

# Mavzu: Kasr chiziqli funksiya

**5. Kasr-chiziqli funksiya grafigi.** Ikki chiziqli funksiyaning nisbatidan iborat

$$y = \frac{ax+b}{cx+d} \quad (1)$$

kasr-chiziqli funksiyaning qaraymiz. Uning grafigi to'g'ri chiziq yoki giperbola bo'lishi mumkin:

1) agar  $c=0$ ,  $d \neq 0$  bo'lsa, (1) munosabat  $y = \frac{a}{d}x + \frac{b}{d}$  chiziqli funksiya aylanadi, uning grafigi to'g'ri chiziqdan iborat;

2)  $c \neq 0$ ,  $\frac{a}{c} = \frac{b}{d} = m$  bo'lsa,  $y = \frac{mcx+md}{cx+d} = m$  ga ega bo'lalimiz. Bu holda (1) funksiya grafigi  $Ox$  o'qqa parallel bo'lgan va  $M(-\frac{d}{c}; m)$  nuqtasi chiqarib tashlangan  $y=m$  to'g'ri chiziq bo'ladi;

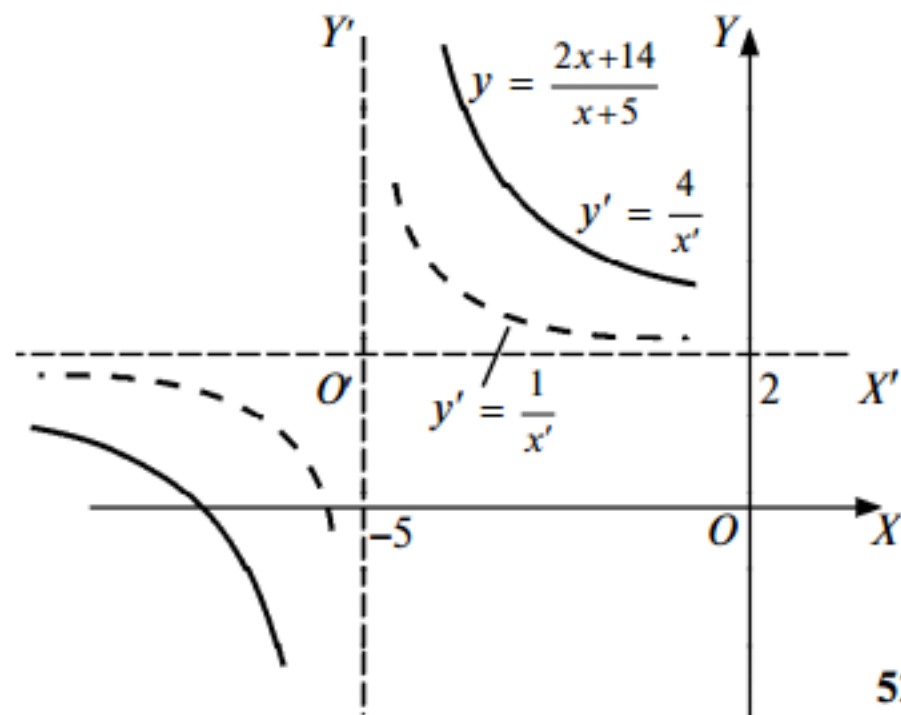
3)  $a \neq 0, \frac{a}{c} \neq \frac{b}{d}$ . Oldin  $\frac{ax+b}{cx+d}$  kasrdan butun qism ajratamiz:

$$\frac{ax+b}{cx+d} = \frac{a}{c} + \frac{b-\frac{ad}{c}}{cx+d} = \frac{a}{c} + \frac{\frac{bc-ad}{c^2}}{\frac{d}{x+\frac{c}{d}}} = \beta + \frac{k}{x-\gamma}, \text{ bunda}$$

$$\beta = \frac{a}{c}, \quad k = \frac{bc-ad}{c^2}, \quad \gamma = -\frac{d}{c}. \quad (2)$$

Bundan ko‘rinadiki,  $y = \frac{x+b}{cx+d}$  funksiya grafigi  $y = \frac{k}{x}$  funksiya grafigi (giperbola)ni parallel ko‘chirishlar bilan hosil qilinadi, bunda koordinatalar boshi  $L(\gamma; \beta)$  nuqtaga o‘tadi.  $\gamma, \beta$  va  $k$  lar (2) formulalar bo‘yicha topiladi.

