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CHAPTER 8

Analytic Geometry in Two and Three Dimensions

The oval-shaped lawn behind the White House in Washington, D.C. is called *the Ellipse*. It has views of the Washington Monument, the Jefferson Memorial, the Department of Commerce, and the Old Post Office Building. The Ellipse is 616 ft long, 528 ft wide, and is in the shape of a conic section. Its shape can be modeled using the methods of this chapter. See page 652.

EXAMPLE 5 Computing with Vectors

(a) $3\langle -2, 1, 4\rangle = \langle 3 \cdot -2, 3 \cdot 1, 3 \cdot 4\rangle = \langle -6, 3, 12\rangle$

(b) $\langle 0, 6, -7\rangle + \langle -5, 5, 8\rangle = \langle 0 - 5, 6 + 5, -7 + 8\rangle = \langle -5, 11, 1\rangle$

(c) $\langle 1, -3, 4\rangle - \langle -2, -4, 5\rangle = \langle 1 + 2, -3 + 4, 4 - 5\rangle = \langle 3, 1, -1\rangle$

(d) $|\langle 2, 0, -6\rangle| = \sqrt{2^2 + 0^2 + 6^2} = \sqrt{40} \approx 6.32$

(e) $\langle 5, 3, -1\rangle \cdot \langle -6, 2, 3\rangle = 5 \cdot (-6) + 3 \cdot 2 + (-1) \cdot 3$

$$= -30 + 6 - 3 = -27$$

Now try Exercises 23–26.

EXAMPLE 6 Using Vectors in Space

A jet airplane just after takeoff is pointed due east. Its air velocity vector makes an angle of 30° with flat ground with an airspeed of 250 mph. If the wind is out of the southeast at 32 mph, calculate a vector that represents the plane's velocity relative to the point of takeoff.

SOLUTION Let **i** point east, **j** point north, and **k** point up. The plane's air velocity is

$$\mathbf{a} = \langle 250 \cos 30°, 0, 250 \sin 30°\rangle \approx \langle 216.506, 0, 125\rangle,$$

and the wind velocity, which is pointing northwest, is

$$\mathbf{w} = \langle 32 \cos 135°, 32 \sin 135°, 0\rangle \approx \langle -22.627, 22.627, 0\rangle.$$

The velocity relative to the ground is $\mathbf{v} = \mathbf{a} + \mathbf{w}$, so

$$\begin{aligned}\mathbf{v} &\approx \langle 216.506, 0, 125\rangle + \langle -22.627, 22.627, 0\rangle \\ &\approx \langle 193.88, 22.63, 125\rangle \\ &= 193.88\mathbf{i} + 22.63\mathbf{j} + 125\mathbf{k}\end{aligned}$$

Now try Exercise 33.

In Exercise 64, you will be asked to interpret the meaning of the velocity vector obtained in Example 6.

Lines in Space

We have seen that first-degree equations in three variables graph as planes in space. So how do we get lines? There are several ways. First notice that to specify the x-axis, which is a line, we could use the pair of first-degree equations $y = 0$ and $z = 0$. As alternatives to using a pair of Cartesian equations, we can specify any line in space using

- one vector equation, or
- a set of three parametric equations.

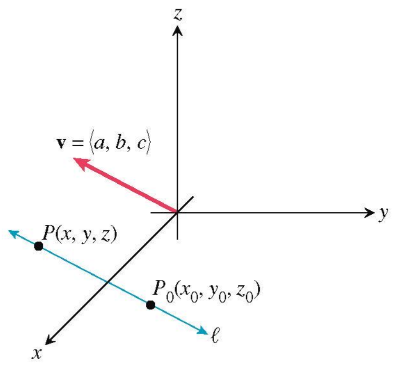

FIGURE 8.51 The line ℓ is parallel to the direction vector $\mathbf{v} = \langle a, b, c\rangle$.

FOLLOW-UP

Ask students to give alternate vector and parametric forms for the line in Example 7.

ASSIGNMENT GUIDE

Day 1: Ex. 1–15, odds
Day 2: Ex. 18–54, multiples of 3

COOPERATIVE LEARNING

Group Activity: Ex. 63

NOTES ON EXERCISES

Ex. 55 and 56 are proof exercises.
Ex. 65–68 introduce the cross product of two vectors.

