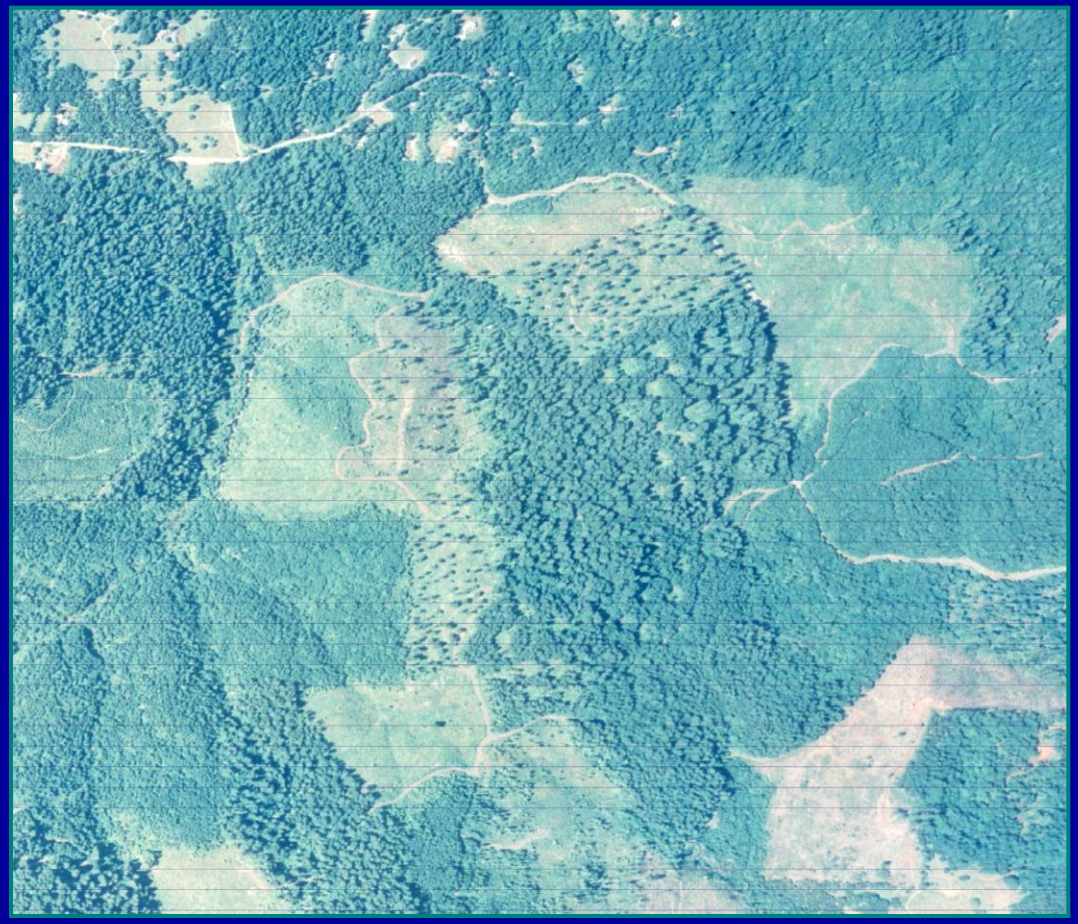
FOR 220 Aerial Photo Interpretation and Forest Measurements



Lecture 9

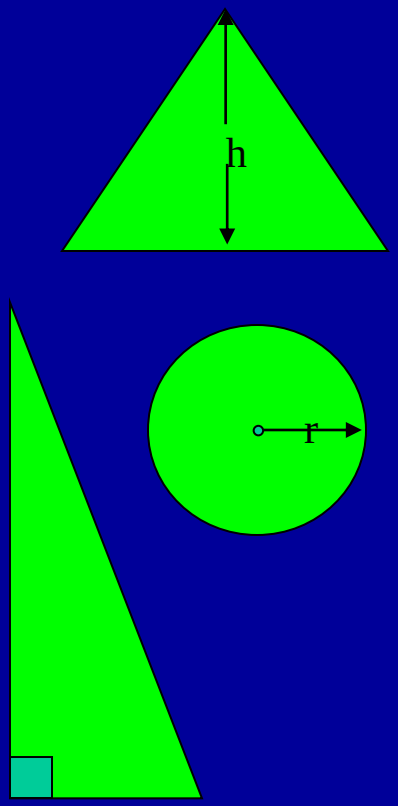
Geometry / Trigonometry Review

Avery and Burkhart
Page 432

Geometry / Trigonometry Review

Concepts used in photogrammetry

1. **Right Triangles / Similar Triangles**
2. **Pythagorean Theorem**
3. **Solution of Right Triangles**
4. **Sum of Interior Angles**
5. **Area Calculations**
6. **Synthesis**



Geometry / Trigonometry Review

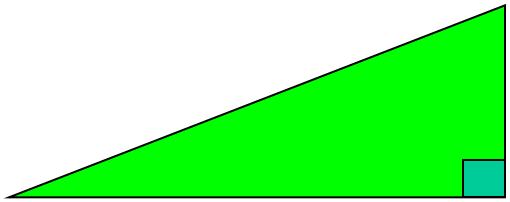
Right Triangles

One of the three interior angles is 90 degrees.

We can know 6 things about right triangles:

3 interior angles
3 side lengths

If we know two things about a right triangle, other than one of the interior angles is 90 degrees, we can figure everything else out.



