Mavzu: Fazoda analitik geometriyaning asosiy tushunchlari va masalalari

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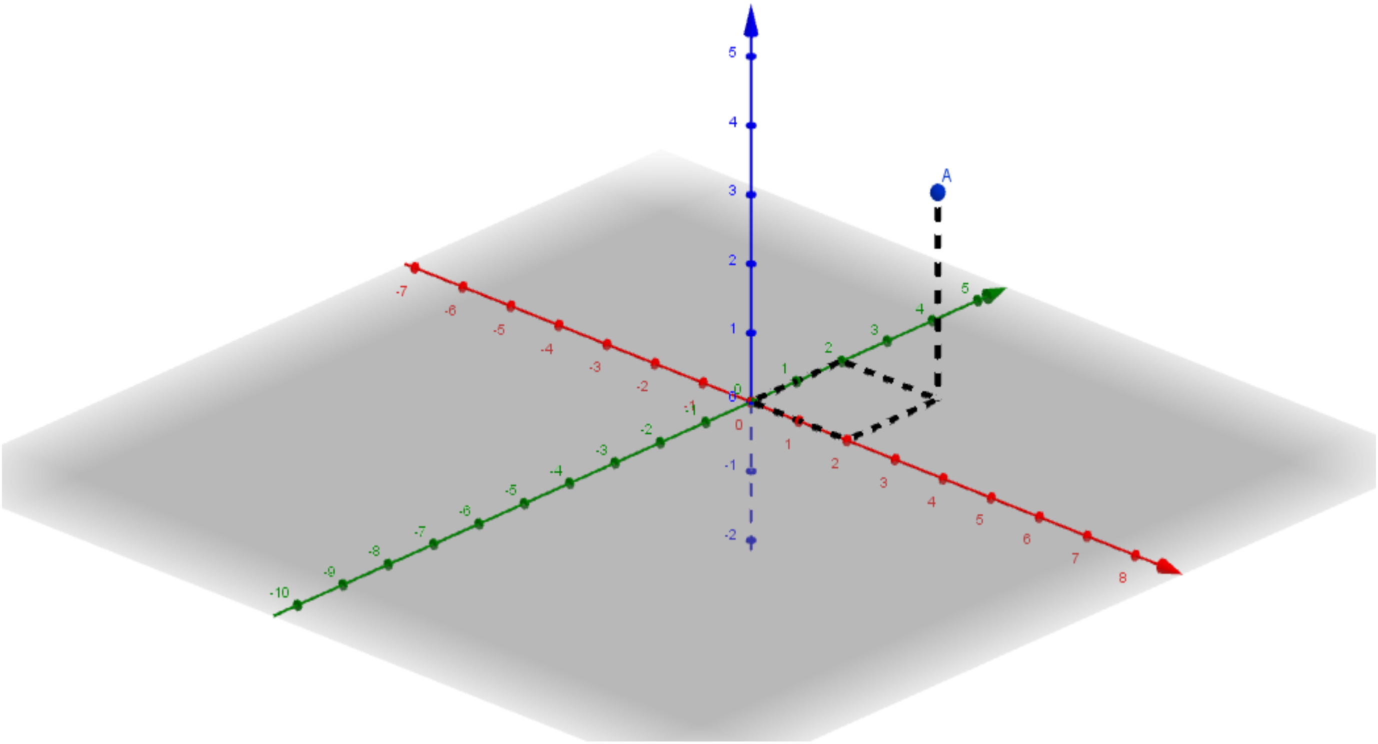
1. Kirish

Biz tekislikda analitik geometriyaning asosiy tushunchalari va sodda masalalari bilan shug'ullandik. Ma'lumki, bizni o'rab turgan borliq (uch o'lchovli fazo) bo'lib, bizga ko'rinib turgan real jismlar shu fazoda ma'lum bir o'rinni egallaydi. Fazoda ularning holatini aniqlash uchun xuddi tekislikdagi kabi Dekart koordinatalar sistemasi kiritiladi. Bizga masshtab birligi bilan berilgan o'zaro perpendikulyar hamda bitta O nuqtada kesishuvchi O_x, O_y, O_z to'g'ri chiziqlar sistemasi berilgan bo'lsin. Odatda bu sistema fazoda *Dekart koordinatalar sistemasi* deyiladi va O_{xyz} kabi belgilanadi.

O nuqta koordinatalar boshi, O_x $-abstsissalar$ $o'qi$, O_y $-ordinatalar$ $o'qi$, O_z $-applikatalar$ $o'qi$ deyiladi. Fazoda biror A nuqtaning holati uning O_x, O_y, O_z o'qlarga proektsiyalari $-(x, y, z)$ uchlik bilan to'la aniqlanadi.

