Differensiyal, ma’nosi , hisoblash qoidalari.

Tarif. Agar y=f(x) funksiya orttirmasini y=A x+ (x) x, bunda A-son, (x)-cheksiz kichik, ko’rinishda yozish mumkin bo’lsa, u differensiyallanuvchi deyiladi.
Funksiya differensyiallanuvchi bo’lishi uchun chekli hosila mavjud bo’lishi zaruriy va etarli shart hisoblanadi, chunki ;
Funksiyani differensiallanuvchi bo’lishi uning uzliksizligini xam keltirib chiqaradi, ani argument va funksiya orttirmalari bir paytda nolga intiladi, bu esa funksiya uzliksizligini bildiradi.
Funksya orttirmasining a ko’rinishida, orttirmaning chiziqli bosh qismi, esa qoldiq qismi deyiladi .
Tarif. Funksiya orttirmasining chiziqli bosh qismi uning differensiyali deyiladi va dy=A tarzida yoziladi .
(x)=A ekanligini hisobga olsak, dy= (x) , agar y=x deyilsa, dx=1 bo’ladi va differensial uchun dy= (x)dx formula hosil qilamiz.
Differensiyal uchun topilgan dy= (x) formula yordamida quyidagi, diferensiyal hisoblash qoidalarini topish mumkin .

1. d($C_1U \pm C_2$V)=($C_1U \pm C_2$V)$'$dx=($C_1U' \pm C_2V'$)dx=$C_1dU \pm C_2d$V

2. d(UV)=(UV)$'$dx=($U'V + UV'$)dx=$VdU + Ud$V

3. d=($\frac{U}{V}$)= ($\frac{U}{V}$)$'$dx=$\frac{VU'-UV'}{V^2}$dx=$\frac{VdU-UdV}{V^2}$.

Agar y=f(x), x=φ(t) funksiyalar yordamida tuzilgan y=f(φ(t)) murakkab funksiya qaralsa, differensial dy=$y'_x t'_x dt = y'_x dx$ ko'rinishida yoziladi, o'z xolatini saqlaydi. Differensial o'z ko'rinishini o'zgartirmaslik xususiyati uning invariyantligi deyiladi .

y=f(x) funksiya biror nuqtadagi birinchi differensialidan shu nuqtada olingan differensial uning ikkinchi differensiali deyiladi,

d^2y=d(dy) ko'rinishida yoziladi. Shunga o'xshash, d^3y=d(d^2y), d^ny=d(d^{n-1}y) lar xam ko'riladi.

Yuqori tartibli hosila, differensiallarini hisoblashda dx ixtiyoriy va x ga bo'g'liqmas son ekanini, uni o'zgarmas ko'paytuvchi sifatida qarash lozimligini yodda tutish zarur.

d^2y=d(dy)=d(y'dx)=d(y')dx=($y''dx$)dx=$y''dx^2$,

d^3y=d(d^2y)=d($y''dx^2$)=d(y'')dx^2=($y'''dx$)dx^2=$y'''dx^3$,

Umuman, $d^n y = y^{(n)} dx^n$,

Agar y=x^n, funksiyaning yuqori tartibli differensiali hisoblansa ,d(x^n), $d^2(x^n)$, ko'rinishida yoziladi.

Yuqori tartibli differensiallarda invariantlik xossasi o'rinli bo'lmaydi chunki, chunki y=f(φ(t)) funksiya uchun d^2y=d(y'_xdx)=d(y'_x)dx+y'_xd(dx)=y''_{x^2} dx^2+$y'_{x^2}d^2x$ hosil bo'ladi.

Biror x=x_0 nuqtada dy$\approx\Delta$y ekanligidan taqribiy hisoblashlarda unimli foydalaniladi.

Δy=f(x_0+Δx)- $f(x_0)\approx f'(x_0)\Delta$x dan f($x_0$+$\Delta$x) $\approx f(x_0)$+$f'(x_0)\Delta$x

taqribiy hisoblash formulasi kelib chiqadi.

Misol. $\sqrt{26}$ taqribiy qiymatini toping.

f(x)= $\sqrt{x}$, x_0=25 deb ,$\sqrt{25+1}\approx\sqrt{25}$+$\frac{1}{2\sqrt{25}}$1=5+ $\frac{1}{10}$=5,1 ekanligini topamiz .

Differensiyal hisob asosiy teoremalari tatbiqlari.

Aniqmasliklarni ochish. Lopital qoidalari.

