Aniq integralning tatbiqlari

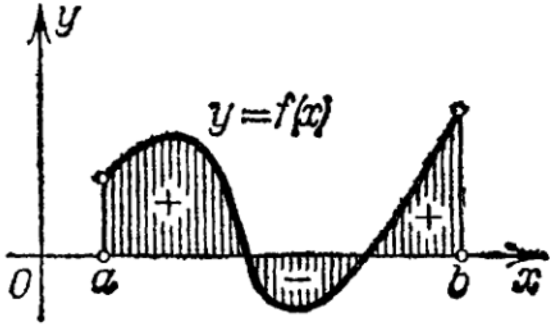
REJA

1. To’g’ri burchak koordinatalar sistemasida yuzalarni hisoblash
2. Qutb koordinatalarda egri chiziqli sektorning yuzi
3. Egri chiziq yoyining uzunligi
4. Aniq integrallarni taqribiy hisoblash

Agar $[a,b]$ kesmada $f(x)\geq 0$ bo'lsa, u holda, $y=f(x)$ egri chiziq, Ox o'q hamda $x=a$, $x=b$ to'g'ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning yuzi

$$Q=\int_a^b f(x)dx \qquad (1)$$

Agar $f(x)\leq 0$ $[a,b]$da bo'lsa, u holda $\int_a^b f(x)dx$ aniq integral ham ≤ 0 bo'ladi.



Absolyut qiymati jihatidan u mos egri chiziqli trapetsiyaning Q yuziga teng:

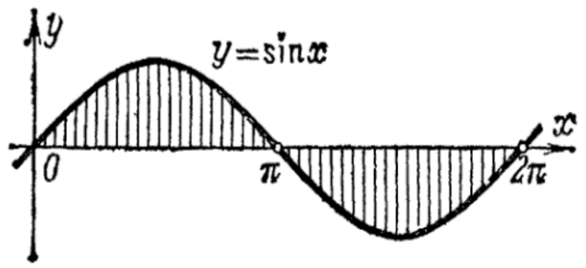
$$-Q=\int_a^b f(x)dx$$

Agar $f(x)$ funksiya $[a,b]$ kesmada chekli marta ishorasini o'zgartirsa, u holda butun $[a,b]$ kesma bo'yicha olingan intervali qism-qism kesmalar bo'yicha integrallar yig'indisiga ajratamiz. Integral $f(x)\geq 0$ bo'lgan joylarda musbat va $f(x)\leq 0$ bo'lganda manfiy bo'ladi. Bunday holda

$$Q=\int_a^b |f(x)|dx$$

bo'ladi.

Misol 1. $y = \sin x$ sinusoid ava Ox o'q bilan $0 \le x \le 2\pi$ bo'lganda chegaralangan Q yuzani toping.



Yechish. $0 \le x \le \pi$ da $\sin x \ge 0$ va $\pi < x \le 2\pi$ da $\sin x \le 0$ bo'lganligi uchun

$$Q = \int_0^{\pi} \sin x dx + \left| \int_{\pi}^{2\pi} \sin x dx \right| = \int_0^{2\pi} |\sin x| dx$$

$$\int_0^{\pi} \sin x dx = -\cos x |_0^{\pi} = -(\cos \pi - \cos 0) = -(-1-1) = 2$$

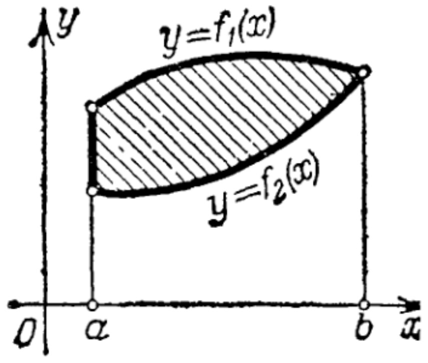
$$\int_{\pi}^{2\pi} \sin x dx = -\cos x |_{\pi}^{2\pi} = -(\cos 2\pi - \cos \pi) = -2$$

Demak, $Q = 2 + |-2| = 4$

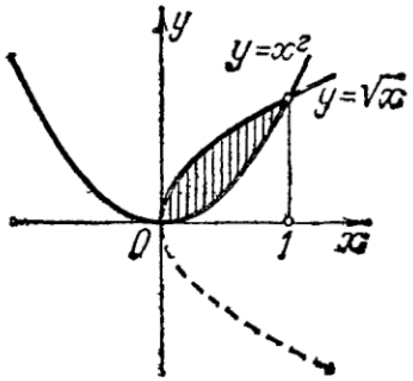
Agar $y = f_1(x), y = f_2(x)$ egri chiziqlar va $x = a, x = b$ ordinatalar bilan chegaralangan yuza $f_1(x) \geq f_2(x)$ shart bajarilganda

$$Q = \int_a^b f_1(x)dx - \int_a^b f_2(x)dx = \int_a^b [f_1(x) - f_2(x)]dx \quad (2)$$

bo'ladi.



