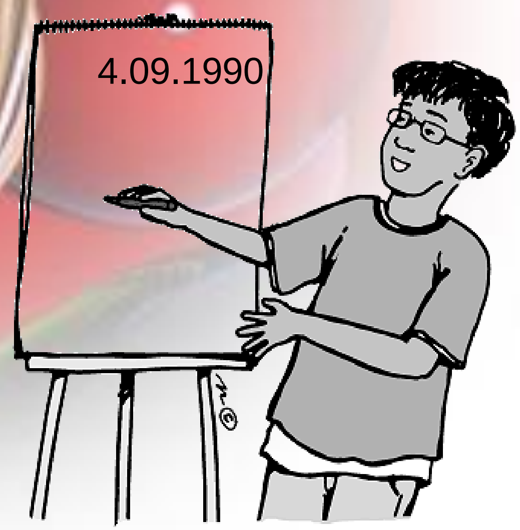
Geron Formulasi. Kosinuslar va Sinuslar Teoremasi

REJA:

1. **Uchburchakning yuzi uchun Geron Formulasi**
2. **Kosinuslar teoremasi va isboti.**
3. **Sinuslar teoremasi va isboti.**



Uchburchakning yuzi uchun Geron formulasi

Uchburchakning yuzi uchun

$$S = \sqrt{p(p-a)(p-b)(p-c)}$$

Bunda a, b, c – uchburchak tomonlarining uzunliklari,

$p = \frac{a+b+c}{2}$ - yarim perimetri.

Yechilishi: $S = \frac{1}{2}ab\sin\gamma$

bunda γ – uchburchakning c tomoni qarshisidagi burchak.

Kosinuslar teoremasiga ko’ra :

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Bundan

$\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$ ekanligi kelib chiqadi.

Demak, $\sin^2\gamma = 1-\cos^2\gamma = (1-\cos\gamma)(1+\cos\gamma) =$

$$= (1-\frac{a^2+b^2-c^2}{2ab})(1+\frac{a^2+b^2-c^2}{2ab}) = \frac{2ab-a^2-b^2+c^2}{2ab}\cdot\frac{2ab+a^2+b^2-c^2}{2ab} =$$

$$= \frac{c^2-(a-b)^2}{2ab}\cdot\frac{(a+b)^2-c^2}{2ab} = \frac{1}{4a^2b^2}\cdot(c-a+b)\cdot(c+a-b)(a+b-c)(a+b+c).$$

$$a+b+c=2p, a+b-c=2p-2c, a+c-b=2p-2b, c-a+b=2p-2a$$

ekanini bilgan holda ushbuga ega bo’lamiz:

$$\sin\gamma = \frac{2}{ab}\sqrt{p(p-a)(p-b)(p-c)}$$

Shunday qilib,

$$S = \frac{1}{2}ab\sin\gamma = \sqrt{p(p-a)(p-b)(p-c)}$$

Kosinuslar teoremasi

Teorema: Uchburchak istalgan tomonining kvadrati qolgan ikki tomoni kvadratlari yig'indisidan shu ikki tomon bilan ular orasidagi burchak kosinusining ikkilangan ko'paytmasini ayirish natijasiga teng.

Isboti: *ABC* – berilgan uchburchak bo'lsin.

$BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cdot \cos A$ **ekanini isbotlaymiz.**

$\overline{BC} = \overline{AC} - \overline{AB}$ **vektor tenglikka egamiz. Bu tenglikni skalyar kvadratga ko'tarib topamiz:**

$$\overline{BC^2} = \overline{AB}^2 + \overline{AC}^2 - 2\overline{AB} \cdot \overline{AC}$$

