3-mavzu.To’g’ri chiziq haqidagi asosiy masalalar. 3.1.Ikki to’g’ri chiziq orasidagi burchak.

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$$y = k_1 x + b_1$$

$$y = k_2 x + b_2$$

$$k_1 = \operatorname{tg} \alpha_1$$

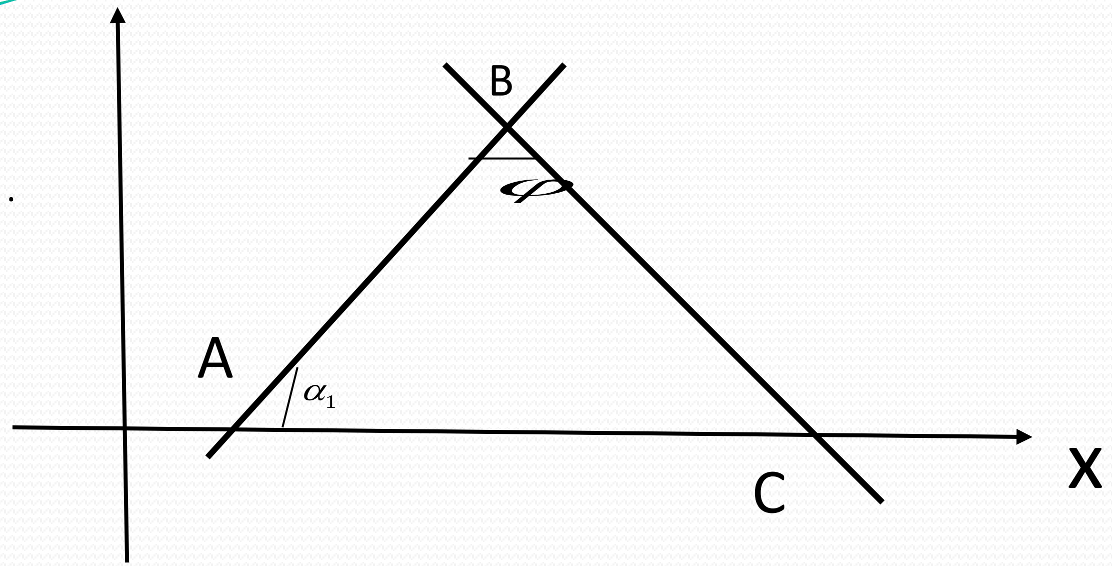
$$k_2 = \operatorname{tg} \alpha_2$$

$$\alpha_2 = \varphi + \alpha_1$$

$$\varphi = \alpha_2 - \alpha_1$$

$$\operatorname{tg}\varphi = \operatorname{tg}(\alpha_2 - \alpha_1) = \frac{\operatorname{tg}\alpha_2 - \operatorname{tg}\alpha_1}{1 + \operatorname{tg}\alpha_1 \cdot \operatorname{tg}\alpha_2} = \frac{k_2 - k_1}{1 + k_1 \cdot k_2}$$

$$\operatorname{tg}\varphi = \left| \frac{k_2 - k_1}{1 + k_1 \cdot k_2} \right|$$



B
φ
A
α_1
X
C

Masalan, y=-2x va y=3x-4 to’g’ri chiziqlar uchun

$$\operatorname{tg}\varphi = \frac{3-(-2)}{1+(-2)\cdot 3} = -1$$

demak ular orasidagi o’tmas burchak $\frac{3\pi}{4}$ ga, o’tkir burchak esa $\frac{\pi}{4}$ ga teng.

Agar to'g'ri chiziqlar parallel bo'lsa, $\varphi = 0$ yoki $\varphi = \pi$

bo'lib $k_2 - k_1 = 0$

kelib chiqadi. Demak, to'g'ri chiziqlar paralellik sharti

$k_2 = k_1$ dir .

- To’g’ri chiziqlar o’zara perpendikulyar bo’lsa,

$$\pi = \frac{\pi}{2}, \qquad tg\frac{\pi}{2} = \infty \quad 1 + k_1 k_2 = 0$$

shart kelib chiqadi. Demak, to’g’ri chiziqlar perpendikulyarlik sharti

$$k_2 = -\frac{1}{k_1}$$ dir.

Agar to’g’ri chiziqlar

$$A_1x + B_1y + C_1 = 0$$

$$A_2x + B_2y + C_2 = 0$$

formulalar bilan berilsa, ularni y ga nisbatan echib

$$k_1 = -\frac{A_1}{B_1} \qquad k_2 = -\frac{A_2}{B_2}$$

bo’lishini topamiz .

