Lesson Menu

5-Minute Check Over Lesson 12–2

CheckPoint

1 Rewrite $\frac{3\pi}{8}$ in degrees.

A. 74.5°

B. 67.5°

C. 58°

D. 47°

5-Minute Check Over Lesson 12–2

CheckPoint

2 Rewrite 135° in radians.

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{2\pi}{3}$

D. $\frac{3\pi}{4}$

5-Minute Check Over Lesson 12–2

CheckPoint

3 **Find one angle with positive measure and one angle with negative measure coterminal with 88°.**

→ **A.** 448°, –272°

B. 358°, –182°

C. 268°, –92°

D. 178°, –2°

5-Minute Check

Over Lesson 12–2

CheckPoint

4 Find one angle with positive measure and one angle with negative measure coterminal with $-\frac{\pi}{5}$.

A. $\frac{4\pi}{5}, -\frac{\pi}{5}$

B. $\frac{9\pi}{5}, -\frac{9\pi}{5}$

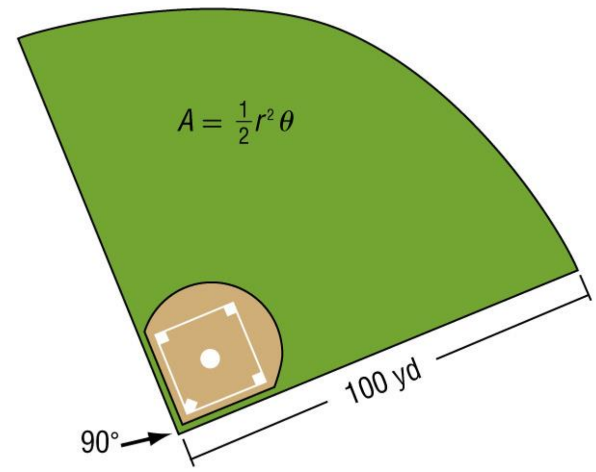
C. $\frac{9\pi}{5}, -\frac{11\pi}{5}$

D. $\frac{11\pi}{5}, -\frac{\pi}{5}$

5-Minute Check Over Lesson 12–2

Standardized Test Practice

5 Use an angle measure in radians to calculate the area *A* of the baseball field. Round to the nearest whole number, if necessary.



A. 7860 yd^2

B. 7854 yd^2

C. 3930 yd^2

D. 3925 yd^2

CCSS

Mathematical Practices

6 Attend to precision.

Then

You found values of trigonometric functions for acute angles.

Now

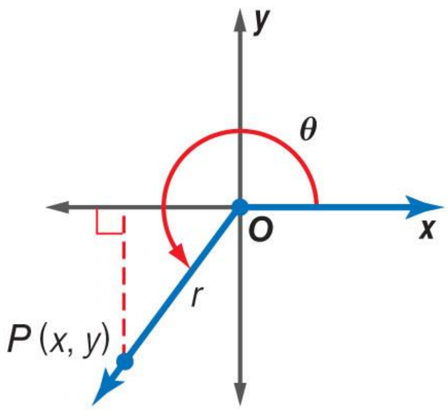
- Find values of trigonometric functions for general angles.
- Find values of trigonometric functions by using reference angles.

New Vocabulary

- quadrantal angle
- reference angle

KeyConcept Trigonometric Functions of General Angles

Let θ be an angle in standard position and let $P(x, y)$ be a point on its terminal side. Using the Pythagorean Theorem, $r = \sqrt{x^2 + y^2}$. The six trigonometric functions of θ are defined below.