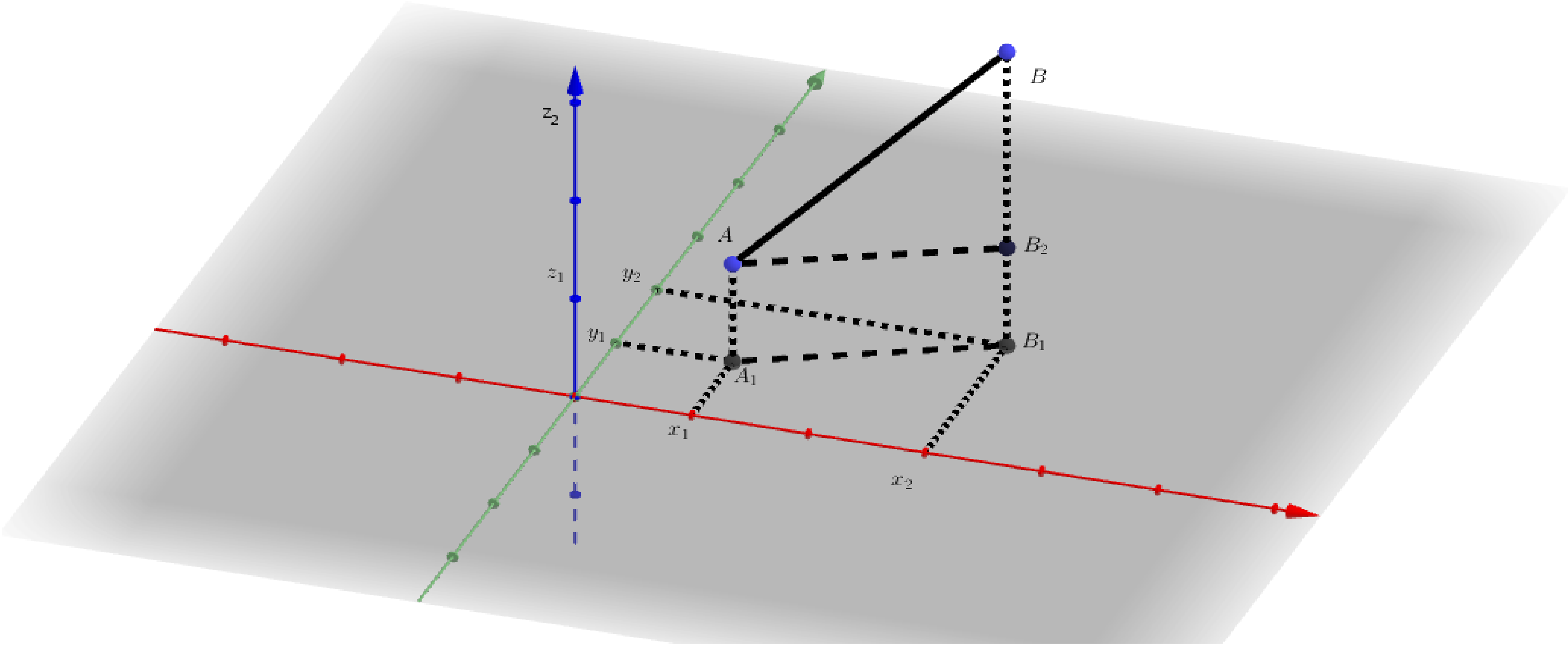


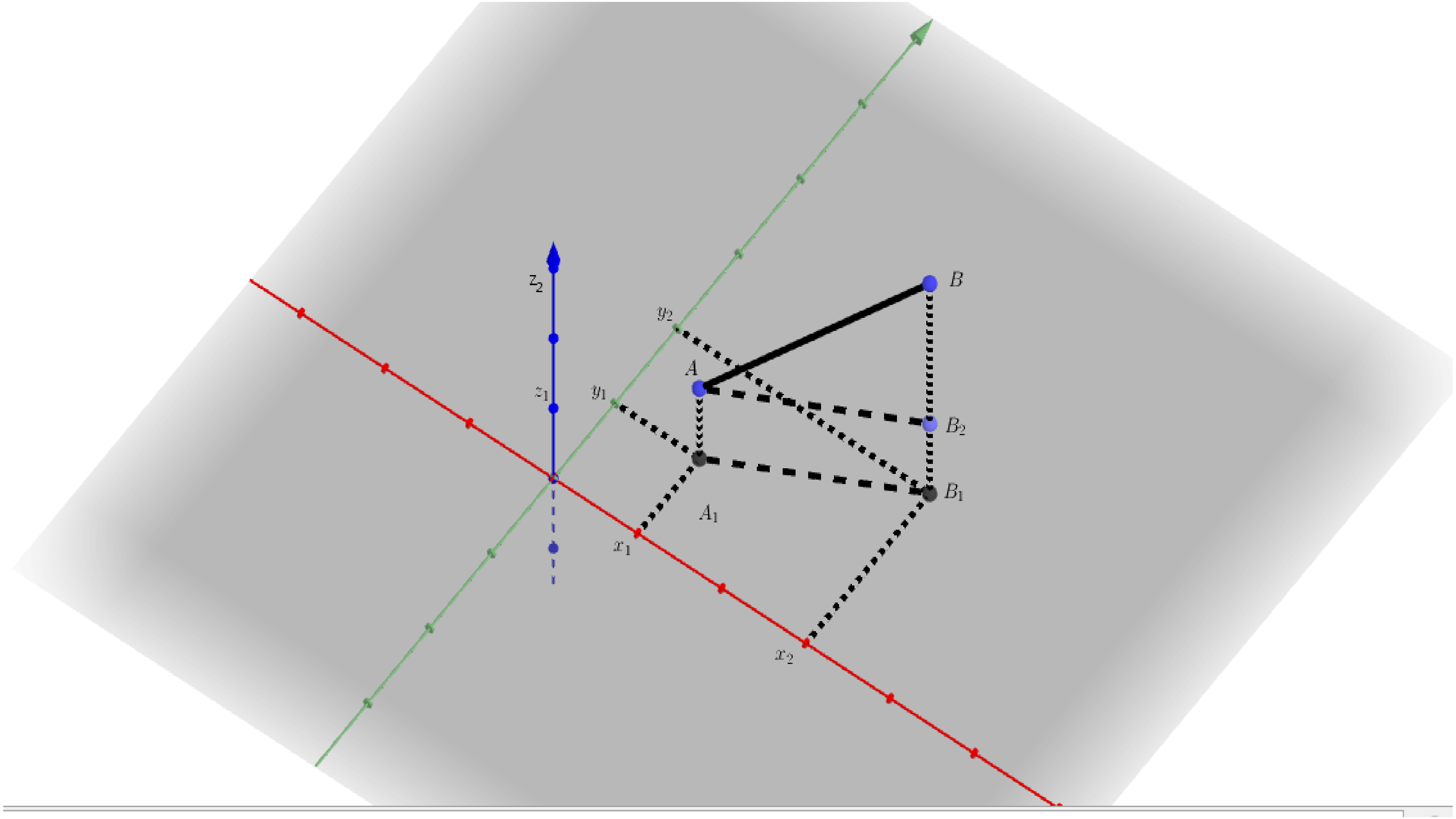
2. Ikki nuqta orasidagi masofa

Fazoda Dekart koordinatalar sistemasi va $A(x_1, y, z_1)$, $B(x_2, y_2, z_2)$ nuqtalar berilgan. Bu nuqtalar orasidagi masofani topamiz. A_1 va B_1 nuqtalar mos ravishda A va B ning O_{xy} tekislikdagi proektsiyalari bo'lsin. Tekislikda ikki nuqta orasidagi masofa formulasiga ko'ra

$A_1B_1 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ bo'ladi. A nuqtadan A_1B_1 kesmaga parallel chiziq o'tkazib, uni B_2 bilan belgilaymiz. U holda BB_2 kesmaning uzunligi $z_2 - z_1$ ga teng.

$$AB = \sqrt{(AB_2)^2 + (BB_2)^2} =$$
$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$





z_2
z_1
y_2
y_1
A
B
B_2
B_1
A_1
x_1
x_2

3. Fazoda tekislik va uning tenglamasi

Faraz qilaylik, fazoda Dekart koordinatalar sistemasi, $P(a_1, b_1, c_1)$ hamda $Q(a_2, b_2, c_2)$ nuqtalar berilgan bo'lsin. Bu ikki nuqtadan bir xil masofada joylashgan nuqtalarning geometrik o'rni tekislikni ifodalaydi. Bu tekislikda ixtiyoriy $M(x, y, z)$ nuqtani olaylik. Ikki nuqta orasidagi masofani topish formulasiga ko'ra

$$MP = \sqrt{(x - a_1)^2 + (y - b_1)^2 + (z - c_1)^2},$$

$$MQ = \sqrt{(x - a_2)^2 + (y - b_2)^2 + (z - c_2)^2}$$

bo’ladi. Agar $MP = MQ$ bo’lishini e’tiborga olsak, unda

$$\sqrt{(x-a_1)^2+(y-b_1)^2+(z-c_1)^2}=$$

$$=\sqrt{(x-a_2)^2+(y-b_2)^2+(z-c_2)^2}$$

Bu tenglikning ikkala tomonini kvadratga oshiramiz:

$$a_1^2+b_1^2+c_1^2-2a_1x-2b_1y-2c_1z+x^2+y^2+z^2=$$

$$=a_2^2+b_2^2+c_2^2-2a_2x-2b_2y-\ 2c_2z+x^2+y^2+z^2$$

Bu tenglikni quyidagicha ham yzish mumkin.

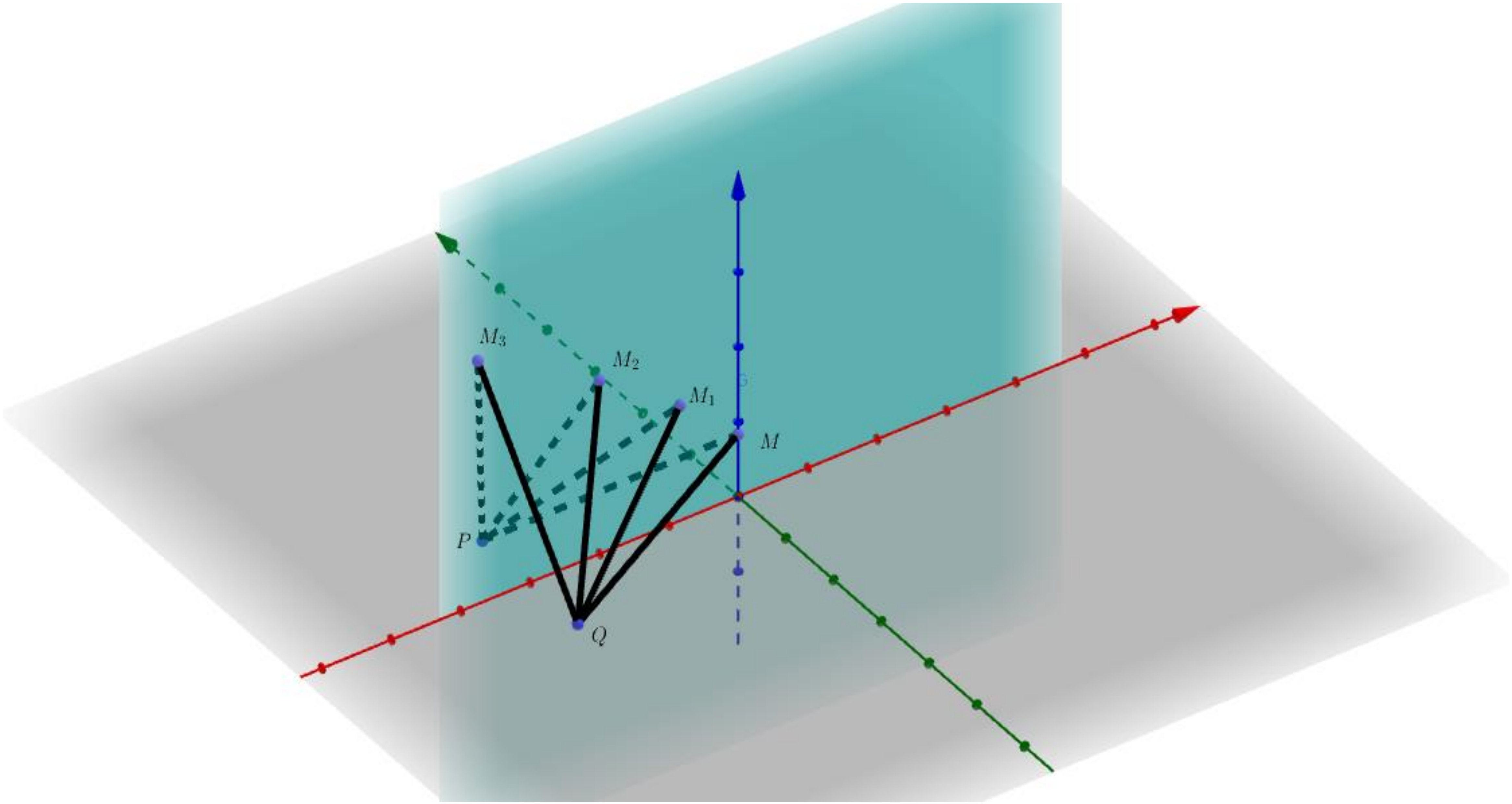
$$2(a_2 - a_1)x + 2(b_2 - b_1)y + 2(c_2 - c_1)z +$$