Misol 2. $y=\sqrt{x}$ va $y=x^2$ egri chiziqlar bilan chegaralangan yuzani toping.



Yechish. Egri chiziqlarning kesishish nuqtasini topamiz: $\sqrt{x}=x^2; x=x^4$, bu yerdan $x_1=0$ va $x_2=1$.

Demak,

$$Q=\int_0^1\sqrt{x}dx-\int_0^1 x^2dx=\int_0^1(\sqrt{x}-x^2)dx=\frac{2}{3}x^{\frac{3}{2}}\Big|_0^1-\frac{x^3}{3}\Big|_0^1=\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$$

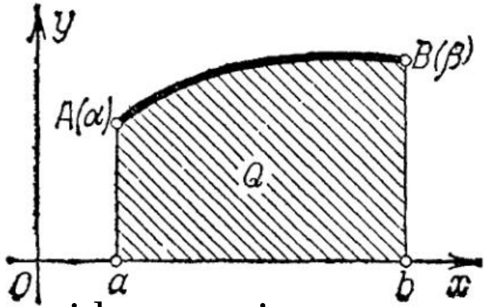
Endi tenglamasi

$$x=\varphi(t),\, y=\psi(t) \qquad (3)$$

parametrik ko'rinishda bo'lgan egri chiziq bilan chegaralangan egri chiziqli trapetsiya yuzasini topamiz, bu yerda $\alpha \le t \le \beta$ va $\varphi(\alpha)=a$, $\varphi(\beta)=b$.

(3) tenglamalar $[a,b]$ kesmada biror $y=f(x)$ funksiyani aniqlash va demak egri chiziqli trapetsiyaning yuzi

$$Q=\int_a^b f(x)dx=\int_a^b y\,dx$$



formula bilan hisoblanishi mumkin.

Bu integralda o'zgaruvchini almashtiramiz: $x=\varphi(t),\, dx=\varphi'(t)dt$. (3) tenglamalar asosida topamiz:

$$y=f(x)=f[\varphi(t)]=\psi(t)$$

Demak,

$$Q=\int_a^b \psi(t)\varphi'(t)dt \qquad (4)$$

Bu parametric ko'rinishda berilgan egri chiziq bilan chegaralangan egri chiziqli trapetsiyani yuzasini topish formulasidir.

Misol. Ellips bilan chegaralangan soha yuzini toping.

$$x = a\cos t, y = b\sin t$$

Yechish. Ellipsning yuqori yarmi yuzasini topamiz va $-a$ dan $+a$ gacha o’zgaradi, demak, t π dan 0 gacha o’zgaradi:

$$Q = 2\int_{\pi}^{0}(b\sin t)(-a\sin t dt) = -2ab\int_{\pi}^{0}\sin^2 t\, dt = 2ab\int_{0}^{\pi}\sin^2 t\, dt =$$

$$= 2ab\int_{0}^{\pi}\frac{1-\cos 2t}{2}dt = 2ab\left[\frac{t}{2} - \frac{\sin 2t}{4}\right]_0^{\pi} = \pi ab$$

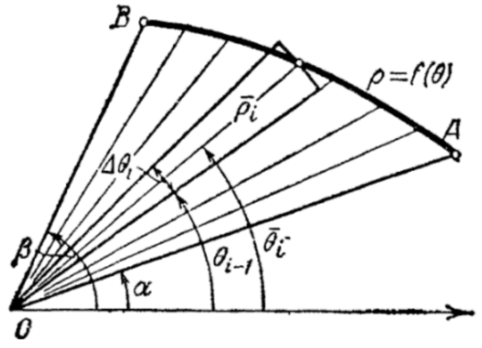
2. Qutb koordinatalarda egri chiziqli sektorning yuzi

Qutb koordinatalar sistemasida egri chiziq

$$\rho = f(\theta)$$

tenglama bilan berilgan bo'lsin, bu yerda $f(\theta)$ - $\alpha \le \theta \le \beta$ da uzluksiz funksiya.

$\rho = f(\theta)$ egri chiziq hamda $\theta = \alpha, \theta = \beta$ radius-vektolar bilan chegaralangan OAB sektorning yuzini topamiz.



