12-mavzu. CHIZIQLI OPERATORLAR.KVADRATIK FORMALAR.

CHIZIQLI OPERATORLAR.

n va m o'lchamli R^n, R^m fazolarni qaraymiz.

Tarif: Agar xar bir $\overrightarrow{x}=(x_1;\ x_2;\ \ldots\ x_n)\in R^n$ vektorga biror A qonun yoki qouda yordamida yagona $\overrightarrow{y}=(y_1;\ y_2;\ \ldots\ y_n)\in R^m$ vektorni mos qo'yilsa, bu qonun operator (akslantirish, almashtirish) deyiladi va $\overrightarrow{y}=A(\overrightarrow{x})$ tarzida yoziladi.

A: $R^n \to R^m$ operator , A: $R^n \to R$ funksiyonal, A:R $\to R$ funksiya deyiladi.

Operator chiziqli deyiladi , agar $\overrightarrow{x}, \overrightarrow{y}\in R^n$, $\lambda \in R^n$ uchun

1).$A(\overrightarrow{x}+\overrightarrow{y})=A(\overrightarrow{x})+(\overrightarrow{y})$ (additivlik)

2).$A(\lambda\overrightarrow{x})=A\overrightarrow{x}$, (bir jinslilik)

$\overrightarrow{y}=A(\overrightarrow{x})$ vektor $\overrightarrow{x}$ vektor obrazi (tasviri), $\overrightarrow{x}$ vektor esa $\overrightarrow{y}$ ning proobrazi (asli)deyiladi.

Agar R^n, R^m fazolar ustma-ust tushsa, A aperator R^n ni o'zini o'ziga akislantiradi . Biz aynan shunday operatorlarni qaraymiz. R^n fazoda $\vec{l}_1, \vec{l}_2 \ldots \vec{l}_n$ bazis berilsa, ixtiyoriy $\overrightarrow{x}\in R^n$ uchun $\overrightarrow{x}=x_1\vec{l}_1+x_2\vec{l}_2+\ldots+x_n\vec{l}_n$. A operator chiziqliligidan: A $(\overrightarrow{x})=x_1A(\vec{l}_1)+x_2A(\vec{l}_2)+\ldots+x_nA(\vec{l}_n)$ Lekin $A(\vec{l}_i)$ (i=$\overline{1,n}$) $\in R^n$, ularni ham $\vec{l}_1, \vec{l}_2 \ldots \vec{l}_n$ bazis bo'yicha yoyish mumkin

$A(\vec{l}_i)=a_{1i}(\vec{l}_1)+a_{2i}(\vec{l}_2)+\ldots+a_{ni}(\vec{l}_n)$ (i=$\overline{1,n}$) . U xolda

A $(\overrightarrow{x})=x_1(a_{11}\vec{l}_1+a_{21}\vec{l}_2+\ldots+a_{n1}\vec{l}_n)+x_2(a_{12}\vec{l}_1+a_{22}\vec{l}_2+\ldots+a_{n2}\vec{l}_n)+\ldots$
$x_n(a_{1n}\vec{l}_1+a_{2n}\vec{l}_2+\ldots+a_{nn}\vec{l}_n)=(a_{11}x_1+a_{12}x_2+\ldots a_{1n}x_n)\vec{l}_1+(a_{21}x_1+a_{22}x_2$
$\ldots a_{2n}x_n)\vec{l}_2++(a_{n1}x_1+a_{n2}x_2+\ldots a_{nn}x_n)\vec{l}_n$

Boshqa tomondan, $\overrightarrow{y}=A(\overrightarrow{x})$ vektorning $\vec{l}_1, \vec{l}_2, \ldots, \vec{l}_n$ bazisdagi y_1, y_2,... y_n kordinatalarini

A $(\overrightarrow{x})=y_1\vec{l}_1+y_2\vec{l}_2+\ldots+y_n\vec{l}_n$; ko'rinishida yoziladi . $\overrightarrow{y}$ vektorni $\vec{l}_1, \vec{l}_2, \ldots, \vec{l}_n$ bazis bo'yicha yoyilmasi yagona ekanligidan

$$\begin{cases} y_1 = a_{11}x_1 + a_{12}x_2 + \ldots a_{1n}x_n \\ y_2 = a_{21}x_1 + a_{22}x_2 + \ldots a_{2n}x_n \\ y_n = a_{n1}x_1 + a_{n2}x_2 + \ldots a_{nn}x_n; \end{cases}$$

Bundagi $\Delta=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{3n} \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$ matritsa A operator matritsasi, Δ ning rangi esa A operator rangi deyiladi.

Umuman, har bir n- tartibli matritsaga n- o'lchamli fazodagi bitta chiziqli operator mos keladi va aksincha.