Demak, to'g'ri chiziqlar umumiy tenglamasi bilan berilsa,

$$\operatorname{tg}\varphi = \frac{-\frac{A_2}{B_2} - \left(-\frac{A_1}{B_1}\right)}{1 + \left(-\frac{A_1}{B_1}\right) \cdot \left(-\frac{A_2}{B_2}\right)} = \frac{A_1 B_2 - A_2 B_1}{A_1 A_2 + B_1 B_2}$$

formulaga ega bo'lamiz. Unda to'g'ri chiziqlar parallel bo'lishi uchun

$$A_1 B_2 - A_2 B_1 = 0$$, yani $$\frac{A_1}{A_2} = \frac{B_1}{B_2}$$

bo'lishi, perpendikulyar bo'lishi uchun esa

$$A_1 A_2 + B_1 B_2 = 0$$ bo'lishi kerak.

1). $y=2x-5$, $y=2x+1$, $y= x+5$ to’g’ri chiziqlarning dastlabki ikkitasi parallel, uchunchisi ularga perpendikulyardir.

2). $2x-3y+5=0$, $4x-6y+1=0$, $3x+2y+5=0$, to’g’ri chiziqlarning dastlabki ikkitasi parallel, uchunchisi ularga perpendikulyardir.

3.2.Nuqtadan to’g’ri chiziqqacha bo’lgan masofa.

Normal tenglamasi bilan berilgan x $\cos\alpha + y\sin\alpha - P = 0$

to'g'ri chiziq va unda yotmagan biror Q($x_0; y_0$

) nuqta berilgan bo'lsin. Q($x_0; y_0$

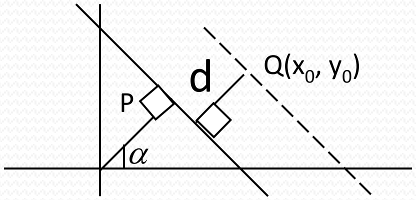
) nuqtadan berilgan to'g'ri chiziq gacha bo'lgan d masofani topish masalasini qaraymiz. Q($x_0; y_0$) dan o'tib, x $\cos\alpha + y\sin\alpha - p = 0$

ga parallel to'g'ri chiziqni X $\cos\alpha + y\sin\alpha - q = 0$

tenglama bilan beriladi, bunda q=p+d, lekin

$$q = x_0 \cos\alpha + y_0 \sin\alpha$$

ekanligida



$$q - p = x_0 \cos\alpha + y_0 \sin\alpha - p$$

kelib chiqadi. Agar q$< p$ bo’lsa d=$p - q$

bo’lishini hisobga olsak, d=$|x_0 \cos\alpha + y_0 \sin\alpha - p|$

formulaga ega bo’lamiz.

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Agar to'g'ri chiziq $Ax+By+C=0$ umumiy tenglamasi bilan berilsa,

, masofa formulasi $d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$

ko'rinishida bo'ladi. Masalan, , A(4;2), B(1;1) dan $3x-4y-4=0$ gacha masofani hisoblaymiz

$$d_A=\frac{|3\cdot4-4\cdot2-4|}{\sqrt{3^2+(-4)^2}}=0 \quad d_B=\frac{|3\cdot1-4\cdot1-4|}{\sqrt{3^2+(-4)^2}}=\frac{|-5|}{5}=1$$

A to'g'ri chiziqqa tegishli, B nuqta esa to'g'ri chiziqdan bir birlik uzoqlikda joylashgan.

Normal tenglamasi bilan berilgan $x\cos\alpha + y\sin\alpha - p = 0$ va $x \cos\alpha + y\sin\alpha - q = 0$ to'g'ri chiziqlar orasidagi masofa $d = |p - q|$ bo'lishi tushunarli. Agar to'g'ri chiziqlar $x\cos\alpha + y\sin\alpha - p = 0$, $\lambda x\cos\alpha + \lambda y\sin\alpha - q = 0$ tenglamalar bilan berilsa, masofa d=$\left|p - \frac{q}{\lambda}\right|$ bo'ladi. Demak ikki parallel $A_1x + B_1y + C_1 = 0$, $\lambda A_1x + \lambda B_1y + C_2 = 0$ to'g'ri chiziqlar orasidagi masofa $d = \left|\frac{C_1 - \frac{C_2}{\lambda}}{\sqrt{A^2 + B^2}}\right|$ formula yordamida topiladi.