$$+a_1^2 + b_1^2 + c_1^2 - a_2^2 - b_2^2 - c_2^2 = 0$$

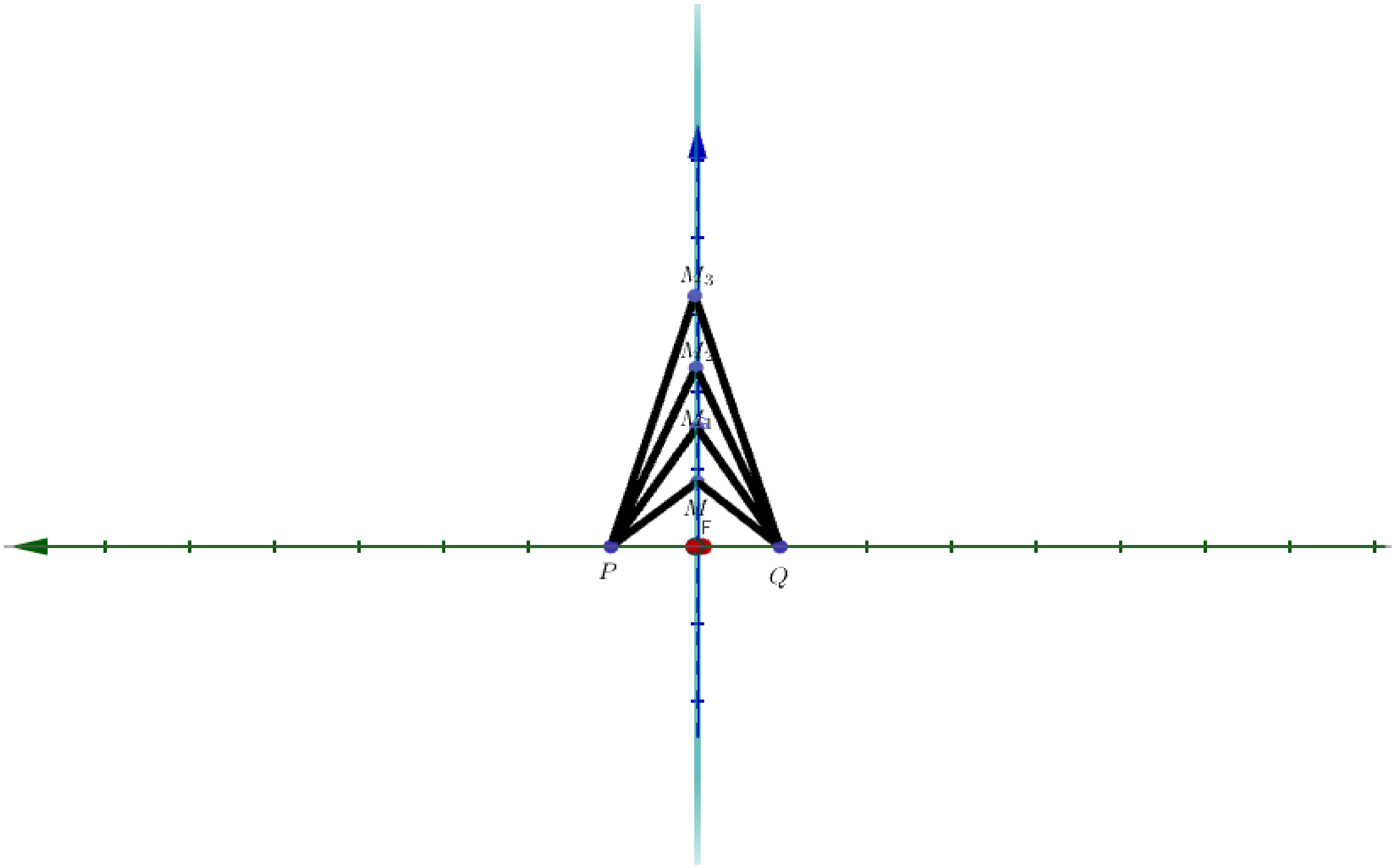
$A = 2(a_2 - a_1), B = 2(b_2 - b_1), C = 2(c_2 - c_1),$
$D = a_1^2 + b_1^2 + +c_1^2 - a_2^2 - b_2^2 - c_2^2$ belgilashlarni kiritsak, ushbu

$$Ax + By + Cz + D = 0 \qquad (1)$$

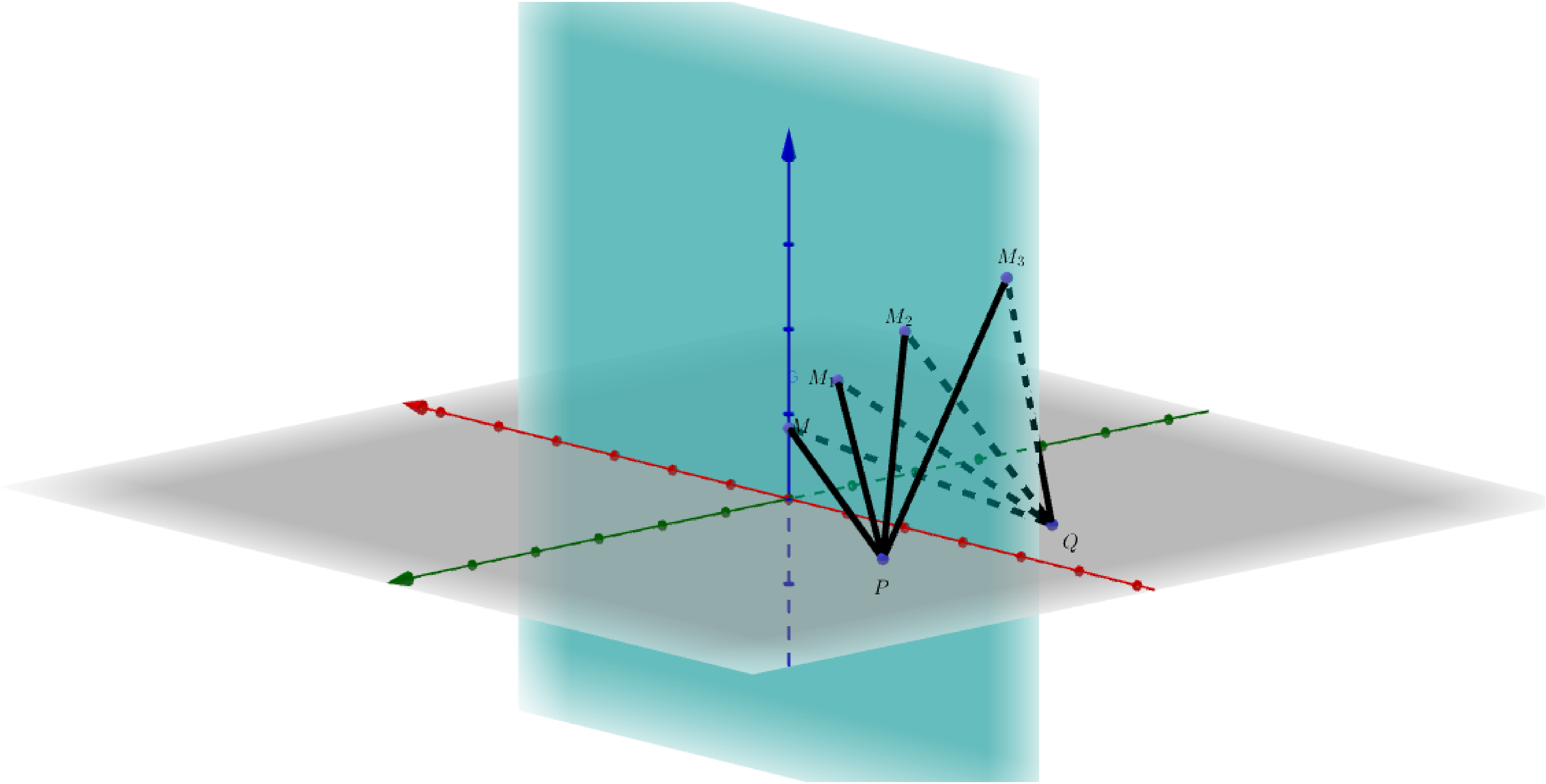
tenglamaga kelamiz. (1) tenglama fazoda *tekisliknig umumiy tenglamasi deyiladi.*