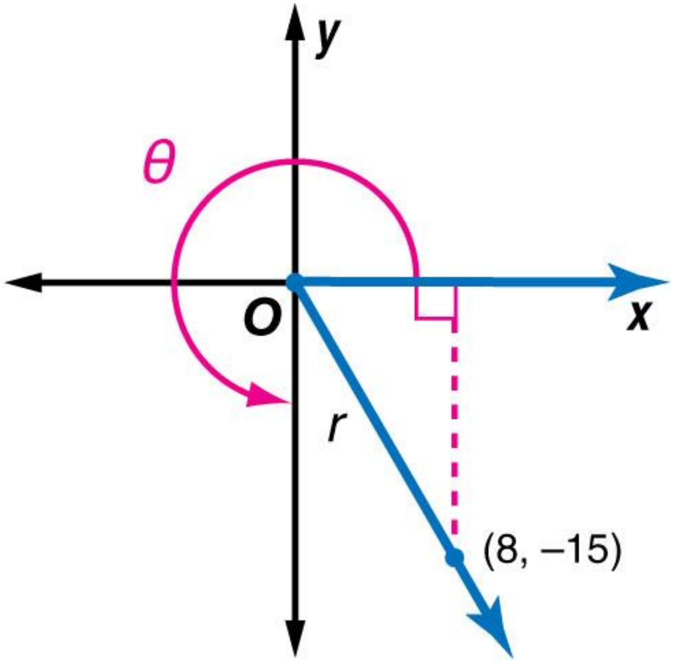


$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}, x \neq 0$

$\csc\theta = \frac{r}{y}, y \neq 0$ $\sec\theta = \frac{r}{x}, x \neq 0$ $\cot\theta = \frac{x}{y}, y \neq 0$

EXAMPLE 1 Evaluate Trigonometric Functions Given a Point

The terminal side of θ in standard position contains the point (8, –15). Find the exact values of the six trigonometric functions of θ.



From the coordinates given, you know that $x = 8$ and $y = -15$.

Use the Pythagorean Theorem to find r.

$$r = \sqrt{x^2 + y^2}$$ Pythagorean Theorem

$$= \sqrt{8^2 + (-15)^2}$$ Replace x with 8 and y with –15.

EXAMPLE 1 Evaluate Trigonometric Functions Given a Point

$= \sqrt{289}$ or 17 Simplify.

Now use $x = 8$, $y = –15$, and $r = 17$ to write the ratios.

$$\sin\theta = \frac{y}{r} = \frac{-15}{17} \quad \cos\theta = \frac{x}{r} = \frac{8}{17} \quad \tan\theta = \frac{y}{x} = \frac{-15}{8}$$

$$\csc\theta = \frac{r}{y} = \frac{-17}{15} \quad \sec\theta = \frac{r}{x} = \frac{17}{8} \quad \cot\theta = \frac{x}{y} = \frac{8}{-15}$$

Answer: $\sin\theta = -\frac{15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = -\frac{15}{8}$,

$\csc\theta = -\frac{17}{15}$, $\sec\theta = \frac{17}{8}$, $\cot\theta = -\frac{8}{15}$

EXAMPLE 1 Check Your Progress

CheckPoint

Find the exact values of the six trigonometric functions of θ if the terminal side of θ contains the point (–3, 4).

A. $\sin\theta = \frac{4}{5}$, $\cos\theta = -\frac{3}{5}$, $\tan\theta = -\frac{4}{3}$, $\csc\theta = \frac{5}{4}$, $\sec\theta = -\frac{5}{3}$, $\cot\theta = -\frac{3}{4}$

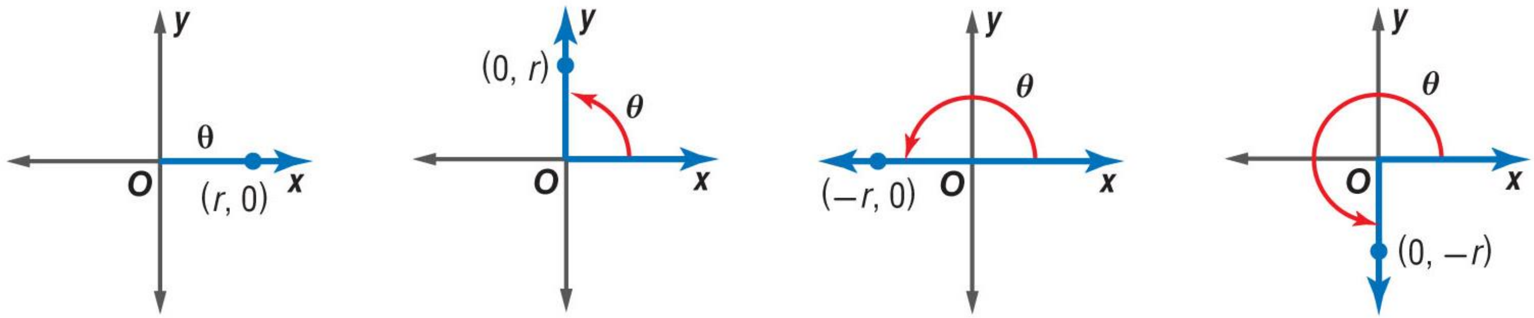
B. $\sin\theta = -\frac{5}{3}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$, $\csc\theta = \frac{5}{3}$, $\sec\theta = \frac{4}{5}$, $\cot\theta = \frac{4}{3}$