Masalan, $6x+8y+7=0$, $3x-4y-7=0$ to'g'ri chiziqlar o'zaro parallel, ularni

$3x+4y+\frac{7}{2}=0$, $3x-4y-7=0$ tarzida yozsak, $d=\frac{\left|\frac{7}{2}+7\right|}{\sqrt{3^2+4^2}}=\frac{21}{10}=2{,}1$ ekanligi kelib chiqadi .

Bazi hollarda birinchi tog'ri chiziqdan biror nuqta tanlab ikkinchisigacha masofani hisoblasa ham bo'ladi, masalan $C(0;\frac{7}{8})$ nuqta birinchi to'g'ri chiziqqa tegishli , undan ikkinchi to'g'ri chiziqgacha masofa esa

$$d=\frac{\left|3\cdot 0-4\cdot\frac{7}{8}-7\right|}{\sqrt{3^2+(-4)^2}}=\frac{\left|-\frac{7}{2}-7\right|}{5}=\frac{21}{10}=2.1\ .$$

Masofa formulasi yordamida ikki kesishuvchi $A_1x+B_1yC_1=0$ va $A_2x+B_2y+C_2=0$ to'g'ri chiziq bissiktrissalari tenglamasini keltirib chiqaramiz.

Bissiktrissadagi ixtiyoriy C(x;y) nuqtadan berilgan to'g'ri chiziqgacha masofalar tengligidan $\frac{|A_1x+B_1y+C_1|}{\sqrt{A^2+B^2}}=\frac{|A_2x+B_2y+C_2|}{\sqrt{A_2^2+B_2^2}}$ yoki $\frac{A_1x+B_1y+C_1}{\sqrt{A_1^2+B_1^2}}=\pm\frac{A_2x+B_2y+C_2}{\sqrt{A_2^2+B_2^2}}$

kelib chiqadi.

3.3.Bitta va ikkita nuqtadan o’tuvchi to’g’ri chiziq tenglamalari.

To'g'ri chiziq y=kx+b tenglama bilan berilib, dastlab, uning bitta $A(x_0;y_0)$ nuqtasi ma'lum bo'lsin, demak , $y_0 = kx_0 + b$. Berilgan tenglamadan topilgan sonli tenglikni ayirsak $y - y_0 = k(x - x_0)$ tenglama hosil bo'ladi . U $A(x_0;y_0)$ nuqtadan o'tuvchi barcha to'g'ri chiziqlar tenglamasidir .

Bu to'g'ri chiziqlar $A(x_0;y_0)$ dan o'tuvchi to'g'ri chiziqlar dastasi deyiladi. Agar dastadagi biror to'g'ri chiziq $B(x_1;y_1)$ nuqtadan ham o'tsa $y_1 - y_0 = k(x_1 - x_0)$ tenglik bajariladi. Undan $k=\frac{y_1-y_0}{x_1-x_0}$ topiladi. Demak A va B nuqtalardan o'tuvchi chiziq tenglamasi $y - y_0 = \frac{y_1-y_0}{x_1-x_0}(x - x_0)$ yoki $\frac{x-x_0}{x_1-x_0} = \frac{y-y_0}{y_1-y_0}$ ko'rinishida bo'ladi.

Masalan, A(2;-1), va B(1;2) nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi

$\frac{x-2}{1-2}=\frac{y+1}{2-(-1)}$ yoki $y=-3x+5$ ko'rinishida bo'ladi.

Endi biror $C(x_0;y_0)$ nuqtadan o'tib , berilgan $y=k_1x+b_1$ to'g'ri chiziqqa parallel (perpendikulyar) tog'ri chiziq tenglamasi formulasini keltirib chiqaramiz.

Izlanayotgan to'g'ri chiziq $C(x_0;y_0)$ dan o'tadi , demak , tenglamasi $y-y_0=k(x-x_0)$ ko'rinishda bo'ladi . Bundan tashqari , agar u $y=k_1x+b_1$ga parallel (perpendikulyar) bo'lsa , $k=k_1(k=-\frac{1}{k_1})$bo'lib tenglamasi

$$y-y_0=k_1(x-x_0)\left[y-y_0=-\frac{1}{k_1}(x-x_0)\right]$$ ko'rinishida bo'ladi .

Masalan , C(2;-1) dan o'tib , y=4x+3 ga parallel (perpendikulyar) bo'lgan to'g'ri chiziq tenglamasi

$y+1=4(x-2)$ $\left[y+1=-\frac{1}{4}(x-2)\right]$ ko'rinishda bo'ladi .

Mavzuga doir masalalar.

