4-MA’RUZA

CHIZIQLI ALGEBRAIK TENGLAMALAR SISTEMASINI MATRITSA USULIDA YECHISH. BIR JINSLI CHIZIQLI ALGEBRAIK TENGLAMALAR SISTEMASI

REJA

1. **CHIZIQLI TENGLAMALAR SISTEMASINI TESKARI MATRITSA YORDAMIDA YECHISH**
2. **BIR JINSLI TENGLAMALAR SISTEMASI**

n ta tenglama m ta noma'lumdan iborat algebraik tenglamalar sistemasi berilgan bo'lsin:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1m}x_m = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2m}x_m = b_2 \qquad (1)$$

$$\vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nm}x_m = b_n.$$

Sistemaning matritsa yordamida ifodalanishi:

$$\boldsymbol{AX} = \boldsymbol{B}. \qquad (2)$$

Bu yerda A sistema koeffitsentlaridan tuzilgan matritsa, x_j noma'lumlardan quyidagi matritsani tuzamiz:

$$\boldsymbol{X} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}$$

va ozod hadlardan

$$\boldsymbol{B} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

matritsa tuzamiz.

Agar A matritsa xosmas matritsa bo‘lsa, u holda (2) tenglama quyidagicha yechiladi. (2) tenglamaning o‘ng va chap qismini A matritsaning teskarisi A^{-1} ga ko`paytiramiz:

$$A^{-1}(AX) = A^{-1}B \quad \text{yoki} \quad (A^{-1}A)X = A^{-1}B,$$

$A^{-1}A = E$ va $EX = X$ bo`lgani uchun tenglamaning

$$X = A^{-1}B \tag{3}$$

ko`rinishidagi yechimiga ega bo`lamiz.

Misol. Ushbu tenglamalar sistemasini matritsalar yordamida yeching.

$$\begin{cases} x_1 - x_3 = 2, \\ 2x_1 - x_2 + 3x_3 = -1, \\ 3x_1 + 2x_2 - 2x_3 = 5. \end{cases}$$

Yechish:

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & 2 & -2 \end{pmatrix};$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix},$$

$$B = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$$

A matritsa determinantini hisoblaymiz:

$$detA = \begin{vmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & 2 & -2 \end{vmatrix} =$$

$$= 2 + 0 - 4 - 3 - 6 - 0 = -11 \neq 0.$$

A^{-1} – mavjud.
A^{-1} ni topamiz:

$$A^{-1} = \frac{1}{15}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 3 \\ 2 & -2 \end{vmatrix} = -4,$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = 13,$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = 7,$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 0 & -1 \\ 2 & -2 \end{vmatrix} = -2,$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} = 1,$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 0 \\ 3 & 2 \end{vmatrix} = -2,$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 0 & -1 \\ -1 & 3 \end{vmatrix} = -1,$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = -5,$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} = -1.$$

Demak, berilgan matritsaga teskari matritsa quyidagi ko‘rinishga bo‘ladi:

$$A^{-1} = \frac{1}{-11} \begin{pmatrix} -4 & -2 & -1 \\ 13 & 1 & -5 \\ 7 & -2 & -1 \end{pmatrix}.$$

$$X = \begin{pmatrix} \frac{4}{11} & \frac{2}{11} & \frac{1}{11} \\ -\frac{13}{11} & -\frac{1}{11} & \frac{5}{11} \\ -\frac{7}{11} & \frac{2}{11} & \frac{1}{11} \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{8}{11} - \frac{2}{11} + \frac{5}{11} \\ -\frac{26}{11} + \frac{1}{11} + \frac{25}{11} \\ -\frac{14}{11} - \frac{2}{11} + \frac{5}{11} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}.$$

$$x_1 = 1;\ x_2 = 0;\ x_3 = -1.$$

Misol. Sistemani yeching.

$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= 4 \\ x_1 + 9x_2 - 2x_3 &= -8 \\ 4x_1 - 8x_2 + 11x_3 &= 15. \end{aligned}$$

$$\boldsymbol{A} = \begin{pmatrix} 2 & -1 & 3 \\ 1 & 9 & -2 \\ 4 & -8 & 11 \end{pmatrix}.$$

$$detA = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 9 & -2 \\ 4 & -8 & 11 \end{vmatrix} = 53 \neq 0.$$

Demak, $detA \neq 0 \implies A^{-1}$ – mavjud. A^{-1} ni

topamiz:
$$\boldsymbol{A}^{-1} = \frac{1}{53}\begin{pmatrix} 83 & -13 & -25 \\ -19 & 10 & 7 \\ -44 & 12 & 19 \end{pmatrix}.$$

$$\boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{B} = \frac{1}{53}\begin{pmatrix} 83 & -13 & -25 \\ -19 & 10 & 7 \\ -44 & 12 & 19 \end{pmatrix}\begin{pmatrix} 4 \\ -8 \\ 15 \end{pmatrix}$$
$$= \begin{pmatrix} 61/53 \\ -51/53 \\ 13/53 \end{pmatrix}.$$

Bir jinsli chiziqli algebraik tenglamalar sistemasini

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1m}x_m = 0$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2m}x_m = 0$$

$$\vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nm}x_m = 0.$$

Sistemadagi a_{ij} koeffitsentlar $\boldsymbol{A} = [a_{ij}]$ matritsa elementlarini tashkil etadi. x_j noma‘lumlardan quyidagi matritsani tuzamiz:

$$\boldsymbol{X} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}$$

Ozod hadlardan $(n \times 1)$ - o‘lchovli **O** – nol matritsa hosil qilamiz va sistemani matritsa ko‘rinishda ifodalaymiz:

$$\boldsymbol{AX} = \boldsymbol{0}.$$

Ushbu bir jinsli chiziqli algebraik tenglamalar sistemasini berilgan bo‘lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = 0 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = 0 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = 0 \end{cases} \qquad (4)$$

Ma’lumki, $x_1 = 0$; $x_2 = 0$; $x_3 = 0$ sonlar (4) sistemaning har bir tenglamasini qanoatlantiradi. Bu yechim (4) sistemaning *trivial* yechimi deyiladi.

Demak, (4) sistema noldan farqli yechimga ega bo‘lishi uchun $detA = 0$ bo‘lishi zarur ekan.

Misol.

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 0 \\ 3x_1 + x_2 + 2x_3 = 0 \\ 2x_1 + 3x_2 + x_3 = 0 \end{cases}$$

Yechish. Sistemaning asosiy determinantini hisoblaymiz:

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = 1 + 8 + 27 - 6 - 6 - 6$$
$$= 36 - 18 = 18 \neq 0.$$

Demak, sistema $x_1 = 0;\ x_2 = 0;\ x_3 = 0$ trival yechimga egadir.

Foydalanilgan adabiyotlar

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