C. $\sin\theta = \frac{5}{3}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = \frac{4}{3}$, $\csc\theta = -\frac{3}{5}$, $\sec\theta = -\frac{4}{5}$, $\cot\theta = \frac{3}{4}$

D. $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = -\frac{3}{4}$, $\csc\theta = \frac{3}{5}$, $\sec\theta = \frac{5}{4}$, $\cot\theta = -\frac{3}{4}$

KeyConcept Quadrantal Angles

$\theta = 0°$ or 0 radians	$\theta = 90°$ or $\frac{\pi}{2}$ radians	$\theta = 180°$ or π radians	$\theta = 270°$ or $\frac{3\pi}{2}$ radians

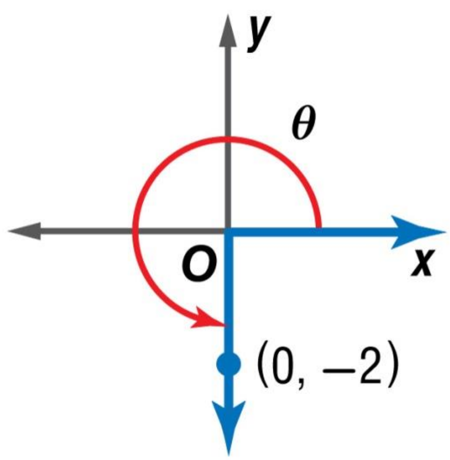


EXAMPLE 2 Quadrantal Angles

The terminal side of θ in standard position contains the point at (0, –2). Find the values of the six trigonometric functions of θ.

The point at (0, –2) lies on the negative *y*-axis, so the quadrantal angle θ is 270°.

Use $x = 0$, $y = -2$, and $r = 2$ to write the trigonometric functions.



EXAMPLE 2 Quadrantal Angles

$$\sin\theta = \frac{y}{r} = \frac{-2}{2} \text{ or } -1$$

$$\cos\theta = \frac{x}{r} = \frac{0}{2} \text{ or } 0$$

$$\tan\theta = \frac{y}{x} = \frac{-2}{0} \text{ or undefined}$$

$$\csc\theta = \frac{r}{y} = \frac{2}{-2} \text{ or } -1$$

$$\sec\theta = \frac{r}{x} = \frac{2}{0} \text{ or undefined}$$

$$\cot\theta = \frac{x}{y} = \frac{0}{-2} \text{ or } 0$$

EXAMPLE 2 Check Your Progress

CheckPoint

The terminal side of θ in standard position contains the point at (3, 0). Which of the following trigonometric functions of θ is incorrect?

A. sin $\theta = 1$

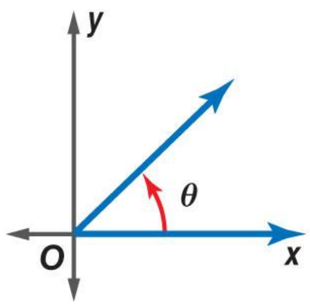
B. cos $\theta = 1$

C. tan $\theta = 0$

D. cot θ is undefined

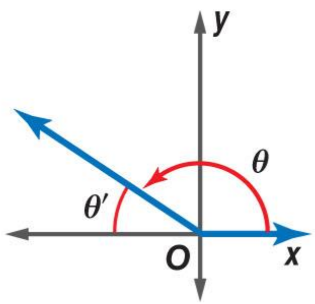
KeyConcept Reference Angles

Quadrant I



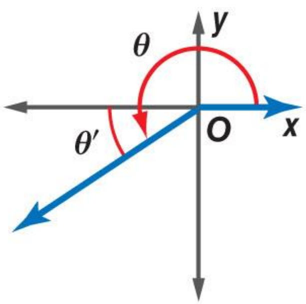
$$\theta' = \theta$$

Quadrant II



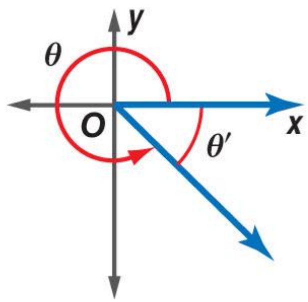
$$\theta' = 180° - \theta$$
$$\theta' = \pi - \theta$$