Similar Right Triangles

1

Geometry / Trigonometry Review

Similar Right Triangles

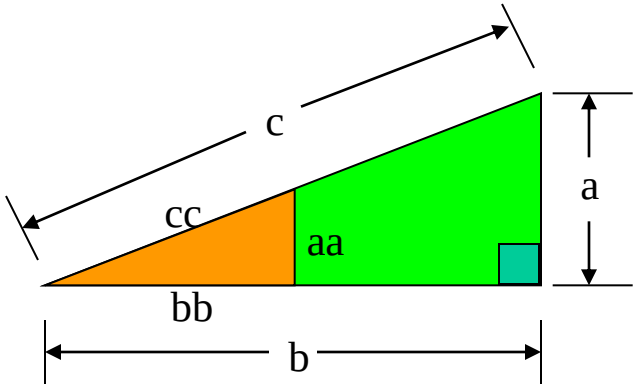
$$\left(\frac{a}{aa}\right) = \left(\frac{b}{bb}\right) = \left(\frac{c}{cc}\right)$$

Example:
b = 100 feet
bb = 30 feet
aa = 20 feet
what is the height of a?

$$\left(\frac{a}{aa}\right) = \left(\frac{b}{bb}\right)$$

$$\left(\frac{a}{20}\right) = \left(\frac{100}{30}\right)$$

$$(a) = \left(\frac{100}{30}\right) 20 = 66.7 \; feet$$



Pythagorean Theorem

2

Geometry / Trigonometry Review

Pythagorean Theorem

$$c^2 = a^2 + b^2$$

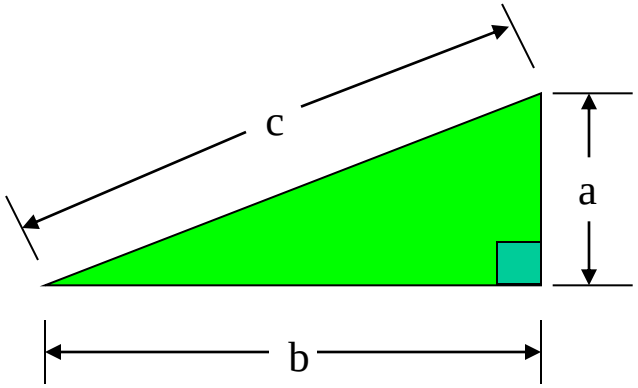
Example:
b = 100 feet
a = 66.7 feet
what is the length of C?

$$c^2 = a^2 + b^2$$

$$c^2 = 66.7^2 + 100^2$$

$$c^2 = 14{,}448.89$$

$$c = 120.2 \ \textit{feet}$$



Geometry / Trigonometry Review

Pythagorean Theorem

$$a^2 = c^2 - b^2$$
$$b^2 = c^2 - a^2$$

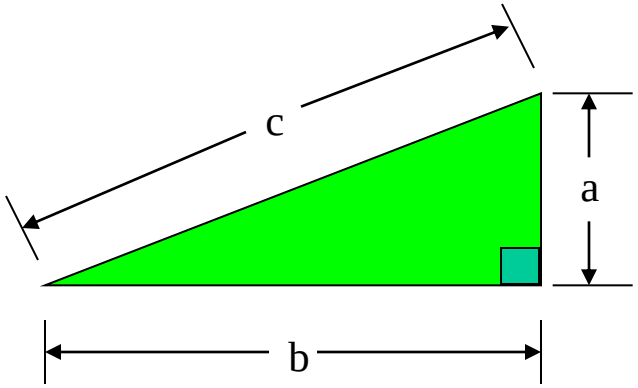
Example:
a = 150 feet
c = 175 feet
what is the length of b?

$$b^2 = c^2 - a^2$$

$$b^2 = 175^2 - 150^2$$

$$b^2 = 8{,}125$$

$$b = 90.1 \ feet$$



Solution of Right Triangles (Trigonometry)

3

Geometry / Trigonometry Review

Solution of Right Triangles

$$Sine(A) = \left(\frac{a}{c}\right) = \left(\frac{opposite\ \ side}{hypotenuse}\right)$$

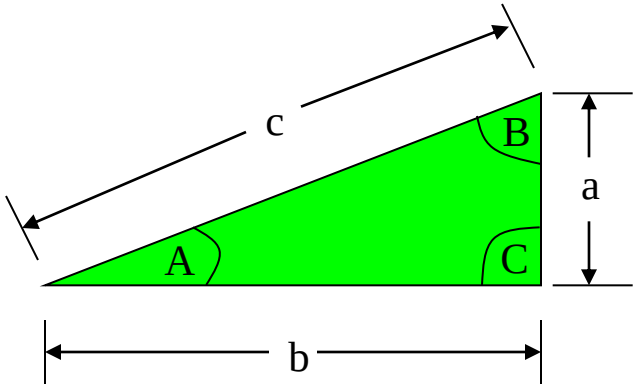
$$Cos(A) = \left(\frac{b}{c}\right) = \left(\frac{adjacent\ \ side}{hypotenuse}\right)$$

$$Tan(A) = \left(\frac{a}{b}\right) = \left(\frac{opposite\ \ side}{adjacent\ \ side}\right)$$

$$Cot(A) = \left(\frac{b}{a}\right) = \left(\frac{adjacent\ \ side}{opposite\ \ side}\right)$$

$$Sec(A) = \left(\frac{c}{b}\right) = \left(\frac{hypotenuse}{adjacent\ \ side}\right)$$

$$Cosec(A) = \left(\frac{c}{a}\right) = \left(\frac{hypotenuse}{opposite\ \ side}\right)$$



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Geometry / Trigonometry Review