1- m i s o l.  $y = \frac{2x+14}{x+5}$  funksiya grafigini yasang (52- rasm).



52- rasm.

Y e c h i s h . Kasrdan butun qismini ajratamiz:  $\frac{2x+14}{x+5} = 2 + \frac{4}{x+5}$ , unda  $k=4$ ,  $\gamma=-5$ ,  $\beta=2$ .  $O'(-5; 2)$  nuqtadan yordamchi  $O'x'$ ,  $O'y'$  koordinatalar o'qlarini o'tkazamiz. Ularda  $y = \frac{1}{x}$  funksiya grafigini, so'ng  $y = \frac{k}{x}$  funksiya grafigini yasaymiz. Bu grafik  $xOy$  koordinatalar sistemasida  $y = \frac{2x+14}{x+5}$  ning grafigi bo'ladi.



## Mashqlar

**7.122.** Funktsiyalarning grafiklarini yasang:

a)  $y = \frac{2x-5}{x+1}$ ;      b)  $y = \frac{-3x+2}{2x-3}$ ;      d)  $y = \frac{4x+1}{2x-3}$ ;

e)  $y = \frac{3x+4}{2x-1}$ ;      f)  $y = \frac{x+9}{-3x+1}$ ;      g)  $y = \frac{6x+1}{4x-2}$ .

**7.123.**  $A$ ,  $B$ ,  $C$  nuqtalar ustidan o'tuvchi  $y = \frac{ax+b}{cx+d}$  funksiya grafigini yasang:

a)  $A(-2; 0)$ ,       $B(1; 4)$ ,       $C(0; 2)$ ;

b)  $A(1; -3)$ ,       $B(3; 2)$ ,       $C(-1; 3)$ ;

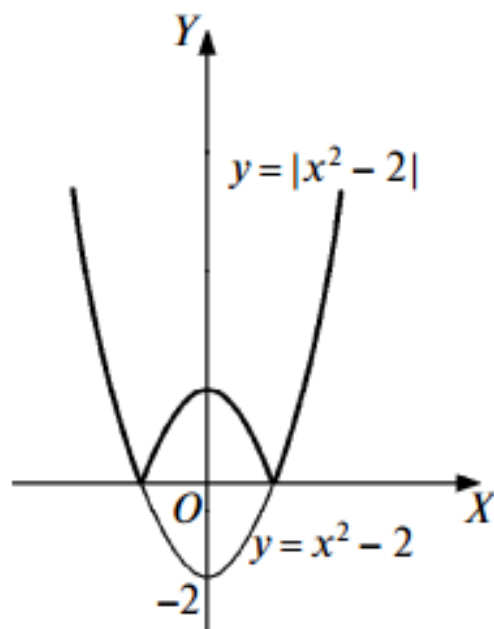
d)  $A(4; -3)$ ,       $B(2; 1)$ ,       $C(3; -4)$ ;

e)  $A(-5; 1)$ ,       $B(-2; 3)$ ,       $C(-1; 5)$ .

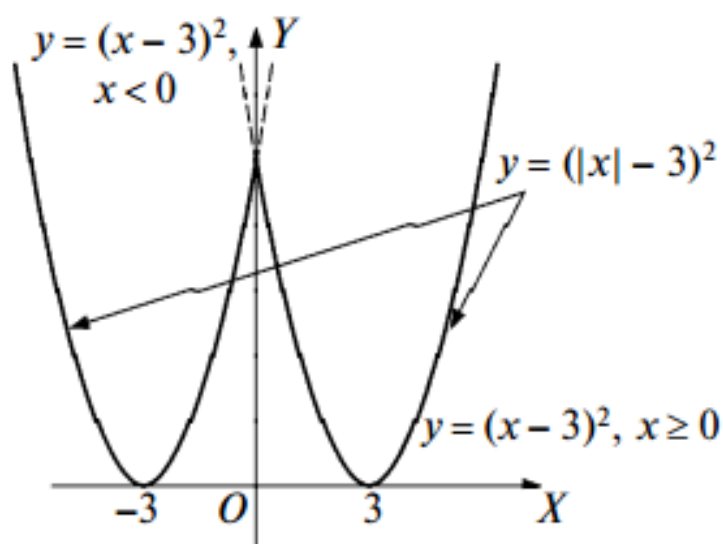
## 6. Ifodasi modul ishorasiga ega funksiyalarning grafigi.

