# Mavzu : To’la orttirma va to’la differensial. To’la differensilaning taqribiy hisobga tatbiqlari. Murakkab va oshkormas funksiyaning hosilasi

# Reja :

1. To’la orttirma va to’la differensial.
2. To’la differensilaning taqribiy hisobga tatbiqlari.
3. Murakkab va oshkormas funksiyaning hosilasi

# To’la orttirma va to’la differensial.

- Ma’lumki, x va y o’zgaruvchilar mos ravishda $\Delta x \ \ va \ \ \Delta y$ orttirmalar olsa, $z = f(x, y)$ funksiya $\Delta z = f(x-\Delta x, y+\Delta y) - f(x, y)$ to’la orttirma oladi. Bu to’la orttirmaning $\Delta x \ \ va \ \ \Delta y$ larga nisbatan chiziqli bo’lgan bosh qismi funksiyaning ***to’la differensiali*** deyiladi va dz bilan belgilanadi. $z = f(x, y)$ funksiyaning to’la differensiali

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy \qquad (1)$$

formula bilan hisoblanadi, bu erda

$$dx = \Delta x, \ \ dy = \Delta y.$$

# FUNKSIYANING TO'LA ORTTIRMASI VA TO'LA DIFFERENSIYALI

$z = f(x, y)$ funksiya uzluksiz $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ xususiy hosilalarga $\Delta z = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta$y+$\alpha_1 \Delta x + \alpha_2 \Delta y$ va d$z$=$\frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta$y bo'lib, cheksiz kichik $\Delta x$, $\Delta$y lar uchun $\Delta z \approx dz$ bo'ladi, shuningdek

$$f(x + \Delta x, y + \Delta y) \approx f(x, y) + \frac{\partial f(x,y)}{\partial x}\Delta x + \frac{\partial f(x,y)}{\partial y}\Delta x$$

bo'ladi

# Funksiyalarning to'la difirensiali topilsin

$1)\ z = x^2y;\ 2)\ u = e^{\frac{s}{t}}\ 3)\ z = \sqrt{x^2 + y^2}$

Yechish:

$1)\ \frac{\partial z}{\partial x} = 2xy;\ \ \frac{\partial z}{\partial x}$$=x^2$ shunda $dz = 2xydx + x^2dy$

$2)\ \frac{\partial u}{\partial s} = e^{\frac{s}{t}} \cdot \frac{1}{t};\ \ \frac{\partial u}{\partial t} = e^{\frac{s}{t}} \left\langle -\frac{s}{t^2} \right\rangle$ shunda

$du = e^{\frac{s}{t}} \left\langle \frac{1}{t} ds - \frac{s}{t^2} dt \right\rangle$ yoki $du = e^{\frac{s}{t}} \left\langle ds - \frac{s}{t} dt \right\rangle$

3) $\frac{dz}{ds} = \frac{2x}{\sqrt{x^2+y^2}} = \frac{x}{\sqrt{x^2+y^2}} = \frac{y}{\sqrt{x^2+y^2}}$

shunda $dz = \frac{xdx+ydx}{\sqrt{x^2+y^2}}$

Misol: $z = xy \cdot e^{5x^2}$ ni to'la ortirmasi topilsin.

Yechim: $\frac{\partial z}{\partial x} = ye^{5x^2} + xy \cdot 10x \cdot e^{5x^2} =$

$$= ye^{5x^2}(1 + 10x^2),$$

$$\frac{\partial z}{\partial y} = xe^{5x^2}$$

Bunda: $\Delta z = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y = y\,(1 + 10x^2)$

$$e^{5x^2}\Delta x + xe^{5x^2}\Delta y.$$

# To'la differensilaning taqribiy hisobga tatbiqlari.

To'la differensialdan funksiyaning taqribiy qiymatlarini hisoblashda foydalanish mumkin, ya'ni $\Delta z \approx dz$ yoki

$f(x_0 + \Delta x,\ y_0 + \Delta y) - f(x_0, y_0) \approx dz$, bundan

$$f(x_0 + \Delta x,\ y_0 + \Delta y) \approx f(x_0, y_0) + z'_x dx + z'_y dy. \qquad (2)$$

Uch argumentli $u = F(x, y, z)$ funksiyaning to'la differensiali

$$du = \frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} dy + \frac{\partial F}{\partial z} dz \qquad (3)$$

formula bilan hisoblanadi.

