1-MA'RUZA

MAVZU:

MATRITSA VA ULAR USTIDA AMALLAR

REJA

1. MATRITSA TUSHUNCHASI
2. KVADRATIK MATRITSA
3. MATRITSALAR USTIDA AMALLAR

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad \text{yoki}$$

$$\begin{Vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{Vmatrix} \quad (1)$$

Misol. $A = \begin{pmatrix} 2 & 1 & \pi \\ 1 & \sqrt{2} & -5 \end{pmatrix}$

2×3 -o‘lchamli ***matritsa.***

$$B = \begin{pmatrix} e^t & 1 & -1 & \cos(t) \\ 0 & 4t & -7 & 1 - t \end{pmatrix}$$

2×4 -o‘lchamli ***matritsa.***

- Matritsalarni A, B, C, … bosh harflar bilan belgilaymiz:

- $$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$ yoki

  $$A = \|a_{ij}\| = (a_{ij})$$

- (i=1,2,…,m, j = 1,2,…,n)
- a_{ij} -lar ***matritsa elementlari*** deyiladi, bu yerda birinchi indeks i – element turgan

- Agar ixtiyoriy *i* va *j* larda $a_{ij} = b_{ij}$ bo'lsa, $A = (a_{ij})$ va $B = (b_{ij})$ ***matritsalar teng*** deyiladi va *A=B* kabi yosiladi.

Resurslar	Iqtisodiyotning tarmoqlari	
	Qishloq xo'jaligi	Suv xo'jaligi
Suv	7,2	8,1
Mehnat	4,1	3,2
Elektroenergiya	5,2	6,3

Matritsalar yordamida ba'zi iqtisodiy bog'liqlarni ifodalash mumkin. Masalan, iqtisodiyotning ba'zi tarmoqlari bo'yicha resurslarning taqsimotini jadvalda ifodalaymiz.

Ushbu jadvalni resurslar taqsimotining ixcham matritsasi ko'rinishida ifodalash mumkin:

$$A = \begin{pmatrix} 7{,}2 & 8{,}1 \\ 4{,}1 & 3{,}2 \\ 5{,}2 & 6{,}3 \end{pmatrix}$$

jadvalga ko'ra, *a11=7,2* – matritsa elementi – qishloq xo'jaligiga qancha suv resursi, *a22=3,2* – element esa – suv xo'jaligiga qancha mehnat resursi sarflanishini ko'rsatadi.

NOL MATRITSA

- Agar (1) matritsaning barcha elementlari nolga teng bo‘lsa,
- $0 = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}$
- u ***nol matritsa*** deyiladi.

KVADRATIK MATRITSA

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

(2)

$$A_1 = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_{nn} \end{pmatrix} \quad \text{yoki}$$

$$B = \begin{pmatrix} b_{11} & 0 & \cdots & 0 \\ b_{21} & b_{22} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{pmatrix}.$$

A_1 – ***yuqori uchburchakli***, B – ***quyi uchburchakli*** matritsadir.

$$\begin{pmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_{nn} \end{pmatrix}$$

(3)

diagonal matritsa, $a_{11} = a_{22} = ... = a_{nn} = 1$

bo‘lsa,

$$E = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

ko‘rinishga keladi, uni **birlik matritsa** deb ataladi.

Matritsani songa ko'paytirish. $A = (a_{ij})$ (i=1,2,…,m, j = 1,2,…,n) matritsaning har bir elementini λ haqiqiy songa ko'paytirganda hosil bo'lgan matritsaga λ son bilan *A* ***matritsa ko'paytmasi*** deyiladi va λA kabi belgilanadi. Demak,

$$\lambda A = \begin{pmatrix} \lambda a_{11} & \lambda a_{12} & \cdots & \lambda a_{1n} \\ \lambda a_{21} & \lambda a_{22} & \cdots & \lambda a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \lambda a_{n1} & \lambda a_{n2} & \cdots & \lambda a_{nn} \end{pmatrix}.$$

Matritsalarni qo'shish xossalari:

1^0. $A+0=0+A=A$

2^0. $A+B = B+A$.

3^0 . $(A+B)+C = A+ (B+C)$ (assosiativ).