$$1) |f(x)| = \begin{cases} f(x), & \text{agar } f(x) \geq 0 \text{ bo'lsa,} \\ -f(x), & \text{agar } f(x) < 0 \text{ bo'lsa,} \end{cases} \text{ ekanini biz bilamiz.}$$

Bundan ko'rinadiki,  $|f|$  grafigini yasash uchun oldin  $f$  grafigini yasash, so'ng uning  $y \geq 0$  yarim tekislikdagi qismini o'z joyida qoldirib,  $y < 0$  yarim tekislikdagi qismini esa  $Ox$  o'qqa nisbatan simmetrik akslantirish kerak. 53- rasmda  $y = |x^2 - 2|$  grafigini  $y = x^2 - 2$  grafigidan foydalanib yasash tasvirlangan.



53- rasm.



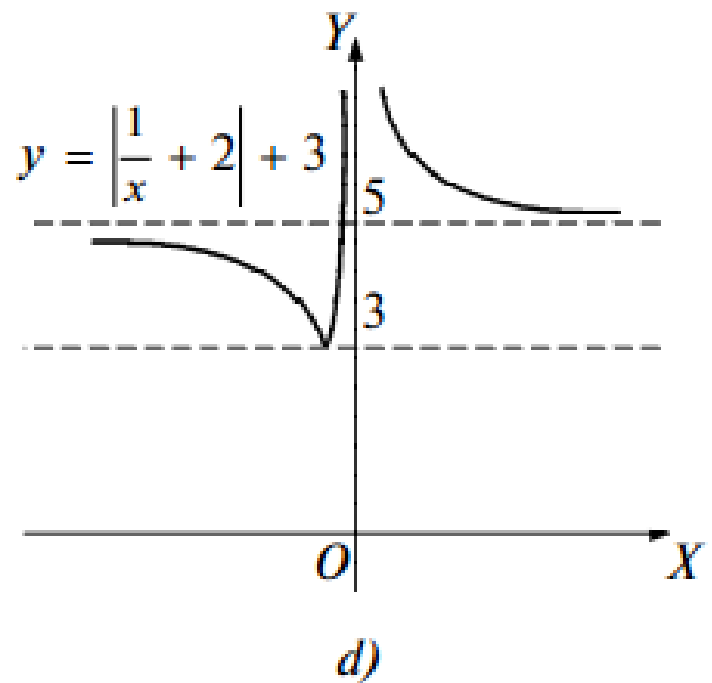
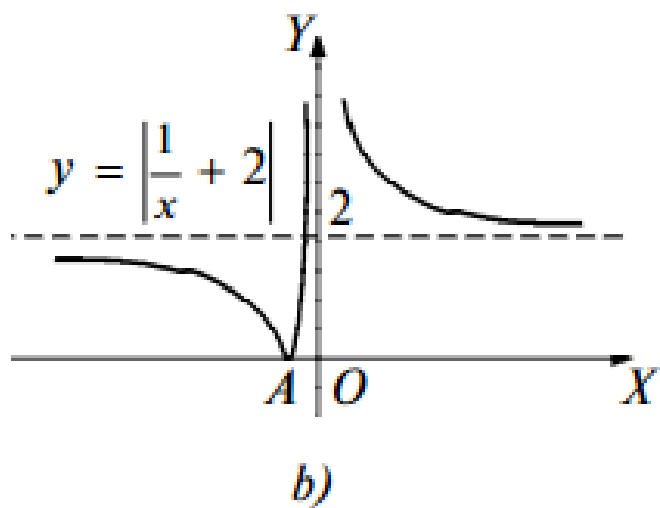
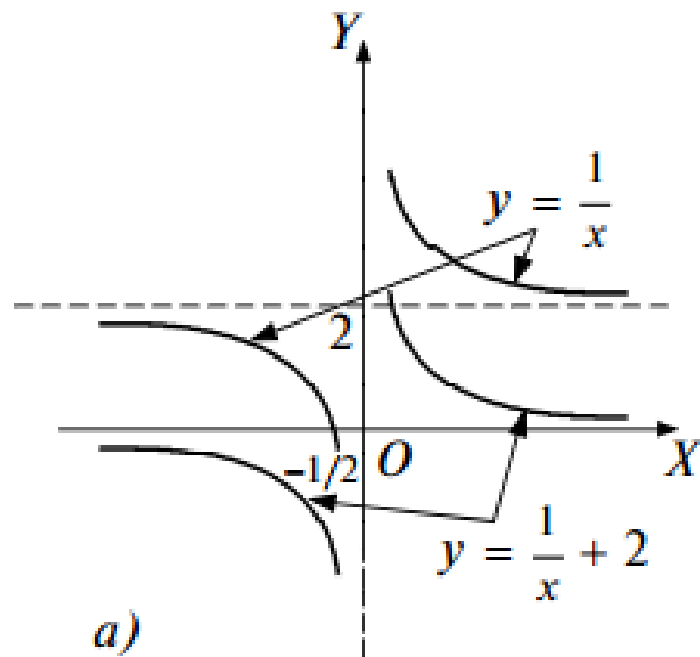
54- rasm.

