

Kompleks sonlar va ularning algebraik ko'rinishi. Nisbat, proporsiya. Protsent va murakkab protsentlar.



M a s h q l a r

3.1. Kompleks son z ning haqiqiy qismi $\text{Re}(z)$ ni va mavhum qismi $\text{Im}(z)$ ni toping:

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|---------------------------------------|----------------------|------------|
| a) $z = -5 + 8i$; | f) $z = 0,5 + 3i$; | j) $8i$; |
| b) $z = 6 + \frac{1}{2}i$; | g) $z = 2 + 0,3i$; | k) 4 ; |
| d) $z = -15 + 2i$; | h) $z = -4,1 + 2i$; | l) 0 ; |
| e) $z = \frac{1}{2} + \frac{3}{2}i$; | i) $z = -3 - 4i$; | m) $-3i$. |

3.2. Agar:

- | | |
|---------------------------|------------------------|
| a) $\text{Re}(z) = -4$, | $\text{Im}(z) = 8$; |
| b) $\text{Re}(z) = 0$, | $\text{Im}(z) = 1,2$; |
| d) $\text{Re}(z) = 1,2$, | $\text{Im}(z) = 0$; |
| e) $\text{Re}(z) = 0$, | $\text{Im}(z) = 0$ |

bo'lsa, z kompleks sonini algebraik shaklda yozing.

3.4. a) Kompleks sonlardan qaysilari teng:

$$3i; -4+5i; \frac{1}{3}+i; -\frac{1}{4}-8i; 0,(3)+i; -\frac{2}{8}-\sqrt{64}i; \sqrt[4]{81}i?$$

b) $(4x-3y) + (3x+5y)i = 10 - (3x-2y-30)i$ bo'lsa, x va y larni toping ($x, y \in R$).

3.5. Agar:

- | | | |
|-------------------|-----------------|------------------------------|
| a) $z = -3 + 5i;$ | f) $z = -3i;$ | j) $z = \frac{1}{3} + 3,4i;$ |
| b) $z = 3 - 5i;$ | g) $z = 4,2;$ | k) $z = 0;$ |
| d) $z = -3 - 5i;$ | h) $z = 4i;$ | l) $z = \sqrt{81} + 4i;$ |
| e) $z = 3 + 5i;$ | i) $z = 4,(3);$ | m) $z = -0,(3) - 2,(3)i$ |
- bo'lsa, $\bar{z}$ ni toping.

3.6. Yig'indini toping:

- | | |
|----------------------------|-------------------------------|
| a) $(-3 + 2i) + (4 - i);$ | e) $4 + (-3 + i);$ |
| b) $(4 + 5i) + (4 - 5i);$ | f) $(1,4 - 3i) + (2,6 - 4i);$ |
| d) $(5 + 2i) + (-5 - 2i);$ | g) $(3 + 8i) + (3 - 8i);$ |

3.7. Yig'indini toping:

a) $\left(\frac{1-\sqrt{2}}{2} + \frac{1+\sqrt{2}}{3}i\right) + \left(\frac{1+\sqrt{2}}{2} + \frac{1-\sqrt{2}}{3}i\right) ;$

b) $(\cos^2 \alpha + i \sin^2 \alpha) + (\sin^2 \alpha + i \cos^2 \alpha), (\alpha \in R) ;$

d) $(0, (3) + i \cdot 1, (5)) + (0, (6) + i \cdot 1, (55));$

e) $(\operatorname{Re}(1 + 2i) + 15i) + (3 - i \cdot \operatorname{Im}(1 + 2i)).$

3.8. Ayirmani toping:

a) $(-5 + 2i) - (8 - 9i);$

f) $(32 + 4, (5)i) - (32 + i);$

b) $(5 + 21i) - (9i + 8);$

g) $\left(\frac{1-\sqrt{2}}{2} + \frac{1-\sqrt{2}}{2}i\right) - (1 + i);$

d) $(4 - (42 - 3i));$

h) $4,8 - \left(\frac{1-\sqrt{2}}{3} - i\right);$

e) $(14 + 3i) - (21 + 3i);$

i) $i - (3i + 8).$

3.9. Ko‘paytmani hisoblang:

a) $(3 + 5i)(2 + 3i)$;

b) $(4 + 7i)(2 - i)$;

d) $(5 - 3i)(2 - 5i)$;

e) $(-2 + i)(7 - 3i)$;

f) $\left(\frac{1}{2} + i\right)\left(\frac{1}{4} - i\right)$;

g) $\left(\frac{4}{7} + 3i\right)\left(\frac{7}{4} + 4,7i\right)$;

h) $(2 + 3i)(2 - 3i)$;

i) $4 \cdot (8,3 - i)$;

j) $(5 - 2i)(2i + 5)$;

k) $(-3 + i)(3 - i)$;

l) $0 \cdot (4,5 - i)$;

m) $\left(\frac{1}{3} - 0,3\right) \cdot i$.