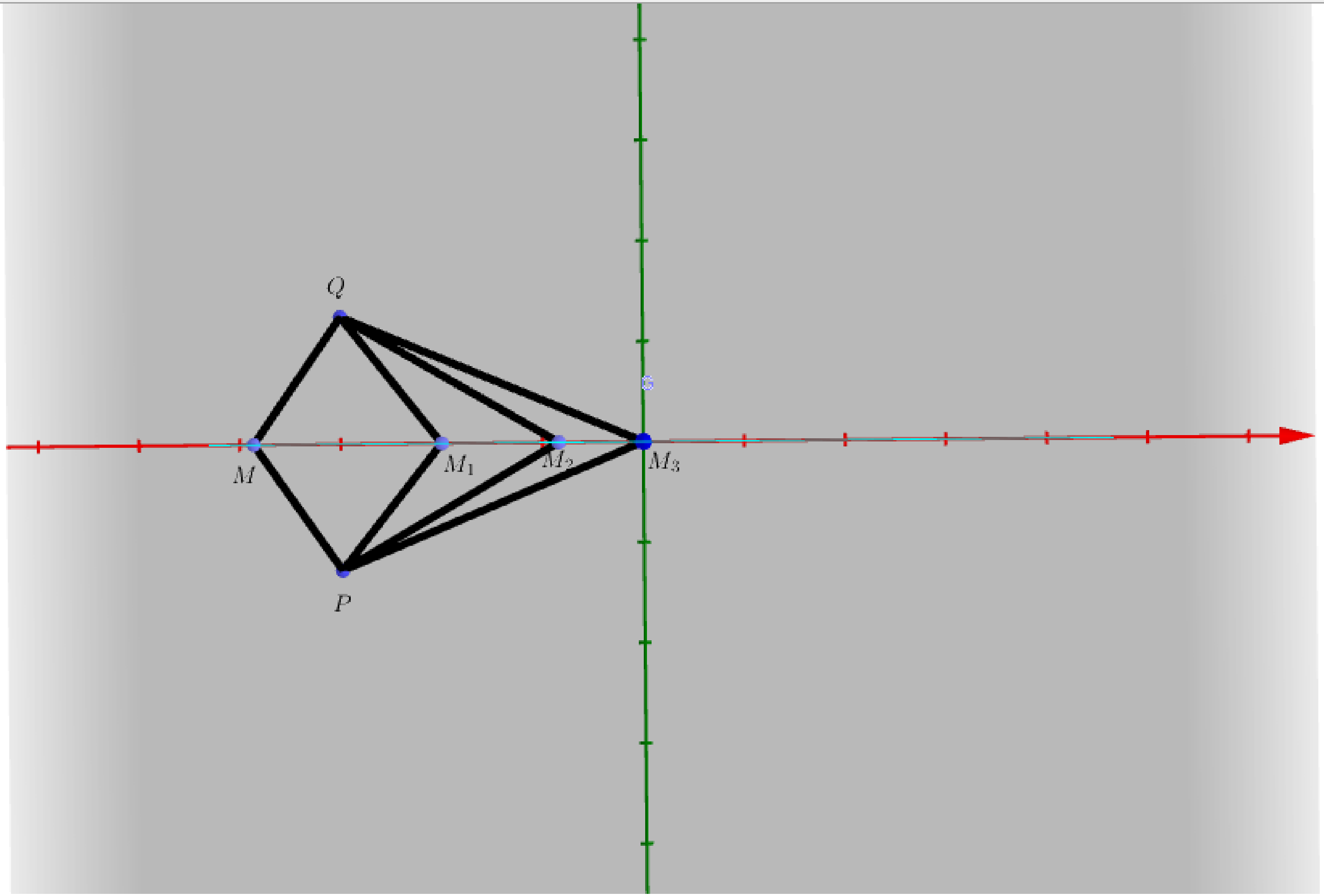
M_3
M_2
M_1
M
P
Q



M_3
M_2
M_1
M
F
P
Q



M_3
M_2
M_1
M
Q
P

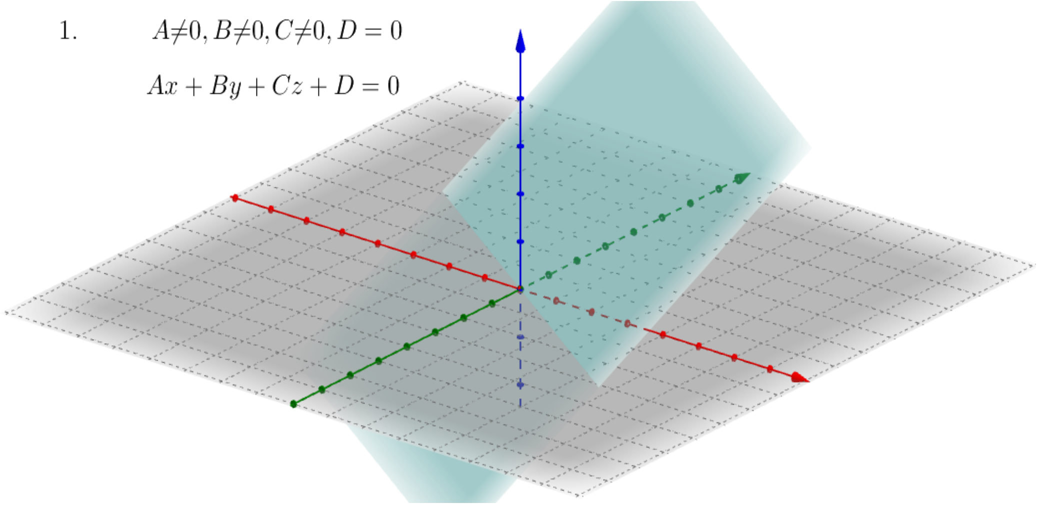

Q
M
M1
M2
M3
P

Bu yerda A, B, C, D o'zgarmas sonlar bo'lib, ular tekislikning fazodagi vaziyatini to'la aniqlaydi.

Endi (1) tenglamaning xususiy hollarini qaraylik.

