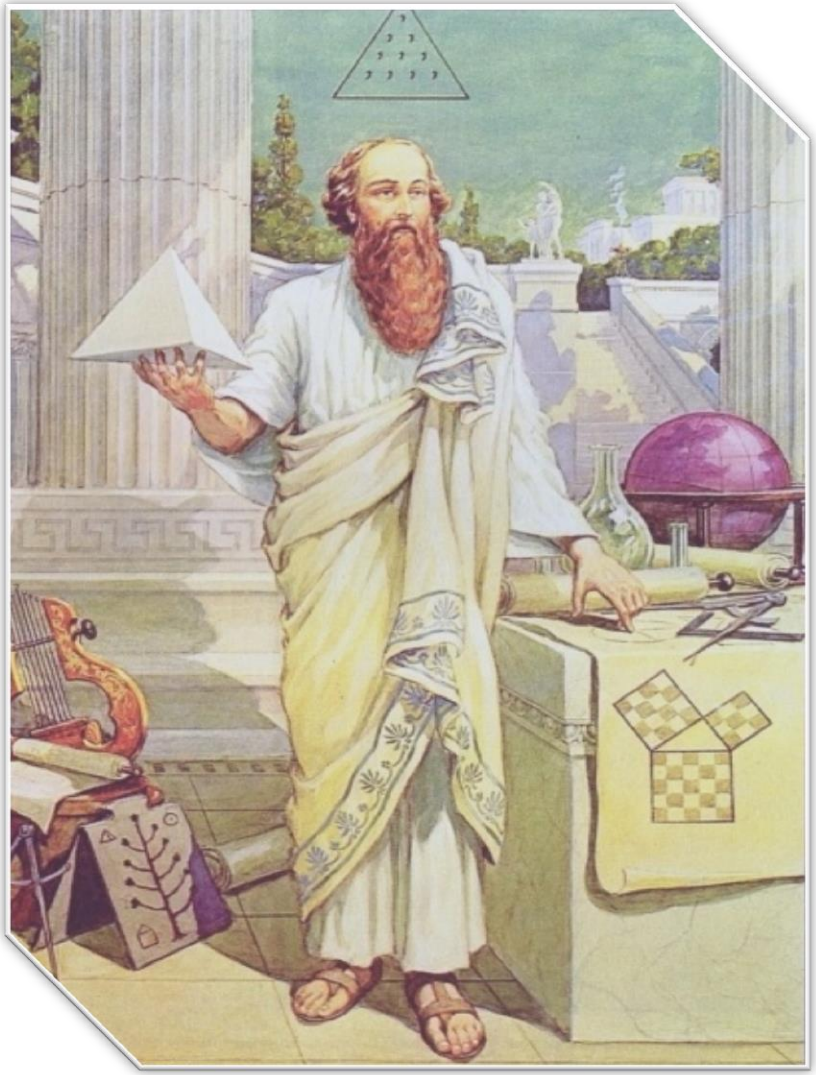
Aniqmas integrallar

Integrallash metodlari

Integral hisob matematika rivojlanishining antik davrlariga borib taqaladi. Qadimgi Yunonistonda sirt va hajmlarni aniqlashning Yevdoks Knidskiy tomonidan ishlab chiqilgan ma'lum metodlari ba'zi sodda integrallarni hisoblash imkonini bergan



