

# MAVZU: Chiziqli tenglamalar sistemasini yechishning teskari matritsa usuli

## **Chiziqli tenglamalar sistemasini matritsalar yordamida yechish.**

Endi matritsalar yordamida chiziqli tenglamalar sistemasini yechishga o'tamiz.

$$\left. \begin{array}{l} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \dots \dots \dots \dots \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{array} \right\} \quad (1)$$

$n$  noma'lumli,  $n$  ta tenglamalar sistemasi berilgan bo'lsin.

$$A = \begin{pmatrix} a_{11} & a_{12} \cdots a_{1n} \\ a_{21} & a_{22} \cdots a_{2n} \\ \hline \cdots & \cdots \\ a_{n1} & a_{n2} \cdots a_{nn} \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{pmatrix}$$

belgilashlarni kiritamiz. Endi (1) sistemani matritsalarni ko'paytirish qoidasidan foydalanib,

$$AX = B \tag{2}$$

ko'rinishda yozish mumkin.  $\det A \neq 0$  bo'lsa, teskari matritsa  $A^{-1}$  mavjud va  $A^{-1}AX = A^{-1}B$  hosil bo'ladi. SHunday qilib, noma'lum  $X$  matritsa  $A^{-1}B$  matritsaga teng bo'ladi, yahni

$$X = A^{-1}B.$$

Bu (1) tenglamalar sistemasini yechishning **matritsavy yozuvini** bildiradi.

Misol. Ushbu tenglamalar sistemasini matritsalar yordamida yeching:

$$\begin{cases} x_1 + x_2 + x_3 = 4, \\ x_1 + 2x_2 + 4x_3 = 4, \\ x_1 + 3x_2 + 9x_3 = 2 \end{cases}.$$

Echish. Quyidagi belgilashlarni kiritamiz:

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix}; \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}; \quad B = \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix}.$$

Bu matritsalar yordamida berilgan tenglamalar sistemasini

$$AX = B \tag{3}$$

ko'rnishda yozamiz.

Endi  $A$  matritsaning determinantini hisoblaymiz.

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{vmatrix} = 1 \cdot 2 \cdot 9 + 1 \cdot 4 \cdot 1 + 1 \cdot 3 \cdot 1 - 1 \cdot 2 \cdot 1 - 1 \cdot 1 \cdot 9 - 1 \cdot 4 \cdot 3 = 2.$$

$A$  matritsaning determinanti 0 dan farqli bo'lganligi uchun, unga teskari yagona  $A^{-1}$  matritsa mavjud va tenglamalar sistemasi yagona yechimga ega bo'ladi. Endi  $A^{-1}$  teskari matritsani topish uchun  $\Delta$  determinant elementlarining hamma algebraik to'ldiruvchilarini hisoblaymiz:

$$A_{11} = \begin{vmatrix} 2 & 4 \\ 3 & 9 \end{vmatrix} = 18 - 12 = 6, \quad A_{12} = -\begin{vmatrix} 1 & 4 \\ 1 & 9 \end{vmatrix} = -5, \quad A_{13} = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 1$$

$$A_{21} = -\begin{vmatrix} 1 & 1 \\ 3 & 9 \end{vmatrix} = -6, \quad A_{22} = \begin{vmatrix} 1 & 4 \\ 1 & 9 \end{vmatrix} = 8, \quad A_{23} = -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -2$$

$$A_{31} = \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix} = 2, \quad A_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} = -3, \quad A_{33} = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1.$$

Teskari  $A^{-1}$  matritsani topish formulasiga asosan,

$$A^{-1} = \frac{1}{2} \begin{pmatrix} 6 & -6 & 2 \\ -5 & 8-3 \\ 1 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -3 & 1 \\ -2,5 & 4-1,5 \\ 0,5-1 & 0,5 \end{pmatrix}$$

(3) tenglikning ikki tomonini chapdan  $A^{-1}$  ga ko'paytirsak,  $A^{-1}AX = A^{-1}B$  yoki  $X = A^{-1}B$  bo'lib, ya'ni

$$X = \begin{pmatrix} 3 & -3 & 1 \\ -2,5 & 4 & -1,5 \\ 0,5 & -1 & 0,5 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \cdot 4 + (-3) \cdot 4 + 1 \cdot 2 \\ -2,5 \cdot 4 + 4 \cdot 4 + (-1,5) \cdot 2 \\ 0,5 \cdot 4 - 1 \cdot 4 + 0,5 \cdot 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$$

tenglik hosil bo'ladi.

Shunday qilib,

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \quad \text{yoki}$$

$$x_1 = 2, x_2 = 3, x_3 = -1.$$

(Topilgan yechimlarni tenglamalar sistemasiga bevosita qo'yib, yechimning to'g'riliгини текширib ko'rish mumkin).



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