Vektorlar algebrasi

REJA:

- Asosiy tushunchalar.
- Vektorlarning skalyar ko’paytmasi va ularning asosiy hossalari.
- Vektorlarning vektor ko’paytmasi va ularning asosiy hossalari.

Asosiy tushunchalar

Matematik kattaliklar

Skalyar miqdor
(son qiymatlariga ko'ra)

Vektor miqdor
(Son qiymati va yo'nalishiga ko'ra)

VEKTORLARNING SKALYAR KO'PAYTMASI

Noldan farqli ikki vektorlarning skalyar ko'paytmasi deb, vektorlar uzunliklarining ular orasidagi burchak cosinusiga ko'paytmasiga aytiladi

Ikki vektor $\bar{a}$ va $\bar{b}$ ning skalyar ko'paytmasi quyidagicha yoziladi:

$$\bar{a}\cdot\bar{b}=|\bar{a}|\cdot|\bar{b}|\cdot\cos\varphi$$

$$\bar{a}\cdot\bar{b}=|\bar{a}|\cdot \Pi P_{\bar{a}}\bar{b}=|\bar{b}|\cdot \Pi P_{\bar{b}}\bar{a}$$

$$\Pi P_{\bar{a}}\bar{b}$$

$$=\Pi P_{\bar{b}}\bar{a}$$

$$\bar{a}\cdot\bar{a}=|\bar{a}|\cdot|\bar{a}|\cdot\cos 0^{o}\Rightarrow \bar{a}^{2}=|\bar{a}|^{2}$$

Vektorning kvadrati uning moduli kvadratiga teng

<u>Skalyar ko'paytma qonuniyatlari</u>

1) $\bar{a}\cdot\bar{b}=\bar{b}\cdot\bar{a}$

2) $(\bar{a}+\bar{b})\cdot\bar{c}=\bar{a}\cdot\bar{c}+\bar{b}\cdot\bar{c}$

3) $\lambda(\bar{a}\cdot\bar{b})=(\lambda\bar{a})\cdot\bar{b}=\bar{a}\cdot(\lambda\bar{b})$

VEKTORLARNING SKALYAR KO'PAYTMASI

Koordinata o'qlarining $\bar{i},\ \bar{j}, \bar{k}$ ortlar uchun quyidagi munosabatlar o'rinli:

$$\bar{i}\cdot\bar{i}=\bar{j}\cdot\bar{j}=\bar{k}\cdot\bar{k}=1 \qquad \bar{i}\cdot\bar{j}=\bar{i}\cdot\bar{k}=\bar{j}\cdot\bar{k}=0$$

To'g'ri burchakli dekart koordinatalar sistemasi yordamida vektorlarning koordinatlarini topish mumkin:

$$\bar{a}=x_1\cdot\bar{i}+y_1\cdot\bar{j}+z_1\cdot\bar{k} \qquad \bar{b}=x_2\cdot\bar{i}+y_2\cdot\bar{j}+z_2\cdot\bar{k}$$

Skalyar ko'paytmani topamiz:

$$\bar{a}\cdot\bar{b}=\left(x_1\cdot\bar{i}+y_1\cdot\bar{j}+z_1\cdot\bar{k}\right)\cdot\left(x_2\cdot\bar{i}+y_2\cdot\bar{j}+z_2\cdot\bar{k}\right)$$

$$=x_1x_2\cdot 1+y_1x_2\cdot 0+z_1x_2\cdot 0+x_1y_2\cdot 0+y_1y_2\cdot 1+$$

$$z_1y_2\cdot 0+x_1z_2\cdot 0+y_1z_2\cdot 0+z_1z_2\cdot 1 \quad\Rightarrow$$

$$\boxed{\bar{a}\cdot\bar{b}=x_1\cdot x_2+y_1\cdot y_2+z_1\cdot z_2}$$

VEKTORLARNING SKALYAR KO'PAYTMASI

Vektorlar orasidagi burchak cosinusini topish formulasi:

$$\cos\varphi = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|} = \frac{x_1\cdot x_2 + y_1\cdot y_2 + z_1\cdot z_2}{\sqrt{x_1^2+y_1^2+z_1^2}\cdot\sqrt{x_2^2+y_2^2+z_2^2}}$$

Vektorlar orasidagi burchak cosinusini topish: $\bar{a} = \bar{i} + 2\bar{j} + 3\bar{k}$

$$\bar{b} = 6\bar{i} + 4\bar{j} - 2\bar{k}$$

$$\bar{a}\cdot\bar{b} = 1\cdot 6 + 2\cdot 4 + 3\cdot(-2) = 8$$

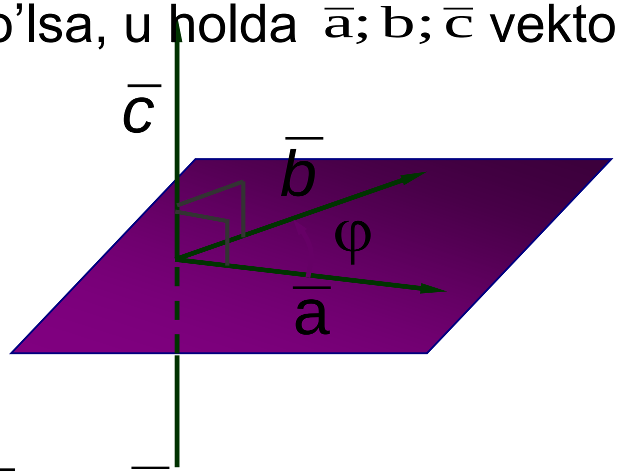
$$|\bar{a}| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$$