Yevdoks Knidskiy

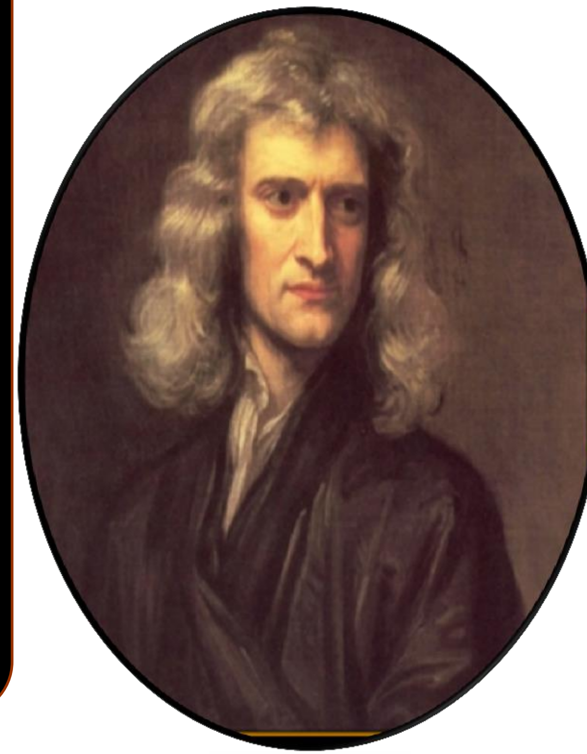
e.av. 408 — 355 yillar.

Leybnits Gotfrid Vilgelm (1646-1716)

$\int$ belgi Leybnits tomonidan 1675 yilda kiritilgan. Bu belgi lotin alifbosidagi S harfining o'zgartirilgan ko'rinishidir. (summa so'zidagi birinchi harf).

Gotfrid Vilgelm Leybnits
(1646—1716)

Nyuton va Leybnitslar bir - birlaridan bexabar ravishda integral uchun Nyuton – Leybnits formulasini ishlab chiqishdi.



Isaak Nyuton
(1643 – 1727)

Koshi va Veyshtrasning ishlari integral hisobning ko’p yillik rivojlanishining cho’qqisi bo’ldi.

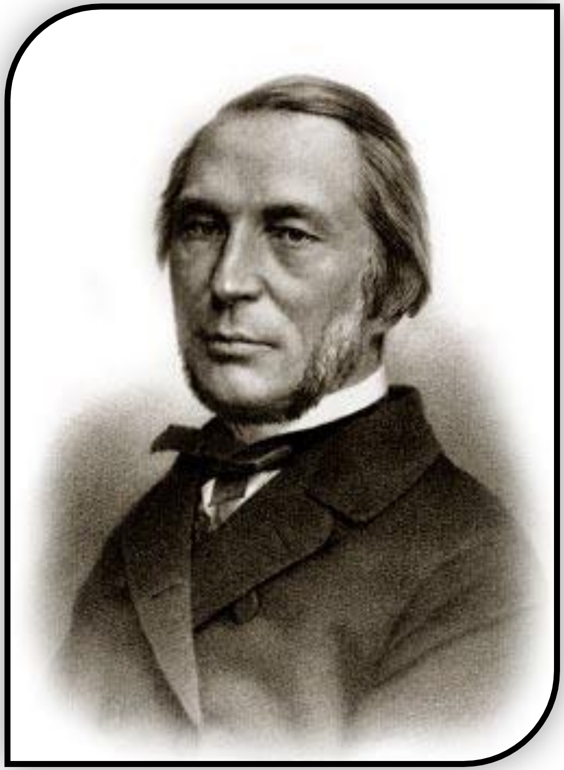
Ostyugen Luyi Koshi (1789 – 1857)

Karl Tedor Vilgelm Veyershtrass (1815 1897)

Rus matematiklardan quyidagi olimlar integral hisoblash usullariga o’z hissalarini qo’shishdi:

B.Bunyakovskiy
(1804 – 1889)

M.B.Ostogradskiy
(1801 – 1862)



P.L.Chebishev
(1821 – 1894)

Aniqmas integral

***f(x)* uzluksiz funksiyaning aniqmas integrali deb – uning ixtiyoriy bir boshlang'ich *F(x)* funksiyalaridan biriga aytiladi.**

$$\int f(x)dx = F(x) + c$$

Bu yerda C – ixtiyoriy o'zgarmas son (const).

Funksiya va uning boshlang'ichi orasidagi bog'lanishni toping.

