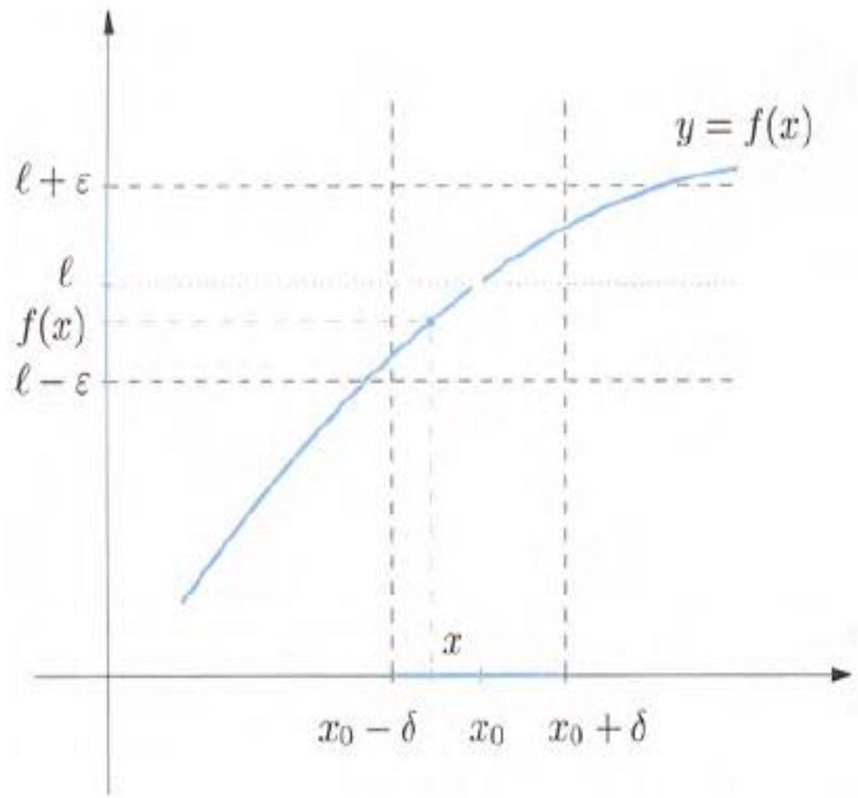
FUNKSIYA LIMITI. FUNKSIYA LIMITINING ASOSIY XOSSALARI

REJA :

- Funksiya limitining ta’rifi
- Asosiy teoremalari
- Misollar

Funksiya limitining ta’rifi



- Agar ixtiyoriy musbat ε son uchun $x = x_0$ nuqtani o’z ichiga olgan shunday interval ko’rsatish mumkun bo’lsaki, bu intervalning $x = x_0$nuqtadan tashqari hamma yerida $|f(x) - l| < \varepsilon$ tengsizlik bajarilsa, l soni $f(x)$ funksiyaning x ning x_0 ga intilgandagi limiti deyiladi va
- $\lim_{x \to x_0} f(x) = l$ ko’rinishda yoziladi.

Misol. $\lim_{x \to +\infty} \frac{x^2 + 2x}{2x^2 + 1} = \frac{1}{2}.$

- $\varepsilon > 0$ uchun $\left|f(x) - \frac{1}{2}\right| < \varepsilon$ yoki $\left|\frac{4x-1}{2(2x^2+1)}\right| < \varepsilon.$ bunda $x > \frac{1}{4}$; deb olishimiz mumkin.
- Funksiyaning xossalariga ko'ra

$$\frac{4x-1}{2(2x^2+1)} < \frac{2x}{2x^2+1} < \frac{2x}{2x^2} = \frac{1}{x} < \varepsilon$$

, agar $x > \frac{1}{\varepsilon}.$

  - U holda $x \in \left(\frac{1}{4}; \frac{1}{\varepsilon}\right)$ qiymatlari uchun bajariladi.

Funksiya limitining asosiy teoremalari

Funksiyalarning kimitlari haqidagi asosiy teoremalarni isbotsiz keltiramiz.

1. O’zgarmasning limiti shu o’zgarmasning o’ziga teng:

$$\lim_{x \to x_0} C = C$$

2. O’zgarmas ko’paytuvchini limit ishorasidan tashqariga chiqarish mumkin:

$$\lim_{x \to x_0} \left(k f(x)\right) = k * \lim_{x \to x_0} f(x)$$

3. Funksiyalar yeg’indisining (ayirmasining) limiti shu funksiyalar limitlarining yig’indisiga (ayirmasiga) teng:

$$\lim_{x \to x_0} [f(x) \pm g(x)] = \lim_{x \to x_0} f(x) \pm \lim_{x \to x_0} g(x)$$

