

Mavzu: Funktsiya tushunchasi va uning limiti.

REJA:

1. Funktsiyaning ta'rif, aniqlanish va qiymatlar sohasi.
2. Funktsiyaning limiti
3. Cheksiz kichik va cheksiz katta funktsiyalar va ularning xossalari
4. Aniqmasliklarni ochish
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1. Funktsiyaning ta'rifi

Aytaylik X va Y haqiqiy sonlar to'plami berilgan bo'lsin.

1-Ta'rif. Agar X to'plamning har bir $x \in X$ elementiga Y to'plamning yagona $y \in Y$ elementi mos qo'yilsa, u holda bu moslik funktsiya deyiladi va uni $y = f(x)$ kabi yoziladi.

Bu yerda x – erkli o'zgaruvchi(argument); y –erksiz o'zgaruvchi (funktsiya); f – x ni y ga mos qo'yuvchi qoida.

2-Ta'rif. Argument x ning berilgan funktsiya ma'noga ega bo'ladigan qiymatlar to'plamiga funktsiyaning *aniqlanish sohasi* deyiladi va uni $D(f)$ bilan belgilanadi.

3-Ta'rif. x ning o'zgarishiga ko'ra y ning qabul qilishi mumkin bo'lgan qiymatlar to'plamiga funktsiyaning qiymatlar sohasi deyiladi va uni $E(f)$ bilan belgilanadi.

2. Funktsiyaning limiti

Berilgan $y = f(x)$ funktsiya $x = a$ nuqtaning shu nuqtani o'zi kirishi shart bo'lmagan biror atrofida aniqlangan bo'lib, A biror haqiqiy son bo'lsin.

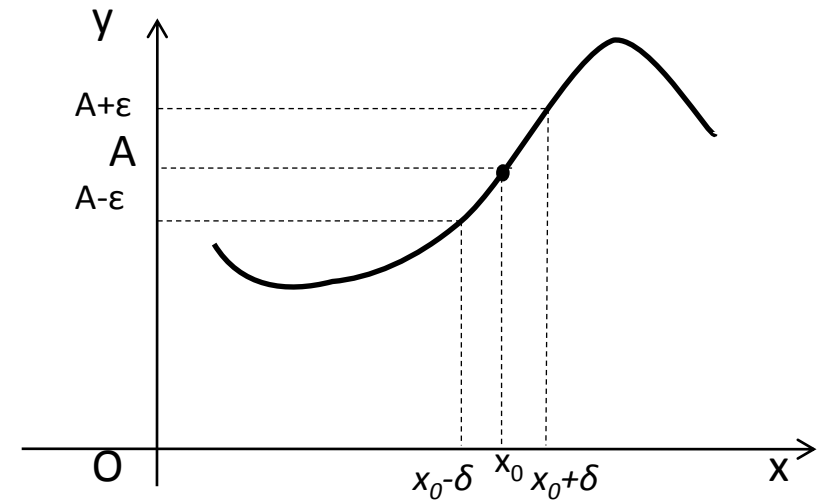
1-Ta'rif. Agar ixtiyoriy $\varepsilon > 0$ son uchun shunday $\delta > 0$ son topilib, $0 < |x - a| < \delta$ tengsizlikni qanoatlantiruvchi barcha x lar uchun $|f(x) - A| < \varepsilon$ tengsizlik bajarilsa, u holda A sonini $f(x)$ funktsiyaning a nuqtagi *limiti* deyiladi va uni

$\lim_{x \rightarrow a} f(x) = A$ ko'rinishda yoziladi.

$$|x - a| < \delta \implies -\delta < x - a < \delta \implies -\delta + a < x < \delta + a$$

oraligini a nuqtaning δ atrofi deyiladi.

Misol. $\lim_{x \rightarrow 3} (x^2 - 5x + 8) = 9 - 15 + 8 = 2$



Umuman x argument a ga o'ngdan

yoki chapdan intilishi mumkin. Agar bu limitlar mavjud bo'lsa, ularni mos ravishda $f(x)$ funktsiyaning o'ng va chap limitlari deyiladi va ularni

$$\lim_{x \rightarrow a-0} f(x) = f(a-0)$$

$$\lim_{x \rightarrow a+0} f(x) = f(a+0)$$

ko'rinishda yoziladi.

Agar $f(a-0) = f(a+0) = A$ bo'lsa, u holda $y = f(x)$ funktsiya $x = a$ nuqtada limitga ega deyiladi.

3. Cheksiz katta va cheksiz kichik miqdorlar

Agar $x \rightarrow a$ ga intilganda

$$\lim_{x \rightarrow a} f(x) = \infty \text{ yoki } \lim_{x \rightarrow a} f(x) = -\infty \text{ bo'lsa,}$$

U holda $f(x)$ funktsiyani *cheksiz katta funktsiya* deyiladi.

Masalan $x \rightarrow 2$ ga intilganda $\lim_{x \rightarrow 2} \frac{x}{x-2} = \infty$ cheksiz katta

funktsiya bo'ladi.

Agar $\lim_{x \rightarrow a} f(x) = 0$ bo'lsa $f(x)$ funktsiyani *cheksiz kichik*

funktsiya deyiladi.