Quadrant III



$$\theta' = \theta - 180°$$
$$\theta' = \theta - \pi$$

Quadrant IV

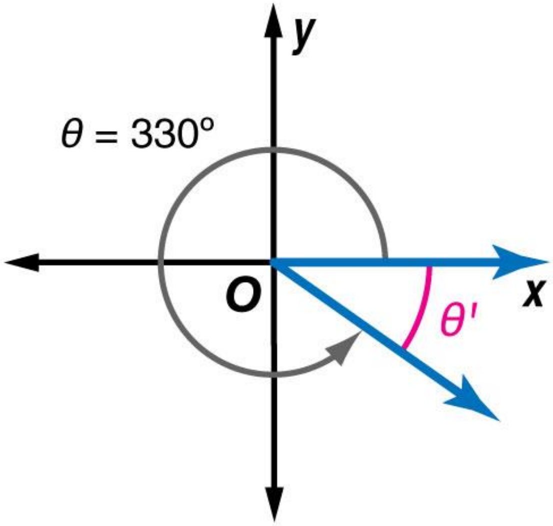


$$\theta' = 360° - \theta$$
$$\theta' = 2\pi - \theta$$

EXAMPLE 3 **Find Reference Angles**

A. Sketch 330°. Then find its reference angle.

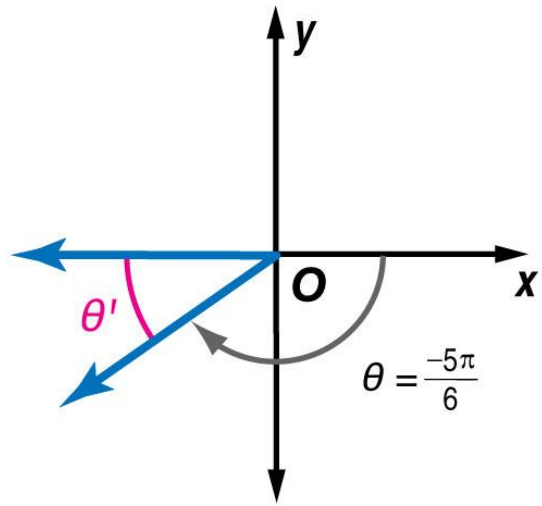
Answer: Because the terminal side of 330° lies in quadrant IV, the reference angle is 360° – 330° or 30°.



EXAMPLE 3 Find Reference Angles

B. **Sketch $\frac{-5\pi}{6}$. Then find its reference angle.**

Answer: A coterminal angle $\frac{-5\pi}{6}$ is $2\pi - \frac{5\pi}{6}$ or $\frac{7\pi}{6}$. Because the terminal side of this angle lies in Quadrant III, the reference angle is $\frac{5\pi}{6} - \pi$ or $\frac{\pi}{6}$.



EXAMPLE 3 Check Your Progress

CheckPoint

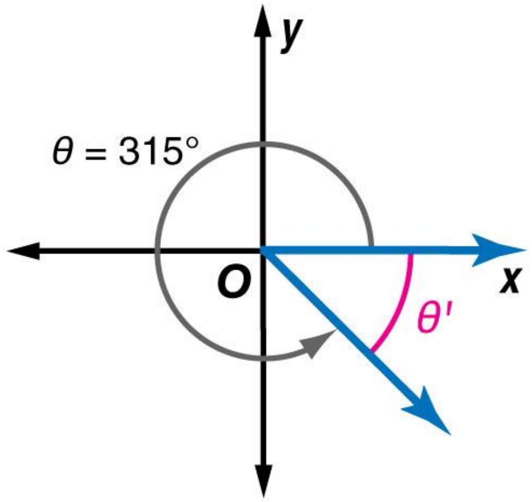
A. Sketch 315°. Then find its reference angle.