Berilgan yuzani $\theta_0 = \alpha, \theta = \theta_1, ..., \theta_n = \beta$ radius-vektorlar yordamida n qismlarga ajratamiz. O'tkazilgan radius-vektorlar orasida burchaklari $\Delta\theta_1, \Delta\theta_2, ..., \Delta\theta_n$ bilan belgilaymiz.

θ_{i-1} va θ_i orasida joylashgan qandaydir $\overline{\theta_i}$ burchakka mos kelgan radius-vektorning uzunligini $\overline{\rho_i}$ bilan belgilaymiz.

Radiusi $\overline{\rho_i}$ va markaziy burchagi $\Delta\theta_i$ bo'lgan doiraviy sektorni qaraymiz. Uning yuzasi $\Delta Q_i = \frac{1}{2}\overline{\rho_i}^2 \Delta\theta_i$ ga teng. Ushbu

$$Q_n = \frac{1}{2}\sum_{i=1}^{n}\overline{\rho_i}^2\Delta\theta_i = \frac{1}{2}\sum_{i=1}^{n}[f(\Delta\theta_i)]^2\Delta\theta_i$$

esa "zinasimon" sektorning yuzini beradi.

Bu yig'indi $\alpha \le \theta \le \beta$ kesmada $\rho^2 = [f(\Delta\theta_i)]^2$ funksiya uchun integral yig'indi bo'lganligi uchun uning $\max\Delta\theta_i \to 0$ bo'lgandagi limiti

$$\frac{1}{2}\int_{\alpha}^{\beta}\rho^2 d\theta$$

aniq integral bo'ladi. U biz $\Delta\theta_i$ burchakning ichida qaysi $\overline{\rho_i}$ radius-vektorni olishimizga bo'gliq emas.

Shunday qilib, OAB sektorning yuzi

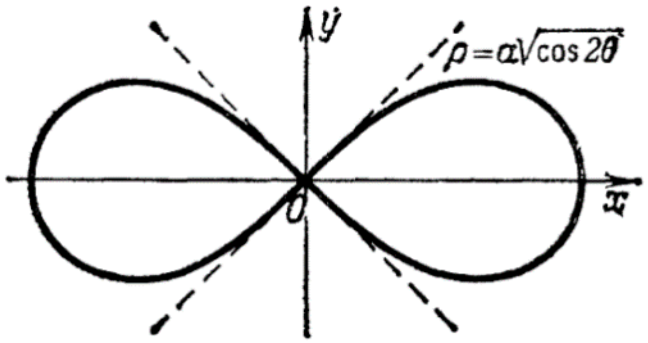
$$Q = \frac{1}{2}\int_{\alpha}^{\beta}\rho^2 d\theta \qquad (1)$$

yoki

$$Q = \frac{1}{2}\int_{\alpha}^{\beta}[f(\Delta\theta_i)]^2 d\theta \qquad (1')$$

formula bilan topiladi.

Misol. $\rho = a\sqrt{\cos 20}$ lemniskata bilan chegaralangan yuzani toping.



Yechish. Agar θ burchak 0 dan $\dfrac{\pi}{4}$ gacha o'zgarsa radius-vektor izlanayotgan yuzaning chorak qismiga teng:

$$\frac{1}{4}Q = \frac{1}{2}\int_0^{\frac{\pi}{4}} \rho^2 d\theta = \frac{1}{2}a^2\int_0^{\frac{\pi}{4}} \cos 20 d\theta =$$

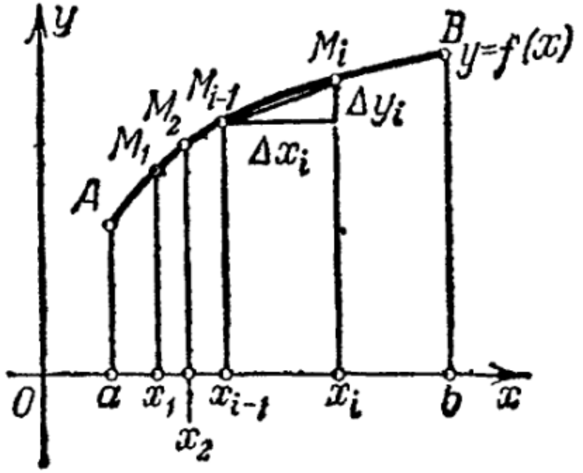
$$= \frac{a^2}{2}\frac{\sin 20}{2}\Bigg|_0^{\frac{\pi}{4}} = \frac{a^2}{4}$$

Demak, $Q = a^2$.

3. Egri chiziq yoyini uzunligi

1.To’g’ri burchakli koordinatalarda egri chiziq yoyining uzunligi. Tekislikda to’g’ri burchakli koordinatalarda egri chiziq $y = f(x)$ tenglama bilan berilgan bo’lsin.