Misol: $z = arctg\frac{y}{x}$ $funksiyaning\ x = 1, y = 3,$

$$dx = 0{,}01, dy = -0{,}05$$

$qiymatlaridagi\ to'la\ differensialini\ toping$.

Yechish: 1- tartibli hususiy hosilalarni topamiz:

$$\frac{\partial z}{\partial x} = \frac{1}{1 + \left(\frac{y}{x}\right)^2} \cdot \left(-\frac{y}{x^2}\right) = -\frac{y}{x^2 + y^2},$$

$$\frac{\partial z}{\partial y} = \frac{1}{1 + \left(\frac{y}{x}\right)^2} \cdot \frac{1}{x} = \frac{x}{x^2 + y^2}$$

Bu funksiyaning 1-tartibli to'la differensiali quyidagicha bo'ladi:

$$dz = -\frac{ydx}{e^2+y^2} + \frac{xdy}{x^2+y^2} = \frac{xdy-ydx}{x^2+y^2},$$

$$dz = \frac{1\cdot(-0,05)-3\cdot 0,01}{1^2+3^2} = -\frac{0,08}{10} = -0,08$$

# Murakkab va oshkormas funksiyaning hosilasi

Z=f (u,v) u=u(x,y) v=v(x,y) x va Δx ortirma bo'lsa, u va v funksiyalar $\Delta_x u \;\; va \; \Delta_x v$ xususiy ortirma deyiladi.

$$\frac{\partial z}{\partial x} = \frac{\partial F}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial F}{\partial v}\frac{\partial v}{\partial x}$$

$$\frac{\partial z}{\partial y} = \frac{\partial F}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial F}{\partial v}\frac{\partial v}{\partial y}$$

Misol: Agar $z = \sqrt{x^2 + y^2}$ funksiyada $x = sint$ va $y = cost$ bo’lganda $\frac{dz}{dt}$ topilsin.

Yechim: $\frac{dz}{dt} = \frac{2x}{2\sqrt{x^2+y^2}} \cdot cost += \frac{2y}{2\sqrt{x^2+y^2}} \cdot (-\sin t)$,

yoki

$$\frac{dz}{dt} = \frac{\sin t \cos t - \sin t \cos t}{\sqrt{sin^2 t + cos^2 t}} = \frac{0}{\sqrt{1}} = 0$$

Misol: Agar $xe^{2y} - ye^{2x} = 0$ bo'lsa $\frac{dy}{dx}$ topilsin.

Yechim: $(xe^{2y} - ye^{2x})' = 0$

Aniqmas funksiyadan kelib chiqib:

$$1 \cdot e^{2y} + x \cdot e^{2y} \cdot 2y' - y'^{e^{2x}} - 2ye^{2s} = 0$$

$y'(2xe^{2y} - e^{2x}) = 2ye^{2x} - e^{2y}$ bunda

$$\frac{dy}{dx} = y' = \frac{2ye^{2x} - e^{2y}}{2xe^{2y} - e^{2x}}$$

Misol:

$z = e^{\frac{x}{y}}$; ni $y = \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial z}{\partial y} - \frac{\partial z}{\partial x}$ da tengligi isbotlansin

$$\frac{\partial z}{\partial x} = e^{\frac{x}{y}} \cdot \frac{1}{y}; \qquad \frac{\partial z}{\partial y} = e^{\frac{x}{y}} \left\langle -\frac{x}{y^2} \right\rangle$$

$$\frac{\partial^2 z}{\partial x \partial y} = -\frac{1}{y^2} e^{\frac{x}{y}} + \frac{1}{y} \cdot e^{\frac{x}{y}} \cdot \left\langle -\frac{x}{y^2} \right\rangle = e^{\frac{x}{y}} \left\langle -\frac{1}{y^2} - \frac{x}{y^3} \right\rangle =$$

$$= e^{\frac{x}{y}} \left\langle \frac{-y-x}{y^3} \right\rangle;$$

Shunda

$$e^{\frac{x}{y}}\left\langle -\frac{x+y}{y^3}\right\rangle = e^{\frac{x}{y}}\left\langle -\frac{x}{y^2}-\frac{1}{y}\right\rangle = e^{\frac{x}{y}}\left\langle -\frac{x+y}{y^2}\right\rangle$$

ga tengligi isbotlandi.