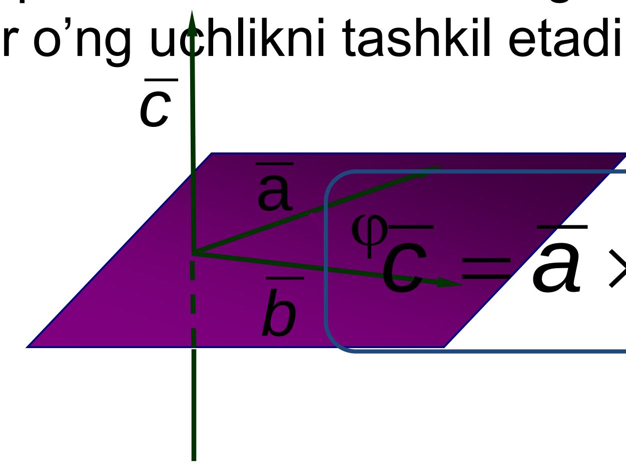
$$|\bar{b}| = \sqrt{6^2+4^2+(-2)^2} = \sqrt{56} = 2\sqrt{14}$$

$$\cos\varphi = \frac{8}{\sqrt{14}\cdot 2\sqrt{14}} = \boxed{\frac{2}{7}}$$

VEKTORLARNI VEKTOR KO'PAYTMASI

Boshlari bitta nuqtaga keltirilgan vektorlar $\bar{a}; \bar{b}; \bar{c}$ uchun o'ng uchlikni kiritamiz: agar $\bar{c}$ vektorning oxiridan $\bar{a}$ va $\bar{b}$ vektorlar yotgan tekislikka qaralganda $\bar{a}$ dan $\bar{b}$ ga eng qisqa burish soat strelkasiga qarama-qarshi bo'lsa, u holda $\bar{a}; \bar{b}; \bar{c}$ vektorlar o'ng uchlikni tashkil etadi, deyiladi.





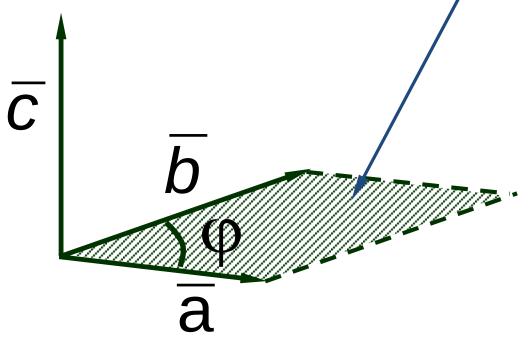
$$\bar{c} = \bar{a} \times \bar{b}$$

$\bar{a}$ va $\bar{b}$ vektorlarning vektor ko'paytmasi deb $\bar{a}\bar{b}$ bilan belgilangan va quyidagi uchta shartlarni qanoatlantiradigan $\bar{c}$ vektorga aytiladi

- $|\bar{c}| = |\bar{a}| \cdot |\bar{b}| \cdot \sin(\bar{a}; \bar{b})$.
- $\bar{c} \perp \bar{a} \times \bar{b}$ $\bar{a} \perp \bar{b} \times \bar{b}$
- $\bar{a}$ $\bar{b}$ $\bar{a} \times \bar{b}$ *lar o'ng uchlik tashkil etadi*

Vektorlarni vektor ko'paytmasi

Parallelogram yuzasi ikkala vektor modullari va vektorlar orasidagi burchak sinusining ko'paytmasiga teng.



$$S = |\overline{c}| = |\overline{a}| \cdot |\overline{b}| \cdot \sin\varphi$$

$$\overline{a} \times \overline{b} = 0 \quad \Leftrightarrow \quad \overline{a} \parallel \overline{b}$$

<u>Vektor ko'paytmaning qonuniyatlari:</u>

1) $\overline{a} \times \overline{b} = -\overline{b} \times \overline{a}$

2) $\overline{a} \times (\overline{b} + \overline{c}) = \overline{a} \times \overline{b} + \overline{a} \times \overline{c}$

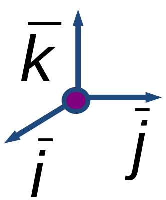
3) $\lambda(\overline{a} \times \overline{b}) = (\lambda\overline{a}) \times \overline{b} = \overline{a} \times (\lambda\overline{b})$