Bu egri chiziqning $x = a$ va $x = b$ vertical to’g’ri chiziqlar orasida joylashgan AB yoyining uzunligini topamiz.



AB yoydan $A, M_1, M_2, ..., M_i, ..., B$ nuqtalarni olamiz, bu nuqtalarning absissalari $x_0 = a, x_1, x_2, ..., x_i, ..., b = x_n$ bo’lsin. $AM_1, M_1M_2, ..., M_{n-1}B$ vatarlarni o’tkazamiz va bu vatarlarning uzunliklarini mos ravishda $\Delta s_1, \Delta s_2, ..., \Delta s_n$ bilan belgilaymiz. Bu holda AB yoyga ichki chizilgan $AM_1M_2...M_{n-1}B$ siniq chiziqqa ega bo’lamiz. Siniq chiziqning uzunligi $s_n = \sum_{i=1}^{n} \Delta s_i$ ga teng.

AB yoyning s uzunligi deb

$$s = \lim_{\max \Delta s_i \to 0} \sum_{i=1}^{n} \Delta s_i \qquad (1)$$

limitga aytiladi. Yuqoridagi kabi mulohazalarni takrorlab topamiz:

$$s = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

yoki

$$s = \int_a^b \sqrt{1 + [\frac{dy}{dx}]^2}\, dx \qquad (2)$$

Misol 1. $x^2 + y^2 = r^2$ aylana uzunligini toping.

Yechish. Avval aylana chorak qismining uzunligini topamiz. Bu holda AB quyidagicha:

$$y = \sqrt{r^2 - x^2}, \text{ bu yerdan } \frac{dy}{dx} = -\frac{x}{\sqrt{r^2 - x^2}}$$

Demak,

$$\frac{1}{4}s = \int_0^r \sqrt{1 + \frac{x^2}{r^2 - x^2}}dx = \int_0^r \frac{r}{\sqrt{r^2 - x^2}}dx = r\arcsin\frac{x}{r}\Big|_0^r = r\frac{\pi}{2}$$

Butun aylananing uzunligi $s = 2\pi r$ ga teng.

Endi egri chiziq parametric ko'rinishida

$$x=\varphi(t), y=\psi(t) \quad (\alpha \le t \le \beta)$$

berilganda yoy uzunlikligini topamiz, bu yerda $\varphi(t)$ va $\psi(t)$ - hosilalari bilan uzluksiz bo'lgan uzluksiz funksiyalar, bunda $\varphi'(t)$ berilgan uchastkada nolga teng emas. Bu holda yoy uzunligi

$$s=\int_{\alpha}^{\beta}\sqrt{[\varphi'(t)]^2+[\psi'(t)]^2}dt \qquad (5)$$

formula bilan topiladi.

Misol 2. $x=a\cos^3 t, y=a\sin^3 t$ giposikloidning uzunliklarini toping.

Yechish. Egri chiziq ikkala koordinata o'qlariga nisbatan simmetrik bo'lganligi uchun avval birinchi chorakda qismining uzunligini topib olamiz:

$$\frac{dx}{dt}=-3a\cos^2 t\sin t, \frac{dy}{dt}=3a\sin^2 t\cos t$$

t parametr 0 dan $\frac{\pi}{2}$ gacha o'zgaradi.

Demak,

$$\frac{1}{4}s=\int_0^{\frac{\pi}{2}}\sqrt{9a^2\cos^4 t\sin^2 t+9a^2\sin^4 t\cos^2 t}dt=3a\int_0^{\frac{\pi}{2}}\sqrt{\sin^2 t\cos^2 t}dt=$$

$$=3a\int_0^{\frac{\pi}{2}}\sin t\cos tdt=3a\frac{\sin^2 t}{2}\Big|_0^{\frac{\pi}{2}}=\frac{3a}{2}$$

$$s=6a$$

$$x=\varphi(t), y=\psi(t), z=\chi(t) \qquad (6)$$

parametrik ko'rinishida berilgan fazoviy egri chiziqning $\alpha \le t \le \beta$ bo'lgandagi uzunligi

$$s=\int_{\alpha}^{\beta}\sqrt{[\varphi'(t)]^2+[\psi'(t)]^2+[\chi'(t)]^2}\,dt \qquad (7)$$

Misol 3. $x = a\cos t, y = a\sin t, z = amt$ vint chiziqning t 0 dan 2π gacha o'zgargandagi yoyi uzunligini toping.