Masalan, $f(x) = \frac{x^2-1}{x+1}$ funktsiya $x \rightarrow 1$ ga intilganda cheksiz kichik funktsiya bo'ladi. Chunki

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x + 1} = \lim_{x \rightarrow 1} (x - 1) = 0$$

Funktsiya limitining xossalari:

$\lim_{x \rightarrow a} f(x) = A$ $\lim_{x \rightarrow a} g(x) = B$ mavjud bo'lsa u holda

$$1) \lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$$

$$2) \lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

$$3) \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$$

$$4) \lim_{x \rightarrow a} kf(x) = k \lim_{x \rightarrow a} f(x)$$

Misollar. $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 8}{x^2 - x + 4}$.

$$\lim_{x \rightarrow 3} (x^2 - 5x + 8) = 9 - 15 + 8 = 2$$

$$\lim_{x \rightarrow 3} (x^2 - x + 4) = 9 - 3 + 4 = 10$$

3-xossaga ko'ra $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 8}{x^2 - x + 4} = \frac{\lim_{x \rightarrow 3} (x^2 - 5x + 8)}{\lim_{x \rightarrow 3} (x^2 - x + 4)} = \frac{2}{10} = \frac{1}{5}$.

$$\lim_{x \rightarrow 3} \frac{x^2 - 5x + 8}{x - 3}.$$

$$\lim_{x \rightarrow 3} (x^2 - 5x + 8) = 9 - 15 + 8 = 2$$

$$\lim_{x \rightarrow 3} \frac{x^2 - 5x + 8}{x - 3} = \infty.$$

4. Aniqmasliklarni ochish

Limitlarni hisoblashda ba'zan quyidagi ko'rinishdagi aniqmasliklarga duch kelamiz:

$$\left(\frac{0}{0}\right), \left(\frac{\infty}{\infty}\right), (\infty - \infty), (1^\infty), (0^\infty), (0^0)(\infty^0).$$

Ushbu ko'rinishdagi limitlarni hisoblash aniqmasliklarni ochish deyiladi.

Misollar.

$$\lim_{x \rightarrow \infty} \frac{2x^2 - 3x - 5}{1 + x + 3x^2} = \frac{\infty}{\infty} = (*)$$

$$\begin{aligned}
 (*) &= \lim_{x \rightarrow \infty} \frac{2x^2 - 3x - 5}{1 + x + 3x^2} = \lim_{x \rightarrow \infty} \frac{\frac{2x^2}{x^2} - \frac{3x}{x^2} - \frac{5}{x^2}}{\frac{1}{x^2} + \frac{x}{x^2} + \frac{3x^2}{x^2}} = \lim_{x \rightarrow \infty} \frac{2 - \frac{3 \rightarrow 0}{x} - \frac{5 \rightarrow 0}{x^2}}{\frac{1 \rightarrow 0}{x^2} + \frac{1 \rightarrow 0}{x} + 3} = \frac{2}{3}
 \end{aligned}$$

$$\lim_{x \rightarrow -1} \frac{2x^2 - 3x - 5}{x + 1} = \frac{2(-1)^2 - 3 \cdot (-1) - 5}{-1 + 1} = \frac{0}{0}$$

$$\lim_{x \rightarrow -1} \frac{2x^2 - 3x - 5}{x + 1} = \frac{0}{0} = (*) \quad (*) = \lim_{x \rightarrow -1} \frac{(x + 1) \cdot (2x - 5)}{x + 1} = (*)$$

$$(*) = \lim_{x \rightarrow -1} (2x - 5) = (*) = 2 \cdot (-1) - 5 = -2 - 5 = -7$$

$$\lim_{x \rightarrow 3} \frac{\sqrt{x+6} - \sqrt{10x-21}}{5x-15} \qquad \lim_{x \rightarrow 3} \frac{\sqrt{x+6} - \sqrt{10x-21}}{5x-15} = \frac{0}{0} = (*)$$

$$(*) = \lim_{x \rightarrow 3} \frac{(\sqrt{x+6} - \sqrt{10x-21}) \cdot (\sqrt{x+6} + \sqrt{10x-21})}{(5x-15) \cdot (\sqrt{x+6} + \sqrt{10x-21})} = (*)$$

$$\begin{aligned}
(*) &= \lim_{x \rightarrow 3} \frac{(\sqrt{x+6})^2 - (\sqrt{10x-21})^2}{(5x-15) \cdot (\sqrt{x+6} + \sqrt{10x-21})} = \\
&= \lim_{x \rightarrow 3} \frac{x+6 - (10x-21)}{(5x-15) \cdot (\sqrt{x+6} + \sqrt{10x-21})} = \\
&= \lim_{x \rightarrow 3} \frac{x+6 - 10x + 21}{(5x-15) \cdot (\sqrt{x+6} + \sqrt{10x-21})} = \\
&= \lim_{x \rightarrow 3} \frac{-9x + 27}{(5x-15) \cdot (\sqrt{x+6} + \sqrt{10x-21})} = (*)
\end{aligned}$$

$$(*) = \frac{1}{6} \lim_{x \rightarrow 3} \frac{-9(x-3)}{5(x-3)} = \frac{1}{6} \lim_{x \rightarrow 3} \frac{-9}{5} = \frac{1}{6} \cdot \left(\frac{-9}{5} \right) = -\frac{3}{10}$$

5. Birinchi va ikkinchi ajoyib limitlar

Birinchi ajoyib limit

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1;$$

Ikkinchi ajoyib limit

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = \lim_{x \rightarrow 0} (1 + x)^{1/x} = e.$$

Misollar

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sin(2x)}{x} &= \left(\frac{0}{0}\right) = \\ &= \lim_{x \rightarrow 0} \frac{2\sin(2x)}{2x} = 2 \cdot \lim_{x \rightarrow 0} \frac{\sin(2x)}{2x} = \\ &= 2 \cdot 1 = 2.\end{aligned}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{3x}\right)^{4x} = 1^\infty = \lim_{x \rightarrow \infty} \left(\left(1 + \frac{1}{3x}\right)^{3x} \right)^{\frac{1}{3x} \cdot 4x} = e^{\frac{4}{3}}$$