A. 105°

B. 85°

C. 45°

D. 35°



EXAMPLE 3 Check Your Progress

CheckPoint

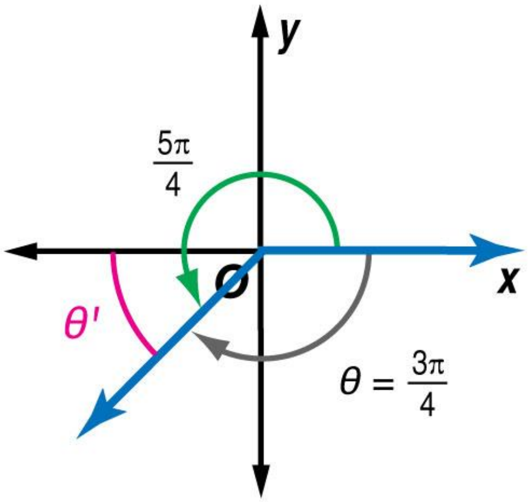
B. Sketch $\frac{-3\pi}{4}$. Then find its reference angle.

A. $-\frac{\pi}{3}$

B. $\frac{3\pi}{4}$

C. $\frac{\pi}{4}$

D. $-\frac{2\pi}{3}$



KeyConcept Evaluate Trigonometric Functions

Step 1 Find the measure of the reference angle θ'.

Step 2 Evaluate the trigonometric function for θ'.

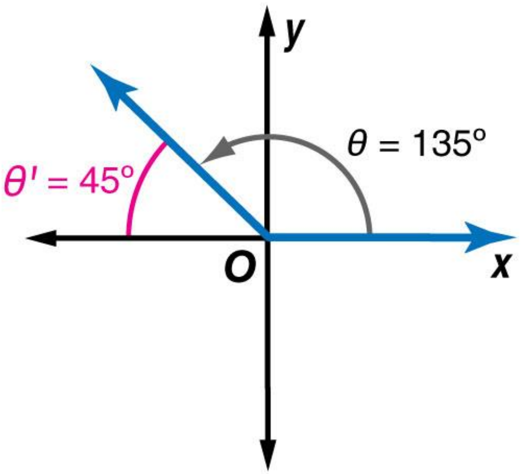
Step 3 Determine the sign of the trigonometric function value. Use the quadrant in which the terminal side of θ lies.

Quadrant II	Quadrant I
sin θ, csc θ: +	sin θ, csc θ: +
cos θ, sec θ: –	cos θ, sec θ: +
tan θ, cot θ: –	tan θ, cot θ: +
Quadrant III	**Quadrant IV**
sin θ, csc θ: –	sin θ, csc θ: –
cos θ, sec θ: –	cos θ, sec θ: +
tan θ, cot θ: +	tan θ, cot θ: –

EXAMPLE 4 Use a Reference Angle to Find a Trigonometric Value

A. Find the exact value of sin 135°.

Because the terminal side of 135° lies in Quadrant II, the reference angle θ' is 180° – 135° or 45°.

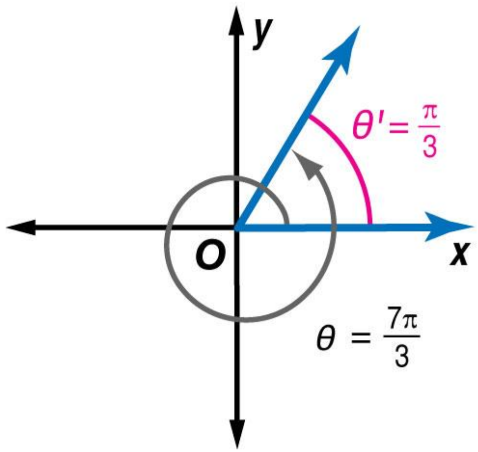


Answer: The sine function is positive in Quadrant II, so, $\sin 135° = \sin 45°$ or $\frac{\sqrt{2}}{2}$.

EXAMPLE 4 Use a Reference Angle to Find a Trigonometric Value

B. Find the exact value of $\cot \frac{7\pi}{3}$.

Because the terminal side of $\frac{7\pi}{3}$ lies in Quadrant I, the reference angle is $2\pi - \frac{7\pi}{3}$ or $\frac{\pi}{3}$. The cotangent function is positive in Quadrant I.