Yechish.

$$dx = -a\sin t dt, dy = a\cos t dt, dz = amdt$$

(7) formulaga qo'yib, topamiz:

$$s = \int_{\alpha}^{\beta} \sqrt{a^2\sin^2 t + a^2\cos^2 t + a^2m^2}\,dt = a\int_{\alpha}^{\beta} \sqrt{1+m^2}\,dt = 2\pi a\sqrt{1+m^2}$$

Qutb koordinatalarida berilgan egri chiziq yoyining uzunlgi.

Egri chiziq

$$\rho = f(\theta) \qquad (8)$$

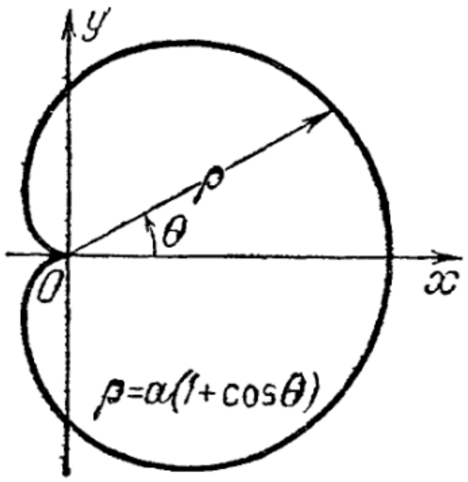
qutb koordinatalarda berilgan bo'lsin, bu yerda ρ - qutb radiusi, θ - qutb burchagi.

(8) egri chiziqning qutb burchagi θ_1 dan θ_2 gacha o'zgargandagi yoyining uzunligi

$$s = \int_{\theta_0}^{\theta_1} \sqrt{\rho'^2 + \rho^2}\, d\theta$$

formula bilan topiladi.

Misol 4. $\rho = a(1+\cos\theta)$ koordinataning uzunligini toping.



Yechish. θ qutb burchagi 0 dan π gacha o’zgarganda chiziqning yarmini olamiz. Bu yerda

$$\rho' = -a\sin\theta$$

Demak,

$$s = 2\int_0^{\pi}\sqrt{a^2(1+\cos^2\theta)+a^2\sin^2\theta}\,d\theta = 2a\int_0^{\pi}\sqrt{2+2\cos\theta}\,d\theta =$$

$$= 4a\int_0^{\pi}\cos\frac{\theta}{2}d\theta = 8a\sin\frac{\theta}{2}\Big|_0^{\pi} = 8a$$

4.Aniq integrallarni taqribiy hisoblash

Ma'lumki, $[a,b]$ intervalda uzluksiz bo'lgan har qanday $y = f(x)$ funksiya shu intervalda boshlang'ichga ega, ya'ni $F'(x) = f(x)$ tenglikni qanoatlantiradigan $F(x)$ funksiya mavjuda. Ammo har qanday boshlang'ich funksiya, hattoki u mavjud bo'lgan holda ham, elementar funksiyalar orqali chekli ko'rinishda ifodalanmaydi. Bunday hollarda aniq integrallarni Nyuton-Leybnits formulasi yordamida hisoblash ancha mushkul ish va aniq integralni hisoblashning turli taqribiy usullar qo'llaniladi. Hozir biz taqribiy integralning bir necha usullarini keltiramiz.

I.To'g'ri to'rtburchaklar formulasi $[a,b]$ kesmada uzluksiz $y = f(x)$ funksiya berilgan bo'lsin. Ushbu

$$\int_a^b f(x)dx$$

aniq integralni hisoblash talab etiladi.

$[a,b]$ kesmani $a = x_0, x_1, x_2, ..., x_n = b$ nuqtalar yordamida uzlukligi Δx bo'lgan n ta teng qismlarga bo'lamiz:

$$\Delta x = \frac{b-a}{n}$$

$y_0, y_1, y_2, ..., y_{n-1}, y_n$ bilan $f(x)$ funksiyaning $x_0, x_1, x_2, ..., x_n$ nuqtalardagi qiymatlarini belgilaymiz:

$$y_0 = f(x_0), y_1 = f(x_1), ..., y_n = f(x_n)$$

Endi

$$y_0\Delta x + y_1\Delta x + ... + y_{n-1}\Delta x$$

$$y_1\Delta x + y_2\Delta x + ... + y_n\Delta x$$

yig'indilarni tuzamiz.