ONGOING ASSESSMENT

Self-Assessment: Ex. 1, 5, 9, 13, 17, 23–26, 33, 41
Embedded Assessment: Ex. 43–48, 53

Suppose ℓ is a line through the point $P_0(x_0, y_0, z_0)$ and in the direction of a nonzero vector $\mathbf{v} = \langle a, b, c\rangle$ (Figure 8.51). Then for any point $P(x, y, z)$ on ℓ,

$$\overrightarrow{P_0P} = t\mathbf{v}$$

for some real number t. The vector $\mathbf{v}$ is a **direction vector** for line ℓ. If $\mathbf{r} = \overrightarrow{OP} = \langle x, y, z\rangle$ and $\mathbf{r}_0 = \overrightarrow{OP_0} = \langle x_0, y_0, z_0\rangle$, then $\mathbf{r} - \mathbf{r}_0 = t\mathbf{v}$. So an equation of the line ℓ is $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$.

Equations for a Line in Space

If ℓ is a line through the point $P_0(x_0, y_0, z_0)$ in the direction of a nonzero vector $\mathbf{v} = \langle a, b, c\rangle$, then a point $P(x, y, z)$ is on ℓ if and only if

- **Vector form:** $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$, where $\mathbf{r} = \langle x, y, z\rangle$ and $\mathbf{r}_0 = \langle x_0, y_0, z_0\rangle$; or
- **Parametric form:** $x = x_0 + at$, $y = y_0 + bt$, and $z = z_0 + ct$,

where t is a real number.

EXAMPLE 7 Finding Equations for a Line

The line through $P_0(4, 3, -1)$ with direction vector $\mathbf{v} = \langle -2, 2, 7\rangle$ can be written

- in vector form as $\mathbf{r} = \langle 4, 3, -1\rangle + t\langle -2, 2, 7\rangle$; or
- in parametric form as $x = 4 - 2t$, $y = 3 + 2t$, and $z = -1 + 7t$.

Now try Exercise 35.

EXAMPLE 8 Finding Equations for a Line

Using the standard unit vectors **i**, **j**, and **k**, write a vector equation for the line containing the points $A(3, 0, -2)$ and $B(-1, 2, -5)$, and compare it to the parametric equations for the line.

SOLUTION The line is in the direction of

$$\mathbf{v} = \overrightarrow{AB} = \langle -1 - 3, 2 - 0, -5 + 2\rangle = \langle -4, 2, -3\rangle.$$

So using $\mathbf{r}_0 = \overrightarrow{OA}$, the vector equation of the line becomes:

$$\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$$
$$\langle x, y, z\rangle = \langle 3, 0, -2\rangle + t\langle -4, 2, -3\rangle$$
$$\langle x, y, z\rangle = \langle 3 - 4t, 2t, -2 - 3t\rangle$$
$$x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = (3 - 4t)\mathbf{i} + 2t\mathbf{j} + (-2 - 3t)\mathbf{k}$$

The parametric equations are the three component equations

$$x = 3 - 4t,\ y = 2t, \text{ and } z = -2 - 3t.$$

Now try Exercise 41.

QUICK REVIEW 8.6 *(For help, go to Sections 6.1 and 6.3.)*

In Exercises 1–3, let $P(x, y)$ and $Q(2, -3)$ be points in the xy-plane.

1. Compute the distance between P and Q.

2. Find the midpoint of the line segment PQ.

3. If P is 5 units from Q, describe the position of P.

In Exercises 4–6, let $\mathbf{v} = \langle -4, 5 \rangle = -4\mathbf{i} + 5\mathbf{j}$ be a vector in the xy-plane.

4. Find the magnitude of $\mathbf{v}$. $\sqrt{41}$

5. Find a unit vector in the direction of $\mathbf{v}$.

6. Find a vector 7 units long in the direction of $-\mathbf{v}$.

7. Give a geometric description of the graph of $(x + 1)^2 + (y - 5)^2 = 25$ in the xy-plane.

8. Give a geometric description of the graph of $x = 2 - t$, $y = -4 + 2t$ in the xy-plane.

9. Find the center and radius of the circle $x^2 + y^2 + 2x - 6y + 6 = 0$ in the xy-plane.

10. Find a vector from $P(2, 5)$ to $Q(-1, -4)$ in the xy-plane.

SECTION 8.6 EXERCISES

In Exercises 1–4, draw a sketch that shows the point.