EXAMPLE 4 Use a Reference Angle to Find a Trigonometric Value

$$\cot \frac{7\pi}{3} = \cot \frac{\pi}{3}$$

$$= \cot 60°$$ $\frac{\pi}{3}$ radians $= 60°$

$$= \frac{\sqrt{3}}{3}$$ $\cot 60° = \frac{\sqrt{3}}{3}$

Answer: $\frac{\sqrt{3}}{3}$

EXAMPLE 4 Check Your Progress

CheckPoint

A. **Find the exact value of sin 120°.**

A. $\frac{\sqrt{3}}{3}$

B. $\frac{2\sqrt{2}}{3}$

C. $\frac{\sqrt{3}}{4}$

D. $\frac{\sqrt{3}}{2}$

EXAMPLE 4 Check Your Progress

CheckPoint

B. Find the exact value of $\cot \frac{11\pi}{2}$.

A. $\sqrt{2}$

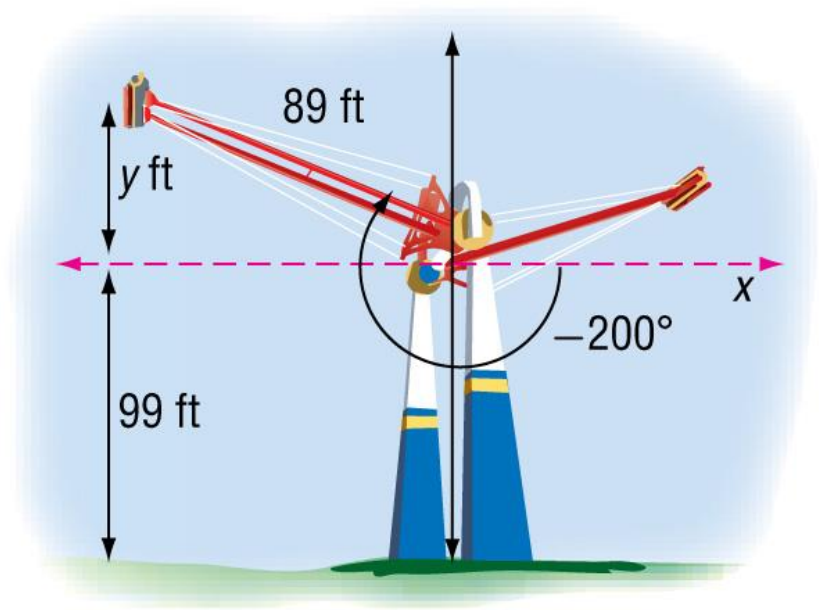
B. 0

C. $\frac{\sqrt{2}}{3}$

D. $-\frac{\sqrt{3}}{3}$

Real-World Example 5 Use Trigonometric Functions

RIDES The swing arms of the ride pictured below are 89 feet long and the height of the axis from which the arms swing is 99 feet. What is the total height of the ride at the peak of the arc?



Real-World Example 5 Use Trigonometric Functions

coterminal angle: –200° + 360° = 160°

reference angle: 180° – 160° = 20°

$\sin \theta = \frac{y}{r}$ Sine function

$\sin 20^\circ = \frac{y}{89}$ θ = 20° and r = 89

89 sin 20° = y Multiply each side by 89.

30.4 ≈ y Use a calculator to solve for y.

Answer: Since y is approximately 30.4 feet, the total height of the ride at the peak is 30.4 + 99 or about 129.4 feet.

Real-World Example 5 / Check Your Progress / CheckPoint

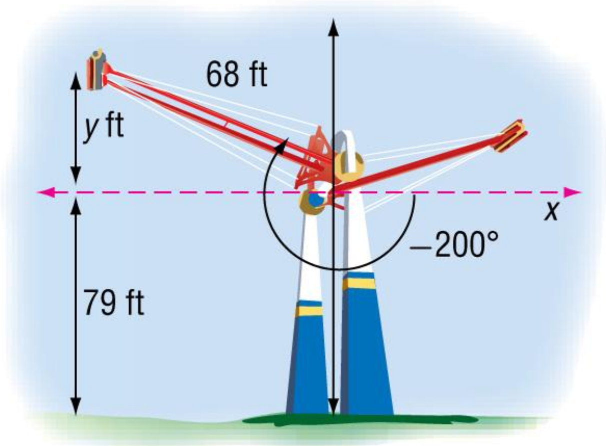
RIDES The swing arms of the ride pictured below are 68 feet long and the height of the axis from which the arms swing is 79 feet. What is the total height of the ride at the peak of the arc?

A. 23.6 ft

B. 79 ft

C. 102.3 ft

D. 110.8 ft



Click the mouse button to return to the lesson menu.