Teorema (Lopital) .Biror aϵx nuqta atrofida f(x),g(x) aniqlanib, hosilalar mavjud bo'lsin. $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x) = 0$, $g'(x) \neq 0$ .U holda, agar $\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$ mavjud bo'lsa, $\lim\limits_{x\to a}\frac{f(x)}{g(x)}$ ham mavjud va $\lim\limits_{x\to a}\frac{f(x)}{g(x)} = \lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$

Isbot. f(a)=g(a)=0 deb qabul qilinsa, masalan, [a;x] oraliqda f,g funksiyalar uchun Koshi teoremasi shartlari o'rinli bo'ladi, shunday cϵ(a;x) mavjudki, $\frac{f(x)-f(a)}{g(x)-g(a)}$ $=\frac{f'(c)}{g'(c)}$ o'rinli, bundan $\lim\limits_{x\to a}\frac{f(x)}{g(x)} = \lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$ kelib chiqadi, chunki x$\to a\ da\ c \to$a bo'lishi tabiiy .

Agar f,g funksiyalar hosilalari ham yuqoridagi shartlarga bo'ysinsa, $\lim\limits_{x\to a}\frac{f(x)}{g(x)} = \lim\limits_{x\to a}\frac{f'(x)}{g'(x)} = \lim\limits_{x\to a}\frac{f''(x)}{g''(x)} = \ldots = \lim\limits_{x\to a}\frac{f^{(n)}(x)}{g^{(n)}(x)}$;

Yani aniqmaslik yo'qolguncha Lopital qodasini qo'llash mumkin.

Misol; 1) $\lim\limits_{x\to 0}\frac{x-sinx}{x^3} \triangleq \lim\limits_{x\to 0}\frac{1-cosx}{3x^2} \triangleq \lim\limits_{x\to 0}\frac{sinx}{6x} \triangleq \lim\limits_{x\to 0}\frac{cosx}{6}=\frac{1}{6}$;

Qarab chiqilgan aniqmaslik $\frac{0}{0}$ tipidagi deyiladi .

Agar $x\to\pm\infty$ bo'lsa ham yuqoridagi teorema o'rinlidir, chunki $x=\frac{1}{t}$ almashtirish o'tkazilsa, $\lim\limits_{x\to\infty}\frac{f(x)}{g(x)}=\lim\limits_{x\to 0}\frac{f\left(\frac{1}{t}\right)}{g\left(\frac{1}{t}\right)} \triangleq \lim\limits_{x\to 0}\frac{f'\left(\frac{1}{t}\right)\left(-\frac{1}{t^2}\right)}{g'\left(\frac{1}{t}\right)\left(-\frac{1}{t^2}\right)} \triangleq \lim\limits_{x\to\infty}\frac{f'(x)}{g'(x)}$.

Misol. 1) $\lim\limits_{x\to\infty}\frac{lnx}{x} \triangleq \lim\limits_{x\to\infty}\frac{1}{x}=0$.

Agar $\lim\limits_{x\to a}f(x)=\lim\limits_{x\to a}g(x)=\infty$ bo'lsa ham, Lopital qoidasi $\lim\limits_{x\to a}\frac{f(x)}{g(x)}=\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$ o'rinli bo'ladi.

Misol. 1) $\lim\limits_{x\to 0}\frac{\ln(sinax)}{\ln(sinbx)} \triangleq \lim\limits_{x\to 0}\frac{asinbxcosax}{bsinaxcosbx}=\frac{a}{b}\lim\limits_{x\to 0}\frac{tgbx}{tgxax}=\frac{a}{b}\lim\limits_{x\to 0}\frac{bcos^2ax}{acos^2bx}=1$;

$0.\infty$, $\infty-\infty$, 0^0, 1^∞ , ∞^0 ko'rinishidagi aniqmasliklar ham $\frac{0}{0}$, $\frac{\infty}{\infty}$ ko'rinishidagi aniqmasliklarga keltiriladi.

Misollar. 1) $\lim\limits_{x\to 0+} x\ln x = \lim\limits_{x\to 0+} \frac{\ln x}{\frac{1}{x}} = \lim\limits_{x\to 0+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim\limits_{x\to 0+} (-x) = 0$

1) $\lim\limits_{x\to \frac{\pi}{2}} \left(\frac{1}{\cos x} - tgx\right) = \lim\limits_{x\to \frac{\pi}{2}} \frac{1-\sin x}{\cos x} \triangleq \lim\limits_{x\to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = 0$

2) $\lim\limits_{x\to 0+} x^x = \lim\limits_{x\to 0+} e^{x\ln x} = e^{\lim\limits_{x\to 0+} \frac{\ln x}{\frac{1}{x}}} = e^0 = 1$

Teylor formulasi .