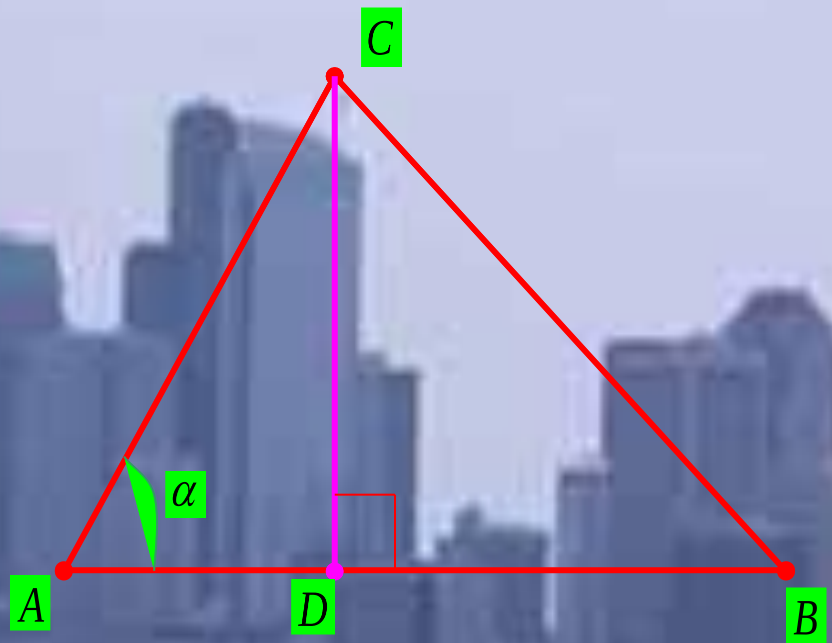
Yoki

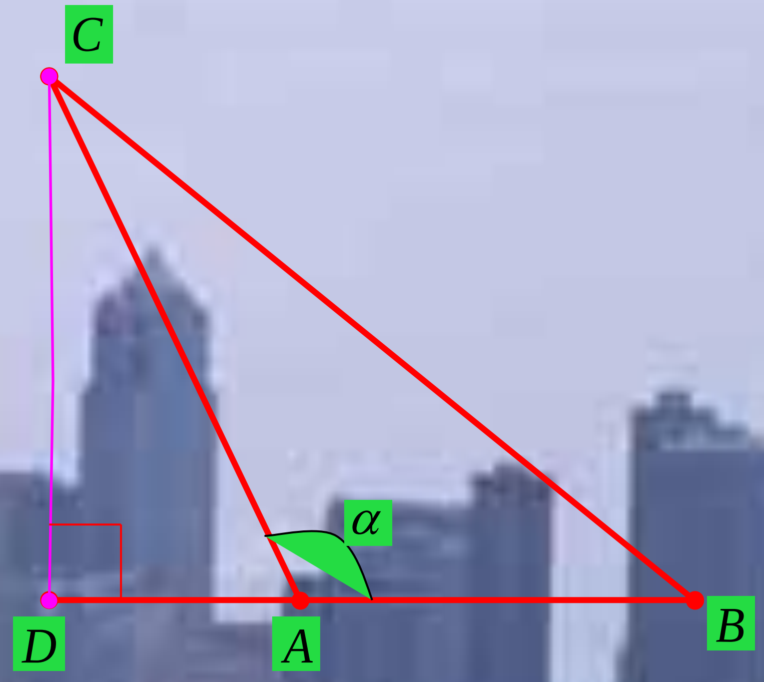
$$BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cdot \cos A$$

Teorema isbotlandi.

Shuni eslatib o'tamizki, $AC \cdot \cos A$ ning absolyut qiymati *AC* tomonning *AB* tomonga tushirilgan *AD* proeksiyasiga yoki *AB* tomonning davomiga tushirilgan proeksiyasiga teng.

$AC \cdot \cos A$ ning ishorasi *A* burchakka bog'liq:





Sinuslar teoremasi

Teorema: *uchburchakning tomonlari qarshisidagi burchaklarning sinuslariga proparsional.*

Isboti: *ABC – tomonlari a, b, c va shu tomonlari qarshisidagi* burchaklarni α, β, γ *bo'lgan uchburchak bo'lsin,*

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

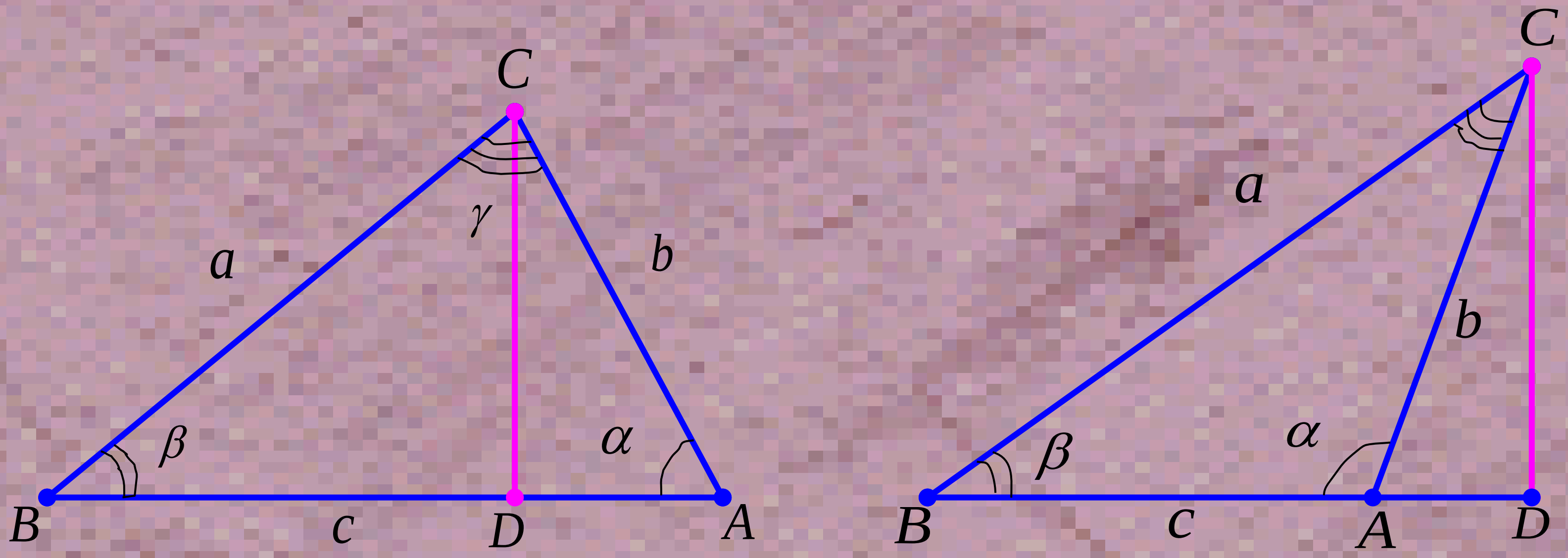
ekanini isbotlaymiz.

