta$

Co function Identities

$\sin\theta = \cos(90^\circ - \theta)$ $\quad$ $\cos\theta = \sin(90^\circ - \theta)$

$\sin\theta = \cos(\pi/2 - \theta)$ $\quad$ $\cos\theta = \sin(\pi/2 - \theta)$

$\tan\theta = \cot(90^\circ - \theta)$ $\quad$ $\cot\theta = \tan(90^\circ - \theta)$

$\tan\theta = \cot(\pi/2 - \theta)$ $\quad$ $\cot\theta = \tan(\pi/2 - \theta)$

$\sec\theta = \csc(90^\circ - \theta)$ $\quad$ $\csc\theta = \sec(90^\circ - \theta)$

$\sec\theta = \csc(\pi/2 - \theta)$ $\quad$ $\csc\theta = \sec(\pi/2 - \theta)$

Quotient Identities

$\tan\theta = \sin\theta/\cos\theta$ $\quad$ $\cot\theta = \cos\theta/\sin\theta$

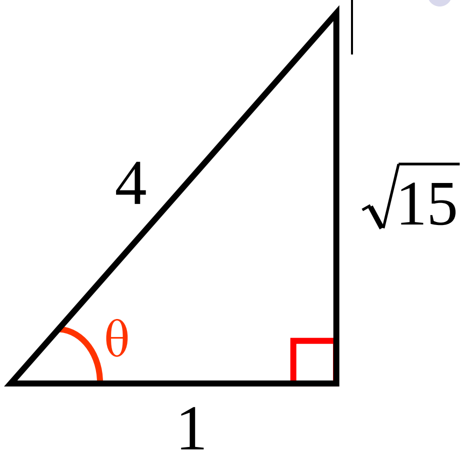
Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\quad$ $\tan^2\theta + 1 = \sec^2\theta$ $\quad$ $\cot^2\theta + 1 = \csc^2\theta$

Example: Given sec $\theta = 4$, find the values of the other five trigonometric functions of θ.

Draw a right triangle with an angle θ such

that $4 = \sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{4}{1}$.



Use the Pythagorean Theorem to solve for the third side of the triangle.

$$\sin\theta = \frac{\sqrt{15}}{4} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{4}{\sqrt{15}}$$

$$\cos\theta = \frac{1}{4} \qquad \sec\theta = \frac{1}{\cos\theta} = 4$$

$$\tan\theta = \frac{\sqrt{15}}{1} = \sqrt{15} \qquad \cot\theta = \frac{1}{\sqrt{15}}$$

Using the calculator

Function Keys

Reciprocal Key

Inverse Keys

Using Trigonometry to Solve a Right Triangle

A surveyor is standing 115 feet from the base of the Washington Monument. The surveyor measures the angle of elevation to the top of the monument as 78.3°. How tall is the Washington Monument?

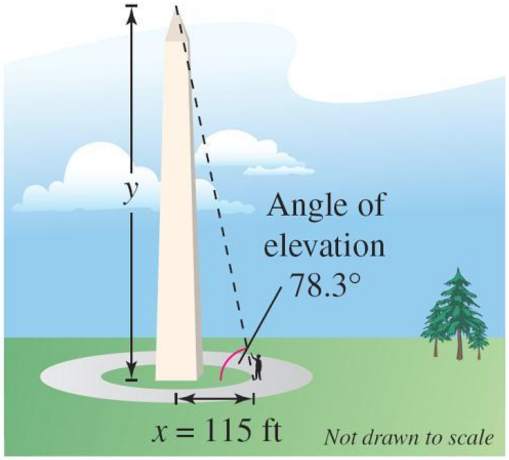
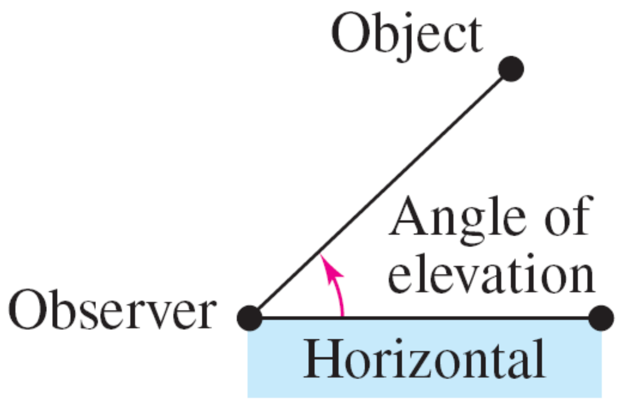


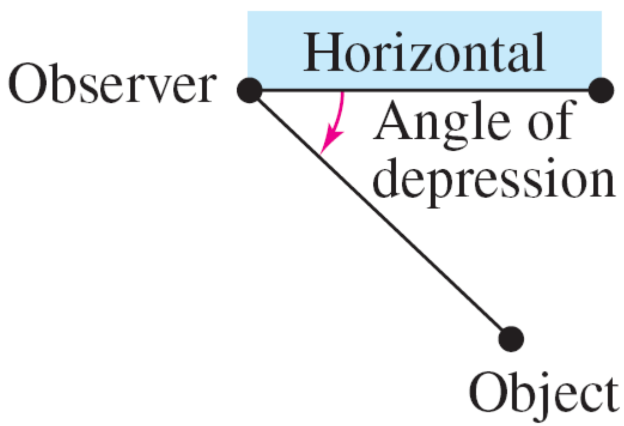
Figure 4.33

Applications Involving Right Triangles

The angle you are given is the **angle of elevation,** which represents the angle from the horizontal upward to an object.



For objects that lie below the horizontal, it is common to use the term **angle of depression.**



Solution

$$\tan 78.3° = \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

where x = 115 and y is the height of the monument. So, the height of the Washington Monument is

$y = x \tan 78.3°$

$\approx 115(4.82882) \approx 555$ feet.

Homework 20

Section 4.3, pp. 287-289:

1, 3, 9-21 odd, 27, 31-39 odd, 47,49