$\overrightarrow{x}=(x_1 ; x_2 ; \ldots x_n)$ va $\overrightarrow{y}=A\overrightarrow{(x)}=(y_1 ; y_2 ; \ldots y_n)$ orasidagi bog'liqlik matrisaviy ko'rinishda

$$\begin{pmatrix} y_1 \\ y_2 \\ y_n \end{pmatrix}=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{3n} \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_n \end{pmatrix};$$

ko'rinishda bo'ladi.

Misol. R^3 da A operator l_1, l_2, l_3 bazisda

$$\Delta=\begin{pmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{pmatrix}$$

matrisa bilan berilgan. $\overrightarrow{x}=4\vec{l}_1-3\vec{l}_2+\vec{l}_3$ vektor obrazi $\overrightarrow{y}=A\overrightarrow{(x)}$ ni toping.

$$\begin{pmatrix} y_1 \\ y_2 \\ y_n \end{pmatrix}=\begin{pmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}=\begin{pmatrix} 10 \\ -13 \\ -18 \end{pmatrix}$$

Demak, y $=10\vec{l}_1-3\vec{l}_2-18\vec{l}_3$.

Chiziqli opratorlar ustida amallar quyidagicha kiritiladi :

1) (A+B)($\overrightarrow{x}$)=A($\overrightarrow{x)}+B(\overrightarrow{y}$) ,2)(λ A)($\overrightarrow{x}$)=λA $\overrightarrow{(x)}$,3) (AB)($\overrightarrow{x}$)=A($B(\overrightarrow{x}$))

Natijaviy operatorlar ham additiv, bir jinsli ,yani chiziqli bo'ladi .

Ixtiyoriy $\overrightarrow{x}=(x_1 , x_2 , \ldots x_n) \in R^n$ uchun O($\overrightarrow{x}$)= $\vec{O}$operatori nol operator,

E($\overrightarrow{x}$)= $\vec{x}$ esa ayniy (birlik)operatori deyiladi .

Turli bazislarda operator matritsalari orasidagi boglanishni quyidagi teorema beradi .

Teorema . A chiziqli operator l_1 , l_2 ,...l_n va f_1 , f_2 ,...f_n bazislardagi matrisalari mos ravishda Δ va Δ^* bo'lsa, $\Delta^*= C^{-1}\Delta C$, bunda C eski bazisdan yangi siga o'tish matritsasi.

Isbot; y=Δx, $y^*=\Delta^*$x matritsaviy tengliklar o'rinli. Agar C o'tish matritsasi bo'lsa x=Cx^*, y=Cy^*. Birinchi tenglikni chapdan Δ ga ko'paytiramiz $\Delta x = \Delta C x^*$, Yani y=Δ Cx^*, yoki C$y^* = \Delta C x^*$, bundan esa $y^*=C^{-1}\Delta C x^*$, yani $\Delta^*=C^{-1}\Delta$C kelib chiqadi.

Misol. $\vec{l}_1, \vec{l}_2$ bazisda A operator $\Delta= \begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix}$

matritsaga ega. A operatorning $f_1 = \vec{l}_1 + 2\vec{l}_2$, $f_2 = -2\vec{l}_1 + \vec{l}_2$ bazisdagi matritsasini toping.

C=$\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$, unga teskari matritsa, $C^{-1}=\frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$ dir.

Demak, $\Delta^* = C^{-1}\Delta C = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix}$.

Ta`rif: Nol bo'lmagan $\vec{x} \neq \vec{0}$ A chiziqli operatorning xos vektori deyiladi, agar shunday λ son topilib, A($\vec{x}$)=λ$\vec{x}$ o'rinli bo'lsa.

Bunda λ soni A operatorning (Δmatritsaning) xos soni deyiladi ($\vec{x}$ vektorga mos).

Demak, xos vektor operator ta`sirida o'ziga kolleniar vektorga o'tadi, o'zi songa ko'payadi, xolos.

Ta`rif matritsaviy yozuvda $\Delta x = \lambda x$ yoki $\begin{cases} a_{11}\, x_1 + a_{12}\, x_2 + \dots + a_{1n}\, x_n = \lambda x_1 \\ a_{21}\, x_1 + a_{22}\, x_2 + \dots + a_{2n}\, x_n = \lambda x_2 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ a_{n1}\, x_1 + a_{n2}\, x_2 + \dots + a_{nn}\, x_{n;} = \lambda x_n \end{cases}$

Soddalashtirsak, $\begin{cases} (a_{11} - \lambda)\, x_1 + a_{12}\, x_2 + \dots + a_{1n}\, x_n = \lambda x_1 \\ a_{21}\, x_1 + (a_{22} - \lambda)\, x_2 + \dots + a_{2n}\, x_n = \lambda x_2 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ a_{n1}\, x_1 + a_{n2}\, x_2 + \dots + (a_{nn} - \lambda)\, x_{n;} = \lambda x_n \end{cases}$