3.10. Ikki kompleks sonning bo‘linmasini toping:

a) $\frac{1+i}{1-i}$;

b) $\frac{3-4i}{2+i}$;

d) $\frac{2+3i}{2-3i}$;

3.14. Hisoblang:

$$\text{a)} \quad \frac{(2 - 3i)(3 - 2i)}{1 + i};$$

$$\text{h)} \quad \frac{13}{1 - 4i} + \frac{11}{1 + 4i};$$

$$\text{b)} \quad \frac{(3 - i)(1 + 3i)}{2 - i};$$

$$\text{i)} \quad \frac{1 - i}{1 + i} + \frac{3 - i}{3 + i};$$

$$\text{d)} \quad \frac{3 - 4i}{(1 + i)(2 - i)};$$

$$\text{j)} \quad \frac{i^{18} + i^{19}}{2 - 3i} + \frac{1}{3 + 4i};$$

$$\text{e)} \quad \frac{2 - 3i}{(1 - i)(3 + i)};$$

$$\text{k)} \quad \frac{2 - 3i}{2 + 3i} \cdot i^{18} + \frac{i}{1 + i};$$

$$\text{f)} \quad \frac{11}{1 - 2i} - \frac{13}{2 - i};$$

$$\text{l)} \quad \frac{4i^8}{9} + i(1 + i^9);$$

$$\text{g)} \quad \frac{3 - 5}{3 + i} + \frac{2 + 3i}{2 - i};$$

$$\text{m)} \quad i^3(1 - i^4) + i^{21}.$$

3.15. Amallarni bajaring:

a) $(3 - 2i)^2$;

f) $(3 + 2i)^2 - (3 - 2i)$;

b) $(4 + 3i)^2$;

g) $(-3 + 5i) + (-3 - 5i)$;

d) $\left(\frac{1-2i}{1+i}\right)^2$;

h) $\left(\frac{i+1}{i-1}\right)^2$;

e) $\left(\frac{1+i}{1-i}\right)^2$;

i) $\left(\frac{4+i}{3-i}\right)^2$.

3.16. Qo'shma kompleks sonlar yig'indisi va ko'paytmasi haqiqiy sonlardan iborat ekanligini isbot qiling.

3.17. $z = a + bi$, $w = c + di$ kompleks sonlar berilgan: a) agar

$z + w = A \in R$ va $zw = B \in R$ bo'lsa, $w = \bar{z}$ bo'ladi; b) agar

$\frac{1}{z} + \frac{1}{w} = C \in R$ va $\frac{1}{z} \cdot \frac{1}{w} = D \in R$ bo'lsa, $w = \bar{z}$ bo'ladi.

Shuni isbot qiling.

e) $\frac{1+2i}{3-2i};$	f) $\frac{5-4i}{-3+2i};$	g) $\frac{-7+2i}{5-4i};$
h) $\frac{3-4i}{-3+2i};$	i) $\frac{14-3i}{3i+2};$	j) $\frac{51}{4-i};$
k) $\frac{4-i}{51};$	l) $\frac{31i}{17+i};$	m) $\frac{14+i}{31i};$
n) $\frac{0}{3i};$	o) $\frac{1+4i}{1-5i};$	p) $\frac{1}{1+5i}.$

3.11. Qo'shma kompleks sonlarning ko'paytmasi shaklida yozing ($a, b \in R$):

a) $a^2 + 4b^2;$	h) $11a^2 + 48b^6;$
b) $9a^2 + 25b^2;$	i) $13a^4 + 29b^8;$
d) $8a^2 + 16b^2;$	j) $a^{2n} + 33b^{2n}(n \in N);$
e) $81a^2 + 5b^2;$	k) $a^{2k} + b^{2n}(k, n \in N);$
f) $3a^2 + 45b^4;$	l) $\sqrt{3}a^2 + b^{18};$
g) $10a^2 + 56b^4;$	m) $9a^2 + \sqrt{5}b^{20}.$

3.12. Mavhum birlik i ning quyidagi darajalarini hisoblang va xulosa chiqaring:

- a) i^1 ; d) i^3 ; f) i^5 ; h) i^7 ; j) i^9 ; l) i^{11} ;
b) i^2 ; e) i^4 ; g) i^6 ; i) i^8 ; k) i^{10} ; m) i^{12} .

3.13. Amallarni bajaring:

- | | |
|--|---------------------------------------|
| a) $-3i + 5 + 8i(3 - i)$; | h) $4(0,5 - 2,5i)(3 + i) + 5i$; |
| b) $(4 + 2i)(-1 - 3i) + 5 - 8i$; | i) $4,2(3 - i)(1 + i) + 2 + 3i$; |
| d) $3i(1 + i) + 3i(3 - i)$; | j) $3 + 5i + 5i^{1999}$; |
| e) $\frac{1}{2}i(5 - 2i) + \frac{1}{3}i(9 - 8i)$; | k) $35 - i^{2000} + i^{1997}$; |
| f) $(5 - 3i)(4 + i) + 15i$; | l) $i^{2001}(3 + 5i^4)$; |
| g) $16 - (15 - i)(1 + i)$; | m) $i^{2002} - i^{2001} - i^{1999}$. |