1. $(3, 4, 2)$ **2.** $(2, -3, 6)$

3. $(1, -2, -4)$ **4.** $(-2, 3, -5)$

In Exercises 5–8, compute the distance between the points.

5. $(-1, 2, 5), (3, -4, 6)$ $\sqrt{53}$

6. $(2, -1, -8), (6, -3, 4)$ $2\sqrt{41}$

7. $(a, b, c), (1, -3, 2)$ $\sqrt{(a-1)^2 + (b+3)^2 + (c-2)^2}$

8. $(x, y, z), (p, q, r)$ $\sqrt{(x-p)^2 + (y-q)^2 + (z-r)^2}$

In Exercises 9–12, find the midpoint of the segment PQ.

9. $P(-1, 2, 5), Q(3, -4, 6)$ $(1, -1, 11/2)$

10. $P(2, -1, -8), Q(6, -3, 4)$ $(4, -2, -2)$

11. $P(2x, 2y, 2z), Q(-2, 8, 6)$ $(x - 1, y + 4, z + 3)$

12. $P(-a, -b, -c), Q(3a, 3b, 3c)$ (a, b, c)

In Exercises 13–16, write an equation for the sphere with the given point as its center and the given number as its radius.

13. $(5, -1, -2), 8$ **14.** $(-1, 5, 8), \sqrt{5}$

15. $(1, -3, 2), \sqrt{a}, a > 0$ **16.** $(p, q, r), 6$

In Exercises 17–22, sketch a graph of the equation. Label all intercepts.

17. $x + y + 3z = 9$ **18.** $x + y - 2z = 8$

19. $x + z = 3$ **20.** $2y + z = 6$

21. $x - 3y = 6$ **22.** $x = 3$

In Exercises 23–32, evaluate the expression using $\mathbf{r} = \langle 1, 0, -3 \rangle$, $\mathbf{v} = \langle -3, 4, -5 \rangle$, and $\mathbf{w} = \langle 4, -3, 12 \rangle$.

23. $\mathbf{r} + \mathbf{v}$ $\langle -2, 4, -8 \rangle$ **24.** $\mathbf{r} - \mathbf{w}$ $\langle -3, 3, -15 \rangle$

25. $\mathbf{v} \cdot \mathbf{w}$ -84 **26.** $|\mathbf{w}|$ 13

27. $\mathbf{r} \cdot (\mathbf{v} + \mathbf{w})$ -20 **28.** $(\mathbf{r} \cdot \mathbf{v}) + (\mathbf{r} \cdot \mathbf{w})$ -20

29. $\mathbf{w}/|\mathbf{w}|$ **30.** $\mathbf{i} \cdot \mathbf{r}$ 1

31. $\langle \mathbf{i} \cdot \mathbf{v}, \mathbf{j} \cdot \mathbf{v}, \mathbf{k} \cdot \mathbf{v} \rangle$ **32.** $(\mathbf{r} \cdot \mathbf{v})\mathbf{w}$ $\langle 48, -36, 144 \rangle$

In Exercises 33 and 34, let $\mathbf{i}$ point east, $\mathbf{j}$ point north, and $\mathbf{k}$ point up.

33. Three-Dimensional Velocity An airplane just after takeoff is headed west and is climbing at a 20° angle relative to flat ground with an airspeed of 200 mph. If the wind is out of the northeast at 10 mph, calculate a vector $\mathbf{v}$ that represents the plane's velocity relative to the point of takeoff. $\mathbf{v} = -195.01\mathbf{i} - 7.07\mathbf{j} + 68.40\mathbf{k}$

34. Three-Dimensional Velocity A rocket soon after takeoff is headed east and is climbing at a 80° angle relative to flat ground with an airspeed of 12,000 mph. If the wind is out of the southwest at 8 mph, calculate a vector $\mathbf{v}$ that represents the rocket's velocity relative to the point of takeoff.

In Exercises 35–38, write the vector and parametric forms of the line through the point P_0 in the direction of $\mathbf{v}$.

35. $P_0(2, -1, 5), \mathbf{v} = \langle 3, 2, -7 \rangle$

36. $P_0(-3, 8, -1), \mathbf{v} = \langle -3, 5, 2 \rangle$

37. $P_0(6, -9, 0), \mathbf{v} = \langle 1, 0, -4 \rangle$

38. $P_0(0, -1, 4), \mathbf{v} = \langle 0, 0, 1 \rangle$