Solution of Right Triangles

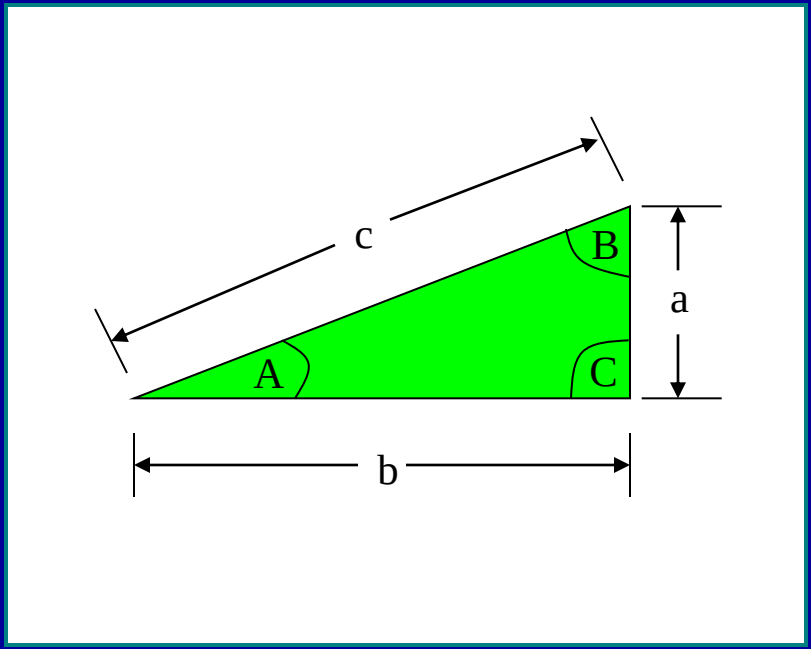
Example:
b = 100 feet
A = 20 degrees
what is the height of "a"?

What do we know?
c is the hypotenuse
(we don't know)
a is the opposite (we want)
b is the adjacent (we know)

$$Tan(A) = \left(\frac{a}{b}\right) = \left(\frac{opposite\ \ side}{adjacent\ \ side}\right)$$

$$Tan(20) = \left(\frac{a}{100}\right)$$

$$0.3639\ (100) = (a) = 36.39\ \ feet$$



Sum of Interior Angles

4

Geometry / Trigonometry Review

Sum of Interior Angles

$$180 = \left(\frac{(Sum\ \ of\ \ angles)}{(number\ \ of\ \ angles - 2)} \right)$$

Example:
C = 90 degree angle
A = 30 degree angle
what is the angle of B?

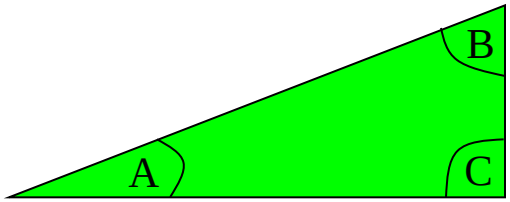
$$180 = \left(\frac{(30 + 90 + B)}{(3 - 2)} \right)$$

$$180 = \left(\frac{(120 + B)}{1} \right)$$

$$180 = (120 + B)$$

$$180 - 120 = B$$

$$60 = B$$



Geometry / Trigonometry Review

Sum of Interior Angles - Proof on a square

$$180 = \left(\frac{(Sum\ \ of\ \ angles)}{(number\ \ of\ \ angles - 2)} \right)$$

Example:
A = B = C = 90 degrees
what is the angle of D?

$$180 = \left(\frac{(90 + 90 + 90 + D)}{(4 - 2)} \right)$$

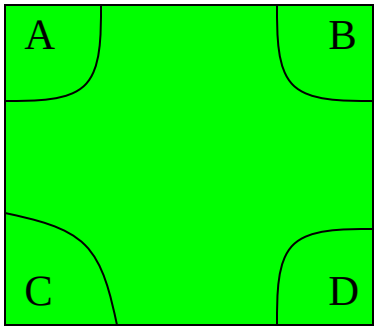
$$180 = \left(\frac{(270 + D)}{2} \right)$$

$$180 = \left(\left(\frac{(270)}{2} \right) + \left(\frac{D}{2} \right) \right)$$

$$180 - 135 = \left(\frac{D}{2} \right)$$

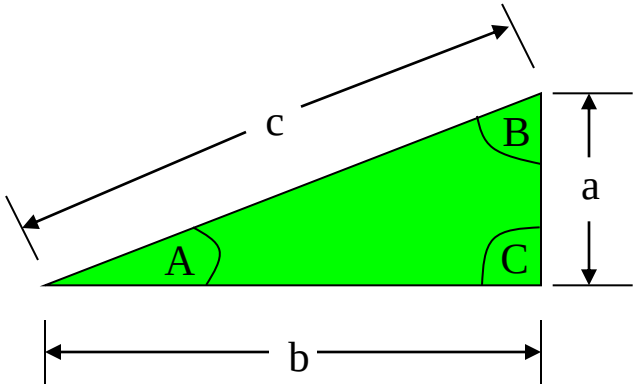
$$45 = \left(\frac{D}{2} \right)$$

$$90 = D$$



BOTTOM LINE - SUMMARY - TAKE HOME MESSAGE

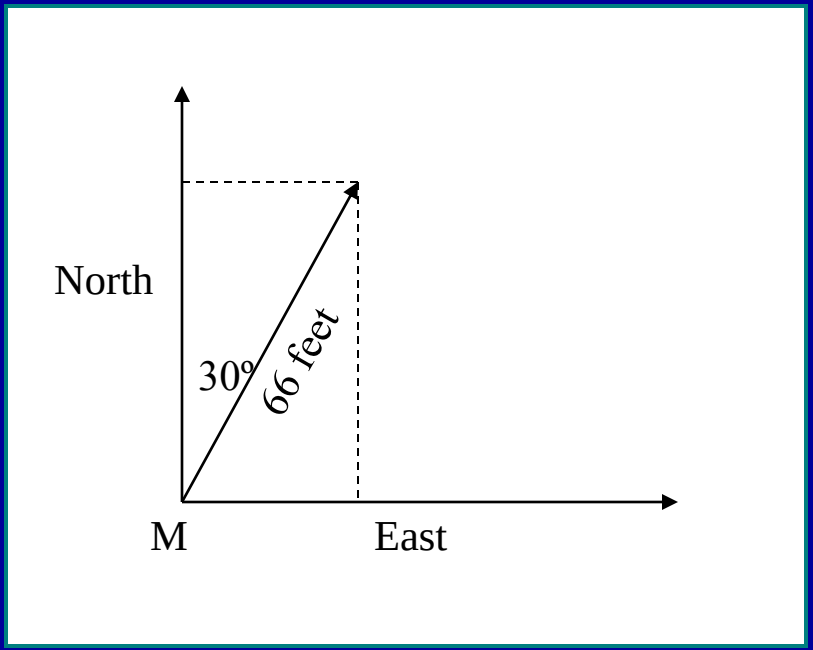
If you know two pieces of information (other than the right angle, which is always 90 degrees), you can figure out everything else about a right triangle.