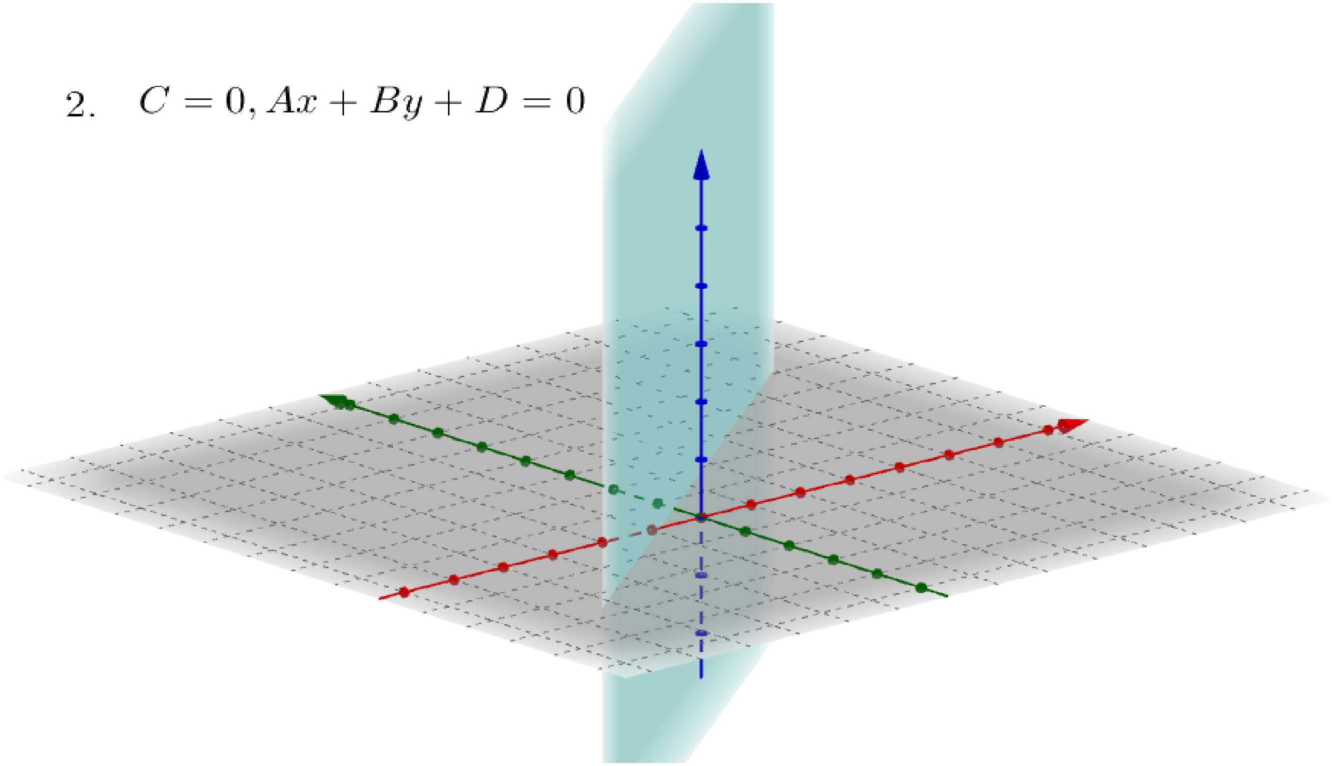
1°. $A \neq 0, B \neq 0, C \neq 0, D = 0$ bo'lsin.

U holda $Ax + By + Cz = 0$ tenglama hosil bo'lib, bu tenglama bilan aniqlangan tekislik koordinatalar boshi $O(0,0,0)$ nuqtadan o'tadi.



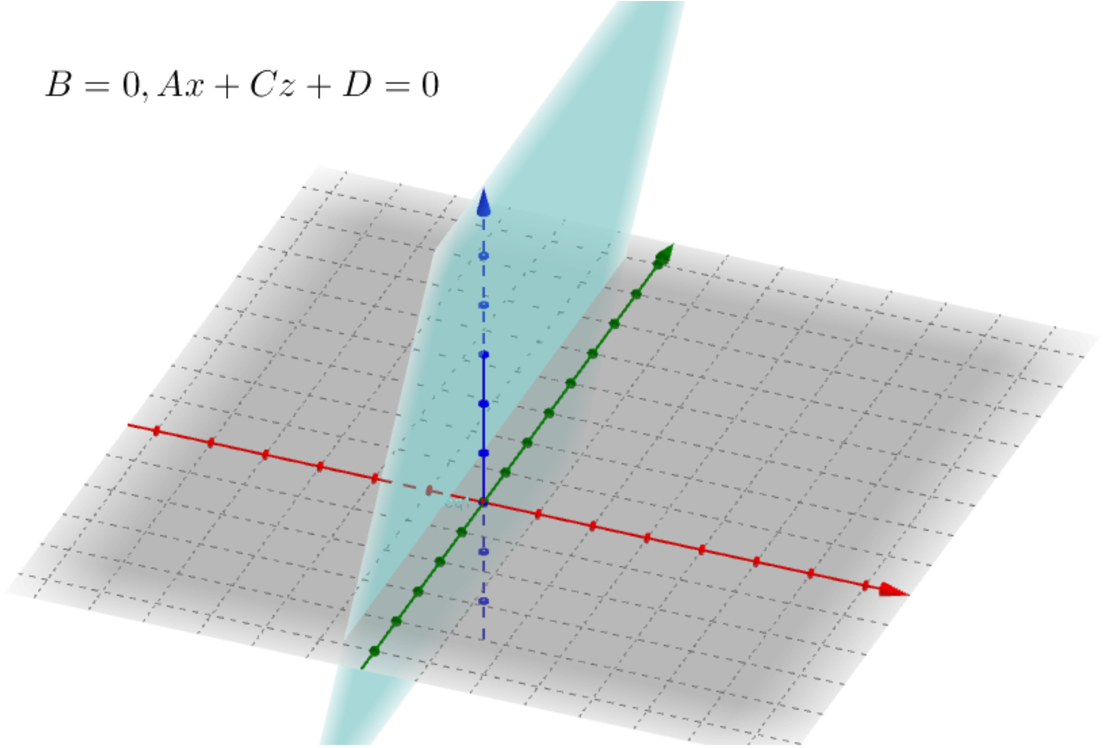
$2^{\circ}.\ A \neq 0, B \neq 0, D \neq 0, C = 0.$

Bu holda biz $Ax + By + D = 0$ tenglamaga ega bo'lamiz. Bu tenglama bilan aniqlangan tekislik Oz o'qiga parallel tekislikdir.



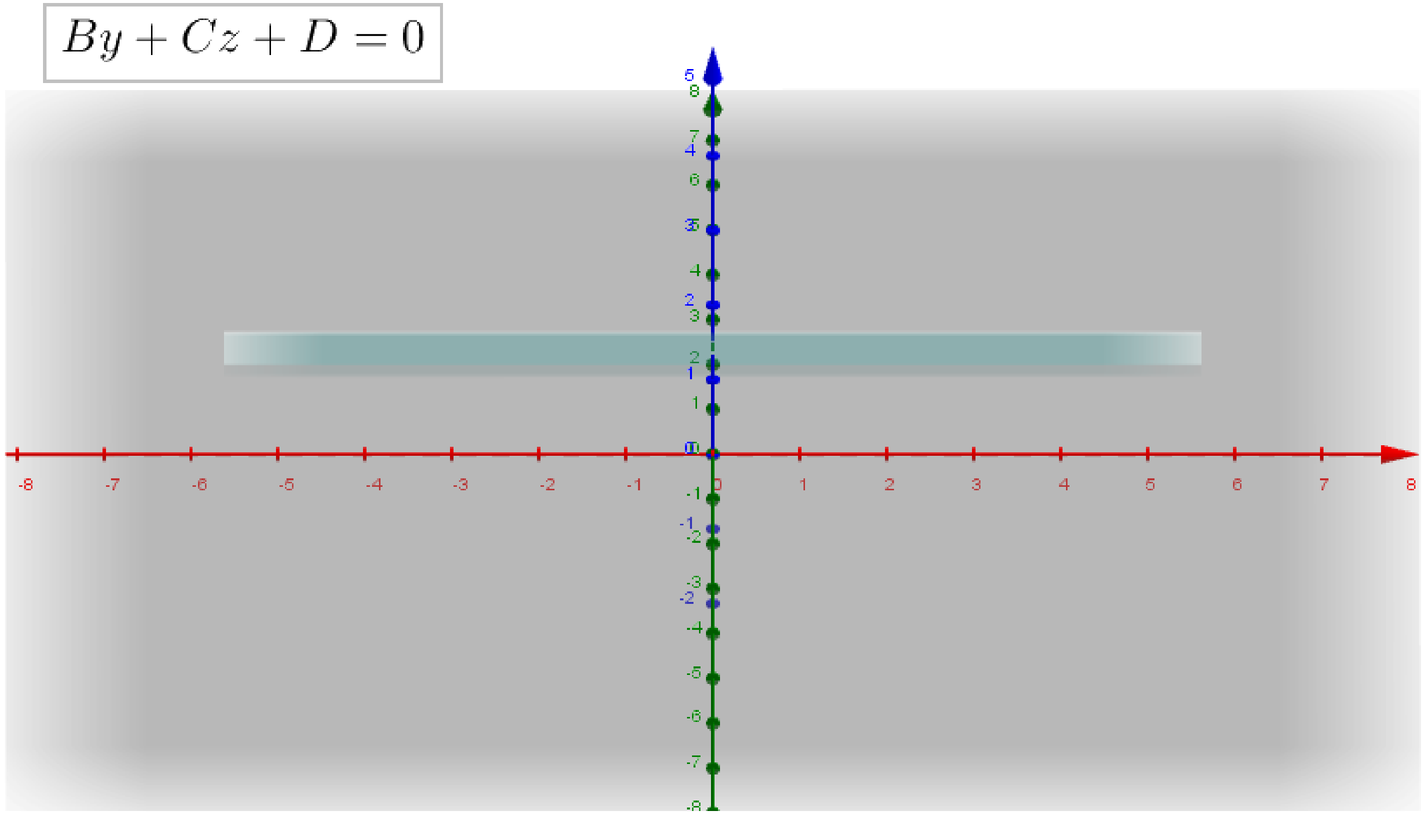
$3^{\circ}.\ A \neq 0, C \neq 0, D \neq 0, B = 0.$

Bu holda $Ax + Cz + D = 0$ tekislik Oy o’qiga parallel tekislikdir.

