Chiziqli tenglamalar sistemasini teskari matritsa yordamida yechish.

Ushbu n ta noma’lumli n ta chiziqli tenglamalar sistemasini qaraylik:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases} \qquad (1)$$

Ushbu belgilashlarni kiritamiz:

$$A = \begin{pmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ \cdots \cdots \cdots & \cdots \cdots \cdots & \cdots \cdots \\ a_{n1} & a_{n2} \cdots & a_{nn} \end{pmatrix}, B = \begin{pmatrix} b_1 \\ b_2 \\ . \\ . \\ b_n \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{pmatrix}. \quad (2)$$

U holda (1) sistemani matritsalarni ko'paytirish qoidasidan foydalanib, ushbu ekvivalent shaklda yozish mumkin:

$$AX = B. \quad (3)$$

Bu yerda A –noma'lumlar oldidagi koeffitsientlardan tuzilgan matritsa, B –ozod hadlardan tuzilgan ustun matritsa,

$X -$ noma’lumlardan tuzilgan ustun matritsa. Agar A matritsa xosmas, ya’ni $detA \neq 0$ bo’lsa, u holda uning uchun A^{-1} teskari matritsa mavjud. (3) matritsali tenglamaning ikkala qismini A^{-1}ga chapdan ko’paytirib, quyidagini hosil qilamiz:

$$A^{-1}(AX) = A^{-1}B$$

yoki

$$(A^{-1}A)X = A^{-1}B.$$

$A^{-1}A = E,\ EX = X$ ekanini hisobga olib,

$$X = A^{-1}B \qquad (4)$$

ni topamiz. (4) formula A matritsa xosmas bo'lganda n noma'lumli n ta chiziqli tenglamalar sistemasi yechimining matritsali yozuvini beradi.

Misol. Ushbu sistemani yeching:

$$\begin{cases} x_1 - 2x_2 + x_3 = 5 \\ 2x_1 - x_3 = 0 \\ -2x_1 + x_2 + x_3 = -1 \end{cases}$$

Yechish: Bu misolda

$$A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 0 & -1 \\ -2 & 1 & 1 \end{pmatrix}, \ B = \begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix},$$

U holda berilgan tenglamalar sistemasini

$$AX = B$$

ko'rinishda yozish mumkin. (4) formulaga asosan

$$X = A^{-1}B.$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & 1 & \frac{2}{3} \\ 0 & 1 & 1 \\ \frac{2}{3} & 1 & \frac{4}{3} \end{pmatrix} \begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

Bundan $x_1 = 1, x_2 = -1, x_3 = 2.$