Geometry / Trigonometry Review

EXAMPLE #1: Bearings and azimuths

You are starting from point M, on an azimuth of 30 degrees. You travel 66 feet on this azimuth. How far in North and East directions have you moved?



Geometry / Trigonometry Review

EXAMPLE #1: Bearings and azimuths

What do we know?
hypotenuse = d = 66 feet
X angle is 30 degrees

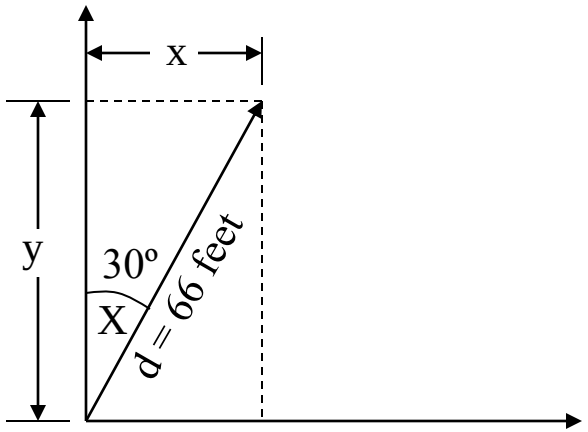
What do we need to know?
We need to determine the opposite (x) and adjacent (y) side distances.

$$Sine(X) = \left(\frac{x}{d}\right) = \left(\frac{opposite\ \ side}{hypotenuse}\right)$$

$$x = 33\ \ feet, or\ 33\ \ feet\ \ to\ the\ \ East$$

$$Cos(X) = \left(\frac{y}{d}\right) = \left(\frac{adjacent\ \ side}{hypotenuse}\right)$$

$$y = 57.2\ \ feet, or\ 57.2\ \ feet\ \ to\ the\ \ North$$

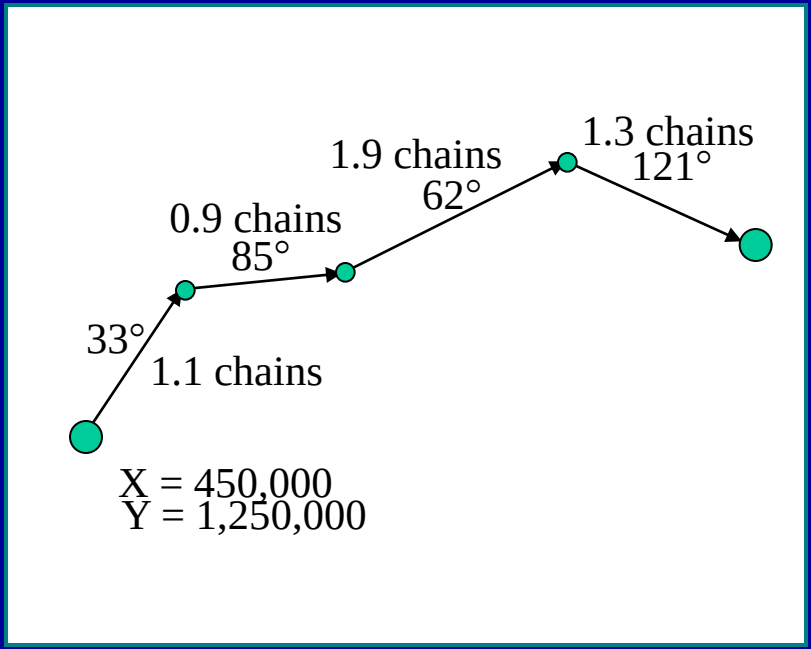


Geometry / Trigonometry Review

EXAMPLE #2: Bearings and azimuths

You start from a known coordinate on a trail, and traverse (compass and pace) a certain number of bearings and distances.

How would you calculate the coordinates at each vertex (station)?



Area Calculations

5

Geometry / Trigonometry Review

Area Calculations for Right Triangles

$$Area = \left(\frac{(a)(b)}{2} \right)$$

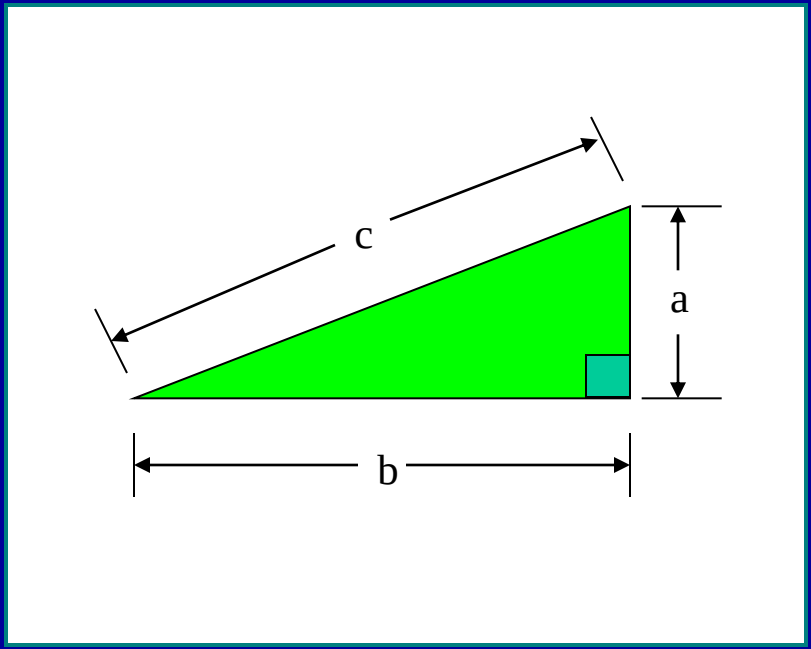
Example:
a = 150 feet
b = 90.1 feet
what is the area of the triangle?