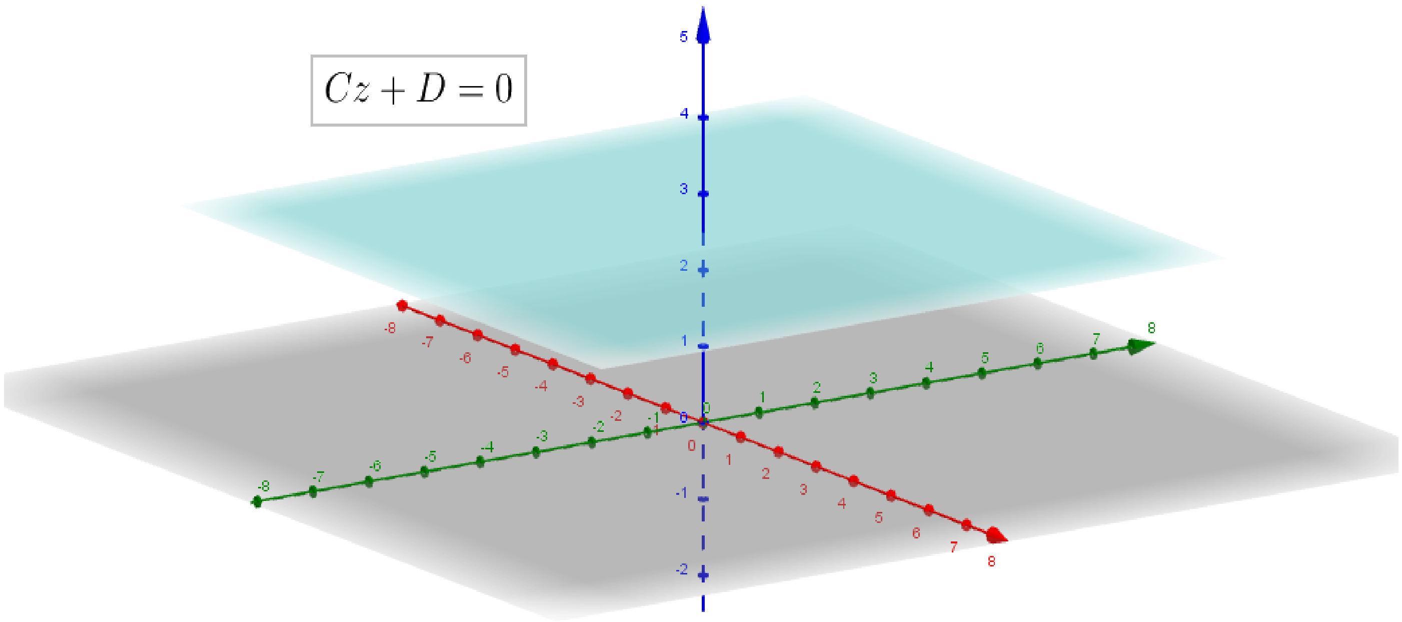


$4^{\circ}.\ A = 0, B \neq 0, C \neq 0, D \neq 0,$

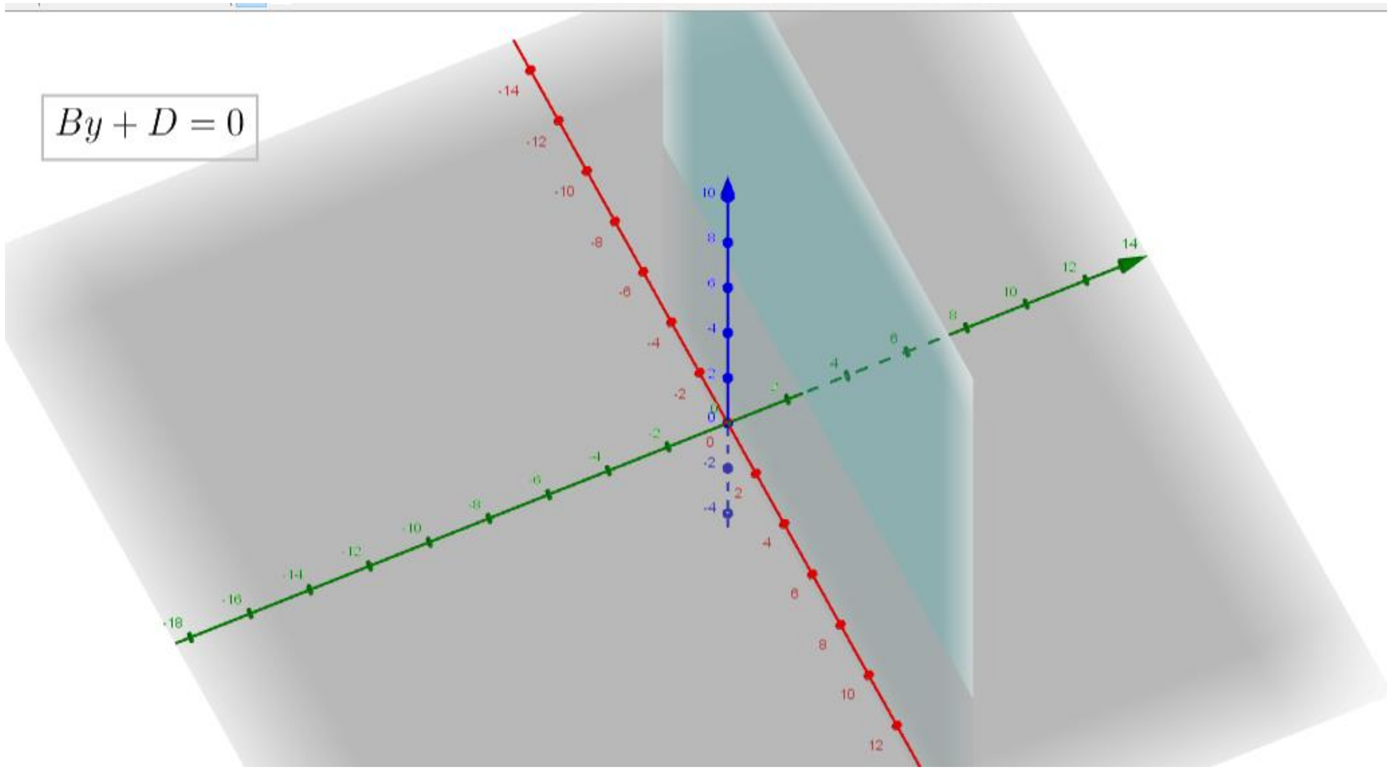
Bu holda $By + Cz + D = 0$ tekislik Ox o'qiga parallel tekislikdir.



