Exploring Analytic Geometry with *Mathematica*®

by Donald L. Vossler

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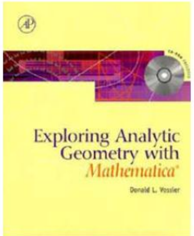

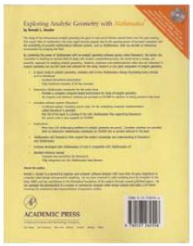

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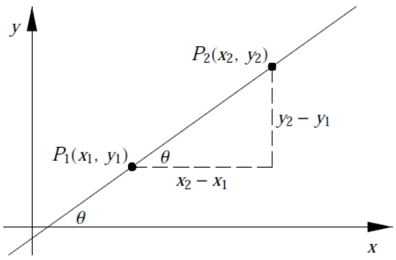

Figure 5.1: Inclination and slope of a line.

Inclination and Slope

The angle, θ, measured counter-clockwise from the $+x$-axis to a line, is called the *inclination* of the line. The tangent of this angle, $\tan\theta$, (generally designated by the letter m) is called the *slope* of the line. It is evident from Figure 5.1 that the slope of line P_1P_2 is given by

$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}.$$

The formula is independent of the position and order of the two points involved.

Let $L \equiv Ax + By + C = 0$ be the general equation of a line. It is clear that the points $(-C/A, 0)$ and $(0, -C/B)$ are on the line since they satisfy the equation of the line. Therefore, the slope of L is given by

$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}.$$

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$$m = \frac{0 - (-C/B)}{(-C/A) - 0} = -\frac{A}{B}$$

and the angle of inclination, $\theta = \tan^{-1}(-A/B)$.

The slope of the line containing a line segment from point (x_0, y_0) to point (x_1, y_1) can be determined directly from the formula given for lines as

$$m = \frac{y_1 - y_0}{x_1 - x_0}.$$

Example. Find the angle of inclination (in degrees) and the slope of the line $x - y + 4 = 0$. Find the slope of the line segment between the points $(-2, 1)$ and $(3, 2)$.

Solution. The *Descarta2D* function `Angle2D[`*line*`]` returns the inclination of a line (in radians); the function `Slope2D[`*line*`]` returns the slope of a line. The function `Slope2D[`*lnseg*`]` returns the slope of the line containing the line segment.

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5.2 Parallel and Perpendicular Lines

If two lines have the same slope they are called *parallel lines*. If two lines share all their points they are said to be *coincident*; coincident lines are also considered to be parallel. Two lines are *perpendicular* if the angle between them is a right angle. Let $m_1 = \tan\theta_1$ and $m_2 = \tan\theta_2$ be the slopes of two perpendicular lines. Since

$$\begin{aligned} \theta_2 &= \theta_1 \pm \frac{\pi}{2} \\ \tan\theta_2 &= \tan\left(\theta_1 \pm \frac{\pi}{2}\right) \\ m_2 &= -\cot\theta_1 \\ &= -\frac{1}{\tan\theta_1} \\ &= -\frac{1}{m_1}. \end{aligned}$$

Therefore, the slopes of two *perpendicular lines* are negative reciprocals of each other related by the equation, $m_1 = -1/m_2$. *Descarta2D* provides functions for querying whether pairs of lines are parallel or perpendicular.

Example. Determine which of the following pairs of lines are parallel:

(a) $2x - 3y + 4 = 0$ and $-4x + 6y - 3 = 0$,

(b) $x + 2y - 3 = 0$ and $-2x + y - 1 = 0$, and

(c) $3x - 4y + 2 = 0$ and $2x + 4y - 1 = 0$.

■

5.3 Angle between Lines

The angle between two non-intersecting (parallel or coincident) lines is zero (radians or degrees). In the case of two intersecting lines, L_1 and L_2, let θ_{12} be the angle between the lines measured counter-clockwise from L_1 to L_2. Since $\theta_{12} = \theta_2 - \theta_1$, it follows that

$$\tan\theta_{12} = \tan(\theta_2 - \theta_1) = \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_1 \tan\theta_2}$$

which, in terms of slopes of the lines, yields

$$\tan\theta_{12} = \frac{m_2 - m_1}{1 + m_1 m_2}.$$

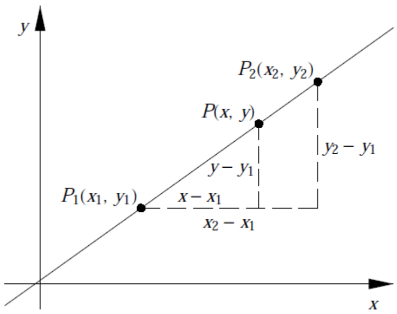

Figure 5.2: Two–point form of a line.

In general form the line is given by

$$-(y_2 - y_1)\,x + (x_2 - x_1)\,y + x_1 y_2 - x_2 y_1 = 0.$$

In determinant form the equation is given by

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \end{vmatrix} = 0.$$

Collinear Points

In a previous chapter it was demonstrated that the three points $P_1(x_1, y_1)$, $P_2(x_2, y_2)$ and $P_3(x_3, y_3)$ are collinear if their coordinates satisfy the determinant equation

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$$

This condition may be stated in a more intuitive form using the two–point form of a line. The line defined by P_1 and P_2 must be satisfied by P_3 yielding the condition

$$-(y_2 - y_1)\,x_3 + (x_2 - x_1)\,y_3 + x_1y_2 - x_2y_1 = 0$$

which can be put into the more symmetrical form

$$y_1(x_2 - x_3) + y_2(x_3 - x_1) + y_3(x_1 - x_2) = 0.$$

5.5 Point–Slope Form

Since $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, the two–point form of a line can be reduced to the *point–slope form*

$$y - y_1 = m\,(x - x_1)$$

as shown in Figure 5.3. In general form the equation of the line is

$$mx - y + (y_1 - mx_1) = 0.$$