$$Area = \left(\frac{(a)(b)}{2} \right)$$

$$Area = \left(\frac{(150)(90.1)}{2} \right)$$

$$Area = 6{,}757.5 \ square \ feet$$

$$Area = 0.155 \ acres$$



Geometry / Trigonometry Review

Area Calculations for Triangles

$$Area = \left(\frac{(b)(h)}{2} \right)$$

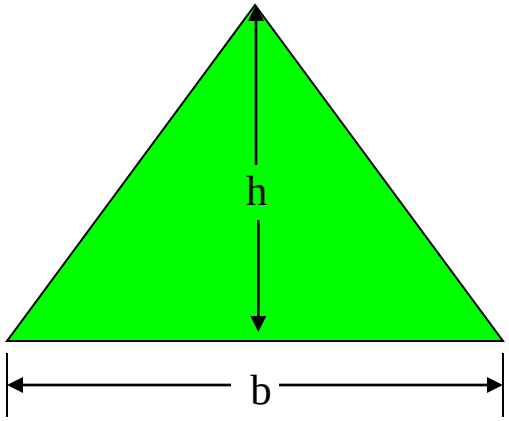
Example:
b = 150 feet
h = 100 feet
what is the area of the triangle?

$$Area = \left(\frac{(b)(h)}{2} \right)$$

$$Area = \left(\frac{(150)(100)}{2} \right)$$

$$Area = 7{,}500 \ square \ feet$$

$$Area = 0.172 \ acres$$



Geometry / Trigonometry Review

Area Calculations for Circles

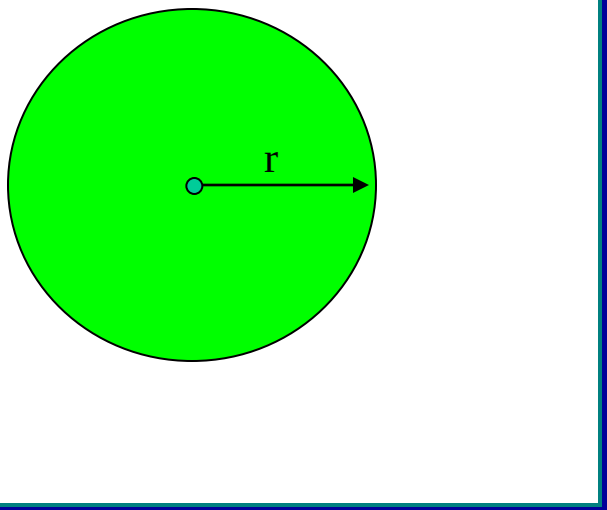
$$Area = \pi \ r^2$$

Example:
r = 50 feet
what is the area of the circle?

$$Area = \pi \ 50^2$$

$$Area = 7{,}853.98 \ square \ feet$$

$$Area = 0.18 \ acres$$



Geometry / Trigonometry Review

Area Calculations for Circles

$$Area = \pi \ r^2$$

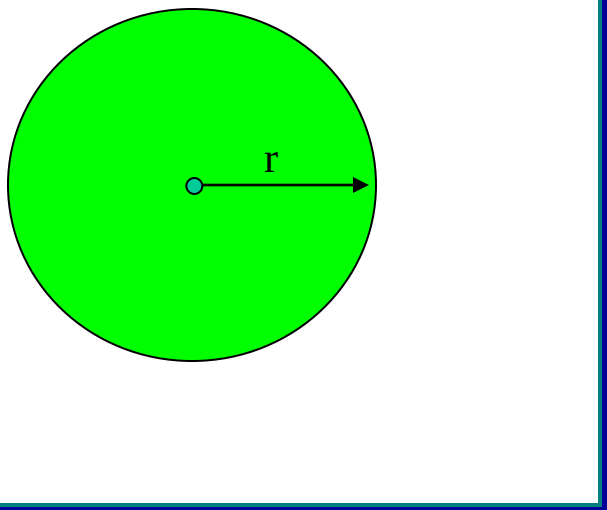
Example:
You are measuring the trees within a 1/10 acre fixed plot. What is the radius (in feet) of the plot?

$$0.10 \ acres = \pi \ r^2$$

$$4{,}356 \ square \ feet = \pi \ r^2$$

$$1{,}386.56 \ square \ feet = r^2$$

$$37.2 \ feet = r$$



Putting it all Together

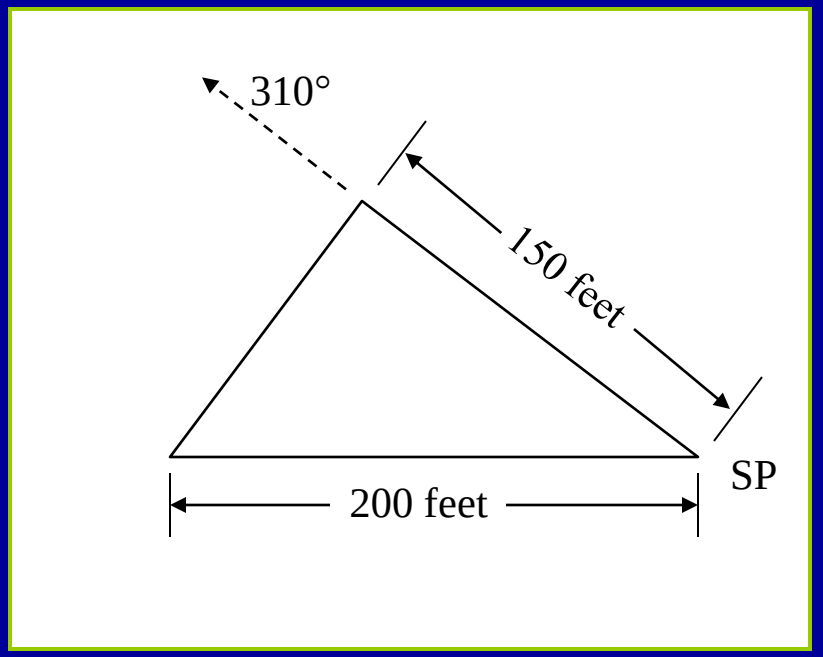
6

Geometry / Trigonometry Review

EXAMPLE #3: Distances and Areas

You only know a little bit about an area, that two sides are a certain number of feet from a starting point, that one side runs 200 ft. directly East-West, and that another is on an azimuth of 310°, and runs for 150 feet. There is only one more side, forming a triangle.