C uchdan CD balandlikni tushuramiz. ACD to'g'ri burchakli uchburchakdan α burchak o'tkir bo'lgan holda topamiz:

$$CD = b\sin\alpha$$

Agar α o'tmas butchak bo'lsa, u holda

$$CD = b\sin(180^0 - \alpha) = b\sin\alpha$$



Shunga o’xshash **BCD** uchburchakdan topamiz:

$$CD = a\sin\beta$$

Shunday qilib,

$$a\sin\beta = b\sin\alpha$$

Bundan,

$$\frac{b}{\sin\beta} = \frac{a}{\sin\alpha}$$

Ushbu

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

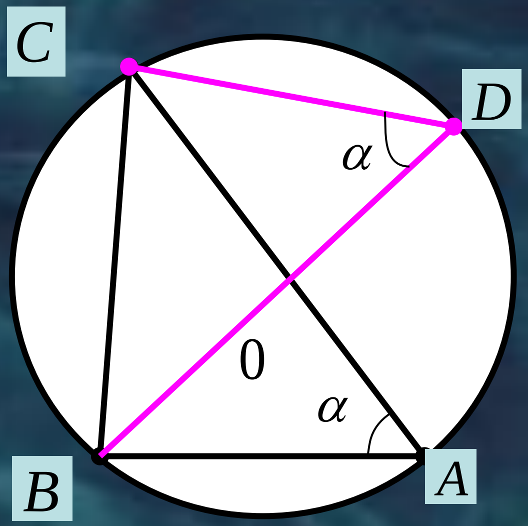
tenglik ham shunga o’xshash isbotlanadi.

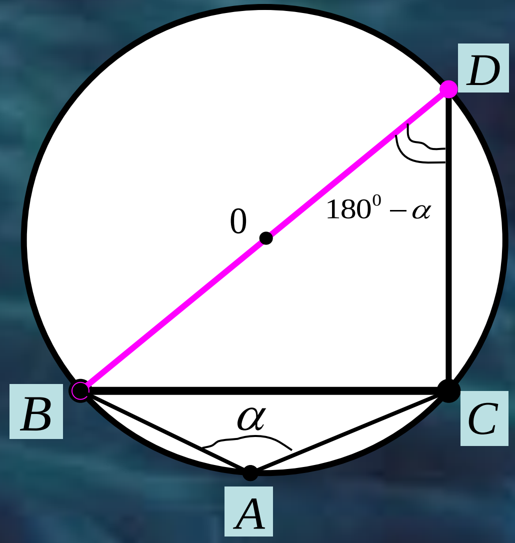
Teorema isbotlandi.

Masala:

Sinuslar teoremasida $\frac{a}{\sin\alpha}, \frac{b}{\sin\beta}, \frac{c}{\sin\gamma}$ *uchta nisbatning har biri* ***2R*** *ga tengligini isbotlaymiz, bunda R – uchburchakka tashqi chizilgan aylananing radiusi.*

Isboti: **BD** *diametrni o'tkazamiz. Aylanaga ichki chuzilgan burchaklarning xossasiga binoan* **BCD** *to'g'ri burchakli uchburchakning* **D** *uchidagi burchagi, agar* **A** *va* **D** *nuqtalar* **BC** *to'g'ri chiziqdan bir tomonda yotsa,* **α** *ga teng, agar bu nuqtalar* **BC** *to'g'ri chiziqdan*





turli tomonda yotsa (yuqoridagi b) rasmga qarang) *180º - α ga teng. Birinchi holda* $BC = BD\sin\alpha$ sinα, *ikkinchi holda* $BC = BD\sin\alpha(180^0 - \alpha)\cdot\sin(180^0 - \alpha) = \sin\alpha$ *bo'lgani uchun har qanday holda ham* $a = 2R\sin\alpha$ *Demak,*

$$\frac{a}{\sin\alpha} = 2R$$

Shuni isbotlash talab qilingan edi.

“**Islom”** Production tayyorlagan “Taqdimot” o’z poyoniga yetdi.

E’tiboringiz uchun rahmat.

Xayr!