1. A(0;1) va B(1;2) nuqtalardan bir xil masofada yotuvchi to'g'ri chiziq tenglamasini yozing.

2. Ordinatao'qidan b=3 kesma ajratib abssissa o'qibilan a)45^0 b) 135^0burchak tashkil etuvchi to'g'ri chiziq tenglamalarini yozing .

3. Koordinatalar boshidan o'tib, abssissa o'qi bilan a)60^0 b)120^0 burchak tashkil etuvchi to'g'ri chiziqlar tenglamalarini yozing.

4. $2x-3y-6=0$ va $12x+5y-60=0$ to'g'ri chiziqlar kesmalar bo'yicha tenglamalarini yozing.

5. A(4;3) nuqtadan o'tib , koordinatalar burchagidan yuzi 30 kv birlikka tenguchburchak ajratuvchi to'g'ri chiziq tenglamasini yozing .

6. $3x-4y-20=0$, $y=kx+b$, $\frac{x}{a}+\frac{y}{b}=1$ to'g'richiziqlar normal tenglamalariniyozing.

7. Koordinatalar boshidan $12x-5y+52=0$ to'g'ri chiziqgacha bo'lgan masofa topilsin.

8. Koeffitsiyentlari noldan farqli $Ax+By+C=0$ to'g'ri chiziq va son o'qlari bilan chegaralangan uchburchak yuzi $S=\frac{1}{2}\frac{C^2}{|AB|}$ formula bilan topilishini isbotlang.

9.Quyidagi to'g'ri chiziqlar orasidagi burchakni toping

1) $5x-y+7=0$ va $3x+2y=0$, 2) $x-2y+4=0$ va $2x-4y+3=0$, 3) $3x-2y+7=0$ va $2x+3y-3=0$

4) $3x+2y-1=0$ va $5x-2y+3=0$

10. Qutb koordinatalar sistemasida berilgan $r_1 = \frac{P_1}{\cos(\varphi-\alpha_1)}$ va $r_2 = \frac{P_2}{\cos(\varphi-\alpha_2)}$ to'gri chiziqlar orasidagi burchakni topishformulasini yozing.

11. Parametrik usulda berilgan $\{x=m\lambda+x_0 , y= n\lambda+y_0 \}$ to'g'ri chiziq va abscissa o'qi orasidagi burchak $tg\varphi=n/m$ formula bilan hisoblanishini isbotdang.

12. Parametrik usulda berilgan $\{x=m_1\lambda+x_1\ ,\ y=n_1\lambda+y_1\}$ va $\{x=m_2\lambda+x_2\ ,\ y=n_2\lambda+y_2\}$ to'g'ri chiziqlar orasidagi burchak $\cos\varphi=\dfrac{|m_1m_2+n_1n_2|}{\sqrt{m_1^2+n_1^2}\sqrt{m_2^2+n_2^2}}$ formula bilan topilishini isbotlang.

Parallellik va perpendikulyarlik shartlarini yozing.

13. Uchburchak tomonlari $x+3y=0$, $x=3$, $x-2y+3=0$ tenglamalar bilan berilgan. Uning uchlari koordinatalari, ichki burchaklari topilsin.

14. $y=kx+5$ to'g'ri chiziq koordinatalar boshidan $d=\sqrt{5}$ masofa uzoqlikda bo'lsa , k qanday qiymatlar qabul qiladi?

15. Berilgan nuqtadan berilgan to'g'ri chiziqgacha masofani toping:

1) $A(2;-1)$, $4x+3y+10=0$; 2) $B(0;-3)$, $5x-12y-23=0$;

3) $C(-2;3)$, $3x-4y-2=0$; 4) $D(1;-2)$, $x-2y-5=0$.

16. Quyidagi parallel to'g'ri chiziqlar orasidagi orasidagi masofani toping:

1) $3x-4y-10=0$, $6x-8y+5=0$; 2) $5x-12y+26=0$, $5x-12y-13=0$;

3) $4x-3y+15=0$, $8x-6y+25=0$; 4) $24x-10y+39=0$, $12x-5y-26=0$.

17. Kvadrat ikki tomoni tenglamalar $5x-12y-65=0$ va $5x-12y+26=0$ bo'lsa, uning perimetri va yuzini toping.

18. $3x-y-4=0$ va $2x+6y+3=0$ to'g'ri chiziqlar hosil qilgan burchak bissektrisalaridan koordinata boshidan o'tuvchi tenglamasini toping.

19. $3x+4y-5=0$ va $5x-12y+3=0$ hosil qilgan o'tkir burchak bissektrisasi tenglamasini yozing.