Misol. 2×3 – o‘lchovli $A = \begin{pmatrix} 1 & 2 & -3 \\ 4 & 0 & 2 \end{pmatrix}$ va

$B = \begin{pmatrix} -1 & 6 & 3 \\ 8 & 12 & 14 \end{pmatrix}$ matritsalar berilgan bo‘lsin.

$A + B$ yig‘indini toping.

$$\begin{pmatrix} 1 & 2 & -3 \\ 4 & 0 & 2 \end{pmatrix} + \begin{pmatrix} -1 & 6 & 3 \\ 8 & 12 & 14 \end{pmatrix} = \begin{pmatrix} 0 & 8 & 0 \\ 12 & 12 & 16 \end{pmatrix}$$

Matritsalarni songa ko‘paytirish xossalari:

$\mathbf{4^0.}(\lambda\mu)A = \lambda(\mu A)$, $(\lambda, \mu = \text{const})$

$\mathbf{5^0.}\lambda(A+B) = \lambda A + \lambda B$

$\mathbf{6^0.}(\lambda+\mu)A = \lambda A + \mu A.$

$\mathbf{7^0}.\ (A\cdot B)\cdot C = A\cdot(B\cdot C)$

$\mathbf{8^0}.\ (A+B)\cdot C = A\cdot C + B\cdot C,\quad A\cdot(B+C) = A\cdot B + A\cdot C$

Misol. Matritsalar uchun berilgan amallarni bajaring.

$$1.\ \boldsymbol{A} = \begin{pmatrix} 1 & -1 & 3 \\ 2 & -4 & 6 \\ -1 & 1 & 2 \end{pmatrix}.\quad \boldsymbol{B} = \begin{pmatrix} -4 & 0 & 0 \\ -2 & -1 & 6 \\ 8 & 15 & 4 \end{pmatrix};\quad 2\boldsymbol{A} - \boldsymbol{3B}$$

$$2.\ \boldsymbol{A} = \begin{pmatrix} -2 & 2 \\ 0 & 1 \\ 14 & 2 \\ 6 & 8 \end{pmatrix},\quad \boldsymbol{B} = \begin{pmatrix} 4 & 4 \\ 2 & 1 \\ 14 & 16 \\ 1 & 25 \end{pmatrix},\ -5\boldsymbol{A} + \boldsymbol{3B}$$

Matritsani songa ko'paytirish.

$A=(a_{ij})$ (i=1,2,…,m, j = 1,2,…,n) matritsaning har bir elementini λ haqiqiy songa ko'paytirganda hosil bo'lgan matritsaga λ son bilan A ***matritsa ko'paytmasi*** deyiladi va λA kabi belgilanadi. Demak,

$$\lambda A = \begin{pmatrix} \lambda a_{11} & \lambda a_{12} & \cdots & \lambda a_{1n} \\ \lambda a_{21} & \lambda a_{22} & \cdots & \lambda a_{2n} \\ \dots & \dots & \dots & \dots \\ \lambda a_{n1} & \lambda a_{n2} & \cdots & \lambda a_{nn} \end{pmatrix}.$$

Misol. A=$\begin{pmatrix} -3 \\ 4 \\ 2t \\ \sin(2t) \end{pmatrix}$ matritsani $\sqrt{2}$ ga ko‘paytiring.

$$\sqrt{2}\begin{pmatrix} -3 \\ 4 \\ 2t \\ \sin(2t) \end{pmatrix} = \begin{pmatrix} -3\sqrt{2} \\ 4\sqrt{2} \\ 2t\sqrt{2} \\ \sqrt{2}\sin(2t) \end{pmatrix}.$$

Misol. **A=**$\begin{pmatrix} 2 & e^t \\ \sin(t) & 4 \end{pmatrix}$ matritsani $\cos(t)$ ga ko‘paytiring.

$$\cos(t)\begin{pmatrix} 2 & e^t \\ \sin(t) & 4 \end{pmatrix} = \begin{pmatrix} 2\cos(t) & e^t\cos(t) \\ \cos(t)\sin(t) & 4\cos(t) \end{pmatrix}$$

4-мisol.

$$\boldsymbol{AB \neq BA}$$

$$\begin{pmatrix} 1 & 0 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} -2 & 6 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 8 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 0 \\ 8 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & -4 \end{pmatrix} = \begin{pmatrix} -14 & 24 \\ -5 & 12 \end{pmatrix}.$$

Demak, $A \cdot B \neq B \cdot A$.

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