What is the length of the third side, the angle associated with all corners, and the area of the triangle.



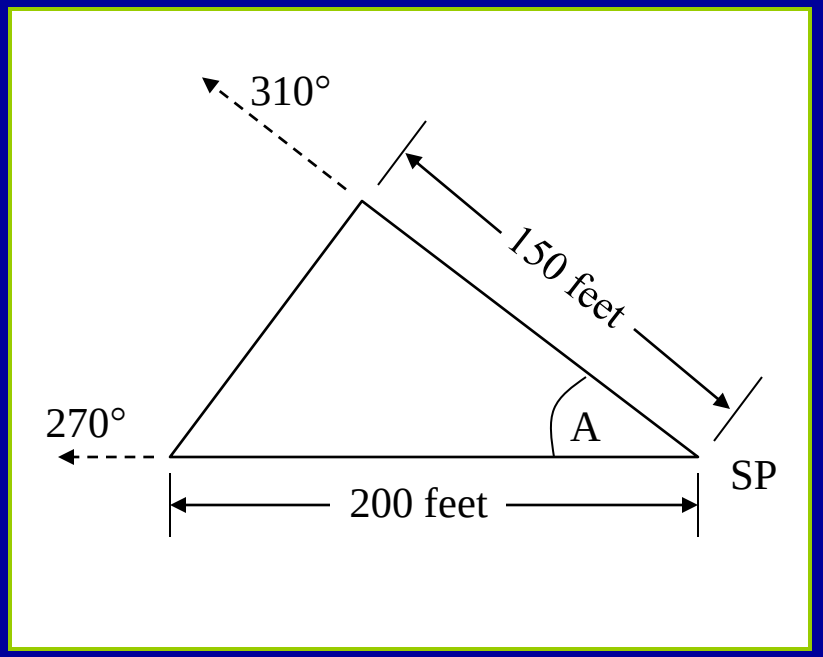
Note:

Do not assume that a triangle that appears to be a right triangle is actually a right triangle. All right triangles in this class will be clearly marked.

Geometry / Trigonometry Review

EXAMPLE #3: Distances and Areas

What do we know?
A = 40 degrees



Geometry / Trigonometry Review

EXAMPLE #3: Distances and Areas

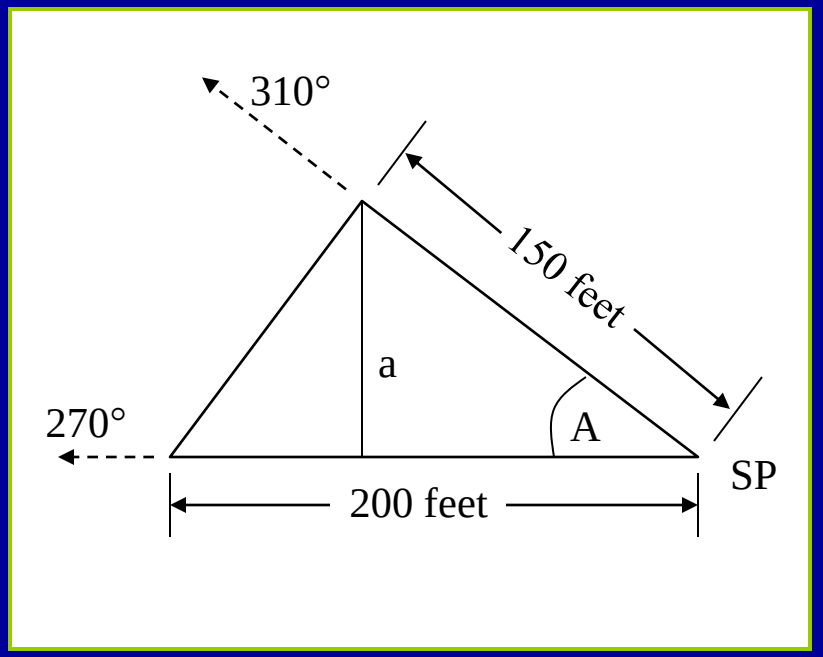
What do we know?

By creating two separate right triangles, we can calculate “a” or the height of the original triangle.

$$Sine(A) = \left(\frac{opposite\ \ side}{hypotenuse} \right)$$

$$0.642788 = \left(\frac{a}{150} \right)$$

$$96.42\ \ feet\ = a$$



Geometry / Trigonometry Review

EXAMPLE #3: Distances and Areas

What do we know?

We can also calculate "b"

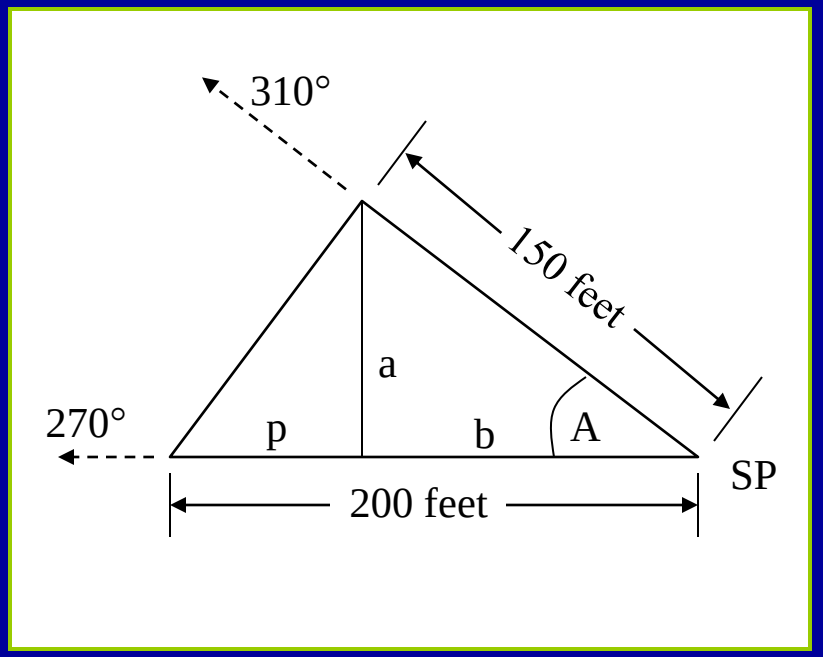
$$Cos(A) = \left(\frac{adjacent \;\; side}{hypotenuse} \right)$$