1- misol.  $y = \left| \frac{1}{x} + 2 \right| + 3$  funksiya grafigini yasaymiz.

Yechish. a) Dastavval  $y = \frac{1}{x}$  funksiya grafigini, so'ngra shu grafik bo'yicha  $y = \frac{1}{x} + 2$  grafigini yasaymiz (56- a rasm);

b)  $x$  ning har qanday qiymatida  $y = \left| \frac{1}{x} + 2 \right| \geq 0$ . Shunga ko'ra,  $y = \frac{1}{x} + 2$  grafigining  $-\frac{1}{2} < x < 2$  da  $Ox$  o'qi ostida turgan qismini  $Ox$  o'qiga nisbatan simmetrik akslantiramiz (56- b rasm). Bunda  $x = -\frac{1}{y}$  qiymat  $y=0$ , ya'ni  $\frac{1}{x} + 2 = 0$  bo'yicha topiladi;

d) talab qilinayotgan  $y = \left| \frac{1}{x} + 2 \right| + 3$  grafikni yasash uchun  $y = \left| \frac{1}{x} + 2 \right|$  grafigi 3 birlik yuqoriga parallel ko'chiriladi (56- d rasm).





## Mashqlar

**7.124.** Funktsiyalarning grafiklarini yasang:

a)  $y = |x^2 - 3x + 2|;$

b)  $y = x^2 - 2|x| - 3;$

d)  $y = |x^2 - 3x| + 2;$

e)  $y = ||x - 2| - 3x|;$

f)  $y = |x - 1| + |x - 3|;$

g)  $y = \left| \frac{x+4}{x+1} \right|;$

h)  $y = \frac{|x|-4}{|x|-2};$

i)  $y = \left| x + \frac{1}{x} - 1 \right|;$

j)  $y = \frac{|x|-4}{x+1};$

k)  $y = \frac{x-3}{|x|+1};$

l)  $y = \frac{1}{|3x-1|+|x|};$

m)  $0 y = \frac{1}{|x|+|x-2|-3}.$

**7.125.** Quyidagi tengliklarni qanoatlantiruvchi  $M(x; y)$  nuqtalar to'plamini yasang:

a)  $x - 2|x| = y - 2|y|;$

b)  $x + 2|x| = y - 2|y|;$

d)  $x - 2|x| = y + 2|y|;$

e)  $x + 2|x| = y + 2|y|;$

f)  $x - 2[x] = y - 2[y];$

g)  $[x] = 2[y].$

**7.126.** Quyidagi tengliklarni qanoatlantiruvchi  $M(x; y)$  nuqtalar to'plamini toping:

a)  $|y| = x^2 - 3x + 2;$

b)  $|y| = \frac{x+1}{x-2};$