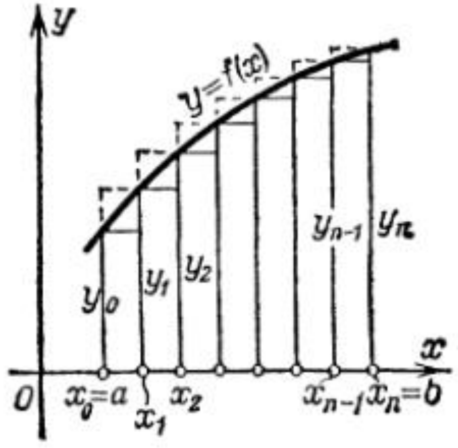
Bu yig’indilardan har biri $f(x)$ funksiya uchun $[a,b]$ kesmada integral yig’indi bo’ladi va shuning uchun

$$\int_a^b f(x)dx \approx \frac{b-a}{n}\left(y_0 + y_1 + y_2 + \ldots + y_{n-1}\right) \qquad (1)$$

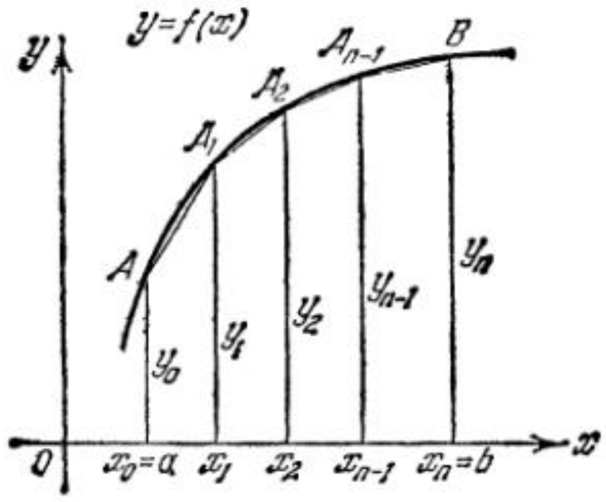
$$\int_a^b f(x)dx \approx \frac{b-a}{n}\left(y_1 + y_2 + \ldots + y_n\right) \qquad (1')$$

Mana shu to’g’ri to’rtburchaklar formulasidir. Rasmdan ko’rinib turibdiki, agar $f(x)$ - musbat va o’suvchi funksiya bo’lsa, u holda (1) formula ichlaridan to’g’ri to’rtburchaklardan tuzilgan zinasimon figuraning yuzasini ifodalaydi. (1’) formula esa tashqariga chiqib turgan zinasimon figurani yuzasini ifodalaydi.

n soni qanchalik kata bo’lsa, (ya’ni $\Delta x = \frac{b-a}{n}$ bo’lishi qadami qanchalik kichik bo’lsa) integralni to’g’ri to’rtburchaklar formulasi yordamida hisoblashdagi hatolik shunchalik kam bo’ladi.



II.Trapetsiyalar formulasi. Agar berilgan $y = f(x)$ egri chiziq o’rniga zinasimon funksiyani emas, balki ichki chizilgan aniq chiziqni olsak (2-rasm) biz aniq integralning yanayam aniqroq qiymatini olamiz. Bu holda $aABb$ egri chiziqli trapetsiyaning yuzasi o’rniga yuqoridan $AA_1, A_1A_2, ..., A_{n-1}B$ vatarlar bilan chegralangan to’g’ri chiziqli trapetsiyalar yuzalarining yig’indisini olamiz. Bu trapetsiyalardan birinchisining yuzasi $\frac{y_0 + y_1}{2}\Delta x$ ga ikkinchisiniki $\frac{y_1 + y_2}{2}\Delta x$ g ava v.h.zga teng bo’lganligi uchun



$$\int_a^b f(x)dx \approx \left(\frac{y_0 + y_1}{2}\Delta x + \frac{y_1 + y_2}{2}\Delta x + ... + \frac{y_{n-1} + y_n}{2}\Delta x \right)$$

yoki

$$\int_a^b f(x)dx \approx \frac{b-a}{n}\left(\frac{y_0 + y_1}{2} + y_1 + y_2 + ... + y_{n-1} \right) \qquad (2)$$

Bu trapetsiyalar formulasidir. (2) formulaning o’ng tomonida turgan son (1) va (1’) formulalarning o’ng tomonlarida turgan sonlarning o’rta arifmetigidir.

n soni ixtiyoriy tanlanadi. Bu son qanchalik kata bo’lsa, demakki, $\Delta x = \frac{b-a}{n}$ qadam shunchalik kichik bo’ladi, (2) taqribiy tenglikning o’ng tomonida turgan yig’indi integralning qiymatini shunchalik aniq beradi.