4. Funksiyalar ko’paytmasining limiti shu funksiyalar limitlarining ko’paytmasiga teng:

$$\lim_{x\to x_0}[f(x)*g(x)] = \lim_{x\to x_0} f(x) * \lim_{x\to x_0} g(x)$$

5. Agar bo’luvchining limiti 0 ga teng bo’lmasa, 2 funksiya nisbatining limiti shu funksiyalar limitlarining nisbatiga teng.

$$\lim_{x\to x_0}\frac{f(x)}{g(x)} = \frac{\lim_{x\to x_0} f(x)}{\lim_{x\to x_0} g(x)}, \qquad \lim_{x\to x_0} g(x)\neq 0$$

Agar

$\lim_{x \to x_0} f(x) = 0 \; yoki \; \lim_{x \to \infty} f(x) = 0$ u holda,

$f(x)$ cheksiz kichik miqdor,

Agar

$\lim_{x \to x_0} f(x) = \infty \; yoki \; \lim_{x \to \infty} f(x) = \infty$ bo'lsa, $f(x)$

cheksiz katta miqdor deyiladi.

Misollar

- №1

$$\lim_{x\to-\infty}\frac{x^3+x^2+x}{2x^2-x+3}=\lim_{x\to-\infty}\frac{x^3\left(1+\frac{1}{x}+\frac{1}{x^2}\right)}{x^2\left(2-\frac{1}{x}+\frac{3}{x^2}\right)}=\lim_{x\to-\infty}\frac{x}{2}=-\infty\,.$$

- №2

$$\lim_{x\to-1}\frac{x+1}{\sqrt{6x^2+3}+3x}=\lim_{x\to-1}\frac{(x+1)(\sqrt{6x^2+3}-3x)}{6x^2+3-9x^2}$$

$$=\lim_{x\to-1}\frac{(x+1)(\sqrt{6x^2+3}-3x)}{3(1-x)(1+x)}=1\,.$$

- №3

$$\lim_{x\to 2}\frac{\sqrt[3]{10-x}-2}{x-2}=\lim_{x\to 2}\frac{10-x-8}{(x-2)\left(\sqrt[3]{(10-x)^2}+2\sqrt[3]{10-x}+4\right)}$$
$$=\lim_{x\to 2}\frac{-1}{\sqrt[3]{(10-x)^2}+2\sqrt[3]{10-x}+4}=-\frac{1}{12}.$$

- №4

$$\lim_{x\to-\infty}\frac{\sqrt{2x^2+3}}{4x+2}=\lim_{x\to-\infty}\frac{|x|\sqrt{2+\frac{3}{x^2}}}{x\left(4+\frac{2}{x}\right)}=\frac{\sqrt{2}}{4}\lim_{x\to-\infty}\frac{-x}{x}=-\frac{\sqrt{2}}{4}.$$

Cheksizlik bilan amallar

$+\infty + s = +\infty$	agar $s \in \mathbb{R}$ va $s = +\infty)$
$-\infty + s = -\infty$	agar $s \in \mathbb{R}$ va $s = -\infty)$
$\pm\infty \cdot s = \pm\infty$	agar $s > 0$ va $s = +\infty)$
$\pm\infty \cdot s = \mp\infty$	agar $s < 0$ va $s = -\infty)$
$\dfrac{\pm\infty}{s} = \pm\infty$	agar $s > 0)$
$\dfrac{\pm\infty}{s} = \mp\infty$	agar $s < 0)$
$\dfrac{s}{0} = \infty$	agar $s \in \mathbb{R} \setminus \{0\}$ va $s = \pm\infty)$
$\dfrac{s}{\pm\infty} = 0$	agar $s \in \mathbb{R})$

Aniqmasliklar:

$$\pm\infty + (\mp\infty), \quad \pm\infty - (\pm\infty), \quad \pm\infty \cdot 0, \quad \frac{\pm\infty}{\pm\infty}, \quad \frac{0}{0}.$$

$$0^0. \quad \infty^0. \quad 1^{\infty}.$$

Ekvivalent funksiyalar

$\sin x \sim x, \quad x \to 0;$

$1 - \cos x \asymp x^2, \quad x \to 0;$ **Aniqrog'i** $1 - \cos x \sim \frac{1}{2}x^2, \quad x \to 0;$

$\log(1 + x) \sim x, \quad x \to 0;$ **yoki** $\log x \sim x - 1, \quad x \to 1;$

$e^x - 1 \sim x, \quad x \to 0;$

$(1 + x)^\alpha - 1 \sim \alpha x, \quad x \to 0.$

Foydalanilgan adabiyotlar

1
- Canuto Tobacco
- “Mathematical analysis”

2
- N.S Piskunov
- “Differensial va integral hisob”

3
- D.A Shteyngardt
- “Высшая математика”

E'tiboringiz uchun rahmat !