1. $f(x) = x^n$	1. $F(x) = Cx + C$
2. $f(x) = C$	2. $F(x) = \frac{x^{n+1}}{n+1} + C$
3. $f(x) = \sin x$	3. $F(x) = \operatorname{tg} x + C$
4. $f(x) = -\frac{1}{\sin^2 x}$	4. $F(x) = \sin x + C$
5. $f(x) = \cos x$	5. $F(x) = \operatorname{ctg} x + C$
6. $f(x) = \frac{1}{\cos^2 x}$	6. $F(x) = -\cos x + C$

Integral xossalari

$$\int (f(x)+g(x))dx =$$

$$\int f(x)dx + \int g(x)dx$$

$$\int Cf(x)dx = C\int f(x)dx$$

Integral xossalari

$$\left(\int f(x)dx\right)' = f(x)$$

$$\int f'(x)dx = f(x) + C$$

$$\int f(kx+b)dx = \frac{1}{k}F(kx+b) + C$$

Integrallashning asosiy metodlari

1.Jadval asosida.

2. Integral ostidagi ifodani jadvalda erilgan funksiyalarga olib kelish.

3. O’zgaruvchini almashtirish (o’rniga qo’yish) metodi

4. Bo’laklab integrallash.

Integrallash jadvali

1. $\int x^{\alpha}dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \quad \alpha \neq -1;$ 2. $\int \frac{dx}{x} = \ln|x| + C, \ x \neq 0;$

3. $\int a^{x}dx = \frac{a^{x}}{\ln a} + C, \quad a > 0, \ a \neq 1;$ $\int e^{x}dx = e^{x} + C;$

4. $\int \frac{dx}{1+x^2} = arctgx + C;$ $\int \frac{dx}{x^2+a^2} = \frac{1}{a} arctg\frac{x}{a} + C, a \neq 0;$

5. $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C;$ $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C \ (|x| < |a|);$

6. $\int \frac{dx}{\cos^2 x} = tgx + C;$ 7. $\int \frac{dx}{\sin^2 x} = -ctgx + C;$

8. $\int \cos x dx = \sin x + C;$ 9. $\int \sin x dx = -\cos x + C;$

10. $\int chxdx = shx + C;$ 11. $\int shxdx = chx + C;$

12. $\int \frac{dx}{ch^2 x} = thx + C;$ 13. $\int \frac{dx}{sh^2 x} = -cthx + C;$

14. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left|\frac{a+x}{a-x}\right| + C, a \neq 0;$

15. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C, (|x| > |a|).$

funksiya boshlang'ichlarini toping:

$F(x) = 5x^2 + C$

1) $f(x) = 10x$

$F(x) = x^3 + C$

2) $f(x) = 3x^2$

$F(x) = -\cos x + 5x + C$

3) $f(x) = \sin x + 5$

$F(x) = 5\sin x + C$

4) $f(x) = 5\cos x$

$F(x) = 2x^3 + C$

5) $f(x) = 6x^2$

$F(x) = 3x - x^2 + C$

6) $f(x) = 3-2x$

O'rniga qo'yish metodi.

f(x) [a;b] kesmada uzluksiz $x=\varphi'(x)$ esa $[\alpha;\beta]$ da uzluksiz va $\varphi(\alpha)=a,\ \ \varphi(\beta)=b.$ o'rinli bo'lsin. unda $dx=\varphi'(t)dt$ ni inobatda olgan holda $\int f(x)dx$ aniqmas integralni quyidagi ko'rinishda ifodalash mumkin:

$$\int f(x)dx=\int f(\varphi(t))\cdot\varphi'(t)dt.$$

Quyidagilar tog'rimi:

а)

$$\int x^5 dx = 5x^4 + C$$

в)

$$\int 3x^2 dx = x^3 + C$$

б)

$$\int 3x^2 dx = 6x + C$$

$$\int x^6 dx = \frac{1}{7} x^7 + C$$

1- Misol.