III.Parabolalar formulasi (Simpson formulasi). $[a,b]$ kesmani juft sondagi $n=2m$ teng bo'laklarga bo'lamiz. Dastlabki ikkita $[x_0, x_1]$ va $[x_1, x_2]$ kesmalarga mos kelgan va berilgan $y=f(x)$ egri chiziq bilan chegaralangan egri chiziqli trapetsiyaning yuzasini $M(x_0, y_0), M_1(x_1, y_1), M_2(x_2, y_2)$ uchta nuqtalar bilan chegaralangan va Oy o'qqa parallel o'qqa ega bo'lgan egri chiziqli trapetsiya yuzasi bilan almshtiramiz. Bunday egri chiziqli trapetsiya parabolic trapetsiya deyiladi.

O'qi Oy o'qqa parallel bo'lgan parabolaning tenglamasi

$$y = Ax^2 + Bx + C$$

ko'rinishida bo'ladi.

A, B, C koeffitsientlar parabolaning berilgan uchta nuqtalardan o'tish shartidan topiladi. Kesmalarning boshqa juftlari uchun ham shunga o'xshagan parabolalarni quramiz. Parabolic trapetsiyalar yuzalarining yig'indisi integralning taqribiy qiymatini beradi.

Avvalo bitta parabolik trapetsiyaning yuzasini hisoblaymiz.

Avvalo bitta parabolik trapetsiyaning yuzasini hisoblaymiz.

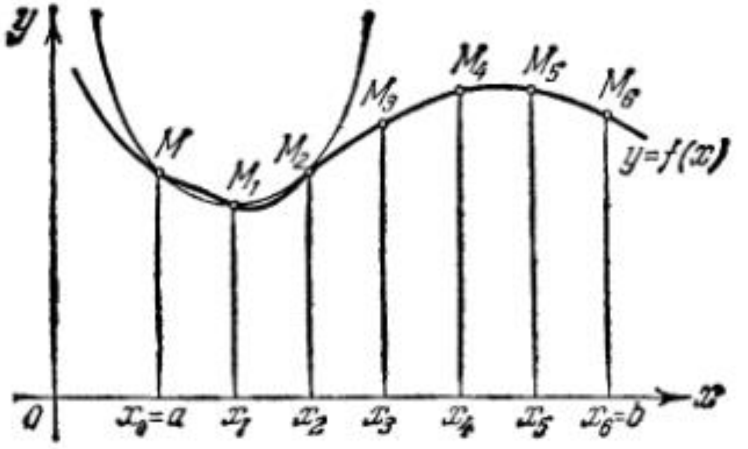
Lemma. Agar egri chiziqli trapetsiya

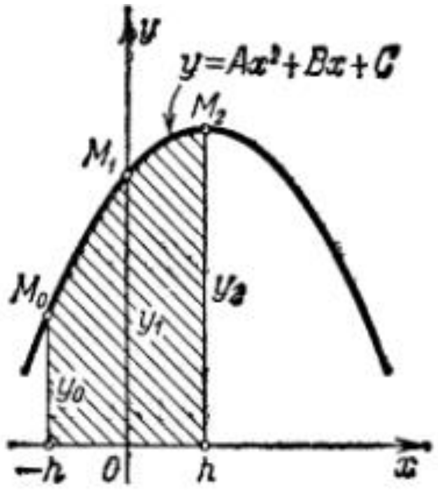
$$y = Ax^2 + Bx + C$$

parabola, Ox o’q va oralaridagi masofalari $2h$ bo’lgan ikkita ordinatalar bilan chegaralangan bo’lsa, u holda uning yuzasi

$$S = \frac{h}{3}\left(y_0 + 4y_1 + y_2\right) \qquad (3)$$

Bu yerda y_0 va y_2 - chetki ordinatalar, y_1-egri chiziqning kesma o’rtasidagi ordinatasi.





Isbot. Yordamchi koordinatalar sistemasini rasmda ko’rasatilgandan joylashtiramiz. $y = Ax^2 + Bx + C$ parabola tenglamasidagi koeffitsientlar quyidagi tenglamalardan topiladi:

Agar $x_0 = -h$ bo'lsa $y_0 = Ah^2 - Bh + C$

Agar $x_1 = 0$ bo'lsa $y_1 = C$ (4)

Agar $x_2 = h$ bo'lsa $y = Ah^2 + Bh + C$

A, B, C koeffitsientlarni ma'lum deb hisoblab, parabolic trapetsiyaning yuzasini aniq integral yordamida topamiz:

$$S = \int_{-h}^{h} \left(Ax^2 + Bx + C \right) dx = \left[\frac{Ax^3}{3} + \frac{Bx^2}{2} + Cx \right]_{-h}^{h} = \frac{h}{3}(2Ah^2 + 6C)$$

Ammo (4) tenglikdan

$$y_0 + 4y_1 + y_2 = 2Ah^2 + 6C$$

Kelib chiqadi. Shunday qilib

$$S = \frac{h}{3}(2Ah^2 + 6C)$$

Shuni isbotlash talab etilgan edi.