$$0.766044 = \left(\frac{b}{150} \right)$$

$$114.91 \;\; feet \;\; = \;\; b$$

and "p"

$$p = (200 - b) = 85.09 \;\; feet$$



Geometry / Trigonometry Review

EXAMPLE #3: Distances and Areas

What do we know?

We can now calculate angle A’

$$Tan(A') = \left(\frac{opposite\ \ side}{adjacent\ \ side}\right) = \left(\frac{a}{p}\right)$$

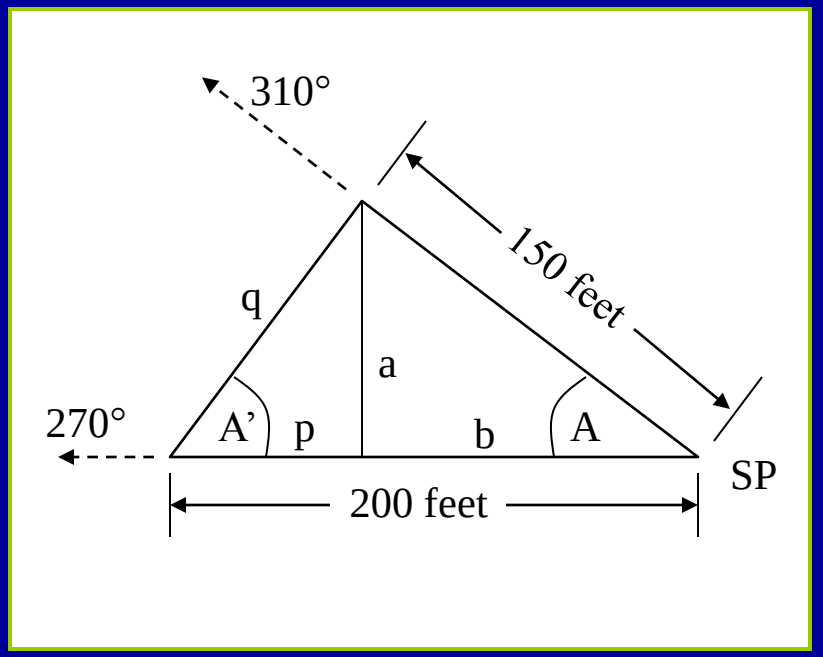
$$Tan(A') = \left(\frac{96.42}{85.09}\right) = 1.1332$$

$$A' = 48.6^{\circ}$$

and side “q”

$$q^2 = p^2 + a^2$$

$$q = 128.6\ \ feet$$



Geometry / Trigonometry Review

EXAMPLE #3: Distances and Areas

What do we know?

Angle A = 40°
Angle A' = 48.6°

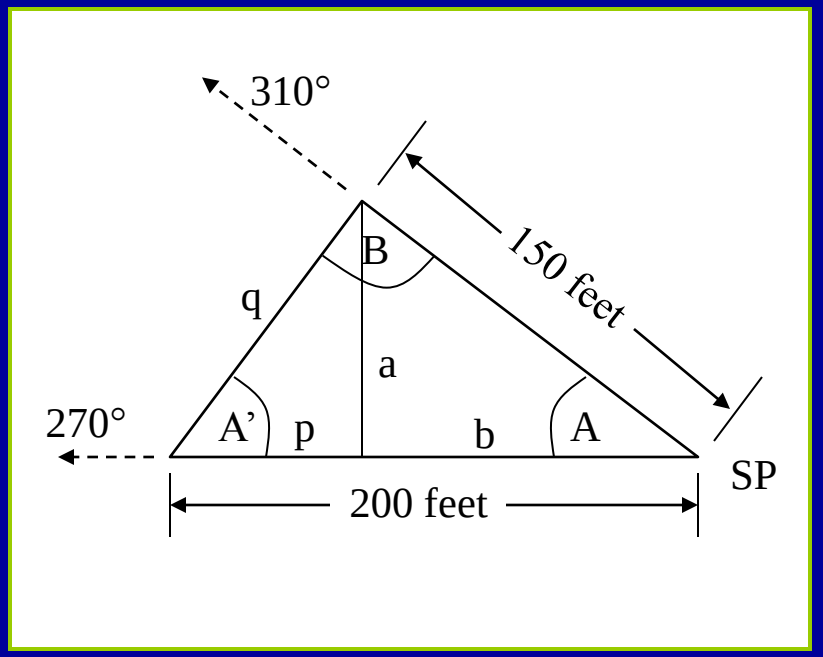
$$180 = \left(\frac{(Sum\ \ of\ \ angles)}{(number\ \ of\ \ angles - 2)} \right)$$

$$180 = \left(\frac{(40 + 48.6 + B)}{(3 - 2)} \right)$$

$$180 = \left(\frac{(88.6 + B)}{(1)} \right)$$

$$180 - 88.6 = B$$

$$91.4^{\circ} = B$$



Geometry / Trigonometry Review

EXAMPLE #3: Distances and Areas

Finally, the area:

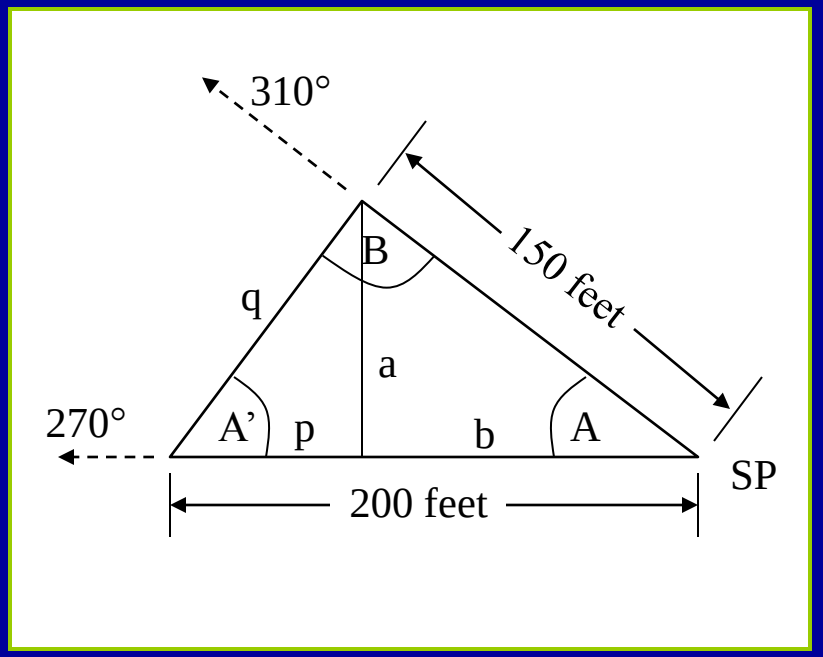
$$Area = \left(\frac{(b)(h)}{2} \right)$$

here:
b = 200 feet
h = a = 96.42 feet

$$Area = \left(\frac{(200)(96.42)}{2} \right)$$

$$Area = 9{,}642 \ square \ \ feet$$

$$Area = 0.22 \ acres$$



Geometry / Trigonometry Review

EXAMPLE #3: Distances and Areas

Summary:

The length of the third side:

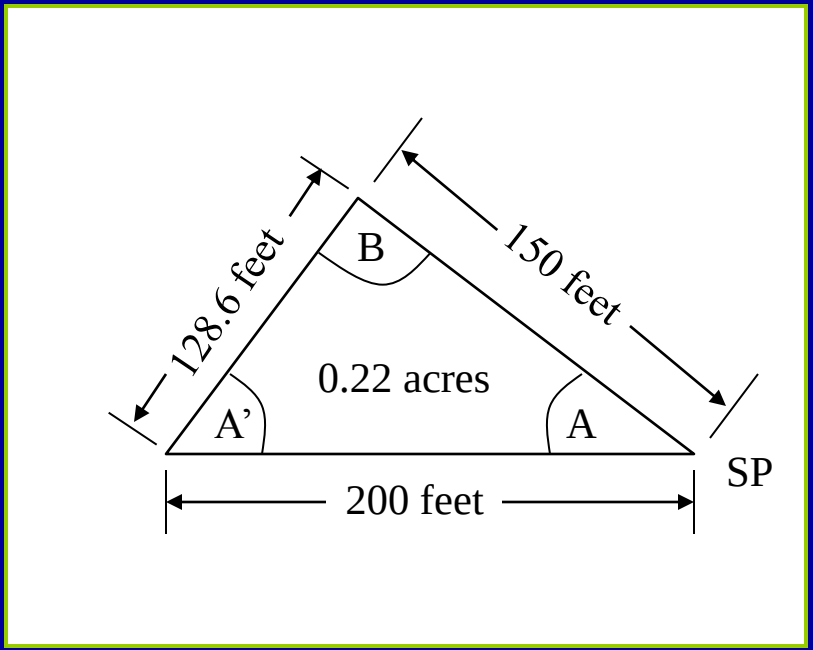
128.6 feet

The angle associated with all corners:

A = 40°, A' = 48.6 °,
B = 91 °

The area of the triangle:

0.22 acres

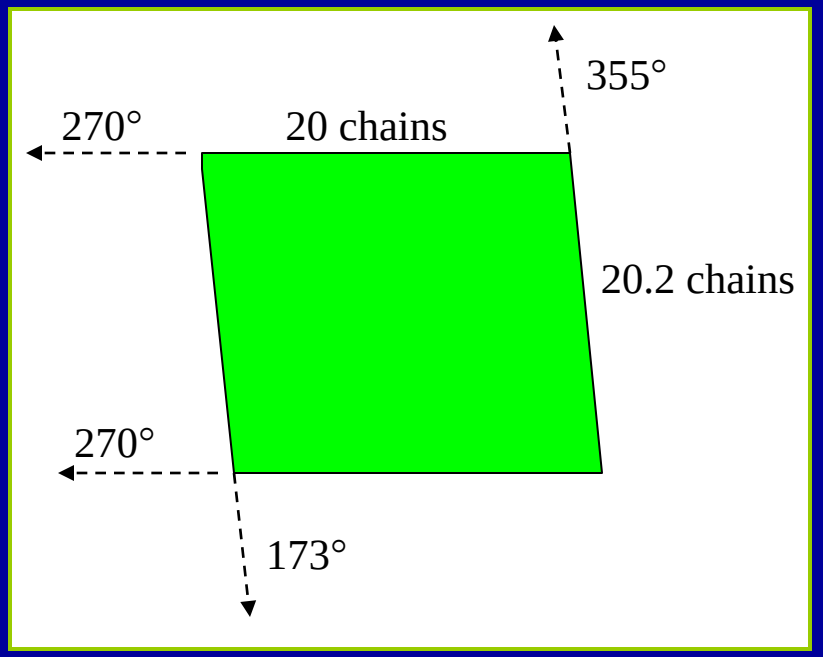


Geometry / Trigonometry Review

EXAMPLE #4: Distances and Areas

Calculate the area of this landscape unit:

How would you proceed?



What is going on here?