4) $\overline{a} \times \overline{a} = 0$

Vektorlarni vektor ko'paytmasi

Koordinata o'qlarining $\bar{i},\ \bar{j},\ \bar{k}$ ortlar uchun quyidagi munosabatlar o'rinli:

$$\bar{i}\times\bar{i} = \bar{j}\times\bar{j} = \bar{k}\times\bar{k} = 0$$



$$\bar{i}\times\bar{j} = \bar{k} \qquad \bar{j}\times\bar{i} = -\bar{k}$$

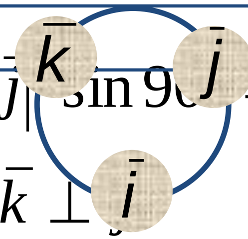
$$\bar{j}\times\bar{k} = \bar{i} \qquad \bar{k}\times\bar{j} = -\bar{i}$$

$$\bar{k}\times\bar{i} = \bar{j} \qquad \bar{i}\times\bar{k} = -\bar{j}$$

$$|\bar{k}| = |\bar{i}|\cdot|\bar{j}|\sin 90^\circ = 1$$

$$\bar{k}\perp\bar{i};\quad \bar{k}\perp\bar{j}$$

$$\bar{i};\ \bar{j};\ \bar{k} - haqiqiy\ uchlik$$



Vektorlarning koordinatalari aniqlansin:

$$\bar{a} = x_1\cdot\bar{i} + y_1\cdot\bar{j} + z_1\cdot\bar{k} \qquad \bar{b} = x_2\cdot\bar{i} + y_2\cdot\bar{j} + z_2\cdot\bar{k}$$

Vektor ko'paytmani topamiz:

Vektorlarni vektor ko'paytmasi

$$\bar{a}\times\bar{b}=\left(x_1\cdot\bar{i}+y_1\cdot\bar{j}+z_1\cdot\bar{k}\right)\times\left(x_2\cdot\bar{i}+y_2\cdot\bar{j}+z_2\cdot\bar{k}\right)$$

$$=x_1x_2\cdot 0+y_1x_2\cdot\left(-\bar{k}\right)+z_1x_2\cdot\bar{j}+x_1y_2\cdot\bar{k}+$$

$$y_1y_2\cdot 0+z_1y_2\cdot\left(-\bar{i}\right)+x_1z_2\cdot\left(-\bar{j}\right)+$$

$$+y_1z_2\cdot\bar{i}+z_1z_2\cdot 0=$$

$$=-y_1x_2\cdot\bar{k}+z_1x_2\cdot\bar{j}+x_1y_2\cdot\bar{k}-z_1y_2\cdot\bar{i}-x_1z_2\cdot\bar{j}+y_1z_2\cdot\bar{i}=$$

$$=\begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix}\cdot\bar{i}-\begin{vmatrix} x_1 & z_1 \\ x_2 & z_2 \end{vmatrix}\cdot\bar{j}+\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}\cdot\bar{k}=$$

$$\bar{a}\times\bar{b}=\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

Vektorlarni vektor ko'paytmasi

Vektorlarning vektor ko'paytmasini topamiz:

$$\bar{a} = 2\bar{i} - 3\bar{j} - \bar{k} \qquad \bar{b} = 3\bar{i} - \bar{j} - 4\bar{k}$$

$$\bar{a} \times \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & -3 & -1 \\ 3 & -1 & -4 \end{vmatrix}$$

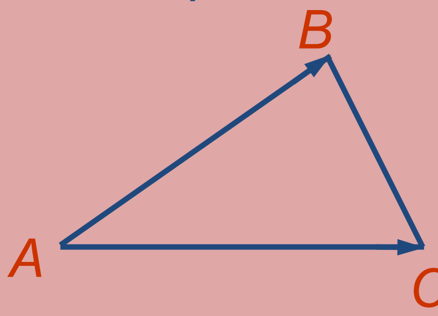
$$= \begin{vmatrix} -3 & -1 \\ -1 & -4 \end{vmatrix} \cdot \bar{i} - \begin{vmatrix} 2 & -1 \\ 3 & -4 \end{vmatrix} \cdot \bar{j} + \begin{vmatrix} 2 & -3 \\ 3 & -1 \end{vmatrix} \cdot \bar{k}$$

$$= (12-1) \cdot \bar{i} - (-8+3) \cdot \bar{j} + (-2+9) \cdot \bar{k} = \boxed{-11\bar{i} + 5\bar{j} + 7\bar{k}}$$

Vektorlarni vektor ko'paytmasi

Uchburchak uchining koordinatalari orqali uning yuzasini topamiz:

$A(2;3;1)$ $B(5;6;3)$ $C(7;1;10)$



Vektorlarning koorditalarini topish:

$$\overline{AB}=\{5-2;6-3;3-1\}=\{3;3;2\}$$

$$\overline{AC}=\{7-2;1-3;10-1\}=\{5;-2;9\}$$

$$S=\frac{1}{2}\left|\bar{a}\times\bar{b}\right|$$

$$\bar{a}\times\bar{b}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 3 & 3 & 2\\ 5 & -2 & 9\end{vmatrix}=31\bar{i}-17\bar{j}-21\bar{k}$$

$$S=\frac{1}{2}\sqrt{31^2+(-17)^2+(-21)^2}=\frac{1}{2}\sqrt{1691}\approx 20.6$$