$$\int (3x^5 + 4\cos x - 2x + 1)dx =$$

Yig'indining integrali integral yig'indilariga tengdir

O'zgarmas son bo'lgan ko'paytuvchini integral belgisidan tashqariga chiqarish mumkin

$$\int 3x^5 dx + \int 4 cosxdx - \int 2xdx + \int 1dx =$$

$$3\int x^5 dx + 4\int cosxdx - 2\int xdx + 1\int dx =$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} +$$

$$\int cosxdx = sinx +$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} +$$

$$\int dx = x + c$$

$$\frac{3x^{5+1}}{5+1} + 4sinx - \frac{2x^2}{2} + x + C \Rightarrow \frac{1}{2}x^6 + 4sinx - x^2 + x + C$$

Misol 2.

$$\int\left(\frac{3}{x^5}-x^4+7e^x-\frac{2}{x}\right)dx$$

$$\frac{1}{a^n}=a^{-n}$$

Yechimni tekshiring

$$\int\left(3x^{-5}-x^4+7e^x-\frac{2}{x}\right)dx$$

$$3\int x^{-5}dx-\int x^4dx+7\int e^xdx-2\int\frac{dx}{x}$$

$$\frac{3x^{-4}}{-4}-\frac{x^5}{5}+7e^x-2lnx+c$$

$$-\frac{3}{4x^4}-\frac{1}{5}x^5+7e^x-2lnx+c$$

Yechimni yozib oling:

Misol 3. $\int\left(\frac{4}{\cos^2 x}+x^3-3\sqrt{x}\right)dx$

Yechimni tekshiring

! $\sqrt[n]{x^m}=x^{\frac{m}{n}}$

Yechimni yozib oling :

$$\int\left(\frac{4}{\cos^2 x}+x^3-3x^{\frac{1}{2}}\right)dx$$

$$4\int\frac{1}{\cos^2 x}dx+\int x^3dx-3\int x^{\frac{1}{2}}dx$$

$$4tgx+\frac{x^4}{4}-3\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+C$$

$$4tgx+\frac{1}{4}x^4-2x\sqrt{x}+C$$

Misol 4.

$$\int \sin(6x+2)dx$$

Javobni tekshiring

Yechimni yozib oling :

Yangi o'zgartiruvchi kiritamiz va differensiallarini hisoblaymiz:

$$6x+2=u$$

$$du=6dx, \qquad dx=\frac{1}{6}du$$

$$\int \sin(6x+2)dx=\int \sin u\cdot\frac{1}{6}du$$

$$=\frac{1}{6}\int \sin u\,du=-\frac{1}{6}\cos u+c$$

$$-\frac{1}{6}\cos u+c=$$

$$-\frac{1}{6}\cos(6x+2)+C$$

Mustaqil ish

Yechimni tekshiring

«A» daraja («3»)

$$1)\ \frac{1}{6}x^6+\frac{3}{2}x^2-4x+C$$

$$2)\ 5x^5+3e^x-4\ln x+C$$

«B» daraja («4»)

$$3)\ \frac{1}{20}(3+4x)^5+C$$

$$4)\ \frac{1}{6}e^{6x-3}+C$$

«C» daraja («5»)

$$5)\ \frac{1}{5}\sin(5x-4)+C$$

$$6)\ 2ctgx+\frac{2}{3}x\sqrt{x}+\frac{3}{5x^5}+C$$

To'g'ri bog'lanishni toping.

1. $f(x) = x + x^3$
2. $f(x) = 2 \cdot \cos x$
3. $f(x) = 8 - 5x + 10x^2$
4. $f(x) = (4 - 3x)^9$

1. $F(x) = \frac{x}{2} + \cos x + C$
2. $F(x) = \frac{x^2}{2} + \frac{x^4}{4} + C$
3. $F(x) = 8x - \frac{5x^2}{2} + \frac{10x^3}{3} + C$
4. $F(x) = 2\sin x + C$
5. $F(x) = 8x + \sin x + \frac{10x^2}{3} + C$
6. $F(x) = -\frac{1}{30}(4 - 3x)^{10} + C$
7. $F(x) = -\frac{1}{3}(4 - 3x)^{10} + C$