Teorema (Teylor Bruk): f(x) funksiya c nuqta va uning atrofida (n+1)- tartibli hosilaga ega bo'lsin. c va x orasida shunday ξ nuqta mavjudki, quyidagi formula o'rinli.

$$f(x)=f(c)+\frac{f'(c)}{1!}(x-c)+\frac{f''(c)}{2!}(x-c)^2+\ldots+\frac{f^{(n)}(c)}{n!}(x-c)^n+\frac{f^{(n+1)}(\xi)}{(n+1)!}(x-c)^{n+1};$$

Agar Teylor formulasida C=0 bo'lsa Makloren K. formulasi f(x)=f(0)+ $\frac{f'(0)}{1!}$x+ $\frac{f''(0)}{2!}x^2$+...+$\frac{f^n(0)}{n!}x^n$+$R_{n+1}(x)$ hosil bo'ladi.

Makloren formulasi bo'yicha quyidagi yoyilmalarni olish mumkin.

1. e^x=1+$\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}$+0($x^n$),

2. sinx=x-$\frac{x^3}{3!}$+$\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots+(-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!}+0(2^{2n})$,

3. cosx=1-$\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots+(-1)^n\frac{x^{2n}}{(2n)!}+0(2^{2n+1})$,

4. $(1+x)^m=1+mx+\frac{m(m-1)}{2!}x^2+\cdots+\frac{m(m-1)\ldots(m-n+1)}{n!}x^n+0(x^n)$,

5. ln(1+x)=x-$\frac{x^2}{2}+\frac{x^3}{3}-\cdots+(-1)^{n-1}\frac{x^n}{n}+0(x^n)$.

Misollar. 1) f(x)=$\sqrt{x}$ funksiyani x-1 darajalari bo'yicha yoyilmasi uchta hadini toping.

Teylor formulasi bo'yicha

f(x)=f(1)+$\frac{f'(1)}{1!}(x-1)+\frac{f''(1)}{2!}(x-1)^2, f'(x)=\frac{1}{2\sqrt{x}}, f''(x)=-\frac{1}{4\sqrt{x^3}}$

Demak, $\sqrt{x}=1+\frac{1}{2}(x-1)-\frac{1}{8}(x-1)^2+0((x-1)^2).$

2)$e^{i\varphi}=cos\varphi+isin\varphi$ Eyler ayniyatini isbotlang.

Funksiyalarning Makloren formulasi bo'yicha yoyilmalaridan foydalanamiz.

$$e^{i\varphi}=1+i\varphi-\frac{\varphi^2}{2!}-\frac{i\varphi^3}{3!}+\frac{\varphi^4}{4!}+\frac{i\varphi^5}{5!}-\frac{\varphi^6}{6!}-\frac{i\varphi^7}{7!}+\cdots=\left(1-\frac{\varphi^2}{2!}+\frac{\varphi^4}{4!}-\frac{\varphi^6}{6!}+\cdots\right)+\frac{\varphi^5}{5!}-\frac{\varphi^7}{7!}+\cdots\Big)=cos\varphi+isin\varphi$$

3)$lim_{x\to0}\frac{e^{-\frac{x^2}{2}}-cosx}{x^2 sinx}=lim_{x\to0}\frac{\left[1-\frac{x^2}{2}+\frac{x^4}{8}+0(x^n)\right]-\left[1-\frac{x^2}{2!}+\frac{x^4}{4!}\right]}{x^2[x-0(x)]}=$

$lim_{x\to0}\frac{\frac{x^4}{8}-\frac{x^4}{24}+0(x^4)}{x^4+0(x^4)}=\frac{1}{8}-\frac{1}{24}=\frac{1}{12}.$

Mavzuga doir misol va masalalar .

1.f(x)=$\sqrt[3]{x^2-1}$ funksiya (-1;1)dan x=0 da eng kichik qiymatiga erishadi,lekin Ferma teoremasi o'rinli emas.Nima uchun?

2. f(x)=x(x^2-1) funksiya uchun [-1;1],[0;1] oraliqlarda Roll teoremasi shartlarini tekshiring.

3.Lagranj teoremasidan foydalanib isbotlang:

1) $\frac{x}{1+x} < \ln(1+x) < x, x > 0$ 2) $e^x > ex, x > 1$

3) $|sinx - siny| \leq |x-y|$.

4. f(x)=x^2,$g(x)$=x^3 funksiyalar uchun [-1;1] oraliqda Koshi teoremasi o'rinlimi?

5. Lopital qoidalari yordamida limitlarni toping.

1). $lim_{x\to 0} \frac{sin^2 ax}{sin^2 bx}$ 2). $lim_{x\to 0} \frac{tgx-x}{x-sinx}$ 3). $lim_{x\to \frac{\pi}{2}} \frac{tg3x}{tgx}$

4). $lim_{x\to 0}. \frac{a^x - a^{sinx}}{x^3}$ $(a > 0)$ 5). $lim_{x\to 0} \frac{\ln(gosax)}{ln(cosbx)}$

6) . $lim_{x\to 0} \frac{\cos(sinx) - cosx}{x^4}$ 7). $lim_{x\to 0} x^{\frac{k}{1+lnx}}$ 8). $lim_{x\to \frac{\pi}{4}} (tgx)^{tg2x}$

9). $lim_{x\to a} \frac{a^x - x^a}{x-a}$(a> 0) 10) $lim_{x\to +\infty} (thx)^x$ 11). $lim_{x\to 0} (\frac{1+e^x}{2})^{cthx}$ 12). $lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$.