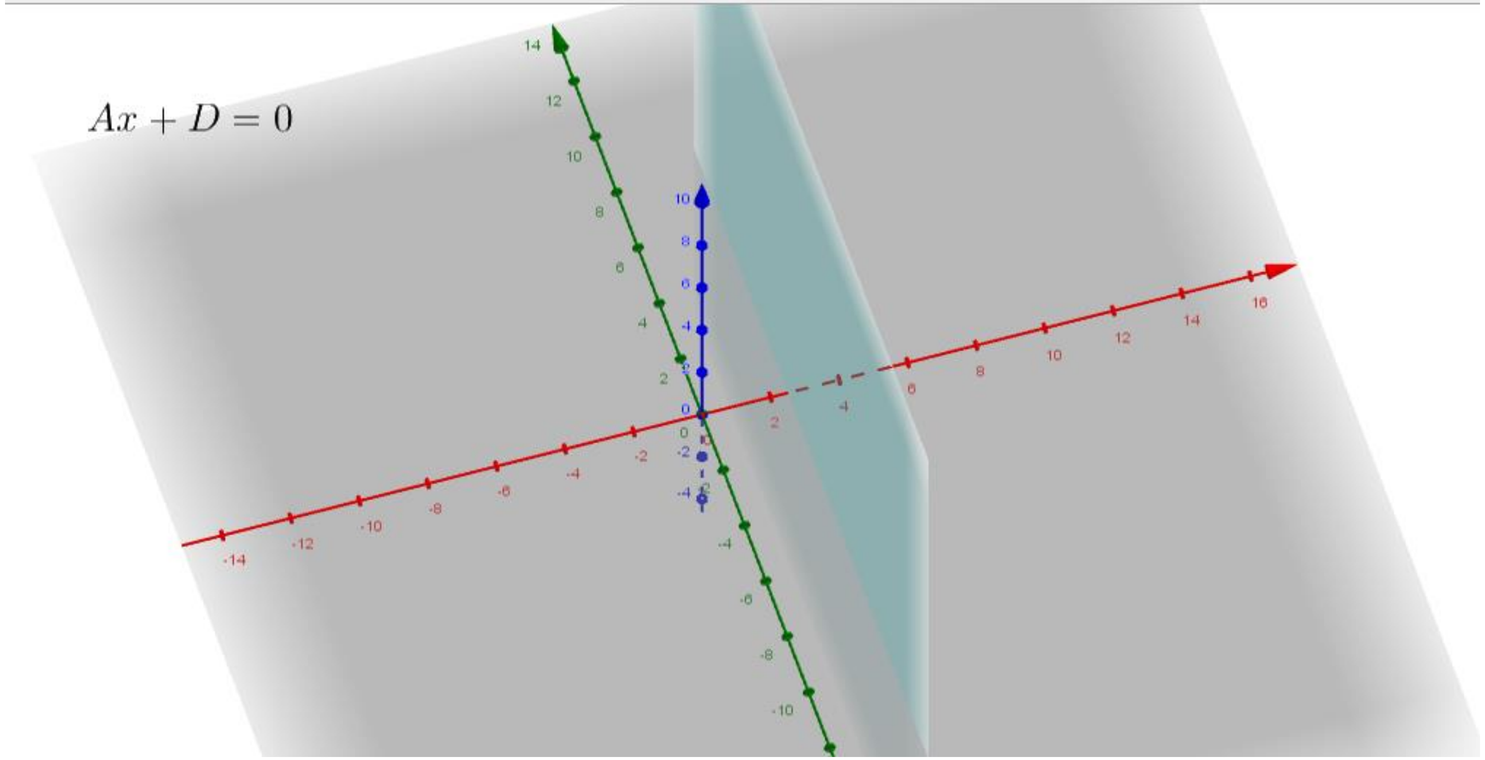
5°. $A = 0, B = 0, C \neq 0, D \neq 0$. U holda (1) tenglama $Cz + D = 0$ ko'rinishga ega bo'lib, u Oxy kordinatalar tekisligiga parallel tekislikdir.



$6^{\circ}.\ A = 0, C = 0, B \neq 0, D \neq 0.$ U holda (1) tenglama $By{+}D = 0$ ko'rinishga ega bo'lib, u Oxz kordinatalar tekisligiga parallel tekislikdir.



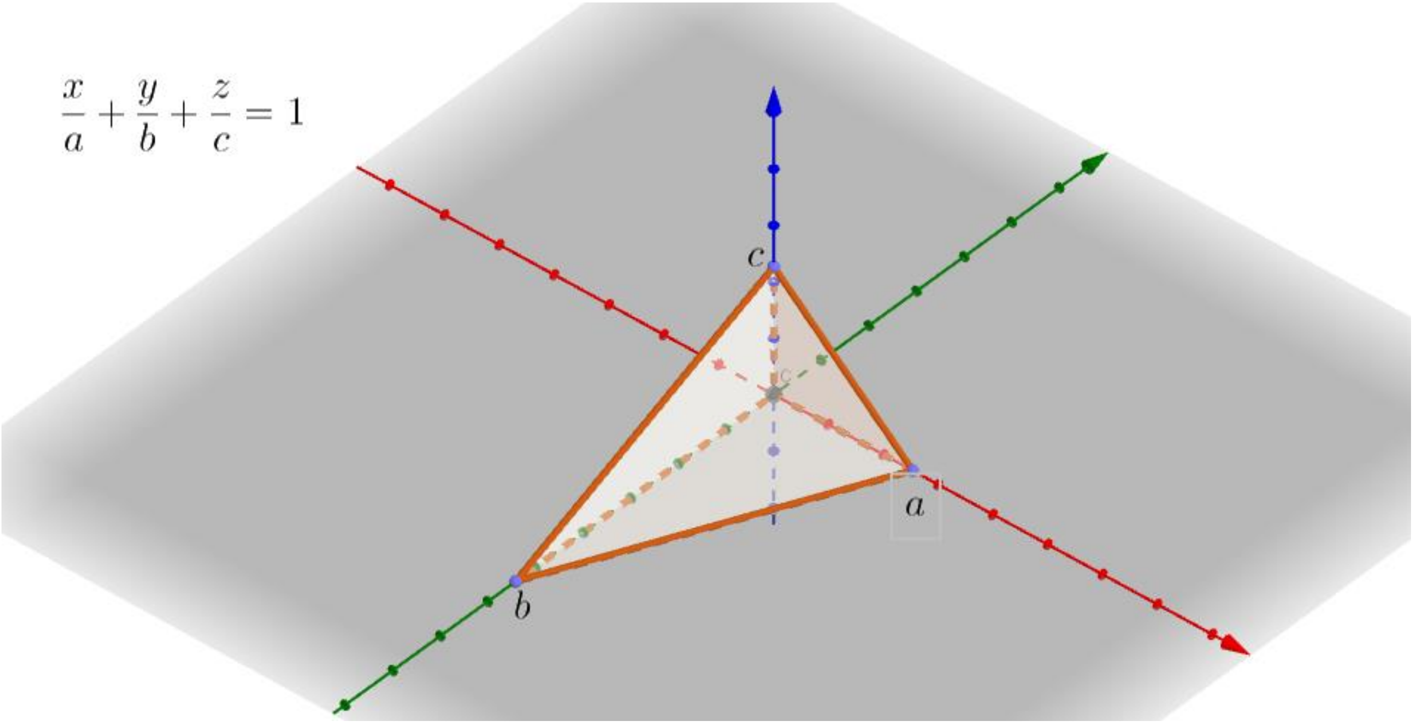
$7^{\circ}.\ B=C=0, A\neq 0, D\neq 0.$ U holda (1) tenglama $Ax+D=0$ ko'rinishga ega bo'lib, u Oyz kordinatalar tekisligiga parallel tekislikdir.



Tekislikning o’qlardan ajratgan kesmalar bo’yicha tenglamasi tenglmasi

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad (2)$$

ko’rinishga ega.



Fazoda

$$A_1x + B_1y + C_1z = 0$$
$$A_2x + B_2y + C_2z = 0 \qquad (3)$$

tenglamalar bilan aniqlangan T_1 va T_2 tekisliklar berilgan bo'lsin. Bu ikki tekislik parallel bo'lishi uchun