Endi o'zimizning asosiy masalamizga qaytamiz. (3) formuladan foydalanib, biz quyidagi taqribiy tengliklarni yozishimiz mumkin ($h = \Delta x$):

$$\int_{a=x_0}^{x_2} f(x)dx \approx \frac{\Delta x}{3}(y_0 + 4y_1 + y_2)$$

$$\int_{x_2}^{x_4} f(x)dx \approx \frac{\Delta x}{3}(y_2 + 4y_3 + y_4)$$

....

$$\int_{x_{2m-2}}^{x_{2m}=b} f(x)dx \approx \frac{\Delta x}{3}(y_{2m-2} + 4y_{2m-1} + y_{2m})$$

Chap va o’ng tomonlarni yig’ib, chap tomonda izlanayotgan integralni chap tomonda esa uning taqribiy qiymatini topamiz:

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + \ldots \quad (5)$$

$$\ldots + 2y_{2m-2} + 4y_{2m-1} + y_{2m})$$

yoki

$$\int_a^b f(x)dx \approx \frac{b-a}{6m}(y_0 + y_{2m} + 2[y_2 + y_4 + \ldots + 2y_{2m-2}] +$$

$$+4[y_1 + y_3 + \ldots + y_{2m-1}])$$

Bu Simpson formulasidir. Bu yerda $2m$ bo’linishlar soni ixtiyoriy, ammo bu son qanchalik kata bo’lsa, (5)ning o’ng tomonidagi yig’indi integralning qiymatini shunchalik aniq beradi.

Misol. Taqribiy hisoblang:

$$\ln 2 = \int_1^2 \frac{dx}{x}$$

Yechish. [1,2] kesmani 10ta teng bo'laklarga bo'lamiz.

$$\Delta x = \frac{2-1}{10} = 0.1$$

deb olib, integral ostidagi funksiyaning qiymatlari jadvalini tuzamiz:

x	$y = 1/x$	x	$y = 1/x$
$x_0 = 1,0$	$y_0 = 1,00000$	$x_6 = 1,6$	$y_6 = 0,62500$
$x_1 = 1,1$	$y_1 = 0,90909$	$x_7 = 1,7$	$y_7 = 0,58824$
$x_2 = 1,2$	$y_2 = 0,83333$	$x_8 = 1,8$	$y_8 = 0,55556$
$x_3 = 1,3$	$y_3 = 0,76923$	$x_9 = 1,9$	$y_9 = 0,52632$
$x_4 = 1,4$	$y_4 = 0,71429$	$x_{10} = 2,0$	$y_{10} = 0,50000$
$x_5 = 1,5$	$y_5 = 0,66667$		

1.To'g'ri to'rtburchaklar (1) formulasi bo'yicha topamiz:

$$\int_1^2 \frac{dx}{x} \approx 0{,}1(y_0 + y_1 + ... + y_9) = 0{,}1 \cdot 7{,}18773 = 0{,}71877$$

To'g'ri to'rtburchaklar (1') formulasi bo'yicha

$$\int_1^2 \frac{dx}{x} \approx 0{,}1(y_1 + y_2 + ... + y_{10}) = 0{,}1 \cdot 6{,}68773 = 0{,}66877$$

Rasmdan bevosita kelib chiqadiki, bu holda birinchi formula integralning qiymatini ortig'i bilan, ikkinchisi esa kami bilan beradi.

II.Trapetsiyalar (2) formulasi bo'yicha

$$\int_1^2 \frac{dx}{x} \approx 0{,}1(\frac{1+0{,}5}{2} + 6{,}18773) = 0{,}69377$$

III.Simpson (5) formulasi bo'yicha

$$\int_1^2 \frac{dx}{x} \approx \frac{0{,}1}{3}[y_0 + y_{10} + 2(y_2 + y_4 + y_6 + y_8) + 4(y_1 + y_3 + y_5 + y_7 + y_9)] =$$

$$= \frac{0{,}1}{3}(1 + 0{,}5 + 2 \cdot 2{,}72818 + 4 \cdot 3{,}45955) = 0{,}69315$$

Aslida $\ln 2 = \int_1^2 \frac{dx}{x} = 0,6931472$ (7xona aniqlikda).

Shunday qilib [1,2] kesmani teng 10ta qismlarga bo’lganda Simpson formulasi bo’yicha 5ta ishonchli raqamlarni; trapetsiyalar formulasi bo’yicha 3ta ishonchli raqamlarni; to’g’ri to’rtburchaklar formulasi bo’yicha faqat 1ta ishonchli raqam oldik.