Bu bir jinsli sistema trivial x=$\vec{0}$=(0;0;...;0) echimga doimo ega .Noldan farqli, notrivial echim mavjud bo'lishi uchun sestema asosiy determinanti nolga teng bo'lishi kerak.

$$|\Delta - \lambda E| = \begin{vmatrix} a_{11} - \lambda & a_{12} & a_{1n} \\ a_{21} & a_{22} - \lambda & a_{2n} \\ \dots & \dots . & \dots .. \\ a_{n1} & a_{n2} & a_{nn} - \lambda \end{vmatrix} = 0$$

Bu determinant λ ga nisbatan n- darajali ko'phad bo'lib, uni A operatop (yoki Δ matritsa) harakteristik ko'phadi deyiladi, $|\Delta-\lambda E|$=0 tenglama A operator harakteristik tenglamasi deyiladi.

Chiziqli operator harakteristik ko'phadi bazis tanlanishiga bog`liq emas.

Misol. $\Delta=\begin{pmatrix} 4 & 12 \\ 3 & 4 \end{pmatrix}$ matritsa bilan berilgan chiziqli operator xos sonlari va xos vektorlarini toping. Harakteristik tenglama tuzamiz : $|\Delta-\lambda E|$=$\begin{vmatrix} 4-\lambda & 12 \\ 3 & 4-\lambda \end{vmatrix}=0$, yoki $(4-\lambda)^2-6^2$=0.

Bundan chiziqli operator xos sonlari λ_1=-2, $\lambda_2=10$. Dastlab, λ_1=-2 ga mos $\vec{x}^{(1)}$ xos vektorni qidiramiz. Buning uchun ($\Delta-\lambda_1 E$) $\vec{x}^{(1)}$ =0 yoki $\begin{pmatrix} 6 & 12 \\ 3 & 6 \end{pmatrix}\begin{pmatrix} \overrightarrow{x_1}^{(1)} \\ \overrightarrow{x_2}^{(1)} \end{pmatrix}$=$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ matritsaviy tenglamani echamiz. $\overrightarrow{x_1}^{(1)}=-2\overrightarrow{x_2}^{(1)}$ munosabatga egamiz. Agar $\overrightarrow{x_2}^{(1)}=C$ desak, $\overrightarrow{x_1}^{(1)}=-2C$ bo'ladi. Demak, λ_1=-2 ga mos xos vektorlar $\vec{x}^{(1)}$ = (-2C;C) ko'rinishida bo'ladi.

C$\neq 0$ $\lambda_2 = 10$ da $\overrightarrow{x_2}^{(2)}$xos vektor uchun $\begin{pmatrix} -6 & 12 \\ 3 & -6 \end{pmatrix} \begin{pmatrix} \overrightarrow{x_2}^{(1)} \\ \overrightarrow{x_2}^{(2)} \end{pmatrix} = 0$ dan $\overrightarrow{x_2}^{(1)}$=$2\overrightarrow{x_2}^{(2)}$ munosabatni olamiz. $\overrightarrow{x_2}^{(2)} = C$ desak, $\overrightarrow{x_2}^{(1)} = 2C$ bo'ladi. Demak, $\lambda_2 = 10$ ga mos xos vektorlar C$\neq 0$da $x^{(2)}$= (2C;C) ko'rinishida bo'ladi.

Agar A chiziqli operator n ta chiziqli erkli $\overrightarrow{l_1}, \overrightarrow{l_2}, \dots, \overrightarrow{l_n}$ xos vektorlarga va $\overrightarrow{\lambda_1}$, $\overrightarrow{\lambda_2}, \dots, \overrightarrow{\lambda_n}$ xos sonlarga ega bo'lsa, $\overrightarrow{l_1}, \overrightarrow{l_2}, \dots, \overrightarrow{l_n}$ vektorlar bazis deb olinsa,

$$A(\overrightarrow{l_1}) = a_{1i}\overrightarrow{l_1} + a_{2i}\overrightarrow{l_2} + \cdots + a_{ii}\overrightarrow{l_i} + \cdots + a_{ni}\overrightarrow{l_n} = \lambda_i \overrightarrow{l_i}$$

Undan i$\neq j$da a_{ij}=0, i=j da esa $a_{ij} = \lambda_i$.

Shunday qilib xos vektorlardan iborat bazisda A operatop matritsasi dioganal ko'rinishdadir.

$$\nabla = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ .. & .. & .. \\ 0 & 0 & \lambda_n \end{pmatrix}$$

va aksincha 'agar A operator matritsasi diogonal ko'rinishda bo'lsa, bu bazis barcha vektorlari xosdir.