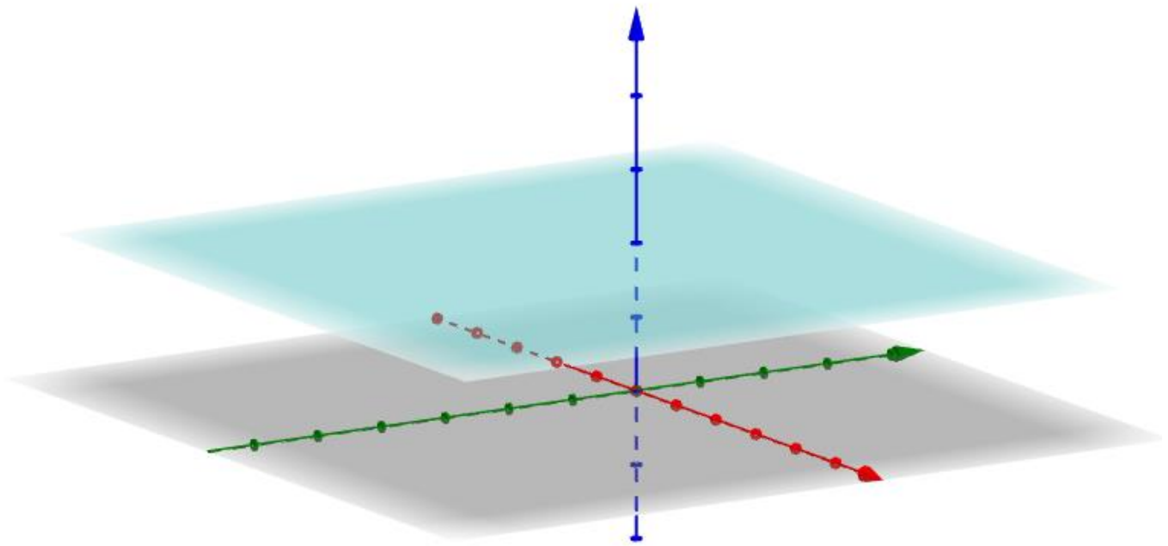
$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$$

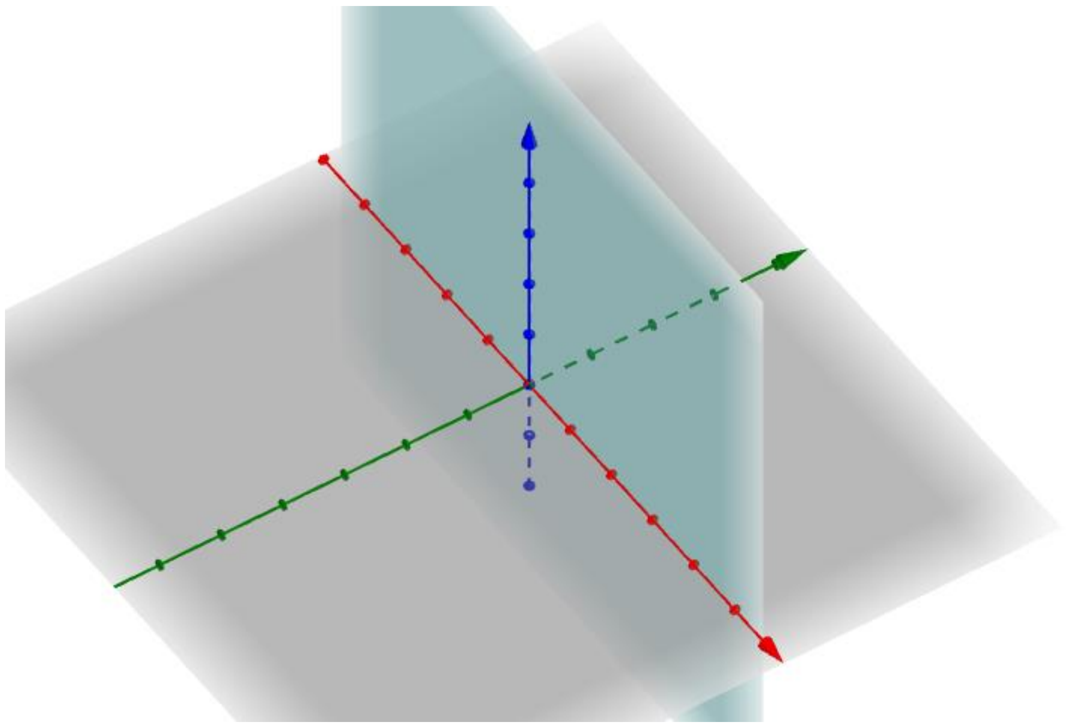
shart bajarilishi zarur va yetarli.

T_1 va T_2 tekisliklar perpendikulyar bo'lishi uchun

$$A_1A_2 + B_1B_2 + C_1C_2 = 0$$

shart bajarilishi zarur va yetarli.





Uch nuqtadan o'tuvchi tekislik tenglamasi.

Fazoda bir to'g'ri chiziqda yotmagan uchta $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$, $P_3(x_3, y_3, z_3)$ nuqtalardan o'tuvchi tekislik tenglamasini keltirib chiqarish mumkin. Bu tenglama

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0 \qquad \text{c}$$

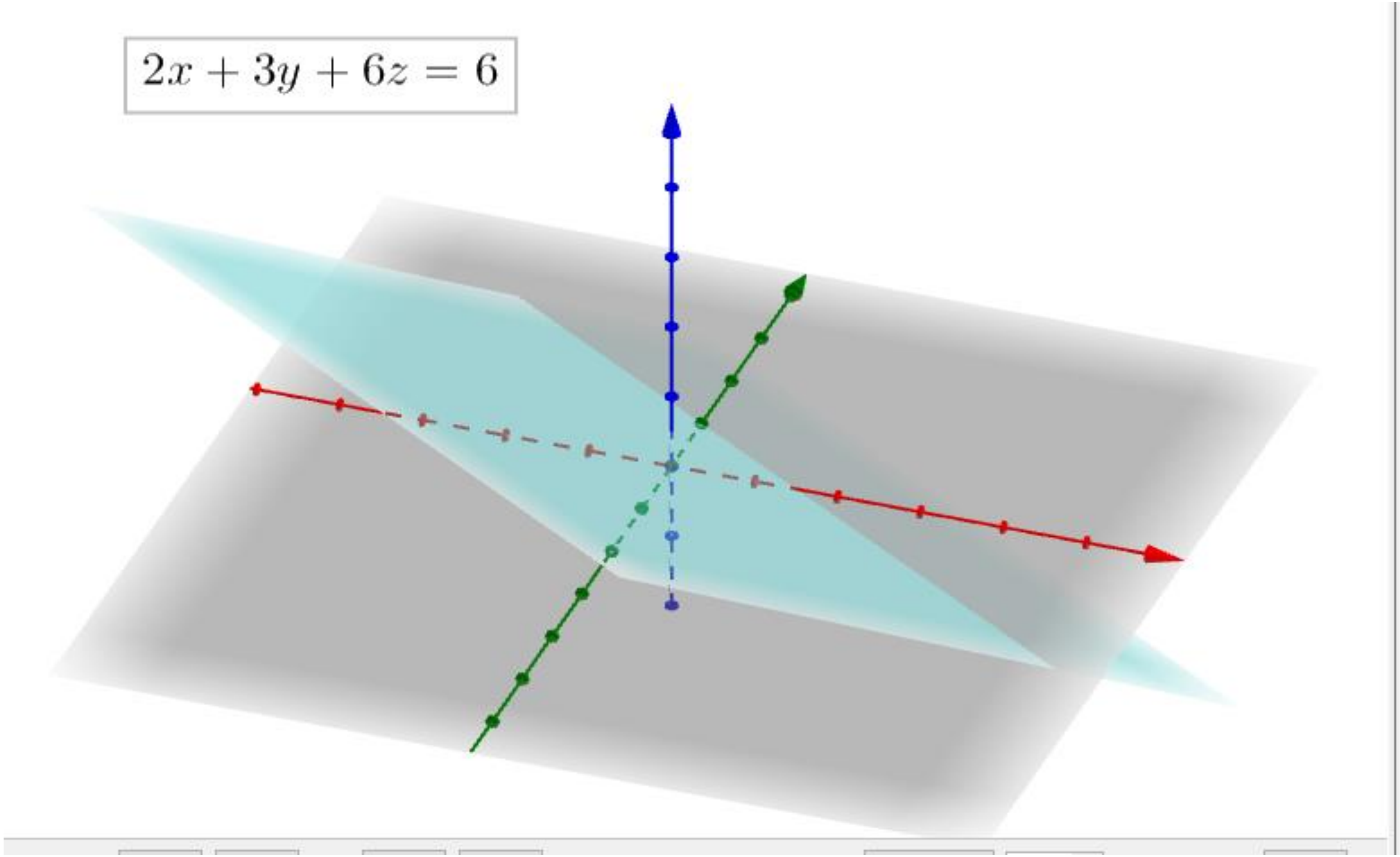
ko'rinishda bo'ladi.

(4) formulaga ko’ra

$$\begin{vmatrix} x-0 & y-0 & z-1 \\ 0-0 & 2-0 & 0-1 \\ 3-0 & 0-0 & 0-1 \end{vmatrix} = 0$$

tenglama bilan ifodalanadi. Bu determinantni hisoblab topamiz

$$2x + 3y + 6z = 6$$


$2x + 3y + 6z = 6$