10-mavzu Fazoda to`g`ri chiziq tenglamalari, asosiy masalalar.

To`g`ri chiziqning kanonik tenglamasi:

$$\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{r}$$

To`g`ri chiziqning parametrik tenglamasi:

$$\begin{cases} x = x_0 + mt \\ y = y_0 + nt \\ z = z_0 + rt \end{cases}$$

Ikki nuqtadan o`tuvchi to`g`ri chiziq tenglamasi:

$$\frac{x - x_0}{x_1 - x_0} = \frac{y - y_0}{y_1 - y_0} = \frac{z - z_0}{z_1 - z_0}$$

$$\begin{cases} x = x_0 + mt \\ y = y_0 + nt \\ z = z_0 + rt \end{cases}$$

Ikki nuqtadan o`tuvchi to`g`ri chiziq tenglamasi:

$$\frac{x - x_0}{x_1 - x_0} = \frac{y - y_0}{y_1 - y_0} = \frac{z - z_0}{z_1 - z_0}$$

To`g`ri chiziq - tekisliklar kesishmasi sifatida. To`g`ri chiziqning umumiy tenglamalari:

$$\begin{cases} A_1 x + B_1 y + C_1 z + D_1 = 0 \\ A_2 x + B_2 y + C_2 z + D_2 = 0 \end{cases}$$

To'g'ri chiziqning kanononik tenglamasi:

$$\frac{x - x_0}{\begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix}} = \frac{y - y_0}{-\begin{vmatrix} A_1 & C_1 \\ A_2 & C_2 \end{vmatrix}} = \frac{Z - Z_0}{\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}}$$

Fazoda to`g`ri chiziq tenglamalariga doir asosiy masalalar.

Ikki to'g'ri chiziq orasidagi burchak:

$$\mathrm{Cos}\varphi = \frac{\overrightarrow{p_1} \cdot \overrightarrow{p_2}}{|\overrightarrow{p_1} \cdot \overrightarrow{p_2}|} = \frac{m_1 m_1 + n_1 n_2 + r_1 r_2}{\sqrt{m_1^2 + n_1^2 + r_1^2} \cdot \sqrt{m_2^2 + n_2^2 + r_2^2}}$$

Nuqtadan to'g'ri chiziqqacha bo'lgan masofa

$$d = \frac{\sqrt{\begin{vmatrix} n_1 & r_1 \\ y_1 - y_0 & z_1 - z_0 \end{vmatrix}^2 + \begin{vmatrix} m_1 & r_1 \\ x_1 - x_0 & z_1 - z_0 \end{vmatrix}^2 + \begin{vmatrix} m_1 & n_1 \\ x_1 - x_0 & y_1 - y_0 \end{vmatrix}^2}}{\sqrt{m_1^2 + n_1^2 + r_1^2}}$$

To`g`ri chiziq va tekislik orasidagi burchak .

Biror $\frac{x-x_0}{x_1-x_0}=\frac{y-y_0}{y_1-y_0}=\frac{z-z_0}{z_1-z_0}$ to`g`ri chiziq va Ax + By +Cz +D = 0 tekislik berilgan bo`lsin. Ular orasidagi burchak φ bo`lsa, yo`naltiruvchi $\vec{p}(m;n;r)$ va normal $\vec{N}(A;B;C)$ vektorlar orasidagi burchak $(90^0-\varphi)$ bo`ladi . Vektorlar orasidagi burchak formulasiga ko`ra $\cos(90^0-\varphi)=\sin\varphi=\frac{\vec{p}*\vec{N}}{|\vec{p}|*|\vec{N}|}==$
$\frac{mA+nB+rC}{\sqrt{m^2+n^2+r^2}*\sqrt{A^2+B^2+C^2}}$

To`g`ri chiziq va tekislikning parallellik sharti mA +nB + rC = 0 bo`ladi. Perpendikulyarlik sharti esa, aksincha, $\frac{m}{A}=\frac{n}{B}=\frac{r}{C}$ ko`rinishidir .

To`g`ri chiziq va tekislik kesishish nuqtasi .

$\begin{cases} x = x_0 + mt \\ y = y_0 + nt \\ z = z_0 + rt \end{cases}$ parametrik tenglamali to`g`ri chiziq va Ax + By +Cz +D = 0 tekislik berilib, ular o`zaro parallel bo`lmasin. Unda to`g`ri chiziq biror nuqtada tekislikni kesadi. Agar o`sha kesishish nuqtasi Q$(x_1; y_1; z_1)$ bo`lsa, uning koordinatalari to`g`ri chiziq tenglamasini ham , tekislik tenglamasini ham qanoatlantiradi .

$\begin{cases} x_1 = x_0 + mt \\ y_1 = y_0 + nt \\ z_1 = z_0 + rt \end{cases}$ larni tekislik tenglamasiga qo`yamiz :

$$A(x_0 + mt) + B(y_0 + nt) + C(z_0 + rt) + D = 0$$

Xosil bo`lgan tenglamada faqatgina parametr t noma`lum bo`lib, uni topish mumkin (faqatgina to`g`ri chiziq va tekislik parallel bo`lgan hol bundan mustasno). Topilgan t ni o`rniga qo`yib, Q$(x_1; y_1; z_1)$ nuqta topiladi .

Masalalar.1)A(2; -4;-1) va $\begin{cases} 3x + 4y + 5z - 26 = 0 \\ 3x - 3y - 2z - 5 = 0 \end{cases}$ to`g`ri chiziqning 5x+3y-4z+11= 0, 5x+3y-4z-41 =0 tekisliklar orasidagi kesmasi o`rtasidan o`tuvchi to`g`ri chiziq kanonik tenglamasini toping.

Dastlab, $\begin{cases} 3x + 4y + 5z - 26 = 0 \\ 3x - 3y - 2z - 5 = 0 \end{cases}$ to`g`ri chiziq parametrik tenglamasini topish kerak.

$z_0 = 2$ desak , $\begin{cases} 3x + 4y = 16 \\ 3x - 3y = 9 \end{cases}$ xosil bo`ladi. Undan $y_0 = 1; x_0 = 4.$

Demak, B(4;1;3) nuqta shu to`g`ri chiziqda yotadi.
To`g`ri chiziq yo`naltiruvchi vektori

$\vec{p} = \overrightarrow{N_1} x \overrightarrow{N_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 4 & 5 \\ 3 & -3 & -2 \end{vmatrix}$ = (7;21;-21) yoki $\vec{p}(1; 3; -3)$ deyish mumkin.

Bu to`g`ri chiziq $\frac{x-4}{1} = \frac{y-1}{3} = \frac{z-2}{-3}$ yoki $\begin{cases} x = 4 + t \\ y = 1 + 3t \\ z = 2 - 3t \end{cases}$ tenglamalarga ega.

Uning parallel tekisliklar bilan kesishgan nuqtalarini topamiz;

a) 5(4+t)+3(1+3t) – 4(2-3t) + 11= 0 dan t= -1 va kesishish nuqtasi $C_1(3; -2; 5)$ bo`ladi.