6. Mokloren formulasi bo'yicha $0(x^2)$ hadgacha yoying:

1.y=e^{tgx} 2.y=lncosx 3.y=ln$\frac{1+2x}{1-x}$

7. Teylor formulsi bo'yicha $0((x - x_0)^2)$ hadgacha yoying .

1) y=$\frac{1}{x}$, x_0=2 2) y=xe^{2x}, x_0=1 3) y=$\frac{2x}{1-x^2}$, x_0=2.

8. Makloren yoyilmalaridan foydalanib limitlarini hisoblang.

1) $lim_{x\to 0} \frac{\ln(1+x) - x}{x^2}$ 2) $lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$ 3) $lim_{x\to 0} \frac{cosx - 1 + \frac{x^2}{2}}{x^4}$

4) $lim_{x\to 0} \frac{cosx - e^{-\frac{x^2}{2}}}{x^4}$ 5) $lim_{x\to 0} \frac{e^x sinx - x(1+x)}{x^3}$ 6) $lim_{x\to 0} (\frac{1}{x} - \frac{1}{sinx})$

7) $\lim_{x\to 0}\frac{a^x+a^{-x}-2}{x^2}$ (a>0) 8) $\lim_{x\to 0}\frac{1-(\cos x)^{\sin x}}{x^3}$

9. Funksiya manotonlik oraliqlarni aniqlang .

1) $y=4+x-x^2$ 2) $y=3x-x^3$ 3) $y=\frac{\sqrt{x}}{x+100}$ 4) $y=x+\sin x$

5) $y=\frac{x^2}{2^x}$ 6) $y=x^2\ln x$ 7) $y=xe^{-3x}$ 8) $y=\operatorname{arctg}x-\ln x$

10. Funksiya ekstrimumlari topilsin .

1) $y=2+x-x^2$ 2) $y=2x^2-x^4$ 3) $y=\frac{x^4}{4}-2x^3+\frac{11}{x}x^2-6x+3$

4) $y=xe^{-x}$ 5) $y=\frac{\ln^2 x}{x}$ 6) $y=x+\sqrt{3-x}$ 7) $y=x^x$

11. Funksiya qavariqlik va botiqlik oraliqlari topilsin .

1) $y=e^x$ 2) $y=\ln x$ 3) $y=x^5-10x^2+3x$ 4) $y=\frac{\sqrt{x}}{x+1}$ 5) $y=e^{-x^2}$ 6) $y=x+\sin x$

12. Funksiyani egilish nuqtalari topilsin .

1) $y=\cos x$ 2) $y=1+x^2+\frac{x^4}{2}$ 3) $y=e^{2x-x^2}$ 4) $y=(x^2-1)^3$

5) y=$\frac{lnx}{\sqrt{x}}$ 6) y=$\sqrt{1-x^3}$

13. ko'rsatilgan sohada funksiyaning eng katta (kichik) qiymatlarini toping .

1) y=2^x [-1;5] 2) y=x^2-3x+2, [-10;10]

3) y= $\sqrt{5-4x}$, [-1;1] 4) y=$6x^2$-x^3; 5) y= y=x^2-6x+13 [0;6]

6) y=2sinx-cos2x, [0;$\frac{\pi}{2}$] 7) y=$\sqrt{\frac{1+x}{lnx}}$, (1;e)

15. Berilgan funksiyani to'liq tekshiring , grafigini chizing .

1) y=$5x^2$-x^4 ; 2) y=$\frac{x^3}{3}$-x^5 ;

3) y=2x^2-8x ; 4) $y=\frac{x}{x^2-1}$;

5) y=x-lnx; 6) y=$\frac{lnx}{x}$

7) y=e^{-x^2} 8) y=$\frac{x^2-1}{x^2+1}$; 9) y=sinx+cos^2x;

10) y=x+arctgx; 11) y=ln(x+$\sqrt{x^2+1}$) ; 12) y=x^x.

16. Ekstrimumga doir quyidagi masalalar echilsin .

1) yig'indisi a bo'lgan ikki musbat son qanday bo'lganda ko'paytmasi eng katta bo'ladi .

2) Yuzasi S bo'lgan uchburchaklar ichida perimetiri eng kichigini toping .

3) moddiy nuqta S(t)= $-t^3+9t^2$-24t-8 qonun bilan harakatlanadi . Uning maksimal tezligini toping .

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